AN ASYMMETRIC SUE MODEL FOR THE COMBINED ASSIGNMENT - CONTROL PROBLEM

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Abstract

In this paper we study the combined assignment-intersection control problem and show that it descends from the more general Equilibrium Network Design Problem (ENDP) in the case of locally optimized control systems (e. g. adaptive traffic-lights). Previous studies deal with this problem using Deterministic User Equilibrium (DUE) assignment models; in this paper we propose a Stochastic User Equilibrium (SUE) model with asymmetric delay functions. In the paper theoretical properties of the model are investigated. Moreover we propose and test some algorithms to solve the problem, with different locally optimized signal control rules, and we estimate their convergence speed.

INTRODUCTION

Traffic signal systems play an important role in the management of urban road networks, on which a great part of travel time is spent as waiting time at intersections (delays). The signalized intersection delay is function of signal settings (effective green times, cycle length, offsets between subsequent intersections) and, obviously, of the traffic flows at the intersections.

If we suppose that the physical configuration of a transportation network (topology and links physical capacity) is known, the signal control problem is to find the values of signal settings that minimize an objective function (or index of performance). In this way it is defined a Global Optimization Signal Settings (GOSS) problem, whose solution is a system's optimum. In the hypothesis of flow-responsive traffic signals, i. e. signals independently responding to local flows following a local control strategy, the problem of simulating network performances under different strategies can be seen as a *combined assignment-intersection control problem*, as it is specified better in the next section; it is a fixed point problem (circular dependence among flows, costs and signal settings). It could be also framed as a Local Optimization Signal Settings (LOSS) problem.

In this paper we study the combined assignment-intersection control problem (LOSS problem) and we propose a Stochastic User Equilibrium (SUE) model to represent the problem as well as some algorithms to solve it. Finally, using a test network, we investigate some properties of the algorithms: convergence, solution's uniqueness and robustness.

GENERAL EQUILIBRIUM NETWORK SIGNAL SETTINGS MODELS

The general network design models seek the topological and functional characteristics of a network (starting from an initial configuration) that minimize an objective function subject to some constraints. The problem's variables, that can be continuous or discrete, are divided into: *decisional*, if they represent the elements on which we can operate to change the network configuration and *descriptive*, if we aren't able to change them directly.

In the urban network design problem the variables are: the **y** vector of the decisional variables different from the signal settings (width, lane number, open or closed road); the **g** vector of the signal settings (effective greens, cycle length, number of the phases), that are decisional continuous variables (for fixed phase plan); the **f** vector of the link flows, that are continuous descriptive variables.

For congested networks (link costs dependent from traffic flows) the link flow vector **f** must be calculated by an equilibrium assignment; in these cases it is formulated an Equilibrium Network Design Problem (ENDP).

Given a transportation network and supposing known and invariant the topologic configuration (**y** vector is made up of constant terms), we want to calculate signal settings (**g** vector) that optimize the network performances. In this way we formulate the *Equilibrium Network Signal Setting* (ENSS) problem (Cantarella and Sforza, 1995), in which the decisional variables are only the signal settings (**g** vector):

$$\text{Min}_g \; Z(\mathbf{g}, \mathbf{f}^*)$$

subject to:

$$\mathbf{g} \in G$$
$$\mathbf{f}^* \in F, \qquad \mathbf{f}^* = \mathbf{f}^* (\mathbf{g})$$

where:
Z is the objective function (generally the total travel time on the network);
G is the set of possible configurations for the **g** vector;
f* is the link flow vector resulting from an equilibrium assignment on the network;
F is the set of possible configurations for the link flows;
f* = **f*** (**g**) is the formal relationship between equilibrium flow vector and decisional variables.

The link flow vector **f**, resulting by an assignment model for fixed O-D demand flows, depends on the link costs and therefore, formally, on the link cost vector **c** (Cascetta, 1990):

$$\mathbf{f} = \mathbf{A}\,\mathbf{P}(\mathbf{A}^T\mathbf{c})\,\mathbf{d} = \mathbf{f}(\mathbf{c}) \tag{1}$$

where:
d is the transportation demand vector (elements of the OD matrix);
A is the link-route incidence matrix, where the element $a_{lk} = 1$ if the link l belongs to route k and $a_{lk} = 0$ otherwise;
P is the route choice probability matrix, where the element p_{ki} is the probability that the users on the O-D pair i choose the route k.

