

# **AN EXTENDED FAMILY OF TRAFFIC NETWORK EQUILIBRIA AND ITS IMPLICATIONS FOR LAND USE AND TRANSPORT POLICIES**

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# **Abstract**

Traffic assignment models have a central place in transport planning, for they provide necessary information on the traffic and congestion loads to be borne across a network and how those loads may vary depending on network configuration, design standard traffic control regime and travel demand distributions. The paper focuses on static equilibrium assignment models because of their relevance in transport policy analysis, as models of congested traffic networks that may be used to examine the effects of policy alternatives for land use-transport interactions and for environmental and energy management, travel demand management, traffic calming, and congestion management and road pricing. The paper formulates and applies a family of equilibrium assignment models with separate objective functions for travel time, travel time reliability, energy and emissions, for both individuals and the community. It also incorporates elastic travel demands and trip timing analysis, enabling the study of alternative network configurations, travel demand management policies, and population and land use distributions.

# **INTRODUCTION**

Traffic assignment models have a central place in transport planning, for they provide necessary information on the traffic and congestion loads to be borne across a network and how those loads may vary depending on network configuration, design standard, control regime and travel demand distribution. Static models, such as the user equilibrium (minimum individual travel time) model, have been used in transport planning for many years. In more recent times, there has been growing interest in dynamic assignment models, for use in network simulation and with route navigation systems and advanced traveller information systems (ATIS).

This paper focuses on static equilibrium assignment models because of their relevance in transport policy analysis, that is as models of congested networks that may be used to examine the effects of policy alternatives in land use-transport interaction, environmental and energy management, travel demand management (TDM), traffic calming, and congestion management and road pricing. To make static assignment models really useful for these broader level studies requires more flexible definition of the assignment models. This broader definition is the principal aim of the paper. In addition, the paper establishes relationships between the resulting models for use in the evaluation and comparison of alternative transport policies.

# **THE BASIC SOLUTION TO THE NETWORK FLOW PROBLEM**

The network flow problem is a basic problem in transport planning. Traffic assignment models to solve this problem for road networks require the following inputs:

- 1. a network description, where the network is a connected graph of nodes and links, e.g. link  $e$ being the one-way link connecting node *a* to node *b;*
- 2. an origin destination matrix  $\{T_n\}$  being the matrix of trips from origins *i* to destinations *j* in the network, which describes the travel demand for the network. On occasions there may be a set of origin-destination matrices, split in terms of factors such as trip type or vehicle class. There may also be separate matrices for time of day, particularly in studies where trip timing or peak spreading is important, and
- 3. descriptions of the physical and traffic-carrying characteristics of network links (e.g. road type, number of lanes, free flow travel time or speed, relationship between link volume and travel time) and (possibly) network nodes (e.g. intersection geometry and control type, traffic signal settings where appropriate, turn penalties and bans as necessary).

The network assignment problem is one of selecting a specific strategy to allocate the trips from the O-D matrices to routes through the network, and thus accumulate the flows on the network. Route choice is assumed to be based on travel times and/or costs between origins and destinations; these times and costs change as traffic volumes on the network build up.

## **Continuity of flow**

The generic solution to the network assignment problem is based on continuity of flow considerations. A given level and pattern of travel demand in a given time period (i.e. a given O-D matrix or set of matrices) has to be loaded on to the network. The trips forming this demand are then routed from origin to destination through the network. Thus we may write the generic network flow solution as given by the following sets of continuity of flow equations.

$$
T_{ij} = \sum_{r} X_{rij} \qquad \forall i, j \qquad (1)
$$

$$
q(e) = \sum_{ijr} \delta_{eijr} X_{rij} \qquad \forall e
$$
 (2)

and

where

 $q(e) \geq 0$ 

 $\delta_{\text{e}}$   $=$  1 *if and only if e* is in path r from i to j, 0 otherwise

 $\forall$  e

 $\dot{T}_{ij}$  is the number of trips from origin *i* to destination *j, q(e)* is the volume on link *e* and  $X_{rij} \ge 0$  is the number of trips using path r between *i* and *j*. Any flow pattern satisfying the equations  $(1)$ -(3) is a feasible solution to the network flow problem. A number of alternative solutions, satisfying different performance standards set in terms of the resulting travel times or costs for network journeys, can be defined. Each of these alternatives can be ascribed to a particular strategy or policy for organising or representing the travel pattern. Further, these different solutions may be evaluated separately or compared with each other, thus providing a means to assess alternative policies.

