th
WCTR

LONG-TERM TREND MODELLING OF ACCIDENT RISK USING TIME SERIES DATA

ROGER JANSSENS,
Belgian Road Research Centre
Boulevard de la Woluwe, 42, 1200 BRUSSELS, BELGIUM

Abstract

An attempt is made to explain the frequently observed law of accident frequencies (rising branch, peak value, descending branch) with its associated law of decreasing risk with increasing mobility, which itself develops approximately linearly with time.
The search for accident potential-defining variables is reviewed and the components of the classical risk triad are considered. Three tentative explanations are discussed, respectively based on the concept of critical density, on the construction of new motorways and on a "dimensional" approach deduced from a physical analogy.
The paper continues with the implementation of a friction model. A "behavioural" lognormal model is examined as an alternative explanation and an empirical law, inspired by Pearsons's frequency curves, is suggested as a purely descriptive tool.

INTRODUCTION

The present paper deals with the first stage of the development of 'theoretical' safety models, as opposed to 'descriptive' models. Proper modelling starts with an adequate problem definition; only at a later stage does the application of specialized statistical techniques become relevant. In their present form our models lack properly defined and tested statistical submodels. This is not a final option. Future research will probably enable us to deal more specifically with statistical calibration and validation issues.

SEARCHING FOR ACCIDENT POTENTIAL-DEFINING VARIABLES

Synthetic review

Characterizing the accident potential of a road has long been a vexing problem to politicians and traffic managers. There is an extensive list of possible candidates for the variables which determine the occurrence of road accidents. To the author's knowledge there does not exist a systematic account of searches for accident determinants, even when focussing exclusively on the aggregate level. Most of the reviews are fragmentary, frequently contradictory, and lack synthesis.

An interesting partial review has been made by Satterthwaite (1981), who discusses several dozens of models mainly based on ***traffic volume*** while mostly neglecting the other variables.

The use of ***traffic density (concentration)*** is also appealing, because it is possible to obtain the same level of traffic volume with two different levels of density and speed. Haight (1973) suggested that the expected number of accidents in a road section should be a quadratic function of density. From Haight's simple model Mahalel (1986) inferred an interesting positive correlation between traffic density and accident frequency: both variables always vary in the same sense, both simultaneously increase or decrease. A similar correlation does not exist in the case of traffic flow.

Critical density is a specific density value which has played a certain role in safety literature, although a precise definition is lacking. We will dwell on it a little longer below when we are seeking an explanation for the observed occurrence of a maximum in the trend of the fatality frequency curve with time - an established fact in many countries.

Historically, ***acceleration noise*** has been an early candidate for an accident-explaining factor. This concept has been discussed by several authors, such as Torres (1970), mostly in the context of general traffic dynamics (internal energy), sometimes in relation to traffic safety, and on other occasions in connection with various topics, e.g. fuel consumption on congested motorways, as treated by Bester (1981). The latter study is specially mentioned here because Bester has combined the acceleration noise theory with a simple SFC model: the model of Greenshields containing a lineair relation between speed and density. By linking the two components Bester finds a cubic relation between acceleration noise and the ratio of measured traffic density to jam density.

Countless analyses, though many in modelling at the microlevel, have focussed on the ***speed variable***. It is common feeling that accident frequency is positively correlated with absolute speed value. Increasing speed seems to cause more accidents, and decreasing speed to cause fewer accidents. However, this does not follow from the static SFC model, which suggests a negative correlation because speed is negatively correlated with concentration in the entire range of variation.

Being deceived by the results obtained when taking absolute speed or some other traffic variable as an accident determinant, some researchers have developed models based on the ***number of speed variations***. As an example we refer to Heimbach *et al.* (1968) who tried to find traffic parameters which are mathematical functions of acceleration noise. One such parameter which appeared to meet this requirement is the mean of the number of absolute 2-mph speed changes per unit of highway travel time. One of the conclusions was the finding of a visual rank order correlation between accident rates and the proposed measure of speed changes.

The internal energy of a traffic stream is a general concept, including some of the preceding ones, to characterize traffic interaction dynamics, which may be implemented in various forms. Drew (1968) introduced the energy concept into traffic flow analysis by considering the traffic stream to be analogous to the flow of a compressible fluid in a constant-area duct. He suggested that the parameter acceleration noise should be used as a measure of internal energy. Although acceleration noise appeared to be a significant concept for studying traffic and related safety aspects, it finally revealed certain shortcomings. Therefore, some researchers examined the relevance of alternative characterizations of internal energy.
We mention a study of Lee *et al.* (1973), who examined four other internal energy parameters: the standard deviation of acceleration, average absolute acceleration, the standard deviation of speed, and finally, the coefficient of variation of speed (related to, but different from, the above-mentioned number of speed variations).
Traffic stream friction being the preferred candidate for our own model development we will consider it further in this paper.

