MODELING LONG-PERIOD TRIP PATTERNS WITH TIME AND MONEY CONSTRAINTS – THE TILT LAND USE-TRANSPORTATION MODEL

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Abstract
TILT is a model for integrated analysis of household location and travel choices. In this paper, we describe the way travel patterns are modeled. The purpose is to develop a way to model trip patterns when time and money constraints are taken into account and when individuals are assumed to prefer variation over destinations. Each individual chooses a travel pattern that maximizes utility subject to budget and time constraints. The travel pattern consists of a set of trip frequencies to different destinations with different modes. When choosing trip frequencies to destinations, individuals are assumed to prefer variation to some extent, measured by a set of parameters to be estimated.

INTRODUCTION

TILT, Tool for Integrated analysis of Location and Transport, is a model for integrated analysis of household location and travel. Households choose location together with a bundle of other choices, such as car ownership, mode choices and trip frequencies to various destinations given the location of activities to visit and prices of housing. Prices are then calculated to make demand equal supply for each zone and each house type. The price-supply functions are exogenous to the model.

The TILT model contains a few somewhat novel ideas on how to calculate location and travel patterns consistently. This paper deals with the way travel patterns are calculated. The entire TILT model is explained more thoroughly in Eliasson and Mattsson (1997), which also contains more references and a brief review of and comparison with similar models.

Traditionally, most land-use models have modeled interaction between the infrastructure system and land-use in a rather aggregate way, most often using some kind of aggregate accessibility measure. Travel models, on the other hand, most often take land-use as fixed, and also calculate trip generation in either a rather crude way, or treating trip frequencies as fixed. While these approaches are suitable for many purposes, there are questions that cannot be treated in a consistent way under these assumptions.

Both land-use and travel models often restrict themselves to dealing only with accessibility to employment and work trips, respectively. But when dealing with household location, accessibility to other things than jobs is obviously an important determinant for the location choice. It is also obvious that we, if we want to analyze how changes in land-use affect the total travel pattern in a region, cannot restrict ourselves to modeling work trips.

If we generalize a little, it is reasonable to say that most traffic models deal with the decisions involved with a *single trip*. This means that there is no way of modeling the individual's desire for varying her destination choice, and that the "trip frequency", if modeled at all, is simply the choice of whether or not to make this single trip. Land-use models, if we generalize again, do not deal with *trips* but rather with *accessibility*. There is usually no connection between how many trips (or, more generally speaking, which "trip pattern") that are generated and the level of accessibility.

TILT is an attempt to model an *explicit connection between location and trip generation*, where "trip generation" includes frequency, destination and mode choice.

The assumption of the model is that households make a joint choice of location (zone and house type), car ownership and travel pattern. This travel pattern includes trip frequencies, destination choices and mode choices for a number of trip types. All of these choices are treated in a coherent, microeconomic framework, which makes it possible for households to trade, for example, accessibility for cheaper or better housing. "Accessibility" is not treated as a single, aggregate measure, but is handled directly through its actual components: time and money devoted to trips, and precisely those trips that are made (described by their destinations and modes).

Land-use/transportation interaction models draw from several modeling traditions. This is natural, since the field encompasses so much of human activities in general. The three most influential approaches or traditions have been urban economics, spatial interaction (gravity) models and discrete choice models.

A brief review of these modeling traditions is given in Eliasson and Mattsson (1997).

A SUMMARY OF THE MODEL

We will here briefly summarize how the TILT model operates.

The region is divided into zones $i = 1, ..., I$. The housing stock is divided into house types $a = 1, ..., A$. The cost for a household to live in zone i and house type a is p_{ia}. This price will be endogenously determined so that demand equals supply for all zones and house types. The location of firms is assumed to be fixed and known. This influences the households' choice of location through accessibility to service and employment. The households are divided into groups $h = 1, ..., H$, and the number of h-households is denoted by N^h. For each price vector **p**, the number of households choosing to live in zone i and house type a is denoted by $N^h{}_{ia}(\mathbf{p})$. For each price vector **p**, the supply of housing of type a in zone i is denoted by $S_{ia}(\mathbf{p})$. This function is assumed to be known. In the short run, the supply can be assumed to be totally inelastic. For most prediction purposes, the housing supply for each zone and house type has natural upper and lower bounds (due to e.g. planning restrictions), with a price-elastic part in-between. The model then finds a price vector that clears all markets:

$$S_{ia}(\mathbf{p}) = \sum_h N^h_{ia}(\mathbf{p}) \quad \forall\, i,a \tag{1.}$$

Such an equilibrium price vector **p** exists and is unique under mild assumptions. Details can be found in Eliasson (1997).

