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RECURSIVE ESTIMATION OF DYNAMIC ORIGIN - DESTINATION MATRICES ON MULTI-ROUTE NETWORKS

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Abstract

A model for recursive estimation of dynamic O-D matrices from traffic counts is developed which is applicable to freeway networks or urban networks where several routes connect each O-D pair. The model requires no additional information such as a target O-D matrix or a prior O-D pattern. The estimation problem of dynamic O-D matrices is formulated as a bilevel problem, where the upper-level problem is a recursive estimation problem of O-D proportions and the lower-level problem is a static user equilibrium assignment problem. Experiments with simulated data suggest that the proposed model is useful to estimate dynamic O-D matrices even under time varying conditions.

INTRODUCTION

Origin-destination (O-D) matrices are indispensable sources of traffic demand information for much transportation planning. Similarly, O-D matrices are necessary for many traffic operations. In particular, for traffic operations which anticipate time variations of traffic flows, dynamic O-D matrices are the most important information. Therefore, many procedures to estimate O-D matrices have been developed.

In situations where a large number of vehicle detectors are located widely on the road network, estimation of O-D matrices from traffic counts has attracted attention of researchers. Various estimation methods of O-D matrices based on this approach have been proposed. Cascetta and Nguyen (1988) comprehensively reviewed such estimation methods. Many of these methods may be suitable for the estimation of a static O-D matrix from single set of traffic counts. Therefore, a prior O-D matrix is required or the path choice proportions are to be presumed in order to obtain a unique solution by the use of these procedures. These approaches are considered to be appropriate only for estimating static O-D matrices.

Since traffic counts directly reflect the time variations of traffic flows, the estimation procedures of dynamic O-D matrices from time series traffic counts were proposed (Cremer and Keller, 1981, 1984). Subsequently, Cremer and Keller (1987) presented nonrecursive, recursive and Kalman filtering procedures to estimate dynamic O-D matrices from input/output counts for intersections or small networks, and at just about the same time Nihan and Davis (1987) presented recursive and Kalman filtering procedure to estimate dynamic O-D matrices. In these procedures, however, the travel times to traverse the system such as an intersection or a network can not be considered in detail. Such travel times can be assumed either to be small as compared with the sampling unit time period, or to be equal to the same constant value for all O-D pairs. Hence, these dynamic procedures are applicable only for individual intersections or small networks.

For estimation of dynamic O-D matrices, especially under the situation in which different travel times between O-D pairs exist, the dynamic relations between the measurements of traffic flows at entrances and exits of the system should be described and the travel time to traverse the system between each O-D pair should be regarded as time-dependent values. Bell (1991) presented two methods for the dynamic estimation of O-D matrices when the travel time was considered to be time-varying. One method is the recurrence model of platoon dispersion which assumes a geometrical distribution of travel times taken by the vehicles and is appropriate for a single intersection and a small network. The other method in which a free distribution of travel times is assumed is suitable for a motorway network. Chang and Wu (1994) paid attention to some unique properties in a freeway corridor that provides useful information for dynamic estimation of O-D matrices, such as the time-varying flows on each freeway segment in addition to the input and output counts at each ramp, and presented dynamic models which made use of an extended Kalman filtering procedure. Sherali *et al* (1997) proposed estimation models of split parameters that describe dynamic O-D matrices when freeway segments having differing time-dependent transfer lags between different pairs of entrances and exits were considered. The models based on a least-squares estimation approach and a least absolute difference approach were developed. However, these dynamic procedures are applicable only to a small network where there is only one route between each origin and destination, rather than a freeway network and a part of road network where several routes exist between each O-D pair. Chang and Tao (1996) proposed the estimation model of dynamic O-D matrices on a multi-route network from entry, exit and cordonline flows. However, this method does not explicitly deal with the routes connecting an origin and a destination. Therefore, the concept of this method may be equivalent to the concept when only one route exists between an O-D pair.

In the present paper, we propose a new model for recursive estimation of dynamic O-D matrices from measurements of link flows and entering flows on the network. The proposed model is applicable to freeway networks or urban networks where several routes exist between each O-D pair, and requires neither target O-D matrix nor information about a prior O-D pattern. The distribution of travel times is made without any specific assumptions and travel times are supposed to be flow-dependent and time-varying. The estimation problem of dynamic O-D matrices is formulated as a bilevel programming problem, where the upper-level problem is recursive estimation of O-D proportions with given path choice proportions and the lower-level problem is a user equilibrium assignment problem with given O-D flows.

DYNAMIC RELATIONS BETWEEN TRAFFIC COUNTS

The dynamic relations between traffic counts of vehicles entering the network and link traffic counts of vehicles traveling on the network are formulated by the use of O-D proportions as parameters. First, the following notation is introduced:

- $q_i(t)$: the number of vehicles entering the network from origin i during time interval t,
- $v_l(t)$: the number of vehicles passing an observation point on link l during time interval t,
- $f_{ij}(t)$: the number of vehicles entering the network from origin i during time interval t that travel to destination j,
- $b_{ij}(t)$: the proportion of the traffic entering the network from origin i during time interval t that travels to destination j,
- $h_{ijk}(t)$: the number of vehicles entering the network from origin i during time interval t that travel to destination j along path k,
- $p_{ijk}(t)$: the proportion of the traffic entering the network from origin i during time interval t that travels to destination j using path k.