The **P** matrix can be built in different ways according to the used route choice model which can be stochastic or deterministic.

The generic link cost c_i is generally assumed as the sum of two terms: the running cost (or time), that depends entirely on the flow of the same link, and the waiting cost (or time) at the intersection that, for signalized intersections, depends on the signal settings too. Formally we can write:

$$\mathbf{c} = \mathbf{c}(\mathbf{f}\,,\mathbf{g}) \tag{2}$$

Replacing eqn (2) into eqn (1) we obtain the following equation:

$$\mathbf{f} = \mathbf{f}(\mathbf{c}(\mathbf{f}\,,\mathbf{g})) \tag{3}$$

that relates flows, costs and signal settings.

Fixed a signal settings vector **g**, the traffic flows vector **f*** at equilibrium can be expressed as:

$$\mathbf{f}^* = \mathbf{f}\,(\mathbf{c}(\mathbf{f}^*, \mathbf{g})) \tag{4}$$

When **g** is fixed and the cost functions are assumed separable, it can be shown under some assumptions the existence and uniqueness of the solution, whatever is the route choice model (either stochastic or deterministic) (Sheffi, 1985; Cascetta, 1990). The univocal relation (application) between **f*** and **g** can be written as:

$$\mathbf{f}^* = \mathbf{f}^*(\mathbf{g})$$

In this context two different ENSS problems can be defined depending on the level of network optimization and the type of traffic control system.

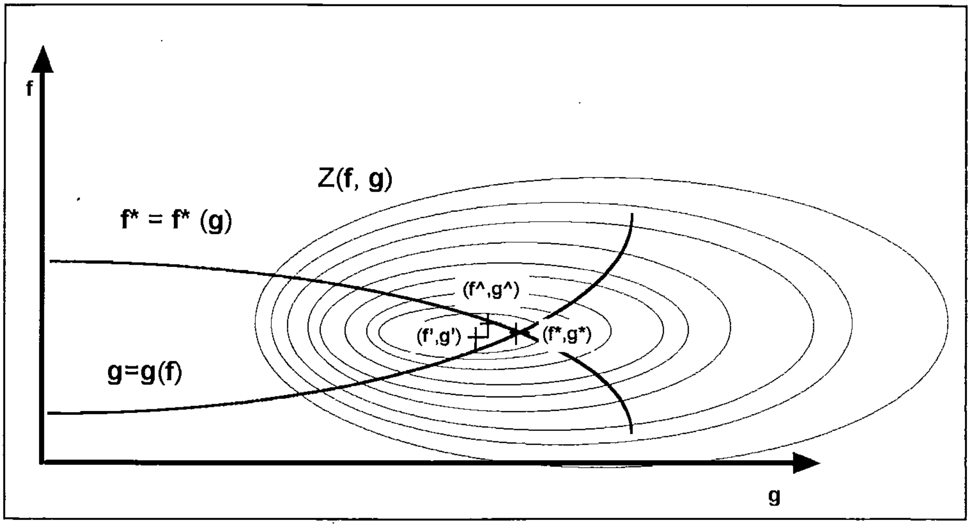

Figure 1 - Graphic representation of GOSS and LOSS solutions

The *Global Optimization Signal Settings* (GOSS) problem is to find the **g**^ vector, and the corresponding **f**^ = **f***(**g**^), such that:

$$\mathbf{g}^\wedge = \mathrm{Min}_{\mathbf{g}}\, Z(\mathbf{f}^*(\mathbf{g}), \mathbf{g})$$

The **g**^ vector is a constrained optimum, where the constrain is the relation **f*** = **f***(**g**) (see fig. 1).