An hypothetical 'complete' MIN-MAX fatality risk model

Combining the elements of the models discussed by Satterthwaite (1981) with those defining density and critical density as genuine risk determinants, we may suggest a synthetic 'complete' risk picture of the MIN-MAX type in which a less pronounced risk minimum is followed by a more pronounced risk maximum, as illustrated in Fig. 1.

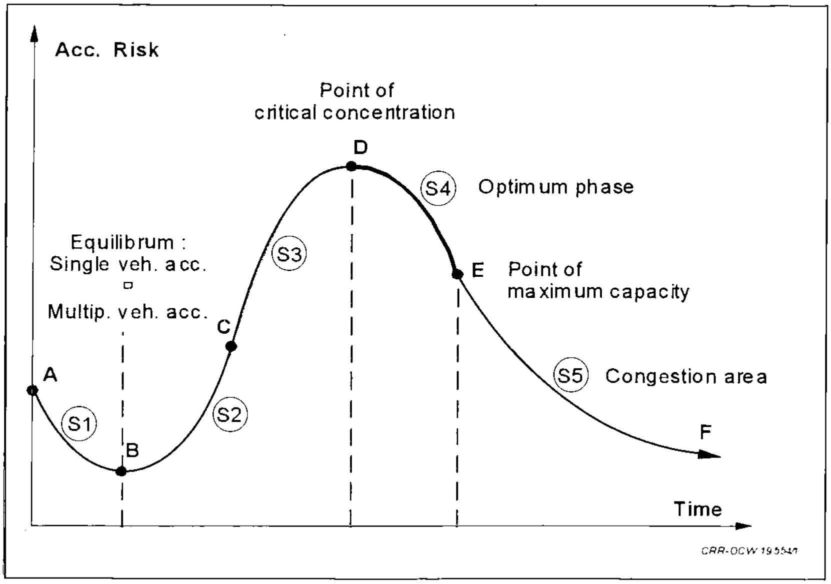

Figure 1 – A "complete" MIN-MAX risk model

The components of such a MIN-MAX risk model would be: a first decreasing branch, a minimum, a rising branch, a maximum, and a second asymptotically decreasing branch. These components are almost never simultaneously present. Each element is characteristic of specific road types.
It seems to us that a complete MIN-MAX model might only occur in some very specific cases. The model is unlikely to be representative of an entire country, since in almost all countries interregional and urban two-lane roads represent more than 90 % of the road infrastructure, the other roads being motorways or highways with three, four or more lanes. The MIN-MAX model does not conveniently describe two-way roads because the exposure dependency of single and multiple accident frequencies on such roads is basically different from that on other types of road.

Friction as a realistic aggregate variable

Friction definitions

The friction concept was introduced by May (1959). He applied this general physical concept to highway traffic flow. The friction caused by increased traffic flow in the same direction is *internal friction*, and the friction caused by road features to traffic is *external friction*. It has been suggested that road features which cause *external* friction can logically be classified into *three groups*: medial friction between vehicles moving in opposite directions, marginal friction along the edge of the road and intersectional friction between vehicles moving at approximately right angles.
It is obvious that certain friction combinations do not exist on motorways. This hints at the *favourable safety status of motorways* among all other types of road.
May proposed to take ***total travel time*** as a unique scalar measure of total traffic friction. His travel time submodel will be presented in the Subsection on "A friction model".

A possible pitfall: distinguishing between frictional effects and speed effects

It is important to distinguish carefully between frictional effects and speed effects.
Over the past decades *decreasing accident risk* has been accompanied, in at least a dozen West European countries, by a gradual *increase* of countrywide *average traffic speed*, which has been observed to be of the order of 1 to 2 km/h during a series of years.

Should we conclude from this that increasing average speed enhances safety ? Certainly not ! Increased speed and increased safety are strongly positively correlated, but that does not mean that there is a causal relationship between the two. They are rather the common consequence of several infrastructural effects and 'behavioural' adaptations as described further on. Moreover the positive correlation *only exists for low speeds* (say, < 50 km/h). Obviously, in the high speed range speed and safety are negatively correlated.