The time interval t spans the time period $[t \cdot u, (t+1) \cdot u)$ denoted as continuous time where u is a unit time of estimation. The above variables are interconnected by the following fundamental relations

$$f_{ij}(t) = q_i(t) b_{ij}(t) \quad (i,j) \in \Omega, \tag{1}$$

$$h_{ijk}(t) = p_{ijk}(t) f_{ij}(t) \quad k \in K_{ij}, (i,j) \in \Omega, \tag{2}$$

$$= p_{ijk}(t) q_i(t) b_{ij}(t) \quad k \in K_{ij}, (i,j) \in \Omega, \tag{3}$$

where Ω is a set of O-D pairs and K_{ij} is a set of paths connecting origin i and destination j. Definitional constraints for the O-D proportions must be satisfied, namely

$$\sum_{j \in J} b_{ij}(t) = 1 \quad i \in I, \tag{4}$$

$$0 \le b_{ij}(t) \quad (i,j) \in \Omega, \tag{5}$$

$$b_{ij}(t) = 0 \quad (i,j) \notin \Omega, \tag{6}$$

where I is a set of origins. The path choice proportion also must satisfy the following constraints

$$\sum_{k \in K_{ij}} p_{ijk}(t) = 1 \quad (i,j) \in \Omega, \tag{7}$$

$$0 \le p_{ijk}(t) \quad k \in K_{ij}, (i,j) \in \Omega. \tag{8}$$

The time differences between the time when vehicles enter the network and the time when vehicles are measured on the link must be considered explicitly in order to describe the dynamic relations of traffic counts.

Consider the observations made on a certain link. Figure 1 shows the simple network where two paths have a common link and the conceptual time-space diagram. The vehicles which pass the observation point on the link at the beginning of the time interval t entered this network earlier by an amount $\tau(t)$, the travel time taken from the origin to the observation point. Similarly, the vehicles which pass the observation point at the beginning of the time interval t+1 entered this network earlier by an amount $\tau(t+1)$. Now, it is assumed that vehicles do not overtake each other, i.e. FIFO discipline is observed. The number of vehicles which pass the observation point during time interval t whilst traveling on path k connecting O-D pair (i, j) is the number of vehicles which enter the network through origin i from $t \cdot u - \tau(t)$ to $(t+1) \cdot u - \tau(t+1)$ and will arrive at destination j along path k.

Let $z_{ijkl}(t)$ denote the path flow based on the measured time, namely, the number of vehicles which travel along path k connecting origin i and destination j and are measured on link l during time interval t. This path flow is expressed as follows

$$\begin{aligned} z_{ijkl}(t) = & \sum_{n=t-\eta_{ijkl}(t)-1}^{t-\eta_{ijkl}(t+1)} h_{ijk}(n) - \left\{1 - \left(\frac{\tau_{ijkl}(t)}{u} - \eta_{ijkl}(t)\right)\right\} h_{ijk}(t - \eta_{ijkl}(t) - 1) \\ & - \left(\frac{\tau_{ijkl}(t+1)}{u} - \eta_{ijkl}(t+1)\right) h_{ijk}(t - \eta_{ijkl}(t+1)) \quad l \in L_{ijk}, k \in K_{ij}, (i,j) \in \Omega, \end{aligned} \tag{9}$$

where L_{ijk} is a set of links on path k connecting origin i and destination j. $\tau_{ijkl}(t)$ is the travel time from origin i to link l along path k on the way to destination j and $\eta_{ijkl}(t)$, the corresponding whole number of time increments is defined as follows

$$\eta_{ijkl}(t) \equiv \mathrm{int}\left[\frac{\tau_{ijkl}(t)}{u}\right].$$

By summing up eqn (9) by all O-D pairs and all paths, the link traffic count on link l during time interval t is obtained as the following

$$v_l(t) = \sum_{ij \in \Omega} \sum_{k \in K_{ij}} \delta_{ijkl} z_{ijkl}(t) \quad l \in L, \tag{10}$$