If we draw the function **f*** = **f***(**g**) in a plane[1] (fig. 1) and the contours of the system objective function Z(**f**, **g**) (Cantarella *et al.*, 1991), the constrained system optimal point (**f**^, **g**^) must belong to the **f***(**g**) curve. In general, it will be different from the unconstrained optimal point (**f'**, **g'**) that isn't, instead, a feasible solution of the problem. All the points of the **f***(**g**) curve are feasible solutions: they represent the unique equilibrium link flow vector **f** that corresponds to each signal settings vector **g** by an equilibrium assignment. For a study of the GOSS problem see Cascetta *et al.* (1998).

The *Local Optimization Signal Settings* (LOSS) problem arises when the signal control parameters **g** are optimized locally, typically in response to intersection flows. In flow-responsive traffic-lights, the signal settings adapt themselves to the flows at the single intersection. A fixed flow-responsive control policy[2] calculates the locally optimal signal settings vector **g** in function of the link flow vector **f**, therefore we can write:

$$\mathbf{g} = \mathbf{g}(\mathbf{f}) \qquad (5)$$

so the relations (2) and (3) become respectively:

$$\mathbf{c} = \mathbf{c}(\mathbf{f}, \mathbf{g}(\mathbf{f})) \tag{6}$$

$$\mathbf{f} = \mathbf{f}(\mathbf{c}(\mathbf{f}, \mathbf{g}(\mathbf{f}))) \tag{7}$$

In this second case the problem is to calculate an $\mathbf{f}^*$ vector that reproduces itself in the eqn (7), once defined the control policy (eqn (4)). At the equilibrium the eqn (7) becomes:

$$\mathbf{f}^* = \mathbf{f}(\mathbf{c}(\mathbf{f}^*, \mathbf{g}(\mathbf{f}^*))) \tag{8}$$

and the $\mathbf{f}^*$ vector is named equilibrium flow vector.

The vector $\mathbf{g}^* = \mathbf{g}(\mathbf{f}^*)$ is the signal settings vector at the equilibrium, solution of the ENSS problem for flow-responsive signals and a solution of the Local Optimization Signal Settings (LOSS) problem. In this case we formulate the combined assignment-intersection control problem.

If we draw in the plane of fig. 1 also the $\mathbf{g} = \mathbf{g}(\mathbf{f})$ curve, $(\mathbf{f}^*, \mathbf{g}^*)$ is a mutually consistent point, and it is in general different from the system optimal solution $(\mathbf{f}^\wedge, \mathbf{g}^\wedge)$. The relation between signal settings and link flows ($\mathbf{g} = \mathbf{g}(\mathbf{f})$) represents the system planner's decision; instead, the relation between the same variables $\mathbf{f}^* = \mathbf{f}^*(\mathbf{g})$ expresses the result of users' choices.

The fixed point problem so defined doesn't have a trivial solution, because the cost dependence on signal settings involves the fact that the cost functions aren't separable and therefore this is an asymmetric equilibrium problem, that presents some difficulties relating to the existence and uniqueness of solution.

The GOSS and LOSS problems can be seen in the framework of game theory. In fact the interaction between the flow vector $\mathbf{f}$ and signal settings vector $\mathbf{g}$ can be seen, by game theory, like a game between two players: the travelers and the system planner (Fisk, 1984). The firsts tend to minimize their individual perceived costs, while the second wants to optimize the objective function Z ($\mathbf{f}$, $\mathbf{g}$) on the whole network. The GOSS problem can be seen as a "Stackelberg game" and the LOSS problem as a "Nash game".

PREVIOUS RESEARCH ON LOCAL OPTIMIZATION SIGNAL SETTINGS EQUILIBRIUM MODELS

Several authors studied the combined assignment-intersection control problem (LOSS problem) under a DUE assignment model. Smith (1979a) formulated conditions guaranteeing existence, uniqueness and stability of the equilibrium. For a detailed treatment on the uniqueness conditions see Meneguzzer (1990). In real situations such conditions aren't respected with conventional control policy (equisaturation and delay minimization).

Florian and Spiess (1982) gave a sufficient condition for the local convergence of the diagonalization algorithm of a general asymmetric (non diagonal) user equilibrium assignment model; this condition is even more restrictive than the one required for the uniqueness. Proposed conditions are only sufficient, but not necessary, for the solution uniqueness and algorithmic convergence.