# **THE WARDROP-JEWELL PRINCIPLES**

For planning and evaluation purposes traffic assignment and route choice modelling is perhaps most successfully undertaken by the formulation of the assignment-route choice problem as a mathematical programming problem. Three basic principles are in common usage: Wardrop's first and second principles (Wardrop, 1952), and Jewell's principle (Jewell, 1967). These three principles may be seen as particular members of a family of traffic network equilibria. Equilibrium assignment involves the solution of a mathematical programming problem in which the objective function is non-linear but the constraints are linear. The constraints are concerned with conservation of flows in the network, and are set to ensure that the travel demand described by the origin-destination (O-D) matrix or matrices is satisfied. The objective function represents the strategy adopted by drivers in selecting the routes for their journeys.

### **Wardrop's first principle**

Under this principle, the journey times on all of the routes used for travel between an origin and a destination will be equal at the equilibrium point, and will be less than those times experienced on any other route. No individual driver can gain an advantage by a unilateral change of route. The principle yields a competitive solution point under the mathematical theory of games, where the objective is individual travel time minimisation, and should provide a stable solution.

### **Wardrop's second principle**

This principle is concerned with the overall minimisation of the travel task represented by the total travel time (vehicle-hours of travel, VHT) in the network. In this case drivers select their routes to produce the minimum VHT which is necessary for the travel demand to be satisfied, i.e. for all of the trips in the O-D matrices to reach their destinations. The solution to this problem implies a degree of cooperation between drivers to attain this result. It is a Pareto solution in the theory of games, and is unstable. It does, however, define a datum in terms of the best distribution of flows that could occur if the overall minimisation of 'travel effort' (e.g. VHT) were to be achieved, and other solutions (e.g. for user travel time minimisation) may be compared to it on those grounds.

 $(3)$ 

### **Jewell's principle**

This principle may be seen as a generalisation of the two Wardrop principles. It is that the assigned flow pattern should optimise some overall economic objective for the network. This objective may be the minimisation of travel time, either by individuals (Wardrop's first principle) or for the system as a whole (Wardrop's second principle). Other definitions of economic objective may be chosen, e.g. objectives such as minimum fuel consumption, generalised cost or pollutant emissions.

### **CONGESTION**

Traffic congestion presents a common if not inevitable facet of traffic activity, particularly in urban areas. The spread, duration and intensity of congestion, the processes that lead to it, and the consequences of it are of special concern in urban policy making and transport planning.

# **What is congestion?**

If knowledge about congestion and its extent and intensity is important, then the first consideration is to define just what congestion is. Congestion is an integral part of a transport system, but its specific definition is not immediately obvious. The following definition of congestion can he proposed for use in traffic studies: `traffic congestion is the phenomenon of increased disruption of traffic movement on an element of the transport system, observed in terms of delays and queuing, that is generated by the interactions amongst the flow units in a traffic stream or in intersecting traffic streams. The phenomenon is most visible when the level of demand for movement approaches or exceeds the present capacity of the element and the best indicator of the occurrence of congestion is the presence of queues'. This definition is an extension of that given in Taylor (1992). The extension recognises that the capacity of a traffic systems element may vary over time, e.g. when traffic incidents occur.

For strategic transport planning purposes a satisfactory definition of the level of congestion is the excess travel time incurred. Excess travel time is the additional travel time over and above the free flow travel time  $(c<sub>o</sub>)$ . Travellers may be able to trade-off excess travel time (or indeed total travel time) for other components of the overall cost of travel on a trip. This requires the introduction of the concept of generalised cost of travel. The economic basis for the trade-off is illustrated by the theory of road pricing (e.g. May *et al,* 1996).