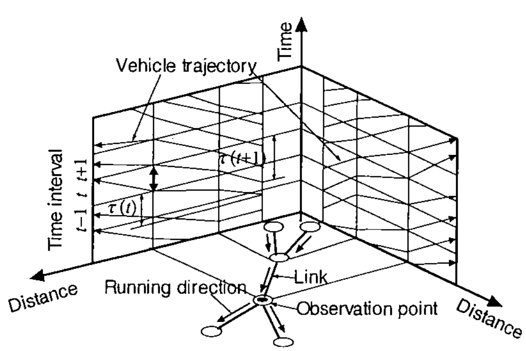

Figure 1 - A simple network and a conceptual time-space diagram

where L is a set of links on the network and δ_{ijkl} equals 1 if link l is an element of a set L_{ijk} and 0 otherwise. The dynamic relations between entering flows and link flows are established by substituting eqn (3) and eqn (9) into eqn (10). These relations mean that, in spite of only one time interval when vehicles pass the observation point on link l, time intervals when vehicles entering the network from origin i extend for some time intervals, namely, from $t-\eta_{ijkl}(t)-1$ to $t-\eta_{ijkl}(t+1)$. However, for the convenience of the estimation problem, an average of the O-D proportions during these time intervals will be estimated. In practice, this does not raise any problems so long as the selected unit time is relatively small. Therefore, the dynamic relations can be expressed as follows

$$v_l(t)=\sum_{ij\in\Omega}\sum_{k\in K_{ij}}\delta_{ijkl}\left[\sum_{n=\theta_{ijkl}(t)}^{\theta_{ijkl}(t+1)} p_{ijk}(n)q_i(n)-\left(1-\xi_{ijkl}(t)\right)p_{ijk}(\theta_{ijkl}(t))q_i(\theta_{ijkl}(t))\right.$$
$$\left.-\xi_{ijkl}(t+1)\,p_{ijk}(\theta_{ijkl}(t+1))q_i(\theta_{ijkl}(t+1))\right]\bar{b}_{ij}(\theta_{ijkl}(t))\quad l\in L, \tag{11}$$

where

$$\theta_{ijkl}(t)\equiv t-\eta_{ijkl}(t)-1,$$
$$\xi_{ijkl}(t)\equiv\frac{\tau_{ijkl}(t)}{u}-\eta_{ijkl}(t),$$

and $\bar{b}_{ij}(\theta_{ijkl}(t))$ denotes the mean O-D proportion during time intervals from $\theta_{ijkl}(t)$ to $\theta_{ijkl}(t+1)$.

MODEL FORMULATION FOR ESTIMATING DYNAMIC O-D MATRICES

In this study, we propose the model for estimating dynamic O-D matrices when the path choice behavior of drivers is assumed to follow the instantaneous user equilibrium principle. By introducing a user equilibrium principle, we avoid the need to have traffic counts on all links of the network. The model can be applied when all entering flows and some link traffic flows on the network are measured in time series. Travel times from the origin to the observation point are assumed to be measured directly or to be estimated indirectly from these and other traffic counts.

The path choice behavior of drivers based on user equilibrium is treated statically, because an assignment principle which can decide the path choice behavior using information only for one time interval is needed in order to estimate O-D matrices recursively and a static assignment principle is practical and easy to apply. Consequently, in order to treat static and dynamic concepts simultaneously, we make two assumptions. Namely, a driver selects the path based on instantaneous travel conditions when he enters the his network and does not change path while travelling. Moreover, the vehicle entering the network can pass several observation points within a single time interval.

The problem for estimating dynamic O-D matrices can be decomposed into two sub-problems. One is the upper-level problem of estimating the O-D proportions from traffic counts with given path choice proportions. The other is the lower-level problem to assign the estimated O-D flows to the network based on the user equilibrium principle.

Formulation of the upper level problem

Generally, additional information such as a target O-D matrix must be given in order to obtain a unique solution from the problem for estimating O-D matrices from traffic counts because the problem is usually underdetermined. However, it would be unusual to get accurate additional information when we estimate dynamic O-D matrices. Hence, it is assumed that O-D proportions do not vary rapidly. The traffic counts in time series which reflect directly the time variations of traffic flows are used. The relation between a link traffic flow and an entering flow is established as the following

$$v_l(t) = \sum_{ij\in\Omega} \gamma_{ijl}(t) b_{ij}(T) + \varepsilon_l(t) \quad l \in L, t = 1,2,\cdots T, \tag{12}$$

where

$$\gamma_{ijl}(t) \equiv \sum_{k\in K_{ij}} \delta_{ijkl} \left[\sum_{n=\theta_{ijkl}(t)}^{\theta_{ijkl}(t+1)} p_{ijk}(n) q_i(n) - \left(1 - \xi_{ijkl}(t)\right) p_{ijk}(\theta_{ijkl}(t)) q_i(\theta_{ijkl}(t)) \right.$$
$$\left. - \xi_{ijkl}(t+1) p_{ijk}(\theta_{ijkl}(t+1)) q_i(\theta_{ijkl}(t+1)) \right] \quad l \in L, (i,j) \in \Omega .$$

In eqn (12), $\varepsilon_l(t)$ denotes the error including all errors such as measurement errors, errors of travel times and stochastic variations in link flows. The parameter T keeps up with progress of a time interval.