THE CLASSICAL RISK TRIAD AND ITS COMPONENTS

The risk triad: frequency exposure risk rate

In accident modelling, use is often made of a triad of objective functions: the frequency function, the exposure function, and the risk function. In general, these three functions depend on time, a set of exogenous variables, and a set of parameters. For the sake of simplicity we shall consider time as the only independent variable.
The ***accident frequency function***, AccFreq(t), describes the development of a representative accident scalar as a function of time. The scalar involved generally relates to an important strategic policy variable. The nature of this variable may vary: number of accidents, number of persons (involved, killed immediately or later, seriously or slightly injured), etc.
The ***exposure function***, Exp(t), describes the development of some chosen measure of exposure. A frequently chosen measure is the number of vehicle-kilometres travelled during the year or some substitute for it, such as the fuel consumption of the vehicle fleet.
The form of the exposure function is generally increasing (linearly, parabolic law, exponential law, logistic growth). The majority of models does not consider the possibility of a decreasing trend, although in the future this could become indispensable in the context of sustainable development, mobility restrictions (road pricing), etc.
The ***risk function***, or accident rate function, is, by definition, the quotient of the frequency function and the exposure function: Risk(t) = AccFreq(t) / Exp(t). By combining various frequency functions with different measures of exposure it is possible to consider many different risk functions, although not all of them are equally relevant. We mainly consider two frequency scalars (yearly numbers of accidents and fatalities within 30 days) and a single measure of exposure (number of vehicle-kms travelled).

We use the following prefixes as *abbreviated notations*: 'AF' (accident frequency), 'FF' (fatality frequency), 'AR' (accident risk) and 'FR' (fatality risk).

A basic empirical fact

In the seventies a basic empirical fact was observed in many European countries: *the occurrence of a maximum ini the FF curve.* To illustrate this observation we refer to Fig. 2 for Belgium and to Figs. 3 and 4 for Germany and France, as examples for analogous trends in Great Britain, Italy, The Netherlands and Norway. This shape of the FF curve is typically accompanied by a *hyperbolically decreasing FR curve* (the last branch of a theoretically 'complete' FR curve, also with a maximum value). Many researchers have been looking for some logical explanation for the phenomenon.

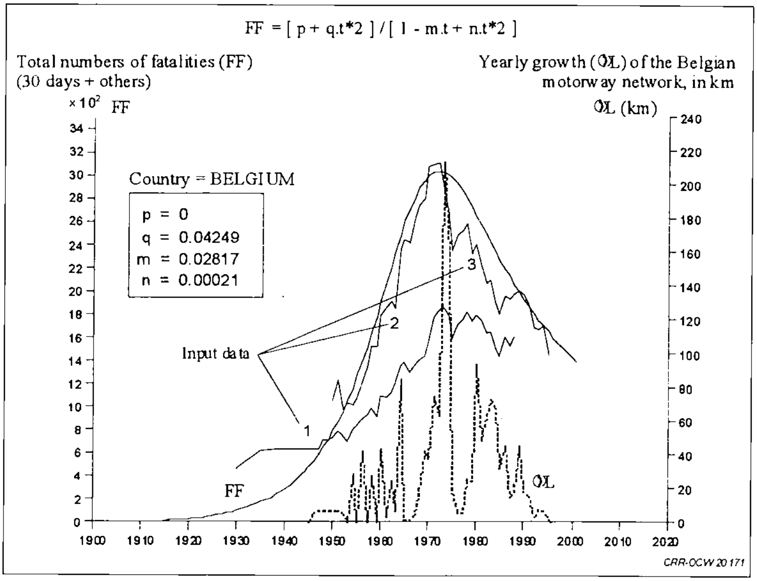

Figure 2 - A comparison between the Belgian 20th century FF-curve and the evolution of the extension of the Belgian motorway network

ACCIDENT FREQUENCY FUNCTIONS OF THE FORM:
Y = [p + q.t*2] / [1 - m.t + n.t*2]

