ON THE IMPORTANCE OF DISTRICTING AND ITS POTENTIAL IMPACT ON ROUTING

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Abstract

We define districting as a partition of a large area into sub-areas (districts) such that each district is responsible for the operations performed within its borders. In the O.R. literature, the problem of districting is either ignored or assumed to be solved apriori. The purpose of this paper is to highlight the importance of districting as a major building block for achieving efficient operations, to set guidelines for good districting, and to emphasise its potential to instigate significant savings at a very low cost. An example demonstrates the potential of achieving better operation schemes by performing good districting.

INTRODUCTION

We define districting as a partition of a large area into sub-areas (districts) such that each district is responsible for the operations performed within its borders. Typically each district contains at least one local centre (depot) which is independently responsible for the performance of a series of operations. Many of the operations within each district frequently involve routing where routes start and end at the depot.

Naturally, the problem of districting is strongly related to both location decisions (regarding location of depots), and routing procedures (which perform the desired operations from the given depots).

Problems of location on one hand, and routing on the other hand have been receiving continuous and extensive attention both in theory and in practical applications. Location analysis deals with the problem of locating one or several facilities such that a certain economical criterion is optimized. Facilities can be plants, warehouses, schools, hospitals, administrative buildings, waste material dumps, ambulance and fire engine depots, etc. Labbé *et al* (1995) provided a survey on location on networks.

As for routing, the most popular problems are vehicle routing problems in which a fleet of vehicles makes deliveries to customers from a central depot. The vehicle problem is then to determine routes for the vehicles, where a route is a tour that begins at the depot, traverses a subset of the customers in a specified sequence and returns to the depot. Typically routes are chosen such that total travel cost is minimized. Routing problems can be roughly divided into node routing problems in which customers are located on nodes of the network (such as: goods delivery to retail outlets, milk collection from farms, etc.), and arc-routing problems in which arcs need to be serviced (such as: mail delivery, waste collection, snow removal, salt spreading, etc.). A lot of literature exists on both types of routings and their variants, for node-routing see for example reviews by Laporte (1992) and Fisher (1995), as for arc-routing Eiselt (1995a, 1995b), and Assad *et al* (1995) provided comprehensive reviews.

Not surprisingly, the state of the art on combined location-routing problems (LRP) is much less developed than that associated with either pure location or pure routing problems. In his survey on LRP, Laporte (1988) claims that this deficiency is mainly due to lack of recognition that in many cases the two problems should be dealt with simultaneously, and due to the increased complexity of LRP (LRP are, in general, NP-hard). Furthermore, there is often an inherent difference between location and routing problems: whereas location decisions are of more static and strategic nature, routings are more operational and dynamic, hence the distinction between the two is in many cases natural.

Districting should be viewed as an integral part of the LRP problem, and consequently is of a static and non-operational nature. District borders will not change frequently since each small change usually involves a lot of administration and operational adjustments. The problem of districting, especially in the context of location and routing, is hardly addressed in the literature. Determining borders of district is typically treated as a separate problem which is either assumed to be a-priori given based on political or administrative considerations, or is done after location and routing have been determined. Following the later scheme, one approach is to assume that districts are determined by the location of the depots, and thus partition into districts is done by assigning nodes and links to their closest depot. Another approach is to perform multi-depot routing and to associate nodes and links to the depot from which the tour containing them emanates.

Consequently, it is not surprising to find that in practice, especially in the public sector, districting for a series of responsibilities is based on intuition, or worse, on political or juridical boundaries. For example, in Belgium in which partitions into sub-areas exist for many years, borders of communes are used to delineate salt spreading operations, canton borders are used for medical intervention teams, and province borders are used for public bus services.

Usually a lot of attention is given to the problem of determining efficient operations within given district borders. However, many times significant savings could be achieved if more careful attention had been devoted to the determination of district borders at the time of their conception. It is the purpose of this paper to highlight the importance of districting and to demonstrate its potential to improve routing.

DISTRICTING AND ITS INTERACTIONS WITH LOCATION AND ROUTING

There are important interactions among definition of district boundaries, location of depots, and the resulting routings (Figure 1). It is important to bear in mind that typically not all the arrows in the Figure are being actually considered. The sequence for dealing with the three components of districting, location and routing is typically one of the following:

- Districts are pre-determined, and location and routings are done for the a-priori given districts.
- Location of depots determines the districts.
- Routing determines the districts.

The definition of districts' boundaries strongly affects the overall combined location-routing problem. Moreover, since relocating main depots is operationally very cumbersome and thus not realistic, adjustment of districts' boundaries to support the existing location of main depots could further improve the efficiency of the resulting routings.