In the present study, because the observation points are assumed to be located where vehicles entering the network can pass at least one point within a single time interval, the nearest time interval of $\theta_{ijkl}(t)$ to a current time interval T is identical with the time interval T. In order to track time-varying O-D proportions, a memory factor is introduced. Therefore, the problem for estimating dynamic O-D proportions from traffic counts with given path proportions is expressed in mathematical programming terms as follows

$$\min_b \sum_{t=1}^{T} d^{T-t} \sum_{l\in L} \left(\sum_{ij\in\Omega} \gamma_{ijl}(t) b_{ij}(T) - v_l(t) \right)^2 , \tag{13}$$

$$\text{s.t.} \quad 0 \le b_{ij}(t) \le 1 \quad (i,j) \in \Omega , \tag{14}$$

$$b_{ij}(t) = 0 \quad (i,j) \notin \Omega . \tag{15}$$

In this problem, inequality constraints are used instead of an equality constraint (4) in order to contain the complexity of the problem.

Formulation of estimation problem

The requirement that the path choice behavior of drivers follows the instantaneous user equilibrium principle imposes a relationship between the O-D matrices and the modelled link flow. This can be expressed as the constraint that the link flows satisfy the sub-problem

$$\min_v \sum_{l\in L} \int_0^{v_l(t)} \pi_l(\omega) d\omega , \tag{16}$$

$$\text{s.t.} \sum_{k\in K_{ij}} h_{ijk}(t) = f_{ij}(t) \quad (i,j) \in \Omega , \tag{17}$$

$$h_{ijkl} \ge 0 \quad k \in K_{ij}, (i,j) \in \Omega , \tag{18}$$

where

$$f_{ij}(t) = q_i(t) b_{ij}(t) \quad (i,j) \in \Omega , \tag{19}$$

$$v_l(t) = \sum_{ij\in\Omega}\sum_{k\in K_{ij}} \delta_{ijkl} h_{ijk}(t) \quad l \in L\,, \tag{20}$$

and $\pi_l(\omega)$ denotes the link-cost function.

SOLUTION OF BILEVEL PROBLEM

The problem described in the previous section can be expressed by a Stackelberg game problem with an upper-level problem (13)-(15) and a lower-level problem (16)-(20), namely

$$\min_b \quad F_1(b, v(b))\,, \tag{21}$$

$$\text{s.t.} \quad g_1(b, v(b)) \le 0\,, \tag{22}$$

$$v(b) = \arg\min_v \quad F_2(b, v)\,, \tag{23}$$

$$\text{s.t.} \quad g_2(b, v) \le 0\,. \tag{24}$$

In this problem, the upper-level problem is composed of strictly convex functions with respect to b and v, and the lower-level problem is the user equilibrium problem which is strictly convex with respect to its decision variables v. Furthermore, both constraints of the upper-level and lower-level problems are convex. Bard (1988) called such problem a convex bilevel optimization problem. We note, however, that problems of this kind are not necessarily convex and can have multiple local minima. In this study, a heuristic solution approach where the upper-level problem and the lower-level problem are solved iteratively is considered for practical use and for convenience.

The lower-level problem can be solved by the well-known equilibrium assignment (Frank-Wolfe) method. However, the path flows are not uniquely determined in an equilibrium assignment. Therefore, the convex combination method of Frank- Wolfe algorithm shown in Tobin and Friesz (1988) is adopted in order to obtain a certain equilibrium path flow.

Recursive solution for the upper-level problem

The Lagrangean problem, where $\boldsymbol{\lambda}(T)$ and $\boldsymbol{\mu}(T)$ denote vectors of Lagrangean multipliers for inequality constraints on the O-D proportions, corresponding to the upper-level problem is expressed as follows

$$\min_{\mathbf{b}} F = \sum_{t=1}^{T} d^{T-t}\left(\boldsymbol{\Gamma}'(t)\mathbf{b}(T) - \mathbf{v}(t)\right)'\left(\boldsymbol{\Gamma}'(t)\mathbf{b}(T) - \mathbf{v}(t)\right) - \boldsymbol{\lambda}'(T)\mathbf{b}(T) + \boldsymbol{\mu}'(T)\left(\mathbf{b}(T) - \mathbf{1}\right), \tag{25}$$

where the prime denotes transposition and a vector and a matrix notation are introduced.

For fixed $b(t)$, the first term in the right-hand side of eqn (25) is quadratic and the other two terms are linear, so the necessary and sufficient condition for an optimum can be obtained as follows

$$\frac{\partial F}{\partial \mathbf{b}(T)} = 2\sum_{t=1}^{T} d^{T-t}\Gamma(t)\left(\Gamma'(t)\mathbf{b}(T) - \mathbf{v}(t)\right) - \boldsymbol{\lambda}(T) + \boldsymbol{\mu}(T) = \mathbf{0}\,, \tag{26}$$

$$\frac{\partial F}{\partial \boldsymbol{\lambda}(T)} \le \mathbf{0}, \quad \boldsymbol{\lambda}(T) \ge \mathbf{0} \quad and \quad \lambda_{ij}(T)\frac{\partial F}{\partial \lambda_{ij}(T)} = 0 \quad (i,j) \in \Omega\,, \tag{27}$$

$$\frac{\partial F}{\partial \boldsymbol{\mu}(T)} \leq \mathbf{0}, \quad \boldsymbol{\mu}(T) \geq \mathbf{0} \quad \textit{and} \quad \mu_{ij}(T)\frac{\partial F}{\partial \mu_{ij}(T)} = 0 \quad (i,j) \in \Omega . \tag{28}$$