Given the prices of housing for each zone and house type, each household makes a joint choice of residential zone, house type, car ownership level and travel pattern. This choice is modeled as a nested choice with four steps: choice of residential zone and house type, choice of car ownership, choice of trip frequencies to all destinations and mode choice for each trip.

All of these choices are outcomes of a single optimization process. This has the advantage of making all choices consistent with each other; for example, the marginal utility of money will be the same both when choosing travel mode and choosing location. The consistency of the valuations stems from using a single pair of budget and time constraints for the entire problem; unfortunately, it is not uncommon that different valuations of e.g. money or time are induced at different steps of a model. This consistency seems to be a desirable feature in the present context, when we are studying long-term choices. Clearly, it may be a less realistic assumption if we were looking at short-term behavior.

Through the choice of distribution of the stochastic terms in the optimization problem, the first, second and fourth choices (which are discrete choices) will be specified as logit models. The third step is a continuous choice.

The purpose is not necessarily to construct a framework for predicting households' actual travel choices as precisely as possible. Our primary concern is rather to reflect the considerations of a household choosing a location, and conversely the impact of locational changes on a region's overall travel pattern. Most travel choice models typically model the choices involved with a single trip or a single day. But when choosing a location, a more realistic assumption is that the household decides for a "typical" travel pattern for a longer period of time.

CHOOSING A TRAVEL PATTERN

We will here explain how the choice of a travel pattern is modeled.

Consider an individual about to choose a travel pattern for a certain time period, for example a month. At her disposal is a monetary budget Y and a time budget T. The travel pattern consists of the elements x_{jm}, which is the number of trips to destination j with mode m. We will deal with only one trip type here; we can think of this as for example shopping trips. The model is easily extended to several trip types; this is done in Eliasson and Mattsson (1997). To reduce the number of indices, we will also suppress the index of the fixed residential zone i. For example, we will denote the travel cost from i to destination zone j with b_j, rather than b_{ij}.

A fairly general way to state this problem would be the following maximization problem:

$$\max_{y,t,\mathbf{x}}\{\alpha \ln y+\beta \ln t+u(\mathbf{x})\} \tag{2.}$$

$$\text{subject to} \qquad y+\sum_{jm} b_{jm}x_{jm}=Y$$

$$t+\sum_{jm} t_{jm}x_{jm}=T$$

$$y,t,x_{jm}\geq 0 \qquad \forall j,m$$

where we have introduced

α, β	parameters
y	money left for other consumption
t	time left for other activities
b_{jm}	monetary cost for a round trip to j with mode m (from home)
t_{jm}	travel time for a round trip to j with mode m (from home)
$\mathbf{x}$	the vector of trip frequencies $\{x_{jm}\}$
$u(\mathbf{x})$	utility of making the trips $\mathbf{x}$

The advantage of stating the problem in this way is that the disposable time and income are explicitly taken into account. Furthermore, we are able to deal with trip patterns over long time periods. This will be clear in the next section.

When choosing a location, the households are assumed to solve this problem for each possible location (zone and house type), and then choose the location with the maximal utility. This location choice is modeled by a nested logit model, with house types, zones and car ownership at different choice levels.

CHOOSING A TRIP UTILITY FUNCTION *u(x)*

We will now consider the following problem: Construct a function $u(\mathbf{x})$ that measures the utility of making the trips $\mathbf{x} = \{x_{jm}\}$. The travel pattern $\mathbf{x}$ consists of all trips a household will make during a fixed time period. First, we make three observations:

1. The utility of making one additional trip to zone j depends on which other trips one has made. To be specific, the marginal utility of a j-trip should decline with the number of j-trips, and also (to a lesser extent) with the total number of trips (remember that we are only considering one trip type here).
2. People differ in their valuation of different destinations, both across individuals, but also across time. This means that we cannot hope to predict behavior exactly, and that some kind of stochastic model is appropriate. But it is important to realize that this variation over individuals and time has a principally different cause than the strive to vary the destination choice that we argued for above.
3. Apart from the time and cost nuisance, travel is also a disutility "in itself". This means that even if travel were very cheap and very fast, people would not travel infinitely much. This is because the marginal utility of travel is decreasing, but this disutility component is constant.

Several functions fulfilling the conditions suggested above can be imagined. We will choose a type of constant elasticity of substitution (CES) function, since this is a fairly flexible function. Another attractive feature is the possibility to construct "nested" CES-functions (Varian, 1992).