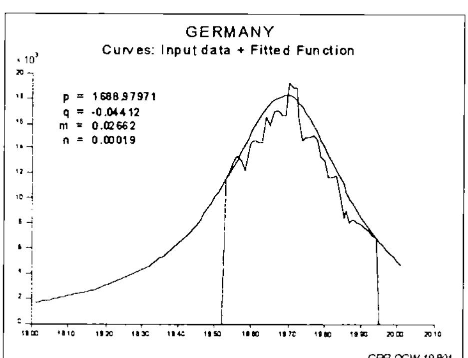

Figure 3

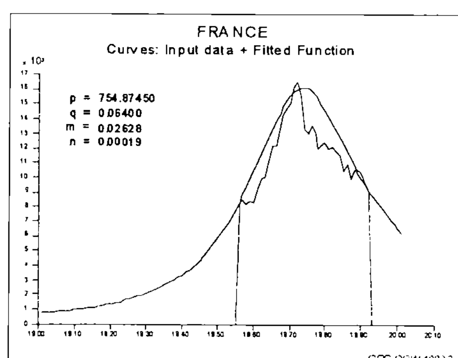

Figure 4

EXPLAINING THE FF MAXIMUM AND THE LAW OF DECREASING RISK

Various suggestions for an explanation of the maximum value

There is a variety of competing theories which could be used to explain the appearance of the maximum FF and/or FR value. We emphasize the need to identify a quantifiable set of parameters rather than to lapse into qualitative speculations and fuzzy metaphors.
Congestion seems to play a crucial role and is in any case relevant. Does congestion decrease the risk (lowered speed, reduced accident severity in terms of number of fatalities) or does it increase the risk (more friction) ? It probably diminishes the number of fatalities but increases the number of accidents.
Let us examine a few examples of explanations more closely.

Maximum theories based on critical density

We refer to two theories which explicitly look for relationships between critical density and accident occurrence. A third theory will also be mentioned, because it will serve as an introduction to our own friction model.

A Japanese safety index

About forty years ago two Japanese authors, Kometani and Sasaki (1959), studied the relationship between traffic density and accident risk. They analysed the frequency response and stability of propagation of a sinusoidal disturbance in a one-lane traffic stream, as a function of the speeds of the lead car and the following car. Moreover, they were among the first researchers to link speed amplitude spectra to accident risk and derived this relationship in the form of a quantitative safety index.
Ferrari (1994) too - in the context of stability analysis and critical density - stresses the importance of the amplitude spectrum of the speed distribution of the lead car, however, without linking it to safety aspects.

A German critical density model

Kaczor (1980) proposes a very specific MIN-MAX model of the type mentioned above. Moreover, he specifies a concrete range of density values characterizing the location of the minimum and the maximum of his model, in relation to link capacity:
"Mit Anwachsen des Belastungsgrades der Strassen verringert sich die Kennziffer d_3^1.
Bei x = 0,3 - 0,4 liegt das Minimum, danach erfolgt ein Anstieg, und bei etwa x = 0,7 - 0,8 wird das Maximum erreicht, danach tritt eine Senkung ein."
The variables used are:

x : "aktuelle zu maximaler Intensität"
d_3^1: "relative Zahl der Getöteten, der Verletzten oder der sonstigen Unfälle pro 1 oder 100 Mill. Fahrkilometer."

A critical density model using friction terminology

Still another version of a critical density theory providing specific critical density values comparable with those of Kaczor (1980), is described by Baass *et al.* (1988) and Manar *et al.* (1996). A peculiarity of this approach - which does not deal with accident risk - is the establishment of a relationship between platoon dispersion, critical density and traffic friction.
The importance of the friction concept lies in the fact that it is more general than that of critical density, because the former is applicable to all types of traffic regime (stationary and transient) whereas the latter is restricted to heavy and continuous traffic flow (e.g. on motorways).

The construction of motorways as an explaining factor

The favorable safety status of motorways

It is not difficult to understand the *crucial impact of motorways on safety* when considering the basic observational fact that the risk level for fatalities (or accidents, or ...) *on motorways is 3 to 4 times lower than on other types of road.* Increasing the proportion of motorways to other roads in successive shocklike steps has created a series of developmental impulses which, in many countries, have turned down the fatality trend already after the second developmental shock.
It should be realized that in the case of Belgium as illustrated in Fig. 2, the construction of motorways has been accompanied and followed by *a systematic annual shift of traffic* from regional and local roads to motorways. To put it more exactly: traffic has increased on each of the three subnetworks, but very heavily on motorways and much more slowly on the other roads.

Empirical evidence from the USA

The above observations were already made in the sixties (USA) and the suggestion for a drastic 'freeway remedy' was formulated there by Bellis (1967).

Empirical evidence in Europe

To find an explanation for the occurrence of the FF maximum we introduce a concept which expresses the relative magnitude of the growth of the length of the motorway subnetwork as compared to the growth of the sum of the lengths of all other types of road.
There are strong indications that a peak value for this relative magnitude (in the case of Belgium it can hardly be denied) *explains the appearance of a maximum* of fatalities (observed in the seventies) *in 7 of the 11 countries* that we have examined (Belgium, Germany, France, Great Britain, the Netherlands, Italy, and Spain).