#### **Congestion functions**

A congestion function describes the relationship between the amount of traffic using a network element and the travel time and delay incurred on that element. For most transport planning applications the network link is the typical level at which congestion functions are applied, but for traffic engineering applications function for lanes and movements may be more appropriate. A number of common forms of the congestion function exist (Ran *et al,* 1997), but for present purposes two particular functions are considered. The first is the hyperbolic function derived from queuing theory by Davidson (1966), and commonly known as the `Davidson function'

$$
c = c_0 \left( 1 + J \frac{q}{C - q} \right) \tag{4}
$$

in which *J* is an environmental parameter that reflects road type, design standard and abutting land use development, and *C* is the absolute capacity for the link. The Davidson function has become popular in economic analysis and travel demand modelling for road networks, mainly because of its flexibility, and its ability to suit a wide range of traffic conditions and road environments. However, equation (4) has one serious flaw. It cannot define a travel time for link volumes which exceed the capacity *(C).* This provides computational problems in (say) a traffic network model which determines link volumes in an iterative manner, and may consequently occasionally overload some links in computing its intermediate solutions. A modification involving the addition of a linear extension term as a second component to the function has thus been used in transport planning practice (Taylor, 1984). The modified Davidson function is then

$$
c = c_0 \left( 1 + J \frac{x}{1 - x} \right)
$$
  
\n
$$
c = c_0 \left( 1 + J \frac{x_0}{1 - x} + J \frac{x - x_0}{1 - x^2} \right)
$$
  
\n
$$
x \ge x_0
$$
  
\n
$$
x \ge x_0
$$

where  $x = q/C$  is the 'volume-capacity ratio' (or 'degree of saturation'), and  $x<sub>0</sub>$  is a user-selected proportion, usually in the range (0.85, 0.95) as discussed by Taylor (1984). This proportion sets a value of *q* after which the travel time increases as a linear function of volume, and it removes the computational difficulties associated with the original function. Note that all of the above congestion functions are `steady-state' functions, as they assume that the flow *q* will persist indefinitely. There is then a problem of how to deal with oversaturated conditions, which in real traffic systems will exist only for finite time periods (such as a rush hour).

Akcelik (1991) compared the Davidson function to the delay equations found in traffic signals analysis, and proposed a new link congestion function for transport planning and traffic impact analysis that is better able to model link travel time when intersection delay provides a significant part of the total link travel time - the common situation in most urban areas! The time-dependent form of Akcelik's congestion function is

$$
c = c_0 \left\{ 1 + \frac{1}{4} r_f \left( (x - 1) + \sqrt{(x - 1)^2 + \frac{8A}{Cc_0 r_f} x} \right) \right\}
$$
 (6)

where A is Akcelik's environmental delay coefficient,  $r_f$  is the ratio of the flow period  $(T_f)$  to the minimum travel time  $t<sub>o</sub>$  on the link.

# **Fuel and emissions functions for network elements**

 $1-x_0$   $(1-x_0)^2$ 

Taylor (1996) indicated how these congestion functions could be combined with fuel consumption and emissions models to generate link-based fuel and emissions functions for use in transport network modelling, by relating fuel usage and emissions to link volume-capacity ratios. Taylor and Young (1996) used the Biggs-Akcelik four-level hierarchy of fuel consumption and emissions modelling. These models are:

- 1. an *instantaneous model,* that indicates the rate of fuel usage or pollutant emission of an individual vehicle continuously over time;
- 2. an *elemental model,* that relates fuel use or pollutant emission to traffic variables such as deceleration, acceleration, idling and cruising, etc. over a short road distance (e.g. the approach to an intersection);
- 3. a *running speed model,* that gives emissions or fuel consumption for vehicles travelling over an extended length of road (perhaps representing a network link), and

4. an *average speed model,* that indicates level of emissions or fuel consumption over an entire journey.

The instantaneous model is the basic (and most detailed) model. The other models are aggregations of this model, and require less and less information but are also increasingly less accurate. The running speed model is suitable for application in strategic networks, for it can be used at the network link level.

# **Road pricing**

Congestion provides a natural but partial restraining mechanism on travel demand. The additional costs (delays, queuing and inconvenience) resulting from congested conditions can act as a form of deterrent to the generation of further travel demand. However, there is widespread belief amongst transport planners that the congestion `price' of itself is inefficient as a demand management tool. Individual drivers may not be fully aware of the true costs that they impose on other travellers and the transport system on the basis of congestion delays alone. Some other pricing signal is required to this end. Assuming that travellers will respond to a composite generalised cost by trading-off the different cost components in their travel decision making, the further step is to impose a congestion tax, toll or road pricing charge on travellers in an intelligent, selective fashion (e.g. for travel on some parts of a network at some times of day)).