Division into districts is strongly compound with location of main depots, since all tours must start and end at the same depot. Hence it is natural to expect that depots would not be located near boundaries of districts. Furthermore, in order to avoid dead-mileage and long tours, it is natural to locate main depots close to the centre of districts. However, this is not necessarily always the case. This phenomenon could be the result of evolutionary gradual changes, that were made to districts' boundaries while keeping the location of depots fixed.

Districting is typically done after the location of the depots has been determined, and before routings are fixed, hence districting can be viewed as an intermediate step between location and routing. Naturally there are strong connections and interactions among location, districting and routing. Location of depots has strong implications on the nodes and links that would be serviced from the depots. Location of depots by itself could determine which nodes and links would be associated with each depot (following a certain criterion such as: shortest distance, minimal maximum distance etc.).

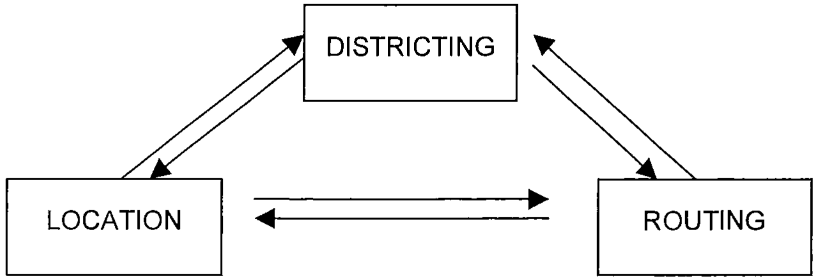

Figure 1 - Interactions among districting, location and routing

Districting is also very much related to the types of routing that would need to be performed in each of the resulting districts. For example, districts that would later support handling of emergency pick-ups would try to ensure tours with minimal maximum distance from the depot (timewise). Whereas districts for performing waste collection, would try to ensure a potential for long balanced tours within capacity limits and with minimal dead-mileage (dead-mileage is defined as mileage traveled without performing service). Furthermore, good districting should be flexible in handling (projected) future operations that would utilize the same depots. However, districts should not be constructed based on the subsequent routings only, but rather many other aspects must be considered.

An appropriate main depot location requires a suitable plot of land, that should satisfy logistical and environmental considerations, such as easy access to operations, minimal disturbance to neighbouring facilities, etc. It should also be noted that relocating existing depots is considered feasible but not very practical, and hence should be recommended only if major improvements and savings are involved.

The problem of districting has not received a lot of attention because typically it is assumed to be taken care of by putting an effort on locating the depots, and once the depots are determined, by performing a single type of routing with a single objective function. Thus often the problem of districting is equated with the problem of routing, and districts are determined solely based on routings. For example, when several depots are involved, a multi-depot routing algorithm is implemented, and districts are determined based on the routes emanating from each depot.

In the literature, some studies are concerned with problems related to routings and location of depots for efficient routings. Consequently, these studies address indirectly the question of constructing districts on which the routings will be determined. For example, Levy *et al* (1989) suggest an algorithm for arc oriented location routing problem, which follows the following three major, steps: location of depots, allocation of arcs into 'partitions', and the creation of tours. Their partitions, in general, are related to the concept of districts since they require that each partition would be associated with one depot, each arc would be associated with one partition, each partition would include an Euler cycle, and that the total workload in each partition would be within pre-specified bounds. However, the partitions are completely motivated by the subsequent routing, and hence a partition is evaluated according to: total dead-mileage time, its violation of bounds regarding total workload, and number of depots, which are all measures related to efficient routing for a specific operation.

Male *et al* (1978) have addressed a problem which they denote by 'districting' in the context of arc-routing, but on a very limited scope. They define a district to be the collection of arcs of each tour, and hence they equate tours with districts. After creating initial tours, they use a procedure, denoted by 'districting', to improve them. Their procedure does not aim at facilitating districts for distributed operations, rather serves as a tool to improve routes by changing the *order* in which arcs are serviced within a given tour. Their definition of district is technical and does not correspond to actual geographical distribution, hence several districts can be associated with the same depot, and the partition into districts is sensitive to the parameters of the routing problem (vehicle capacity, dead-mileage etc.).

From a planning point of view, there is a fundamental difference between districting and routing: whereas districting should be performed at the strategic and tactical level, routing is performed at the operational level. Thus districting should involve a more global view and is often related to the managerial and administrative levels. Apart from being a frame for routing, districts often serve administrative purposes. It is at the depot of a district that, over many years, useful, interrelated data is collected. Thus districts borders should not be changed too frequently. They should be modified only when major changes take place, such as the construction of an important new road, the addition of secondary depots, the introduction of new district responsibility, etc.

Furthermore, the same districts should be able to support *different* types of routings and possibly other operations. Hence whereas routings are more sensitive to specific constraints (capacity, time, distance etc.), districting should be more robust and not influenced by minor changes in the characteristics of the operations that need to be performed within the districts. Therefore, different guidelines should be used for districting and for routing while keeping in mind the important interactions between the two operations.