From eqn (26), the following can be obtained

$$\mathbf{b}(T) = \left(\sum_{t=1}^{T} d^{T-t} \Gamma(t) \Gamma'(t) \right)^{-1} \left(\sum_{t=1}^{T} d^{T-t} \Gamma(t) \mathrm{v}(t) + \frac{1}{2} \boldsymbol{\lambda}(T) - \frac{1}{2} \boldsymbol{\mu}(T) \right). \tag{29}$$

The variables calculated from traffic counts are expressed as follows

$$\begin{aligned} \mathbf{c}(T) &\equiv \sum_{t=1}^{T} d^{T-t} \Gamma(t) \mathrm{v}(t), \\ &= \mathbf{c}(T-1)d + \Gamma(T)\mathrm{v}(T), \end{aligned} \tag{30}$$

$$\begin{aligned} \mathbf{P}(T)^{-1} &\equiv \sum_{t=1}^{T} d^{T-t} \boldsymbol{\Gamma}(t) \boldsymbol{\Gamma}'(t), \\ &= \mathbf{P}(T-1)^{-1} d + \boldsymbol{\Gamma}(T) \boldsymbol{\Gamma}'(T). \end{aligned} \tag{31}$$

Obviously, these variables can be calculated recursively from traffic counts at current time interval T and variables calculated at time interval $T-1$. Therefore, this model is especially suitable for on-line estimation of dynamic O-D flows.

Substituting eqns (30) and (31) into (29) yields

$$\mathbf{b}(T) = \mathbf{P}(T)\left(\mathbf{c}(T) + \frac{1}{2}\boldsymbol{\lambda}(T) - \frac{1}{2}\boldsymbol{\mu}(T) \right). \tag{32}$$

Therefore, the O-D proportions can be estimated from eqn (32) with arbitrary feasible solutions of Lagrangean multipliers.

Updating Lagrangean multipliers

The dual problem for the Lagrangean problem which estimates O-D proportions will be made in order to obtain a solution satisfying the necessary and sufficient condition for the optimal Lagrangean multipliers of the inequality constraints. By solving such dual problem, an optimal solution satisfying conditions (27) and (28) can be obtained.

The dual problem for the Lagrangean problem formulated previously can be expressed as follows

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \inf_{\mathbf{b}} \{F\}, \tag{33}$$

$$\text{s.t.} \quad \boldsymbol{\lambda}(T) \geq \mathbf{0}, \tag{34}$$

$$\boldsymbol{\mu}(T) \geq \mathbf{0}. \tag{35}$$

If an optimal solution for this dual problem can be obtained, the solution of eqn (32) calculated with it is an optimal solution for the primal problem, because of the convexity of the problem. If any directional derivative of the objective function (33) is not equal to zero, then this means that the objective function of the dual problem can be increased. However, if a certain Lagrangean multiplier is equal to zero and the corresponding directional derivative is negative, updating the solution along this direction will violate the positivity conditions for Lagrangean multipliers. Then the directions for updating the solution of the objective function can take the following form

$$\sigma_{ij} = \begin{cases} -b_{ij}(T), & \text{if} \quad \lambda_{ij}(T) > 0 \\ \max(0, -b_{ij}(T)), & \text{if} \quad \lambda_{ij}(T) = 0 \end{cases} \quad (i,j) \in \Omega\,, \tag{36}$$

$$\rho_{ij} = \begin{cases} b_{ij}(T) - 1, & \text{if} \quad \mu_{ij}(T) > 0 \\ \max(0, b_{ij}(T) - 1), & \text{if} \quad \mu_{ij}(T) = 0 \end{cases} \quad (i,j) \in \Omega\,. \tag{37}$$

Furthermore, Lagrangean multipliers are updated along these directions as follows

$$\lambda_{ij}(T) = \lambda_{ij}(T) + \alpha\sigma_{ij} \quad (i,j) \in \Omega\,, \tag{38}$$

$$\mu_{ij}(T) = \mu_{ij}(T) + \alpha\rho_{ij} \quad (i,j) \in \Omega\,, \tag{39}$$

where α is a parameter of the line search. Finally, the following form can be obtained from the necessary condition for the dual problem

$$\alpha = 2\frac{-\mathbf{b}'(T)\boldsymbol{\sigma} + (\mathbf{b}(T) - \mathbf{1})'\boldsymbol{\rho}}{\boldsymbol{\sigma}'\mathbf{P}(T)\boldsymbol{\sigma} + \boldsymbol{\rho}'\mathbf{P}(T)\boldsymbol{\rho} - 2\boldsymbol{\sigma}'\mathbf{P}(T)\boldsymbol{\rho}}\,. \tag{40}$$

The algorithm proposed by Bell (1991) corresponds to when each Lagrangean multiplier is updated individually in our proposed algorithm. However, in Bell's case, the interrelationship between multipliers is ignored. Because the interrelationship between multipliers and the upper and lower bounds of inequality constraints are simultaneously taken into account in our proposed algorithm, the Lagrangean multipliers can be updated more efficiently than Bell's algorithm.