The conceptual model for a congestion tax or price considers the difference between the average travel cost curve (i.e. the congestion function) and the marginal cost curve. Marginal cost indicates the additional travel cost imposed by each new driver using the facility. The unified definition of traffic congestion says that congestion is any additional travel cost above the minimum cost to traverse the system element. The average cost  $(G<sub>i</sub>)$  is generally less than the marginal cost, so the motorists do not meet their full marginal cost. A congestion charge *(AG).* could then be imposed to this end. The actual congestion charge is not easily determined. It requires detailed knowledge of the characteristics of the facility and its average and marginal cost curves, and the level of traffic flow. If a mathematical relationship (such as the Davidson or Akcelik congestion functions) is available, then the marginal cost can be found from the derivative of this function. The marginal travel cost on a link is  $g_{\mu}$  where

$$
g_m = \frac{\partial G_T}{\partial q} = \frac{\partial (c(q)q)}{\partial q} \tag{7}
$$

and  $G_r$  is the total travel cost on the link, given by  $G_r = cq$ . It then follows that for the Davidson function  $c(x)$  defined by equation (5), the marginal cost is given by

$$
g_m = c_0 \left( 1 + \frac{2Jx}{(1-x)^2} - \frac{Jx^2}{(1-x)^2} \right) = c(x) + \frac{c_0Jx}{(1-x)^2}
$$
  
\n
$$
g_m = c_0 \left( 1 + \frac{2Jx_0}{(1-x_0)^2} - \frac{J(2x-x_0)}{(1-x_0)^2} \right) = c(x) + \frac{c_0Jx}{(1-x_0)^2}
$$
  
\n
$$
x \ge x_0
$$
  
\n(8)

For the time-dependent Akcelik function (equation (6)), the marginal travel cost is given by

$$
g_m = c(x) + \frac{r_f}{4} x \left( 1 + \frac{x - 1 + \theta}{\sqrt{(x - 1)^2 + 2\theta x}} \right)
$$
(9)

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where  $\theta = 4A/ C c_0 r_f$ . Note that equations (8) and (9) enable the 'congestion tax' or 'road price'  $(AG)$  to be identified explicitly, given that  $g_m(x) = c(x) + \Delta G$ .

# **POLICY QUESTIONS AND•OPTIONS**

Consider a set of alternative policies that seek to optimise different characteristics of network travel performance, such as minimum overall travel (i.e. minimum overall VHT), or minimum total travel cost, overall delay, fuel usage or pollutant emissions, or peak hour trip spreading, or optimised transport/land use plans for given levels and distribution of travel demand. As indicated below, each of these policies can be represented by a particular equilibrium assignment model, and the resulting network flow distributions can be compared with each other and with the individual travel time minimisation flow pattern. The resulting solutions thus indicate the relative transport performance between the alternative policies, and the degree of similarity or difference between them can be examined.

A variety of models may be developed by applications of Jewell's principle. In the case of road pricing systems, for instance, use of marginal travel costs (e.g. equations (8) and (9)) rather than average travel costs (the corresponding equations are (5) and (6)) for all links in the network provides an appropriate network equilibrium model including individual choice in the presence of a perfect road pricing regime, although as indicated below this model is merely to the system-wide equilibrium model minimising total VHT in the network (Wardrop's second principle). Other cases of road pricing implementations may be more interesting? For example, what if the road pricing is only imposed on a subset of the links (e.g. in a downtown area or regional activity centre), and not across the whole network? In addition, there is the question of how a practical road pricing system might be implemented? Technological developments not withstanding, it seems unlikely that a perfect, real-time road pricing system, in which marginal costs are adjusted continuously in response to traffic flow variations, can be readily employed. Some simplified systems of imposing the 'congestion charge'  $\Delta G$  are more likely to be used.

May *et al* (1996) described a number of alternatives for applying road pricing to real networks:

- 1. road pricing based on charges for usage of road space, e.g. in a given sub-area ('road pricing zone') corresponding to the application of marginal travel costs in that area;
- 2. cordon based road pricing, in which drivers are charged for entering the road pricing zone;
- 3. travel distance-based road pricing, in which a per unit distance charge is levied on each vehicle travelling along the links in the road pricing zone, and
- 4. travel time-based road pricing, in which a per unit time charge is levied on each vehicle travelling along the links in the road pricing zone.

## **FAMILY MEMBERS - MEET THE Z's**

Using the Wardrop-Jewell principles we can define a family of equilibrium assignment models, in which each model represents a particular strategy or policy. The individual models form a family because they have a common structure, through the continuity of flow constraint equations (equations  $(1)-(3)$ ). Each model has its own version of the objective function  $(Z)$ , but all of the models include the same constraint equations and lie in the same decision space.