The trip utility function

We choose a function of the following general form:

$$u(\mathbf{x}) = \frac{1}{r}\ln\left(\sum_j w_j x_j^{\,r}\right) - \sum_{jm} q_{jm} x_{jm} \qquad (3.)$$

where

x_{jm}	number of jm-trips
x_j	number of j-trips ($x_j = \Sigma_m x_{jm}$)
w_j	utility of visiting zone j
q_{jm}	disutility of making a jm-trip
r	relative similarity between destinations

The first term is a CES function, and describes the utility of our visit. Note that this term does not depend on which modes we use to make our j-trips. All that matters is that we get there. The second term is a sum of travel disutilities. This term does depend on which modes we use. These disutilities can be further separated into components like number of changes for transit modes and so on. It could also be used to introduce different weights on different time components for e.g. public transit. Note that this term does not include physical travel times or out of pocket travel costs; these are handled through the time and money constraints.

The attractiveness measure w_j is a standard tool, and can be constructed in many different ways. It is often taken to be proportional to the share of floor space or the number of employed in the zone, and can also include agglomeration effects.

To reflect that people have different preferences, we let $\mathbf{w} = \{w_j\}$ and $\mathbf{q} = \{q_{jm}\}$ be stochastic. We assume that $\mathbf{w}$ is fixed for each individual, but varies across individuals. $\mathbf{w}$ is hence stochastic only from the modeler's point of view.

On the other hand, we assume that $\mathbf{q}$ is stochastic also from the individual's point of view. More precisely, we assume that once a particular trip is about to be made, when the individual has already made the choice of destination and frequency, then (but not until then), she knows the outcome of $\mathbf{q}$. The choice of destination and frequency (i.e., the choice of the vector $\{x_j\}$) is thus made with the knowledge that once the trip is about to be made, we choose the optimal mode given the outcome of $\mathbf{q}$. The choice of $\{x_j\}$, on the other hand, is made ex ante, only knowing the probability distribution of $\mathbf{q}$. A new drawing of $\mathbf{q}$ is then performed each time a trip is made.

The mode choice

Consider the situation where we are about to make a trip. This means that we are looking at problem (2) with $u(\mathbf{x})$ taken from eqn (3), but with $\{x_j\}$ given. It still remains to choose how to distribute these trips across modes.

It turns out that for each j, there will only be one optimal mode, namely the m that minimizes the generalized cost c_{jm}:

$$c_{jm} \equiv \lambda b_{jm} + \sigma t_{jm} + q_{jm} \qquad (4.)$$

where λ and σ are the Lagrange multipliers corresponding to the budget and time constraint, respectively. We assume that the q_{jm}:s have means $\overline{q}_{jm}$, and are independent and distributed such that $-q_{jm}$ are Gumbel distributed with dispersion parameter η_j. This means that over a longer period of time, a certain (stochastic) share of the j-trips will be made with each mode m. The expected share of j-trips made with mode m becomes

$$P_{m|j} = \frac{\exp\left(-\eta_j\left(\lambda b_{jm} + \sigma t_{jm} + \overline{q}_{jm}\right)\right)}{\sum_m \exp\left(-\eta_j\left(\lambda b_{jm} + \sigma t_{jm} + \overline{q}_{jm}\right)\right)} \tag{5.}$$

and we have

$$\mathrm{E}(x_{jm}) = x_j P_{m|j} \tag{6.}$$

Our individual now solves the optimization problem in a probabilistic sense, maximizing the expected value of the objective function while requiring the constraints to hold in a probabilistic sense:

$$\max_{y,t,x} E\left\{\alpha \ln y + \beta \ln t + \frac{1}{r}\ln\left(\sum_j w_j x_j^{\,r}\right) - \sum_{jm} q_{jm} x_{jm}\right\} \tag{7.}$$

$$\text{subject to} \qquad E\left[y + \sum_{jm} b_{jm} x_{jm}\right] = Y$$

$$E\left[t + \sum_{jm} t_{jm} x_{jm}\right] = T$$

$$y, t, x_j \geq 0 \qquad \forall j$$

Recall the ex ante decision situation: we choose the trip frequencies $\{x_j\}$ and the expected residual time and money t and y, knowing that when the trip is about to be made, we choose the optimal mode knowing the outcome of **q**. Problem (7) reduces to

$$\max_{y,t,x}\left\{\alpha \ln y + \beta \ln t + \frac{1}{r}\ln\left(\sum_j w_j x_j^{\,r}\right) - \sum_j \overline{q}_j x_j\right\} \tag{8.}$$

$$\text{subject to} \qquad y + \sum_j \overline{b}_j x_j = Y$$

$$t + \sum_j \overline{t}_j x_j = T$$

$$y, t, x_j \geq 0 \qquad \forall j$$

where we have introduced expected costs, times and disutilities:

$$\overline{b}_j = \sum_m b_{jm} P_{m|j} \tag{9.}$$

$$\overline{t}_j = \sum_m t_{jm} P_{m|j} \tag{10.}$$

$$\overline{q}_j = c_j(\lambda, \sigma) - \lambda \overline{b}_j - \sigma \overline{t}_j \tag{11.}$$

and where $c_j(\lambda,\sigma)$ is the *expected generalized cost*

$$c_j(\lambda, \sigma) \equiv E \min_m\left[\lambda b_{jm} + \sigma t_{jm} + q_{jm}\right] = -\left\{\frac{1}{\eta_j}\ln \sum_m \exp\left[-\eta_j\left(\lambda b_{jm} + \sigma t_{jm} + \overline{q}_{jm}\right)\right]\right\} \tag{12.}$$

The destination choice

The solution to problem (8) is

$$y = \frac{\alpha}{\lambda} \tag{13.}$$

$$t = \frac{\beta}{\sigma} \tag{14.}$$

$$x_j = \left[\frac{1}{X} \frac{w_j}{c_j(\lambda, \sigma)} \right]^{1/(1-r)} \tag{15.}$$

where we have introduced

$$X = \sum_j w_j x_j^r \tag{16.}$$

Since the x_j:s enter on both sides of eqn (15), this is not really a closed-form solution. Moreover, since we will assume that **w** varies randomly over individuals, we would like to calculate the expected value of **x** to get the aggregate travel pattern. This would clearly involve rather heavy numerical simulation. Instead, we note that there are two values of the parameter r for which we can in fact obtain closed-form solutions: $r = 0$ and $r = 1$. We will calculate these, and approximate eqn (15) with a combination of these two cases.

Consider first the case when $r \to 0$, and the first term in $u(\mathbf{x})$ tends to a Cobb-Douglas function (see for example Varian, 1992). This can be interpreted as if all destinations were regarded as different "products". The objective function in (8) is thus replaced with

$$\max_{y,t,x} \left\{ \alpha \ln y + \beta \ln t + \sum_j w_j \ln x_j - \sum_j \bar{q}_j x_j \right\} \tag{17.}$$

and the solution (15) becomes

$$x_j = \frac{w_j}{c_j(\lambda, \sigma)} \tag{18.}$$

Consider next the case $r = 1$. This means that each individual chooses to make all their trips to one single destination j^*, defined by

$$j^* = \arg\max \left[\frac{w_j}{c_j(\lambda, \sigma)} \right] \tag{19.}$$

Given that all $x_j = 0$ for $j \neq j^*$, the optimal trip frequencies become

$$x_j = \frac{1}{c_j(\lambda, \sigma)} \qquad \text{if } j = j^*; \quad x_j = 0 \text{ otherwise} \tag{20.}$$

Approximate eqn (15) with the convex combination of eqn (18) and eqn (20):

$$x_j \approx \rho \frac{\delta_{jj^*}}{c_j(\lambda, \sigma)} + (1 - \rho) \frac{w_j}{c_j(\lambda, \sigma)} \qquad \delta_{jj^*} = 1 \text{ if } j = j^*, 0 \text{ otherwise} \tag{21.}$$

ρ is a parameter to be estimated, and replaces the previous parameter r. When $r=1$ ($\rho=1$) and $r=0$ ($\rho=0$), the approximation is exact.

Now, remember that we assumed that **w** is stochastic from the modeler's point of view. What the modeler would be interested in is therefore the *expected* trip pattern E(**x**) that we would observe if we picked a *random individual*:

$$E(x_{jm}) = E(x_j)P_{m|j} \approx E\left\{\rho\frac{\delta_{jj^*}}{c_j(\lambda,\sigma)} + (1-\rho)\frac{w_j}{c_j(\lambda,\sigma)}\right\}P_{m|j} \tag{22.}$$

When we introduced the destination attractiveness w_j, we said that they were randomly distributed across individuals, without specifying the distribution. We now assume that the distribution is such that the random error from the modeler's point of view can be written as

$$\frac{w_j}{c_j(\lambda,\sigma)} = \frac{\overline{w}_j}{c_j(\lambda,\sigma)} + \varepsilon_j \tag{23.}$$

and that the ε_j:s are independent and Gumbel distributed with zero mean and dispersion parameter μ.

This is not such a strange assumption as it may seem. When people evaluate the utility of going to j, they in fact evaluate all the "sub-destinations" aggregated into zone j. This means that the utility U_j for each j is the outcome of a maximization process over a large number of alternatives. If we assume that these elementary alternatives have independent and identically distributed additive random terms, U_j will be asympotically Gumbel distributed (under some mild assumptions; Jaïbi and ten Raa (1998)). What *is* strange with this assumption is that we cannot let the dispersion parameter μ depend on λ or σ, and hence not on income Y or total available time T. This is of course possible in principle, but would give a model hopeless to estimate.