The motorway puzzle and its solution

The above criterion does not work in countries where the motorway network is less extensive with respect to the other subnetworks, or where the motorway subnetwork developed slowly without shocks (Finland, Norway, Sweden and Danmark).
In these countries the slow gradual growth of the motorway network did not produce observable effects, whereas a maximum still appeared in the same period.
Consequently, the *explosive growth* of the motorway network seems to be a *sufficient* condition for a maximum value but *not a necessary* one. Its influence is similar to the action of a chemical catalyst: if present in an adequate dose, it triggers the occurrence of an FF peak, but the underlying 'chemical reaction' still progresses when the catalyst is absent.
The preceding facts indicate a need to identify a more general mechanism covering motorway building as a subcase. A candidate explanation with interesting potentialities may be found in the concept of *total traffic friction.*
The general accident-causing mechanism is strongly related to total friction. However, the construction of motorways is only one method - a very effective one - to reduce total friction. There are many other ways, e.g. widening 2- and 3-lane highways to 4-lane highways, building roundabouts at selected places, automatic traffic monitoring, etc.

A 'dimensional' explanation of the law of decreasing risk

Heat losses from buildings

If heat losses per m^3 are plotted as a function of the increasing total volume of a building, a hyperbolically decreasing law (HDL) is found in the ceteris paribus hypothesis (same mean transmission coefficient of the outer surfaces of buildings which are mutually compared). This law bears an astonishing resemblance to the law of decreasing fatality risk.
In the case of heat losses the phenomenon is easily understood as a consequence of the difference in dimensionality of the surrounding surface of the building (two-dimensional) through which the calories escape, and its volume (three-dimensional). The HDL naturally follows from the decreasing proportion surface/volume. For that reason a spherical building having the same volume as a rectangular block (especially one with one small dimension) will lose much less heat per m^3.

Traffic friction as a 'surface' phenomenon

The same type of explanation applies, to a certain degree, to traffic safety on road networks when accident risk is formulated in terms of traffic friction.
Traffic friction and traffic volume (exposure) are also different in dimensionality. Traffic volume is related to the two-dimensional area of the road. External friction is caused by linear elements (shoulders, median barriers, ...). Even internal friction is mainly related to linear phenomena (lane changing, car following). This means that the proportion of total friction to total exposure tends to decrease with increasing exposure. This automatic law provides a natural explanation for the observed law of decreasing fatality risk.
This does not mean that safety actions undertaken by road managers cannot produce results, because all actions are reducible to the minimization of total traffic friction.

SOME TENTATIVE MODELS

A friction model

The mobility submodel

The 'equation of state' (SFC model) at the network link level is: $F = S.C$ (1)
where: F = average traffic flow rate (hourly volume) (in veh/h/lane),
S = average traffic speed (in km/h),
C = average concentration (density) of the traffic stream (in veh/km/lane).

Although the methodology for developing areawide relationships is not well established, we exclusively aim at an areawide, even countrywide, extension of these relations, which are usually applied at the link level.
A starting point for a viable solution is the very general fundamental equation proposed by Pursula in a comment to a paper by Banks (1995). According to Pursula, we have to reformulate the definition of the three basic traffic flow variables in the context of a generalized space-time domain. The definitions of Pursula and the fundamental equation based on them are universally valid for any kind of traffic (congested, uncongested, random, uniform, etc.) in any imaginable space-time domain XT, where X represents the space axis and T the time axis. In our model, the unit of the space axis is a lane-km, whereas the unit of the time axis is a year. The countrywide network is characterized by its total length, expressed by the total number of available lane-kms.
An SFC model may be split into three two-dimensional relations: an FC relation (also called the fundamental diagram), an SF relation and an SC relation. These relations exist in many forms varying from the elementary Greenshields' model (with linear SC law) to more realistic models based on

catastrophe theory. See, e.g., Branston (1976), and Acha-Daza *et al.* (1994). It should be noted that the two relations, SC and SF, cannot be simultaneously linear, because of the existence of the equation of state.
We advocate an elementary mobility model with a lineair SF relation applied at the network level:

$$\text{SF submodel:} \qquad S = a - b.F \tag{2}$$

where: parameter a = free-flow speed,
parameter b = slope of the linear speed-flow relation (> 0).

The accident frequency travel time submodel

According to May (1959), the accident rate expressed in number of accidents per million vehicle-miles of travel is primarily dependent on the amount of road traffic friction and the number and amount of deviations from the average rate of motion (stopped time and speed distribution). Because stopped time and speed distribution can be shown to be related only to travel time, the accident rate is related only to road type and travel time.
An extensive statistical study by May revealed the following relationship:

$$AR = 1.52\ T^{2,25} \tag{3}$$

which may be approximated by the parabolic law:

$$AR = 2.15\ T^2 \tag{4}$$

where:

AR = accident rate (number of accidents per million vehicle-miles)
T = average travel time per mile for a specific route.