### **Models including inelastic travel demands**

Inelastic travel demand is defined as a fixed O-D matrix. The starting point for assignment modelling is the well-known user equilibrium model for fixed travel demand, which is an expression of Wardrop's first principle (individual travel time minimisation). This model formulation provides a useful macroscopic simulation of travel on a metropolitan network. It is written as the following non-linear optimisation problem,

$$
Z_{W1} = \min \left\{ \sum_{e} \int_{0}^{q(e)} c_e(x) dx \right\} \tag{10}
$$

subject to the continuity of flow constraints of equations (1)-(3). The equivalent system-wide travel time minimisation problem, the network flow pattern satisfying Wardrop's second principle, may be written as a similar optimisation problem with objective function

$$
Z_{W2} = \min \left\{ \sum_{e} q(e)c_e(q(e)) \right\} \tag{11}
$$

### *Generalised travel cost models*

Utilising Jewell's principle that the ultimate pattern of flow in a network will satisfy some explicit economic objective, for instance minimum generalised travel cost or minimum fuel consumption, a number of equilibrium assignment models can be derived for both individual traveller or systemwide objectives. For example, direct substitution of link emission/consumption (E/C) functions for the congestion function  $c_e(q)$  would yield assignment models that could generate traffic patterns corresponding to minimum fuel use or minimum pollution generation. Generalised cost functions including travel time, fuel consumption, pollutant emissions, tolls and charges, road pricing, etc can also be proposed and solved. For instance, the user equilibrium assignment problem for overall travel cost based on a generalised travel cost function *(g,)* is

$$
Z_{JU} = \min \left\{ \sum_{e} \int_{0}^{q(e)} g_A(e, x) dx \right\} \tag{12}
$$

while the equivalent system-wide travel cost minimisation problem is

$$
Z_{JS} = \min \left\{ \sum_{e} q(e) g_A(e, q(e)) \right\} \tag{13}
$$

Road tolls and general charges (e.g. vehicle operating costs are included in the models defined by equations (12) and (13). Models to study the network traffic effects of road pricing models are better treated separately, especially when alternative road pricing regimes are contemplated.

#### *Road pricing models*

Equation (7) defined the relationship between the marginal cost of travel and the average cost of travel on a link. Substitution of the marginal cost of travel in the objective function for the userminimisation equilibrium assignment (equation  $(10)$ ) shows that it is equivalent to solving the system-wide travel time optimisation problem (equation  $(11)$ ). This is indicated below, starting with equation (14) which is the user equilibrium objective function based on marginal travel costs.

$$
Z_{RP} = \min \left\{ \sum_{e} \int_{0}^{q(e)} g_m(e, x) dx \right\}
$$
 (14)

**36 VOLUME 4**  8TH WCTR PROCEEDINGS Consider the integral in the righthand side of equation (14). Given the definition of marginal travel cost in equation (7), this can be written as

$$
\int_{0}^{q(e)} g_m(e, x) dx = \int_{0}^{q(e)} \frac{\partial (c_e(x)x)}{\partial x} = [c_e(x)x]_0^{q(e)} = c_e(q(e))q(e)
$$

from which

$$
Z_{RP} = \min \left\{ \sum_{e} \int_{0}^{q(e)} g_m(e, x) dx \right\} = \min \left\{ \sum_{e} c_e(q(e))q(e) \right\} = Z_{W2}
$$

Again, the continuity of flow constraints (equations (1)-(3) apply. Thus the road pricing solution yields the system-wide minimum travel time (VHT) distribution of traffic, as long as marginal travel costs are applied on all links in the network. In the case where road pricing might only be applied to a subset of the roads (e.g. the central business district) whereas other links remained in their `normal' state, then the road pricing solution is found by using a composite objective function (e.g. derived from equation (15) for a user equilibrium formulation). Different models apply to the four road pricing schemes examined by May *et al* (1996) and described above. In the following discussion these models are, for purposes of brevity, described as the Leeds models. Assume that the full set of network links then consists of three subsets:

- $(1)$  links w which lie wholly within the road pricing zone;
- (2) links *l* which cross the cordon around the road pricing zone, and
- $(3)$  all remaining links  $e$ , which are external to the road pricing zone

The first Leeds model, charging for the use of road space in the road pricing zone, is based on the application of marginal costs to all links within that zone. This niodel has the objective function

$$
Z_{M1} = \min \left\{ \sum_{e} \int_{0}^{q(e)} c_e(x) dx + \sum_{l} \int_{0}^{q(l)} g_m(l, x) dx + \sum_{w} \int_{0}^{q(w)} g_m(w, x) dx \right\}
$$
(15)

subject to the continuity of flow constraints (equations (1)-(3)).