Recursive algorithm for estimating dynamic O-D matrices

The algorithm to estimate dynamic O-D matrices under the equilibrium conditions is as follows

- Step0 : Initialize $p_{ijk}^{(0)}(T)$ and set m=0.
- Step1 : Set $\boldsymbol{\lambda}^{(0)}(T) = \mathbf{0}$, $\boldsymbol{\mu}^{(0)}(T) = \mathbf{0}$ and n=1.
- Step2 : Update $\mathbf{c}^{(m)}(T)$ and $\mathbf{P}^{(m)}(T)^{-1}$ using traffic counts and path choice proportions.
- Step3 : Compute $\mathbf{b}^{(n)}(T)$.
- Step4 : If a convergence criterion is met, go to Step6; otherwise, go to next step.
- Step5 : Update $\boldsymbol{\lambda}^{(n)}(T)$ and $\boldsymbol{\mu}^{(n)}(T)$. Set $n=n$+1 and go to Step3.
- Step6 : Calculate $\mathbf{f}^{(m)}(T)$ using $\mathbf{b}^{(n)}(T)$ and perform the equilibrium assignment based on $\mathbf{f}^{(m)}(T)$. This yields $p_{ijk}^{(m)}(T)$.
- Step7 : If a convergence criterion is met, go to next time interval; otherwise, set $m=m$+1 and go to Step2.

The vector $\mathbf{f}^{(m)}(T)$ is a column form where each element denotes $f_{ij}(T)$.

Although the upper-level problem for estimating O-D proportions with given path choice proportions and the lower-level problem of an equilibrium assignment are optimized iteratively in proposed algorithm, the lower-level decision variables are not fixed but rather are updated based on the upper-level decision variables for each solution of the upper-level problem. This means that the Stackelberg leader-follower structure is explicitly taken into account in the present algorithm.

EXAMPLES

In order to examine the efficiency of the present algorithm, data were simulated for a test network represented in Figure 2. This network consists of 12 nodes and 17 links which include three origins, labeled 1, 2 and 3, and three destinations, labeled 4, 5 and 6. Therefore, there are nine O-D pairs in this network. The link property used for this network is indicated in Table 1.

Table 1 - Link property used for examples

Link	Length (km)	Capacity (veh/5-min)	Free speed (km/h)
1,2,3	2	180	60
5,8,10	10	180	60
11,13,16	5	180	60
4,6,7,9,12,14,15,17	2	100	60

Data generation

Data used for analysis of the application and effectiveness of the present approach were simulated in order to produce dynamic traffic conditions. The traffic flows entering the network from each origin node for 1-minute were generated as Poisson random variables. The mean rates for each time interval were supposed to vary with time as shown in Figure 3. By way of example, the traffic flow entering the network from origin node 1 for each 1-minute interval is also drawn in Figure 3.

The means of O-D proportions were assumed to be the following

$$\begin{bmatrix} 1-4 & 1-5 & 1-6 \\ 2-4 & 2-5 & 2-6 \\ 3-4 & 3-5 & 3-6 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.25 & 0.6 & 0.15 \\ 0.22 & 0.38 & 0.4 \end{bmatrix}.$$

The deviations of O-D proportions were given by normal random variables with zero mean and a standard deviation set equal to 5 percent of the corresponding mean. Then, O-D flows for 1-minute were obtained, after rounding, from eqn (1). Similarly, another O-D flows were generated when standard deviations of normal random variables were set equal to 10 and 15 percent of their

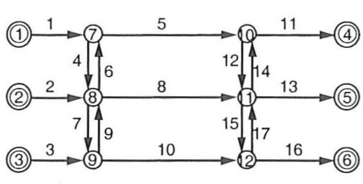

Figure 2 - A test network

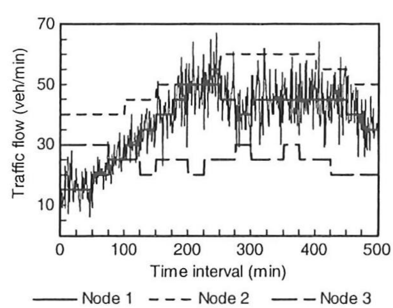

Figure 3 - Means of Poisson random variables and a simulated entry flow

corresponding means.