Using other units (replacing a million vehicle-miles by a milliard vehicle-kms), the above law may be written as:

$$AR = u.T^2 \tag{5}$$

where the value of parameter u may be determined by some calibration procedure (or calculated from May's constant and the converted units).

The in-motion time travel time submodel

May has found the following relationship between average running time, t (the time a vehicle is in motion per mile of travel), and average total travel time, T:

$$t = T / [0.782 + 0.132\ T] \tag{6}$$

Generalizing and converting the units from miles to kms, we adapt this model and define this relation to be:

$$t = T / [s + r.T] \tag{7}$$

where r and s are parameters to be calibrated from experimental data.

The accident severity submodel

It is important to distinguish between the occurrence of an accident and its consequences in terms of fatalities, injuries and material damage. In our accident severity model we only consider the relation between the number of accidents and the resulting number of fatalities (say within 30 days).

The number of accidents as a function of increasing exposure (congestion) is an increasing function, which in a heavy congestion situation may become saturated or even reverse to a slightly decreasing function.
It is intuitively apparent that the number of fatalities (accident severity) is a monotonically decreasing function of exposure. This may be explained by a number of factors.
Firstly, there is the fact that traffic speed always decreases with increasing congestion. But everybody knows that the probability of a fatality is much smaller in the case of a collision at low speed (say 30 km/h) than in the case of high speed (say 120 km/h).
An additional explanation may be the increasing technological progress achieved in road design, vehicle construction and traffic safety measures.
Last but not least, improved medical skills and the increasing efficiency of emergency services may also explain the gradual reduction observed in the number of fatalities.
We propose the following mathematical form (decreasing function with increasing T or increasing congestion) for the accident severity model:

$$Sev = 1 / [g + h.T^2] \qquad (8)$$

where: Sev: severity function giving the number of fatalities per accident,
g, h: parameters to be evaluated by suitable calibration.

Putting all submodels together: the friction model

We propose an FR model, with an associated FF model, which is a combination of all preceding submodels. Although the appearance of a maximum might already be implied by the features of the accident frequency function alone, a genuine maximum is certainly imposed - even in the case of an ever increasing accident frequency function - by combining the friction function with the ever decreasing severity function.
After manipulating the above relations (1), (2), (5), (7) and (8), we can compute the first derivatives and equate the expression obtained with zero. Results are as follows :

$$FatRisk, FR = u / [h + g.(v - w.F)^2] \qquad (9)$$

where: $v = (a - r) / s$,
$w = b / s$.

The risk function has a maximum for	:	F_{maxFR}	=	v / w.
The value of the maximum is given by	:	$FatRisk_{max}$	=	u / h.
The initial (F = 0) value of the risk is	:	$FatRisk_{ini}$	=	$u / [h + g.v^2]$.

The location of, and function value at, inflection points may be calculated by equating the second derivatives with zero. This calculation is rather complex and the results will not be discussed here.

The derived FF model

The FF function is by definition: FatFreq = FatRisk x F.

Multiplying (9) by F and equating the first derivative with zero gives:

$$F_{maxFF} = SqRoot \{ [v^2 / w^2] + [h / (g.w^2)] \} \qquad (10)$$

Calculating the value of the FF maximum is complex but possible in principle.

The time series FR(Time) and FF(Time)

In order to relate the above FR model and the derived FF model to time series data, it is still necessary to take account of the development of exposure and all other variables with time. If, by way of approximation, we adopt a *linear exposure law* (with slope f) of the form:

$$\text{Exposure(Time)} = F(\text{Time}) = F_0 + f.\text{Time},$$

the expressions for the location and the value of the FR maximum and the FF maximum change. The details of these transformations will not be discussed.

A 'behavioural' lognormal FF model

The headway probability distributions for various traffic regimes have been studied frequently, theoretically and empirically. See e.g. Mei *et al.* (1993). Theoretical analysis has shown that the lognormal mechanism is applicable to individual headways for drivers in a car-following situation. A look at Fig. 2 (Heuristic Empirical-Model HEM) and Fig. 5 reveals the similarity between the general form of the FF curve and the shape of a lognormal distribution. Thus it seems natural to seek an explanation for the shape of the FF curve in the context of lognormal distributions.

The lognormal distribution arises as the result of a multiplicative mechanism acting on a number of factors. For our purpose, a single case is of special interest, that is, *the law of proportionate effect*, established by Aitchison (1963). This law deals with a variable the value of which varies in a step-by-step sequence, as in a time frame.

We assume that these consecutive values constitute the time series of yearly fatalities at the aggregate country level (FF time curve).

Suppose that the FF variable has initially the value X(0), a value X(j) after the *j*th time step (year), and a final value X(n) after n years.