The second Leeds model is cordon-based road pricing, in which drivers are required to pay a fixed charge  $(m_i)$  when they cross the cordon line to enter the road pricing zone. In this case the corresponding objective function is

$$
Z_{M2} = \min \left\{ \sum_{e} \int_{0}^{q(e)} c_e(x) dx + \sum_{l} \left[ \int_{0}^{q(l)} c_l(x) dx + q(l) m_l \right] + \sum_{w} \int_{0}^{q(w)} c_w(x) dx \right\}
$$
(16)

The third Leeds model is that of travel distance-based road pricing. In this case each vehicle is charged for the distance travelled in the road pricing zone, and the objective function  $Z_{\alpha}$  is thus given by

$$
Z_{M3} = \min \left\{ \sum_{e} \int_{0}^{q(e)} c_e(x) dx + \sum_{l} \left[ \int_{0}^{q(l)} c_l(x) dx + q(l) \eta_l L_l \right] + \sum_{w} \left[ \int_{0}^{q(w)} c_w(x) dx + q(l) \eta_w L_w \right] \right\}
$$
(17)

where  $L_{\rm w}$  is the length of link w and  $\eta_{\rm w}$  is the charge per vehicle per unit distance.

The final Leeds model is that of time-based road pricing. In this case each vehicle is charged at a rate of  $\phi_{\nu}$  for each unit of time that it spends in the road pricing zone. The corresponding objective function may be written as

$$
Z_{M4} = \min \left\{ \sum_{e=0}^{q(e)} \int_{0}^{q(e)} c_e(x) dx + \sum_{l} \left[ \int_{0}^{q(l)} c_l(x) dx + q(l) c_l(q(l)) \phi_l \right] + \sum_{w} \left[ \int_{0}^{q(w)} c_w(x) dx + q(l) c_w(q(w)) \phi_w \right] \right\}
$$
(18)

#### *Fuel and emissions models*

The individual assignment models represented by the specific objective functions given above each represent an optimisation of some specific measure of traffic network performance. It is possible to compare the network states between models and to obtain values of the other performance measures from each specific model, allowing comparison of the effectiveness of different policies. For instance, given the flow pattern from any of the above traffic assignment models, the total fuel consumption and emissions generated can be estimated using the link E/C running speed model. In addition, assignment models for direct optimisation of fuel usage or pollutant emissions can also be generated. These models might be used to investigate network flow patterns where (say) drivers adopt route choice strategies to minimise fuel consumption, or a community seeks a flow pattern minimising overall pollutant emissions. The objective function  $Z<sub>E</sub>$  for individual minimum fuel consumption would be

$$
Z_E = \min \left\{ \sum_e L_e \int_0^{q(e)} E_{se}(x) dx \right\} \tag{19}
$$

noting that  $L_{e}$ , the length of link, is needed in the expression because  $E_{se}$  is a rate per unit distance. The objective function for minimum total fuel consumption or total pollution emissions across the network  $(Z_r)$  would be

$$
Z_X = \min \left\{ \sum_e E_{se}(q(e))q(e)L_e \right\} \tag{20}
$$

### **Models involving elastic travel demands**

Three elastic demand equilibrium assignment models are presented here. The first is for time-elastic travel, where the total travel demand over an extended time period is fixed in space, and some drivers can choose their departure times in particular intervals within the overall time period. The second model is where the total number of trips is fixed, but travellers have the ability to set their origins and destinations in response to congestion on the network. This is a long-run space-elastic model, with applications in land use-transport interaction studies. The third is a combined model using demand elasticities to approximate both time-elastic and space-elastic behaviour simultaneously.

#### *Time-elastic travel demand*

Traffic models accounting for trip timing and peak spreading behaviour are useful for studies of the impacts of travel demand management policies and time-dependent road pricing systems. The model derived by Matsui and Fujita (1996) for individual travel time minimisation including departure time choice fits closely to the equilibrium assignment model family described previously. They applied this model to Toyota City, Japan, to study the impacts on congestion and journey times of a flexitime system and a road pricing system. They found a five per cent reduction in total VHT under the flexitime regime, and a two per cent reduction under the road pricing system.