Smith and Van Vuren (1993) demonstrated that "non conventional" control policies could be used to building a general theory; this theory integrates the deterministic user equilibrium assignment and signal settings into a single equilibrium model. In fact, choosing an opportune control policy, the combined assignment-intersection control problem is equivalent to a minimization problem of a convex function.

The algorithms proposed to solve the combined assignment-intersection control problem follow in two different approaches. *Simultaneous* algorithms solve a unique problem, in which the signal settings and the equilibrium traffic flows are considered as unknown variables. *Sequential* algorithms, instead, solve alternatively a problem of signal regulation (with flows known) and an equilibrium network assignment (with signal settings known).

Different simultaneous algorithms have been proposed in literature; for instance Dafermos (1980, 1982), Fisk and Nguyen (1982), Florian and Spiess (1982), Meneguzzer (1995). Sequential algorithms were by Allsop (1974) and Cantarella *et al.* (1991).

AN ASYMMETRIC SUE MODEL AND SOME ALGORITHMS FOR THE LOCAL OPTIMIZATION SIGNAL SETTINGS PROBLEM

In this section we propose a formulation of the LOSS problem as a fixed point problem and some algorithms for its solution. The proposed approach is based on stochastic equilibrium assignment (SUE) models; these models, formalized by Daganzo and Sheffi (1977), are derived under random utility route choice models. For the fixed point formulation of SUE see Cascetta (1990, 1998) and Cantarella (1997).

In this case the elements of the probability choice matrix $\mathbf{P}$ are continuous functions of the route costs and the equilibrium flows could be defined by:

$$\mathbf{f}^* = \mathbf{A}\mathbf{P}[\,\mathbf{A}^T\mathbf{c}\,(\mathbf{f}^*, \mathbf{g}(\mathbf{f}^*))]\,\mathbf{d} \tag{9}$$

where $\mathbf{g}(\mathbf{f})$ expresses the relations between control variables and flows, depending on the local optimization rules.

The existence of a solution is ensured by the continuity of the functions by Brouwer's theorem (1912). This requirement is typically satisfied as the vectorial function $\mathbf{f}$ is continuous under quite mild assumptions. For random utility route-choice models with non-zero variance (det $|\Sigma| \neq 0$) the $\mathbf{P}$ matrix elements are continuous functions of path costs. The elements of the $\mathbf{c}$ vector $[c_i(\mathbf{f}^*, \mathbf{g}(\mathbf{f}^*))]$ are composed functions of continuous functions, being continuous the $c_i(\mathbf{f})$, the $c_i(\mathbf{g})$ (the delay formulas used are always continuous in their feasibility set) and the generic component g_i of the $\mathbf{g}$ vector, which is continuous function of the flows for commonly adopted local control strategies.

A sufficient condition for the uniqueness of the fixed point solution is that the Jacobian of the cost functions is a positive definite matrix (Sheffy and Powell, 1981). This is true if the elements of $\mathbf{P}$ matrix are monotone non increasing with route costs and the link cost functions are derivable, continuous with their first order derivatives and strictly increasing with flows. This last condition usually is not satisfied for the combined assignment-intersection control problem.

The fixed-point problem (9) can be solved by MSA algorithm, that can be seen as a fixed point algorithm (Cantarella, 1997; Cascetta, 1998) or a feasible descent algorithm for an equivalent optimization problem (Sheffy and Powell, 1981); this algorithm is demonstrated to be convergent (Powell and Sheffy, 1982; Cantarella, 1997) in the hypothesis that the Jacobian is symmetric and definite positive. Again the positive definiteness of Jac($\mathbf{c}$) is not usually guaranteed and the MSA algorithm cannot proved to be convergent in the asymmetric SUE model formulated in this paper.

In the following, to solve the fixed point problem eqn (9) we propose three new heuristic algorithms, derived by the MSA structure, but adapted to the specific problem at hand.