This gives us

$$E(x_j) \approx \rho\frac{e^{\mu V_j}}{\sum_j e^{\mu V_j}}\frac{1}{c_j(\lambda,\sigma)} + (1-\rho)\frac{\overline{w}_j}{c_j(\lambda,\sigma)} \tag{24.}$$

where $V_j = \overline{w}_j/c_j(\lambda,\sigma)$.

Interpretation of the trip utility function

To summarize, the expected value of the travel pattern that solves (7) is approximately given by

$$E(x_{jm}) = E(x_j)P_{m|j} \approx \frac{1}{c_j(\lambda,\sigma)}\left(\rho\frac{e^{\mu V_j}}{\sum_j e^{\mu V_j}} + (1-\rho)\overline{w}_j\right)P_{m|j} \tag{25.}$$

The basic idea behind this travel behavior model is that it might be rewarding to try to distinguish between intra-individual and inter-individual variation. For some travel types, like work travel and perhaps daily shopping, the destination is almost always the same for each individual. Inter-individual variation, however, causes the observed "spread" over destinations. This is either because there is no incentive for variation (as in the case with daily shopping), or because the destination choice represents a long-term engagement (as in the work-travel case). The parameter μ measures this inter-individual variation in destination preferences.

For some trip types, on the other hand, like recreational travel or non-daily shopping, there is a strong incentive for variation. People normally don't want to visit the same friend, shoe shop, cinema or whatever over and over again. This is principally a different type of choice situation, since *history* is suddenly a matter of concern. The parameter ρ then measures the influence of "history", or put another way, the taste for destination variation.

Another, slightly different interpretation is that r (and hence ρ) measures the *similarity between destinations*. ρ = 1 then implies that all destinations "are the same". This explains why w_j does not influence the number of trips in this case (eqn 20).

A COMPARISON WITH THE NESTED LOGIT MODEL

For those that find the formulas above a little dry, we will here present some diagrams to show the principal functional form of our proposed model. We will also compare it to the standard nested logit model, and note differences and similarities.

First, we note that the mode choice is a normal logit model. Since this is a well-known model, we will not deal with this part any further, but concentrate on the construction of x_j, the number of trips to j.

A standard nested logit model for this would be

$$x_j = F_0 \frac{e^{\theta V}}{1+e^{\theta V}} \frac{\exp\left(\mu \overline{w}_j - \mu\left(\lambda b_j + \sigma t_j + \overline{q}_j\right)\right)}{\sum_j \exp\left(\mu \overline{w}_j - \mu\left(\lambda b_j + \sigma t_j + \overline{q}_j\right)\right)} \tag{26.}$$

where V is the expected utility of making one trip:

$$V = \mu^{-1} \ln \sum_j \exp\left(\mu \overline{w}_j - \mu\left(\lambda b_j + \sigma t_j + \overline{q}_j\right)\right) \tag{27.}$$

and F_0 is the "saturation trip frequency", the number of trips that is made when V approaches infinity. b_j, t_j and $\overline{q}_j$ is travel cost, time and disutility just as before; λ and σ are parameters reflecting cost and time sensitivity, respectively. $\overline{w}_j$ is the utility of visiting j, just as before.

We remind of the corresponding expression in our model, from eqn (25) (when there is only one mode):

$$x_j = \frac{1}{\lambda b_j + \sigma t_j + \overline{q}_j} \left(\rho \frac{\exp\left(\dfrac{\mu \overline{w}_j}{\lambda b_j + \sigma t_j + \overline{q}_j}\right)}{\sum_j \exp\left(\dfrac{\mu \overline{w}_j}{\lambda b_j + \sigma t_j + \overline{q}_j}\right)} + (1-\rho)\overline{w}_j \right)$$

The similarities are obvious. The greatest difference is perhaps that the cost and time sensitivities λ and σ are not constant; remember that they are calculated to make the time and money constraints hold. Since it is probably not obvious how this function behaves, we will present a few diagrams to illustrate the general behavior.