### *Space-elastic travel demand*

In the case that travel demand (as represented by a O-D trip matrix) is elastic, i.e. the trip distribution (destination choice) may vary depending on the congestion levels in the network, then the combined distribution-assignment model proposed by Evans (1976) and explained by Horowitz (1989) provides an equivalent formulation to the equilibrium assignment model, and may be solved by a similar mathematical programming approach. This model may be treated identically to the equilibrium assignment model for fixed travel demand. More complex models bringing modal choice into this formulation have also been developed, e.g. Tatineni *et al* (1995). Other recent developments include the SUSTAIN model (Roy *et al,* 1996), a land use-transport interaction model designed for studies of transport-land use development behaviour of cities under different economic and planning scenarios. SUSTAIN includes user equilibrium assignment in a combined assignmenttrip distribution-modal split model connected to a housing and employment location model.

# *Use of demand elasticities*

With the growing interest in trip timing decisions, especially for peak spreading and travel demand management considerations, an obvious future step is the integration of the time-elastic and spaceelastic models into a single elastic-demand model. There is also the question of the influences of levels of traffic congestion on the demand to use road space (i.e. on the total VKT), which involves considerations of the phenomenon of 'induced traffic'.

In the modelling of road pricing, elastic travel demand is of great importance. For instance, if road pricing is intended to provide motorists with better cost and impact information on which to base their travel choices, then it is to be expected that some travellers will decide to change their mode of travel, trip timing, or destination choice when faced with a road pricing zone in the network that they are using. One way to model these choices is through the use of demand elasticities. This topic has the subject of some recent research in Australia, e.g. Bray and Tisato (1997). A demand elasticity approach was adopted for use with the equilibrium assignment models for road pricing. This enabled the origin-destination tables to be adjusted for each of the road pricing schemes.

# **WHAT TO DO NEXT - EMPLOYING THE Z's**

An initial application of the model was made to a `scenario-planning' network representing the primary road system for the Melbourne urban region. This coarse network of some 300 nodes, 1200 links and 50 zone centroids represents the principal road corridors for that city. A peak period origin-destination matrix was developed from 1991 census journey-to-work data. The network model considered peak period traffic flows and the air pollutant emissions effects in the network under the following regimes:

- the *status quo* for 1991 (this being the time period which applied to the available travel demand  $\bullet$ data), using a user-optimum equilibrium assignment. This was taken as the datum for the study
- a system-wide assignment minimising the total vehicle-hours of travel (VHT) in the network
- the imposition of full marginal cost road pricing in the entire network, using ITS technology to monitor traffic flows and congestion levels, and to make appropriate charges to all road users
- the definition of a restricted zone (road pricing area) around the Melbourne CBD. The road pricing zone was defined by a 5 km radius circular boundary. A cordon charge of one dollar (AUD) was imposed on vehicles entering the zone in the morning peak
- the imposition of a distance-based road pricing charge of \$(AUD) 0.19 per kilometre on all vehicle journeys within the road pricing zone



**Figure 1: Summary travel statistics (trips, VHT and VKT) for the Melbourne network** 



**Figure 2: Summary emissions data for the Melbourne network under different road pricing** 

- the imposition of a time-base road pricing charge of  $$(AUD) 0.13$  per minute of travel time on all vehicle journeys within the road pricing zone
- the imposition of full marginal cost road pricing in the Melbourne CBD road pricing zone.

In addition, the possible network-wide effects of incidents were modelled by imposing a blockage on the in-bound lanes of an inner-area freeway link in the Melbourne network.

The overall results for the eight assignment models, in terms of broad travel statistics, are summarised in Figure 1. This shows the relative amounts of trips, VKT and VHT in the network under the different road pricing regimes, when compared to the base case. VHT, VKT and total car trips are least in the network-wide full marginal cost road pricing scenario. Marginal cost road pricing in the road pricing zone shows the second least number of car trips, VKT and VHT. The freeway incident shows a small increase in VHT with the other factors unchanged. Figure 2 shows the summary results in terms of air pollutant emissions generated by peak period road traffic, in terms of emissions of carbon monoxide (CO), hydrocarbons (HC) and nitrogen oxides (NOX). Each road pricing scheme has different effects. Full marginal cost road pricing across the network has the largest impact on travel and emissions, whilst marginal cost road pricing and time-based road pricing in the central area road pricing zone are the next most effective schemes. The levels of emissions of the pollutants largely follow the scheme of the travel parameters, except perhaps for NOX. Decreases of between three and six per cent in overall emissions may be seen for the pollutant emissions in the marginal road pricing scenarios. Differences of one to three per cent are apparent for the other road pricing schemes.