At the *j*th time step the change in the variable becomes - according to the law of proportionate effect - a random proportion of the momentary value X(j-1) already attained:

$$X(j) - X(j-1) = \varepsilon(j).X(j-1) \qquad (11)$$

where the members of set { ε(j) } are mutually independent and also independent of those of set {X(j) }.

Expression (11) may be rewritten as:

$$[X(j) - X(j-1)]/X(j-1) = \varepsilon(j) \qquad (12)$$

so, that:

$$\Sigma_{j=1}^{n} [X(j) - X(j-1)]/X(j-1) = \Sigma_{j=1}^{n} \varepsilon(j) \qquad (13)$$

If the effect at each step is assumed to be small, we have:

$$\Sigma_{j=1}^{n} [X(j) - X(j-1)]/X(j-1) \approx \int_{X(0)}^{X(n)} dX/X = \ln X(n) - \ln X(0) \qquad (14)$$

which gives:

$$\ln [X(n)/X(0)] = \varepsilon(1) + \varepsilon(2) + ... + \varepsilon(j) + ... + \varepsilon(n) \qquad (15)$$

The importance of the above law is its link with the central limit theorem:

"Under very general conditions, as the number of variables in a sum becomes large, the distribution of the sum of random variables will approach the normal distribution".

The preceding process, therefore, approximately (≈) meets the requirements of the central limit theorem, so that ln [X(n)/X(0)] is ≈ normally distributed. Random variable X(n)/X(0) is then ≈ lognormally distributed. Consequently, a variable obeying the law of proportionate effect is ≈ lognormally distributed provided that the change in each step is small.

It should be noted that the foregoing model admits of an interpretation of the process dynamics in the context of small 'behavioural' adaptations : the basic dynamic fatality trend is 'corrected' each year by 'random effects' (randomly spread interventions) causing parameter variations which produce yearly downward shifts of the trend.
However, we should realize that the concept of 'behavioural adaptation' is be taken in a very general sense: not only genuine behavioural adaptations of drivers to risk situations, but also technical adaptations of the transportation system by constructing gradually safer vehicles and safer roads.

A heuristic empirical FF model (HEM)

When we summarize the main characteristics of the models discussed above, the following common features can be observed:

- all models start from a small initial value (possibly zero), go through a maximum, and evolve to a zero limiting value;
- they are composed of consecutive branches, convex or concave, delimited by a maximum and one or two inflection points.

A set of similar curves has been studied and classified by Pearson into thirteen types, containing a subset of three 'main types' [I, IV and VI] and a subset of ten 'transition types' ['normal curve', II, VII, III, V, VIII, IX, X, XI and XII]. A typical example of Pearson's transition type III (gamma distribution) is represented in Fig. 6, which again is very similar to the observed FF trend. From a purely empirical point of view - without theoretical justification - we suggest the following heuristic model, admitting a nonzero starting value and a nonzero limiting value. The 'complete' empirical FF model is a quotient of two quadratic terms:

FF model = Numerator / Denominator.
Numerator = $p + q.x^2$
Denominator = $k + m.x + n.x^2$
The initial value (for $x = 0$) equals p / k.
The asymptotic limiting value (for $x = \infty$) = q / n.
The numerator may be thought of as representing the accident frequency curve.
The denominator may be thought of as representing the accident severity curve.
Examples for European countries are given in Fig. 2 for Belgium and in Figs. 3 and 4 for Germany and France.

It should be noted that the model parameter values have by no means been obtained by one of the well-known statistical fitting methods (least squares, maximum likelihood, ...), but rather by a heuristic trial method. The quality of fit may probably be improved at a later stage, while several test measurements may be added.