In order to verify the capability of the estimation under time varying conditions, namely O-D proportions vary with time, the O-D proportion 1-4 was varied slowly according to the half cosine curve between the values from 0.5 to 0.3 and the O-D proportion 1-6 was varied in a complementary manner. Similarly, the O-D proportion 3-5 was varied slowly according to the half cosine curve between the values from 0.38 to 0.28 and the O-D proportion 3-4 was varied in a complementary manner. By way of example, Figure 4 shows two time variations of the simulated O-D (1-4) flows when the standard deviation of O-D proportions is set equal to 10 percent under steady state conditions and also time varying conditions.

When making dynamic network flows, in order to limit the complexity, it was assumed that the path choice proportion was decided at the time when vehicles entered the network and the selected path was not changed after vehicle had entered the network. In each that, the path choice behavior was supposed to follow one of two principles, namely

AN : the shortest path at the time when a vehicle enters the network is selected,
UE: the path based on a user equilibrium condition at the time when a vehicle enters the network is selected.

Multiplying O-D flows by path choice proportions decided from AN or UE produced the path flows for 1-minute based on entering time. The travel speed of each link was calculated from the density-speed function as follows

$$\zeta k^{\beta} v^{\beta+1} + \phi^{\beta}(v - v_0) = 0 \tag{41}$$

where ζ was set equal to 0.15 and β was set equal to 4. The parameter k, v, v_0 and ϕ were density (veh/km), speed (km/h), free speed (km/h) and capacity (veh/min) of the link respectively.

Let $\tau(t)$ denote the link travel time corresponding to the speed calculated from the above density-speed function. It can be considered that the vehicles entering the link in time interval t leave this link during a time period from $t \cdot u+\tau(t)$ to $(t+1) \cdot u+\tau(t+1)$. This is depicted in Figure 5. According to this concept, time intervals for which the vehicles entering the link for time interval t will leave the link can be calculated. Therefore, dynamic network flows were simulated by repeating this procedure by each link and each time interval.

Although data were simulated for each minute, data accumulated for 5 intervals were used for application of the model. Therefore, the unit time interval of estimation was 5-minutes and data were generated for 100 time intervals.

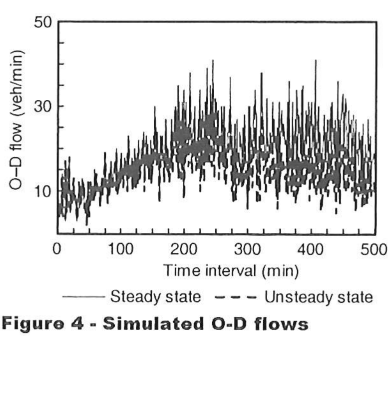

Figure 4 - Simulated O-D flows

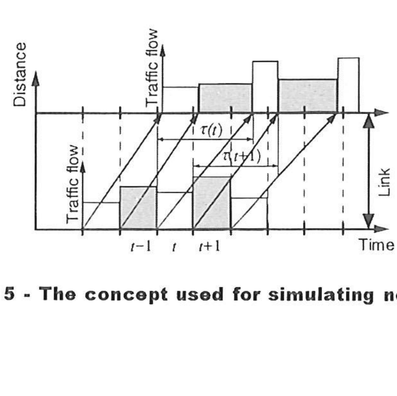

Figure 5 - The concept used for simulating network flows

Estimation result under steady state conditions

The model was applied to data under steady state conditions, namely O-D proportions varied randomly around its means. Three cases which have distinct sets of observation points were considered, namely

case-1 : all links are observed,
case-2 : links except 12, 14, 15 and 17 are observed,
case-3 : links except 5, 8 and 10 are observed.

In order to remove any effect of a transition period from the simulated data, time intervals for which dynamic O-D matrices were estimated were from 10 to 100 time intervals. Because the O-D proportions varied randomly around their means, the memory factor was set equal to 1.0 in the estimation presented here.

Table 2 - RMS errors under steady state conditions

		case-1	case-2	case-3
AN	5%	17.90	11.54	11.52
	10%	18.93	13.57	11.30
	15%	21.08	18.98	12.43
UE	5%	6.86	7.93	4.01
	10%	7.90	9.09	5.53
	15%	10.68	10.76	7.98

Table-2 shows the root mean squared (RMS) errors between the actual and the estimated O-D matrices over all time intervals of estimation. The estimated O-D matrices of UE are more accurate than those of AN, because the lower-level problem of the present model consists of an equilibrium assignment problem which is based on the same principle as is used for data generation. However, even when the path choice behavior does not follow the user equilibrium principle the RMS errors are not too large which shows that the present model can estimate O-D matrices reliably. The variation of a deviation of O-D proportions influences the estimation results directly. The RMS errors may increase in proportion as the variation of O-D proportions becomes large.

The RMS errors of case-3, case-2 and case-1 of AN increase in that order but the RMS errors of case-3, case-1 and case-2 of UE increase in that order. This means that the number of observation points may not be related to the accuracy of estimation results in any simple manner. Information beyond what is necessity may occasionally make the accuracy of estimates worse.