# **CONCLUSIONS**

A family of traffic assignment models was established, suitable for comparative analysis of alternative networks and transport policies. The model family includes with the well-known individual travel time minimisation model. With the addition of generalised and perceived travel cost functions and fuel and emissions relationships, the model family offers a useful means to examine the ways in which variations in vehicle fleet composition, travel demand patterns, vehicle operating costs, road user charges and tolls, and congestion levels affect network performance. Thus traffic network models sensitive to transport and land use planning objectives can be established and applied to examine the effects of alternative policies. Some preliminary results have been found, indicating that there are differences in the flow patterns resulting from the different objectives. Further investigations are needed to explore the wide variety of alternative assignment models made available in the family of equilibrium models, and to make comparisons between the resulting outputs of those models in terms of link flows and network performance parameters such as travel times, fuel consumption and pollutant emissions.

# **REFERENCES**

Akcelik, R (1991) Travel time functions for transport planning purposes: Davidson's function, its time-dependent form and an alternative travel time function. Australian Road Research 21 (3), 49- 59.

Bray, D and Tisato, P M (1997) Broadening the debate on road pricing. Papers of the Australasian Transport Research Forum 21 (2), 599-616.

Davidson, K B (1966) A flow travel-time relationship for use in transportation planning. **Proceedings 3rd Australian Road Research Board Conference 3** (1), 183-194.

Evans, S P (1976). Derivation and analysis of some models for combining trip distribution and assignment. **Transportation Research 10** (1), 37-57

Horowitz, A J (1989). Tests of an ad hoc algorithm of elastic-demand equilibrium traffic assignment **Transportation Research B 23B** (4), 309-313

Jewell,.W S (1967). Models for traffic assignment. **Transportation Research 1,** 31-46.

May, A D, Milne, D, Smith, M, Ghali, M and Wisten, M (1996). A comparison of the performance of alternative road pricing systems. In D A Hensher, **J** King and T Oum (eds), **World Transport Research 3: Transport Policy.** Elsevier, New York, 335-346.

Matsui, H and Fujita, M (1996). Modelling and evaluation of peak hour traffic congestion under flextime and road pricing systems. In D A Hensher, J King and T Oum (eds). **World Transport Research 2: Modelling Transport Systems.** Elsevier, New York, 183-193.

Ran, B, Rouphail, N M, Tarko, A and Boyce, D E (1997). Toward a class of link travel time functions for dynamic assignment models on signalized networks. **Transportation Research B 31B**  (4), 277-290.

Roy, J R, Marquez, L O, Taylor, M A P and Ueda, T (1996). SUSTAIN - a model investigating sustainable urban structure and interaction networks. In Y Hayashi and **J** R Roy (eds). **Transport, Land Use and the Environment.** Kluwer, Dordrecht, 125-145.

Tatineni, M R, Lupa, M R, Englund, D B and Boyce, D E (1995). Transportation policy analysis using a combined model of travel choice. **Transportation Research Record 1452,** 10-17.

Taylor, M A P (1984). A note on using Davidson's function in equilibrium assignment. **Transportation Research B 18B** (3), 181-199.

Taylor, M A P (1992). Exploring the nature of urban traffic congestion: concepts, parameters, theories and models. **Proceedings 16th Australian Road Research Board Conference 16** (5), 83- 106.

Taylor, M A P (1996). Incorporating environmental planning decisions in transport planning: a modelling framework. In Y Hayashi and **J** R Roy (eds). **Transport, Land Use and the Environment. Kluwer,** Dordrecht, 337-358.

Taylor, M A P and Young, T M (1996). Development and application of a set of fuel consumption and emissions models for use in traffic network modelling. In J-B Lesort (ed). **Transportation and Traffic Theory.** Pergamon-Elsevier, Oxford, 289-314.

Wardrop, **J** G (1952). Some theoretical aspects of road traffic research. Road Paper 36, **Proceedings of the Institution of Civil Engineers 1** (2), 325-378.