Imagine that we have three destinations to choose from. We get the same utility from visiting each of them, meaning that $w_1 = w_2 = w_3$. To begin with, we examine the following setup:

Destination	Travel time	Travel cost
1	variable: 0-40	15
2	17	15
3	20	15

and examine what happens when we vary the travel time to destination 1. We get the following:

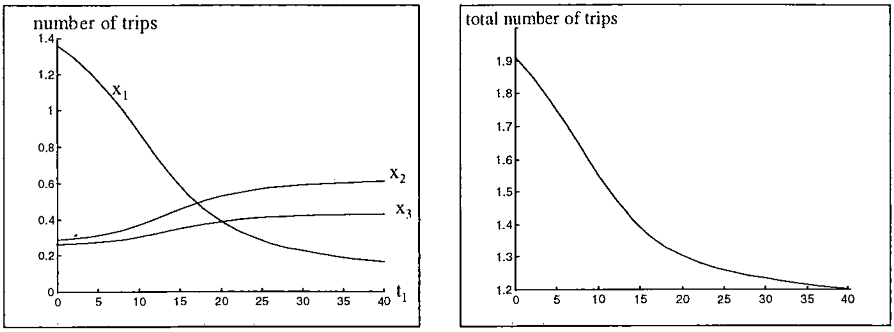

Figure 1. Number of trips as a function of travel time - TILT. Example 1.

The same example with a logit model gives:

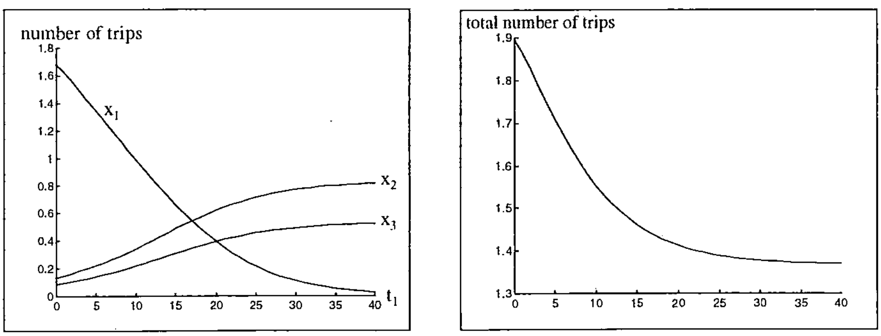

Figure 2. Number of trips as a function of travel time - nested logit. Example 1.

Of course, it makes no sense to compare the numerical results; we are only interested in the principal shape of the curves.

The similarities are obvious, as expected. One noticeable difference is that our function does not exhibit an as strong asymptotic tendency; the function "stays elastic longer". Another difference is that the logit function is almost symmetric around a midpoint, the point with maximum slope. This is not the case with our function, which has a more complicated elasticity variation.

Another important difference becomes clear if we change the example a little (that t_1 is negative has no importance; it is just a convenient scale):

Destination	Travel time	Travel cost
1	variable: -40 – 40	22
2	14	14
3	22	8

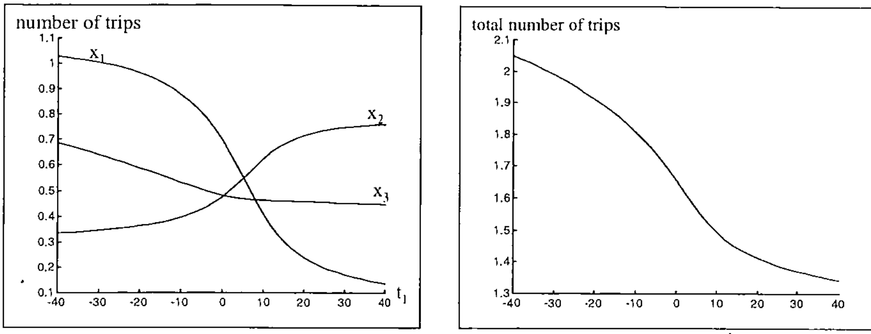

Figure 3. Number of trips as a function of travel time – TILT. Example 2.

We see that destination 3 acts similar to an "inferior good". When t_1 rises, this makes time a more scarce resource and thus changes the time and cost sensitivities λ and σ to make destination 2 be perceived as the "cheapest" destination, measured in generalized cost. This does not only decrease x_1, but also x_3, since this is the destination with highest travel time. This phenomenon would be regarded as an error in a nested logit model. In this context, however, it can be perfectly rational.
The same scenario studied with the logit model gives:

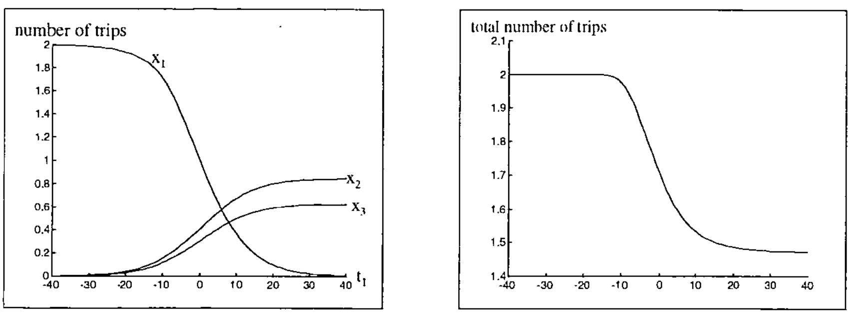

Figure 4. Number of trips as a function of travel time – nested logit. Example 2.