Algorithm with external calculation of signal settings (Al-e-1)

This algorithm is similar to the IOA (Allsop, 1974) but it weighs the result of the assignment at the iteration k with the preceding iterations with an MSA method. It is evident that the Al-e-1 algorithm solves the problem with a *sequential* approach. The steps of the algorithm are:

Step 0 - *Initialization.*
Set iteration counter k = 1 and fix a stop test. Determine a vector of initial settings g^0 and calculate, with a SUE assignment (with link costs corresponding to zero flows: $\mathbf{c}^0 = c(\mathbf{0})$), the flow vector $\mathbf{f}^0_{SUE}$. This is the first iteration flow vector:

$$\mathbf{f}^1 = \mathbf{f}^0_{SUE}$$

Step 1 - *Updating signal settings (**g** vector).*
The new signal setting vector is calculated with the control policy in function ofh current flow vector:

$$g^k = g(\mathbf{f}^k)$$

Step 2 - *Calculation of the support flow vector.*
Using the signal setting vector $\mathbf{g}^k$ a new SUE assignment is carried out and it is indicated with $\mathbf{f}^k_{SUE}$ the resulting flow vector.

Step 3 - *Updating of the current flow vector.*
The new flow vector $\mathbf{f}^{k+1}$ is calculated like weighted average of $\mathbf{f}^k$ and $\mathbf{f}^k_{SUE}$:

$$\mathbf{f}^{k+1} = [(k-1)\,\mathbf{f}^k + \mathbf{f}^k_{SUE}]/k$$

Step 4 - *Convergence test.*
It is applied to the average percentage difference between greens in two following iterations:

$$\text{Average}_{ij}\ \{|g^{k+1}_{ij} - g^k_{ij}| \,/\, g^{k+1}_{ij}\} < \varepsilon$$

If it has come true then *stop*, otherwise k= k+1 and *return to step 1.*

Algorithm with internal calculation of signal settings (Al-i-1)

This algorithm follows a simultaneous approach to the problem that is more suitable for a SUE asymmetric assignment problem. It is similar to the *classic* MSA algorithm but update the signal settings (and resulting costs), according to the control policy, before every SNL assignment. The steps of algorithm are:

Step 0 - *Initialization.*
Set iteration counter k = 1 and fix a stop test ε. Determine a vector of initial settings g^0 and calculate, with a SNL assignment, the flow vector $\mathbf{f}^0_{SNL}$. This is the first iteration flow vector:

$$\mathbf{f}^1 = \mathbf{f}^0_{SNL}$$

Step 1 - *Updating signal settings (**g** vector).*
The new signal setting vector is calculated by the control policy in accordance with current flow vector:

$$\mathbf{g}^k = \mathbf{g}(\mathbf{f}^k)$$

Step 2 - *Updating costs (**c** vector).*
The new link cost vector is calculated in accordance with current flow vector and signal setting vector calculated in the previous step:

$$\mathbf{c}^k = \mathbf{c}(\mathbf{f}^k, \mathbf{g}^k)$$

Step 3 - *Calculation of the support flow vector.*
Using the link cost vector $\mathbf{c}^k$ a new SNL assignment is carried out and it is indicated with $\mathbf{f}^k_{SNL}$ the resulting flow vector.

Step 4 - *Updating of the current flow vector.*
The new flow vector $\mathbf{f}^{k+1}$ is calculated like weighted average of $\mathbf{f}^k$ and $\mathbf{f}^k_{SNL}$:

$$\mathbf{f}^{k+1}=[(k-1)\,\mathbf{f}^k + \mathbf{f}^k_{SNL}]/k$$

Step 5 - *Convergence test.*
It is applied to the average percentage difference between flows in two following iterations:

$$\text{Average}_{ij}\ \{|f^{k+1}_{ij} - f^k_{SNL\,ij}|/\ f^{k+1}_{ij}\} < \varepsilon$$

If it has come true then *stop*, otherwise k= k+1 and *return to step 1*

Modified algorithm with internal calculation of signal settings (Al-i-2)