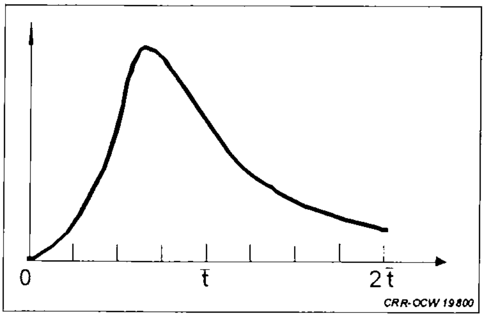

Figure 5- General form of the Lognormal distribution

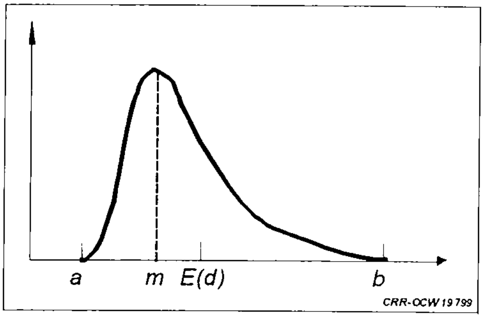

Figure 6- General form of the Gamma distribution

CONCLUSIONS

The development of an adequate theoretical model is a prerequisite for the calibration and validation of a relevant statistical model, at least when one is interested in causal explanations going beyond a pure empirical 'curve fitting' approach.
Behavioural changes of the driver population and structural changes of road infrastructure are among the primary determinants of accident and fatality trends.
The friction concept as defined by May (1959) and embodied in the form of total travel time seems to be a relevant aggregate traffic variable, as it accounts for the impact of many aspects of the transportation system (intersectional effects, uninterrupted flow conditions, ...) on accidents.
There is no unique explanation for the maximum observed - mainly during the seventies - in the number of fatalities in many European countries. Several explanations seem to be logically consistent. Accident modelling suffers from unsolved problems already at the very first stage of proper model definition. There is ample scope for future research.

ACKNOWLEDGEMENTS

The financial support of the Belgian Road Research Centre is gratefully acknowledged. The author also likes to thank a few colleagues at BRRC and some members of the European COST-329 working group on accident time series modelling, for their useful suggestions for this paper.

REFERENCES

Acha-Daza J.A., Hall F.L., (1994), "Application of Catastrophe Theory to Traffic Flow Variables", **Transportation Research B, Vol. 28B, No. 3**, pp. 235-250.

Aitchison J., Brown A.C., (1963). "**The Lognormal Distribution**", Cambridge University Press, Cambridge, pp. 22-23.

Baass K.G., Lefebvre S., (1988). "Analysis of Platoon Dispersion with Respect to Traffic Volume". **Transportation Research Record 1194**.

Banks J.H., (1995). "Another Look At A Priori Relationships Among Traffic Flow Characteristics". **Transportation Research Record 1510**.

Branston D., (1976). "Link Capacity Functions: a Review", **Transportation Research, Vol. 10**, pp. 223-236.

Bellis W.R., (1967). "Operational Effects of Overall Geometries on Highway Safety", **Highway Research Record No. 162**, Highway Research Board, National Research Council, Washington, D.C., pp. 119 - 133.

Bester C.J., (1981). "Fuel Consumption on Congested Freeways". **Transportation Research Record 801**.

Drew D.R., (1968). "**Traffic Flow Theory and Control**", McGraw-Hill, New York.

Ferrari P., (1994). "The Instability of Motorway Traffic", **Transportation Research, Vol. 28-B**, No. 2, pp. 175-186.

Haight F., (1973). "Induced Exposure", **Accident Analysis and Prevention, Vol. 5**, pp. 111-126.

Heimbach C.L., Vick H.D., (1968). "Relating Change of Highway Speed per Unit of Time to Motor Vehicle Accident Rates". **Highway Research Record 225**, Highway Research Board. Washington D.C.

Kaczor M., (1980). "Strassenverkehrssituationen und ihre Sicherheitskoeffizienten". **Forschungshefte zur Verkehrssicherheit, Heft 7,** Dezember 1980.

Kometani E., Sasaki T., (1959). "A Safety Index for Traffic with Linear Spacings", **Operations Research, Vol. 7**, No. 6, pp. 704 - 720.

Lee J., Yu J.C, (1973), "Internal Energy of Traffic Flows", **Higway Research Record No. 456**, Highway Research Board, National Research Council.

Mahalel D., (1986). "A Note on Accident Risk", **Transportation Research Record 1068**, Transportation Research Board, National Research Council, Washington, D.C.

Manar A., Baass K.G., (1996). "Traffic Platoon Dispersion Modelling on Arterial Streets", **Transportation Research Record 1566**.

May A.D. Jr., (1959). "A Friction Concept of Traffic Flow", NAS-NCR Publication 674, Highway Research Board, **Proceedings of the 38th Annual Meeting**, Jan. 5-9, 1959, Washington D.C.

Mei M., Bullen A.G.R., (1993). "Lognormal Distribution for High Traffic Flows". **Transportation Research Record 1398**.

Sattertwaithe S.P., (1981). "A Survey of Research into Relationships Between Traffic Accidents and Traffic Volumes", **Supplementary Report 692**, Transport and Road Research Laboratory, Crowthorne, U.K.

Torres J.F., (1970). "Acceleration Noise, Power Spectra, and Other Statistics Derived From Instrumented Vehicle Measurements Under Freeway Driving Conditions", **Highway Research Record 308**, Highway Research Board. Washington D.C.