We see that the relative shares of destination 2 and destination 3 remain unchanged.

Perhaps the most interesting application is to study what happens when we change the budget and time restrictions. We will examine a budget variation. Varying the time constraint gives similar results. The scenario is the same as the previous example, but with $t_1 = 8$, Y varying from 10 to 70.

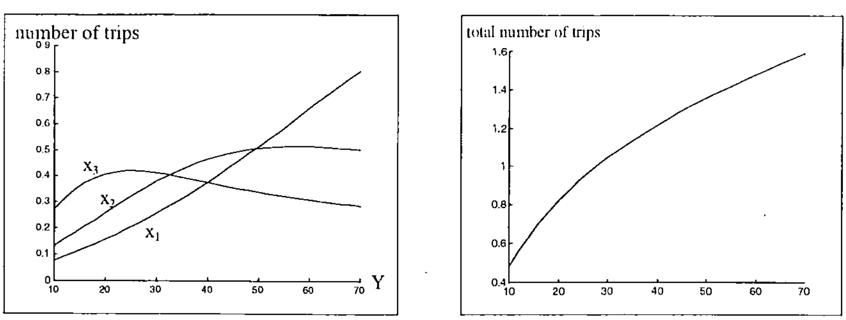

Figure 5. Number of trips as a function of budget – TILT.

As Y grows, λ drops and the trip frequencies eventually converges asymptotically to levels where only travel time matter. The convergence is however rather slow. It is interesting to study time and money devoted to other activities (t and y):

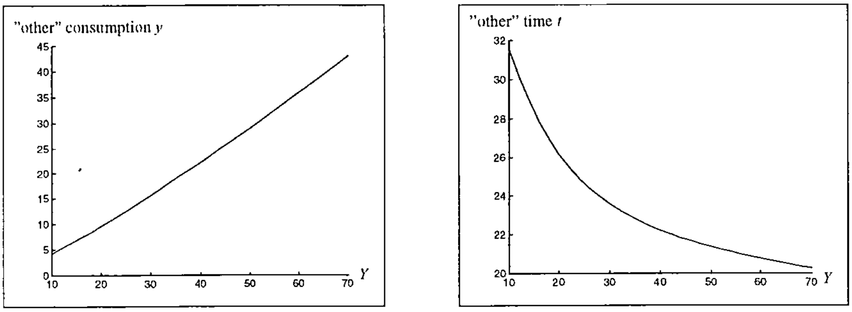

Figure 6. Residual consumption and time as a function of budget – TILT.

Note that t drops as Y increases. This is because the total generalized cost for a trip decreases, which makes it rewarding to use more time for travel. Eventually, this converges asymptotically as price matters less and less.

It is not obvious how we should model an increasing budget restriction in the logit case, but a common and reasonable way is to set $\lambda = 1/Y$. This gives the following:

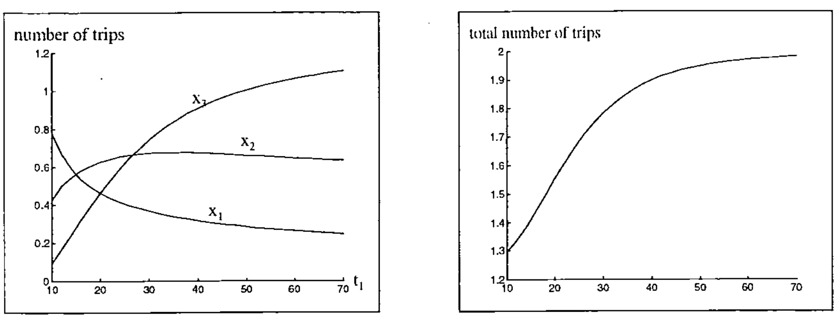

Figure 7. Number of trips as function of budget – nested logit.