The structure of MSA algorithm, reducing the step length as iteration number increases, involves that the initial solution influences the current solution indefinitely, even though with a decreasing weight. This typically causes a low convergence speed, especially if the initial solution is far from the equilibrium point. To improve the performances of the conventional MSA and reduce the influence of initial solutions, we propose a modified MSA algorithm that "refreshes" the memory with a frequency that is decreasing as the number of iterations increases. The modified algorithm will be refereed to as *Method of Successive Averages with Decreasing Refreshing* (MSADR). This algorithm increases the step length in the first iterations, when it is farther from the solution. The steps of algorithm are:

Step 0 - *Initialization.*
Fix an initial iteration counter k_in = 1. Set iteration counter k = k_in, fix a stop test ε and a max number of iterations N_it (for instance 10). Determine a vector of initial settings $\mathbf{g}^0$ and calculate, with a SNL assignment, the flow vector $\mathbf{f}^0_{SNL}$. This is the first iteration flow vector:

$$\mathbf{f}^1 = \mathbf{f}^0_{SNL}$$

Step 1 - *Updating signal settings (**g** vector).*
The new signal setting vector is calculated by the control policy in accordance with current flow vector:

$$\mathbf{g}^k = \mathbf{g}(\mathbf{f}^k)$$

Step 2 - *Updating costs (**c** vector).*

The new link cost vector is calculated in accordance with current flow vector and signal setting vector calculated in the previous step:

$$\mathbf{c}^k = \mathbf{c}(\mathbf{f}^k, \mathbf{g}^k)$$

Step 3 - *Calculation of the support flow vector.*
Using the link cost vector $\mathbf{c}^k$ a new SNL assignment is carried out and it is indicated with $\mathbf{f}^k_{SNL}$ the resulting flow vector.

Step 4 - *Updating of the current flow vector.*
The new flow vector $\mathbf{f}^{k+1}$ is calculated like weighted average of $\mathbf{f}^k$ and $\mathbf{f}^k_{SNL}$:

$$\mathbf{f}^{k+1} = [(k-1)\,\mathbf{f}^k + \mathbf{f}^k_{SNL}]/k$$

Step 5 - *Convergence test.*
It is applied to the average percentage difference between flows in two following iterations:

$$\text{Average}_{ij}\ \{|f^{k+1}_{ij} - \text{SNL } f^k_{ij}|/\ f^{k+1}_{ij}\} < \varepsilon$$

If it has come true then *stop*, otherwise k= k+1 and *go to step 6.*

Step 6 - *Max number of iterations test*
If k < N_it *then go to step 1.*

Step 7 - *"Refreshing memory".*
$\mathbf{g}^1 = \mathbf{g}^k$
$\mathbf{f}^1 = \mathbf{f}^k$
k_in = 2 k_in
N_it = 2 N_it
k = k_in
go to step 1.

SOME NUMERICAL RESULTS

To experiment the proposed algorithms we used the test network depicted in fig. 2. The proposed algorithms could be used with any stochastic route choice model, in the tests we used the C-Logit model (Cascetta *et al.*, 1996) with explicit enumeration of all feasible routes. This model overcomes the main shortcoming of Multinomial Logit, i. e. unrealistic choice probabilities for paths sharing a number of links, making possible a closed analytical structure.

The tests were run in order to: evaluate and compare algorithms performances; evaluate the influence of the initial solution on the final result (uniqueness of the solution); compare algorithms' behavior with different control policies.

In the tests we used the delay formula of Doherty (1977), modified prolonging linearly the delay curve when the saturation degree exceeds 0.95 (Cascetta, 1990).

All the intersections are controlled and with two phases, the cycle length C is fixed at 90 seconds; initial green times were calculated in proportion to the accesses widths and we used the equisaturation control policy (Webster, 1958).