Figure 6 shows actual and typical estimated O-D flows of pair 1-4 and 1-6 over all time intervals of estimation. The estimates in case of UE can track the variations of actual O-D flows more accurately than the estimates in case of AN.

Estimation result under time varying conditions

Estimation under time varying conditions, namely O-D proportions vary with time, was conducted in the same three cases as for the steady sate estimation, case-1, case-2 and case-3. Since O-D proportions vary with time in this situation, the value of the memory factor was changed from 1.00 to 0.70 in step of 0.02.

Table 3 shows the RMS errors between the actual and estimated O-D matrices for all time intervals of estimation. These are the smallest values in all estimation with memory factors from 1.0 to 0.7. The best memory factors, which yield these RMS errors are also shown in parenthesis in Table 3.

Table 3 - The most accurate RMS errors under time varying conditions and values of memory factor

		case-1	case-2	case-3
AN	5%	19.01 (1.00)	17.07 (1.00)	12.57 (1.00)
	10%	20.09 (1.00)	17.40 (1.00)	13.00 (1.00)
	15%	22.22 (1.00)	21.82 (1.00)	14.72 (1.00)
UE	5%	6.63 (0.86)	9.64 (0.88)	11.13 (1.00)
	10%	8.32 (0.88)	12.42 (0.94)	12.59 (1.00)
	15%	9.36 (0.74)	14.11 (0.96)	13.52 (1.00)

When the RMS errors are examined in the light of the values of the memory factor shown, it can be seen that estimation results show a tendency to increase according to the value of the memory factor, namely, the RMS errors become larger as the memory factor proportionately approaches 1.0. The present model can inherently track time variations of O-D proportions with the memory factor less than 1.0. However, when the factors with the exception of time variations of O-D proportions have strong effect on the estimates of O-D flows, namely in situation of AN and case-3 of UE, the best memory factor may be close to 1.0. Since the factors of quantity of information used for estimation, the variations of O-D proportions varying randomly around its mean and a difference of path choice behavior are large in comparison with the time variations of O-D proportions, the model may have the memory factor 1.0 in order to absorb these factors. Nevertheless, It can be seen that the RMS errors are proper regardless of the value of the memory factor.

Figure 7 shows actual and typical estimated O-D flows of pair 1-4 and 1-6 over all time intervals of estimation. The estimates track the time variations of actual O-D flows even when the O-D proportions vary with time.

CONCLUSION

A method for the estimation of dynamic O-D matrices from link traffic counts is proposed. This method is applicable to freeway networks and parts of urban road networks with several routes between each origin and destination. Travel times are allowed to vary depending on the traffic flow throughout the entire route. Time delays between the time at which vehicles enter the network and that at which they reach each observation point are taken into account explicitly.

The estimation problem of dynamic O-D matrices has been formulated as a bilevel programming problem, where the upper-level problem is a recursive estimation problem for O-D proportions with given path choice proportions and the lower-level problem is a user equilibrium assignment problem with given O-D flows. The dual problem for the primal problem to estimate the O-D proportions with inequality constraints is constructed. The algorithm to obtain a solution that can satisfy the

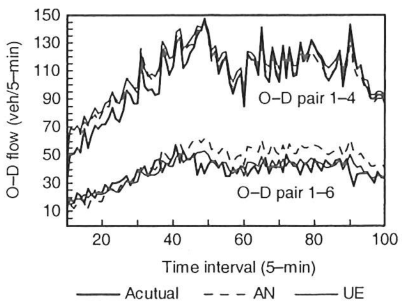

Figure 6 - Actual and estimated O-D flows under steady state condtions

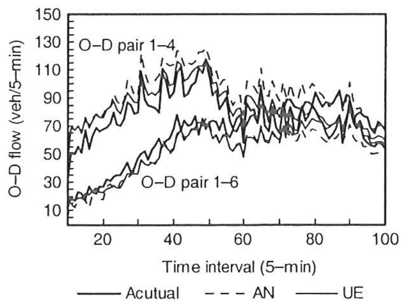

Figure 7 Acutual and estimated O-D flows under time varying conditions

inequality constraints is presented by solving not the primal problem but this dual problem. Furthermore, the heuristic algorithm of bilevel programming is proposed in consideration of practical try and convenience.

Experiments with simulated data suggest that the proposed model is useful to estimate dynamic O-D matrices. When the O-D proportions vary randomly around their means, the estimates of O-D flows are accurate. The proposed model can precisely estimate dynamic O-D flows even when the path choice behavior does not follow the user equilibrium principle. Furthermore, when O-D proportions vary with time, the memory factor value between about 0.74 and 1.0 seems to be appropriate for good estimation. Namely, the value of the memory factor should be determined in consideration of the situation to which the model is applied. Moreover, even when the O-D proportions vary in time, accurate estimates can be obtained by this model.

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