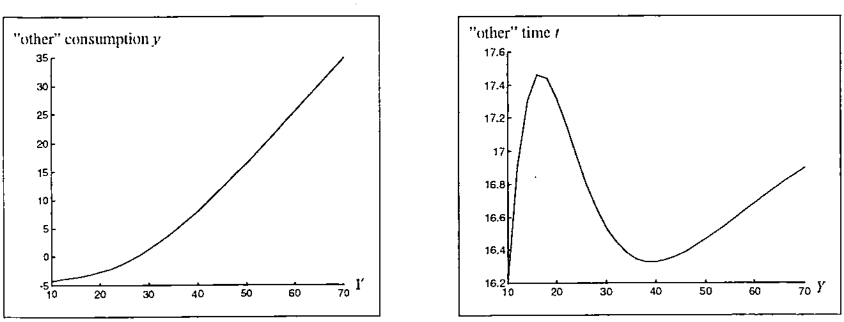

Figure 8. Residual consumption and time as a function of budget – nested logit.

Once again, the similarities are obvious. The logit function shows once again a faster convergence towards the asymptote; that the total trip frequency converges so fast causes the peculiar shape of the t-curve in the fourth frame.

Our last experiment is to examine variation in ρ. Remember that this parameter describes the relative similarity of destinations. If $\rho = 1$, the destinations are perceived as identical, and only differences in individual taste (measured by μ) will cause any spread over the destinations. In the other extreme, when $\rho = 0$, destinations are perceived as completely separate, and each individual will try to spread his traveling over them, proportional to the generalized cost and the utility of each destination.

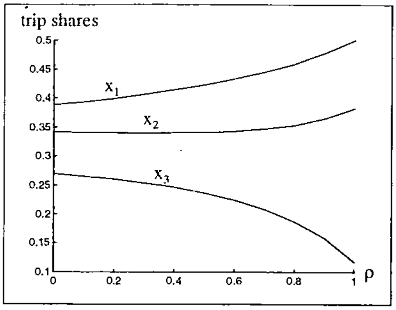

Figure 9. Trip shares as a function of similarity parameter – TILT.

This looks as expected: the more similar destinations are, the less spread is observed.

To summarize, the nested logit model and our model have many features in common. There are mainly two differences.

The first is that the elasticity of the logit curve tends to its asymptotes faster outside the curves most elastic middle part. Our model has a wider interval where it is still elastic, and also shows a more complicated elasticity variation. Whether this is good or not is an irrelevant question; the proper way to choose between the two would be to test them on a real data set. Still, we might guess that the logit model should perform better when we consider short time periods, say one or a few days, where the discrete nature of trips is still very apparent. Similarly, we might guess that our model should perform well when we consider longer time spans, where the trip frequency more resembles a continuous good.

The second difference is more important: we are able to treat time and money constraints consistently, and to model changes in them explicitly. We thus know that the time and cost sensitivity will be the same at all levels of the model. This is an attractive feature when we want to model long-term choices like location or employment choices. On the other hand, it might in some cases actually be restrictive, if we deal with time spans where we know that some choices (like where to work) are fixed. Here, a logit model could be more appropriate.

CONCLUSIONS

The purpose of this work is to construct the framework of an operational model that establishes an explicit connection between location choice and trip pattern, including choices of trip frequencies, destinations and modes, using a coherent microeconomic framework.

This connection is modeled by letting people choose an optimal travel pattern for each alternative location, and then choose the location that gives the highest expected utility. Since this utility is different across households, a logit location choice model follows.

Two problems arise. The first one is that when choosing destinations for the trips, people can be expected to have a certain taste for variation. This rules out the standard logit destination choice. However, it is not reasonable to assume (a priori) that all destinations are regarded as entirely different products. It is more realistic to assume that different destinations to some extent are similar, although they in some regards are different, and that this similarity between destinations vary with trip purpose. This makes the standard Cobb-Douglas function unsuitable.

The second problem is that the mode choice, as opposed to the destination/frequency choice, is a discrete choice, i.e. a choice between mutually exclusive alternatives. This makes the standard logit model a suitable choice. Combining these two types of models, a continuous choice and a discrete choice model, is not straightforward, but requires some care to be consistent and mathematically correct.

We believe that these two difficulties are solved by our approach. The first problem is handled by introducing the function $u(\mathbf{x})$, which has the logit model and the Cobb-Douglas model as limiting cases, and where the parameter r measures to what extent "history" should influence the destination choice. The second problem is handled by introducing a random cost associated with each mode.

When compared to the standard nested logit model, our model exhibits a fairly similar general behavior. The differences are mainly that the time and cost elasticities decline slower, and that the time and cost sensitivities are not constant, which among other things affects the cross-elasticities. Our model is also possible to use to examine changes in the time and budget constraints.

Hopefully, models of this type will allow practitioners to investigate the connections between infrastructure, location, travel demand and the resulting travel pattern with greater confidence in the results, and also making new interesting areas, such as the environmental impact of an emerging IT society, open for research.

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