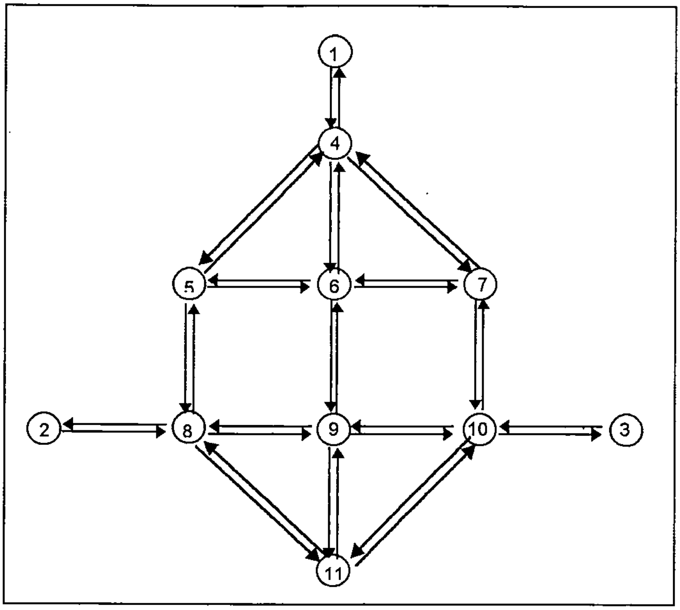

Figure 2 - Test network

The base transportation demand **d** is reported in table1.

Table 1 - Transportation demand vector d

O-D	1-2	2-1	1-3	3-1	2-3	3-2
Flows	850	1250	750	1000	600	700

Table 2 - Mean and maximum saturation degrees (sd) connected with different demand levels

Demand	sd - Max	sd - mean
0.7 d	0.851	0.452
1.0 d	1.040	0.733
1.2 d	1.420	0.934

Various level of network congestion were obtained by scaling vector **d** by a constant. A general idea of the resulting congestion level can be deduced by table 2 where mean and maximum saturation degrees connected with different demand levels are reported.

Evaluation and comparison of the proposed algorithms

Initially we compared the MSA algorithms with "internal" (Al-i-1 and Al-i-2) and with "external" (Al-e-1) calculation of signal settings, counting the number of SNL assignments for convergence. This comparison showed (see table 3) that the internal algorithms converge more quickly than the external. These results led us to test afterwards only the internal algorithms.

The "classical" version of internal algorithm (Al-i-1) has a speed of convergence not sufficiently high for low coefficients of variation (see tab. 4). For instance, with coefficient of variation 0.2 and transportation demand level **d**, it requires 15 iterations (SNL assignments) to reach a value of convergence test of 10 %, while requires 1308 iterations to reduce this value under 5%.

Table 3 - Number of SNL iteration for different algorithms

Cv	Demand	Algorithm Al-e-1 ε_{int} = 2% ε_{est} = 0.5%	Algorithm Al-i-1 ε = 2%	Algorithm Al-i-2 ε = 2%
	0.7 d	78	5	5
0.4	1.0 d	80	41	14
	1,2 d	64	31	13

The Al-i-2 performs clearly better than the other two algorithms also for low coefficient of variation (see table 4). For instance the Al-i-2 algorithm for Cv = 0.2 and demand **d** converges under the 5% of the percentage error in 50 SNL iterations against 1308 of the Al-i-1 algorithm.

In conclusion, the comparison among the algorithms shows that the *simultaneous* approach (internal algorithms) is better than the *sequential* one as for the speed of convergence; besides, between internal algorithms, the Al-i-2 (with the *refreshing memory method*) makes possible to reduce significantly the calculation time especially for low Cv coefficient.

Table 4 - Number of SNL iterations of internal algorithms (Al-i-1 and Al-i-2)

		Algorithm Al-i-1			Algorithm Al-i-2		
Cv	Demand	ε = 10%	ε = 5%	ε = 2%	ε = 10%	ε = 5%	ε = 2%
0.1	1.0 d	1686	>100.000	-	68	1021	1848
	0.7 d	3	7	60	3	7	18
0.2	1.0 d	15	1308	-	23	50	99
	1.2 d	138	1183	-	28	45	75
	0.7 d	2	3	8	2	3	8
0.3	1.0 d	17	28	-	11	13	171
	1.2 d	8	498	-	8	35	66
	0.7 d	2	3	5	2	3	5
0.4	1.0 d	5	16	41	5	12	14
	1.2 d	5	10	31	5	10	13

Influence of the initial solution

To estimate the uniqueness of solution, the modified algorithm (Al-i-2) was applied starting from different initial solutions. The used starting points were: green proportional to approach widths (PG) and green equal to half the cycle time (EG).

Table 5 - Comparison of different starting point (PG and EG) in terms of SNL

Cv	Demand	ε = 2% (PG)	ε = 2% (EG)	Average difference among flows (%)	Max difference among flows (%)
	0.7 d	18	20	0.546	2.254
0.2	1.0 d	99	99	0.067	0.086
	1.2 d	75	77	0.107	1.434
	0.7 d	8	7	0.934	2.841
0.3	1.0 d	171	168	0.082	0.632
	1.2d	66	67	0.085	0.716
	0.7d	5	4	0.956	2.669
0.4	1.0 d	16	13	0.027	1.344
	1.2 d	13	13	0.089	0.256

The results of these assignments (see table 5) seem to show the uniqueness of the solution for the test network; only for low demand the percentage differences between equilibrium flows are appreciable. These results on the uniqueness obviously cannot be generalized to all networks as there are not theoretical sufficient conditions ensuring it.

Comparison among different control policies

All tests reported in the previous subsections were carried out using the Webster's method as control policy for single intersections. In table 6 the results obtained with the delay minimization method are reported; in this case the updating of signal settings are computed, at each iteration, optimizing the parameters of single intersections in order to minimize the total delay with constant flows f^k, using a monodimensional optimization method (in our tests we used the "golden section" method).

The results show that with the minimization control policy the algorithm needs less iterations (SNL assignments) than the Webster's method, but the total calculation time is almost the same, because the time due to the local optimizations (updating signal settings) is longer.

Table 6 - Comparison of total calculation time for the Al-i-2 algorithm convergence

Algorithm Al-i-2; Doherty delay formula; Calculation time for the convergence; ε = 2%					
		Webster's Method		**Delay minimization control policy**	
Cv	**Demand**	*SNL*	*Time*	*SNL*	*Time*
	0.7 d	18	0' 19"	2	0' 28"
0.2	1.0 d	99	1' 43"	8	1' 44"
	1.2 d	75	1' 26"	8	1' 44"
	0.7 d	8	0' 08"	1	0' 15"
0.3	1.0 d	171	3' 04"	6	1' 18"
	1.2 d	66	1' 16"	7	1' 32"

In table 6 the total travel time for the two control policies is compared. The results obtained show that the differences between total travel times for the two different control policies are modest; for low demand levels the Webster's method seems better while, with increasing demand, better values are obtained with the total delay minimization policy.

Table 7 - Comparison of total travel time between control policies

Cv	**Demand**	**Total travel time (sec.)** *Webster's method*	**Total travel time (sec.)** *Total delay minimization method*
	0.7 d	532,448	889,551
0.2	1.0 d	2,457,276	2,313,767
	1.2 d	5,613,436	5,285,566
	0.7 d	613,883	940,106
0.3	1.0 d	2,368,672	2,492,564
	1.2 d	5,749,692	5,627,685

CONCLUSIONS

In this paper we proposed an asymmetric SUE model for the combined assignment-control problem under the hypothesis of locally optimized signal control strategies. This problem was treated in literature almost exclusively with deterministic models of route choice giving rise to a Deterministic User Equilibrium assignment model; the use of stochastic models permits to demonstrate easily the existence of equilibrium solution and to propose some algorithms with a high speed of convergence.

In the paper three algorithms were proposed, based on modifications of standard MSA algorithm. Among the proposed algorithms, the Al-i-2 algorithm (MSA with refreshing memory) resulted to be the most effective as it sometimes reduced the speed of convergence over 10 times with respect to a classic asymmetric MSA (Al-i-1 algorithm).

The comparison of delay minimization and equisaturation local control strategies showed that the former produces smaller total travel time for very congested networks, while the latter was more effective for relatively uncongested networks.

ENDNOTES

[1] It is possible only if is unique the signal setting to determine and the flow that influences it.

[2] For isolate intersections three control policies are usually referred in the literature: Webster's equisaturation method (Webster, 1958), local delay minimization and Smith's P_0 policy (Smith 1980).

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