In the literature, specification of objective functions for evaluating good location and routing decisions are pre-specified and usually relate to the optimization of some economic criterion. While location theory typically deals with minimizing maximal or total distance from the depot, routing schemes normally aim at optimizing the following measures: number of tours, distance traveled, time spent on service and travel, dead-mileage, compliance with capacity and time restrictions, etc. It appears to be more difficult to specify exact economical measures for performing good districting. Obviously good districting should be able to support good routings, however, there are other requirements which do not relate directly to routings. Good districting should result in balanced and compact districts whose borders define clear geographical sub-areas. Thus it is reasonable to expect that districts would not overlap each other, whereas routings can cross each other.

There are many variants to the problem of districting which affect the inter-relations among location, districting and routing. For example, it is not always the case that the depots are centrally located within the district borders, and sometimes districting without association of nodes and links to particular depots are needed. Consider for the example, the problem of public waste collection as described in Gelders *et al* (1991). For this case, there exists only one depot from which the waste collection trucks start their routes, and one waste site to which they bring the waste. Hence districting for this application requires a good partition of the area that needs to be serviced into sub-areas according to the day of service, for example. Thus for this case, districting is strongly related to the routings and is less influenced by the location of the depot and the waste site.

AN EXAMPLE: DISTRICTING FOR ROAD GRITTING IN THE PROVINCE OF ANTWERP

In Lotan *et al* (1996) the problem of salt spreading or winter gritting for the province of Antwerp was described in detail, with emphasis on its location aspects . In Cattrysse *et al* (1997) the potential savings associated with addressing the problem of districting in connection with routing was demonstrated. In this section we demonstrate the importance of districting using the same example of road gritting for the province of Antwerp in Belgium. Road gritting is a common practice in wintertime in many countries with a moderate climate and involves the spreading of salt on roads when frost, ice, or snow have made them slippery. Several authors have dealt with the problem of road gritting (see for example Eglese *et al*, 1992, Cook *et al*, 1976) but have not dealt directly with districting.

Routings for salt spreading can be categorized into two main types: preventive and curative gritting. Preventive gritting is done *before* roads have become icy, and curative gritting is done *after* icy conditions have occurred and requires double the amount of salt. Consequently, routings for preventive gritting can differ from those of curative gritting, however the same depots organize both types of operations within the same district borders. Moreover, in summer time other activities (such as road maintenance) are conducted from the same depots to the same districts.

From the above, it is clear that districting requires a multi-criteria approach. An important requirement for good districting should be a careful enumeration of all planned activities and responsibilities to be performed within the boundaries of a single district, and their related objectives. In the province of Antwerp mainly three types of operations are being conducted from the same depots:

- preventive gritting
- curative gritting
- road maintenance.

When dealing with multiple operations it is important to identify the most critical operation in order to ensure that it could be conducted without interference. For the districts of Antwerp it is clear that curative gritting, which has the most demanding requirements in terms of capacity, should be considered as the most critical operation. Preventive gritting, which requires half the amount of salt as curative gritting, could be done based on combining two tours into one, for example. Good districting for road maintenance operations is achieved by ensuring compact and balanced districts in which the depot is centrally located. Note that even if we focus only on the operational curative gritting, we still have a multicriteria problem (e.g. minimize number of trucks, minimize dead-mileage, minimize total mileage). However, by performing a careful partition into compact districts and ensuring that all nodes are of even degree in order to facilitate any kind of arc-routing, we can concentrate on the most important objective (minimize number of trucks) for the most critical operation (curative gritting).

In the province of Antwerp there is a trend to reduce the number of manned depots and consequently the number of districts. Indeed in January 1996 the number of districts has been reduced from nine to six, motivated mainly by the need to reduce operation costs within the districts' managerial force by reducing the permanent workforce. However, the new partition into districts did not take explicitly into consideration arguments related to the routing operations. In practice, 'intuition' was used, and district borders coincide, to a large extent, with irrelevant borders of communes.

Figure 2 depicts some part of the border area of the districts of: Vosselaar, Grobbendonk, and Brecht. Arcs are road segments belonging to either Vosselaar, Grobbendonk, or Brecht; nodes A, B, and C are crossings. It can be seen that the current partition does not take into account degree of nodes, and hence the resulting routing are likely to include unnecessary dead-mileage or excessive total mileage. For example, the six arcs emanating from node A, which is common to the three districts in the Figure, are divided as follows: two arcs for Vosselaar, three for Grobbendonk, and one for Brecht, hence causing unavoidable mileage of 3km for Brecht (from A to B), and at least 7 km for Grobbendonk (from B to C and from C to A). Clearly the adjusted partition in the Figure has a better potential not to include unnecessary dead-mileage, that is save at least 10 km of dead-mileage, and at the same time it does not interfere with the balance among districts (in terms of total mileage).

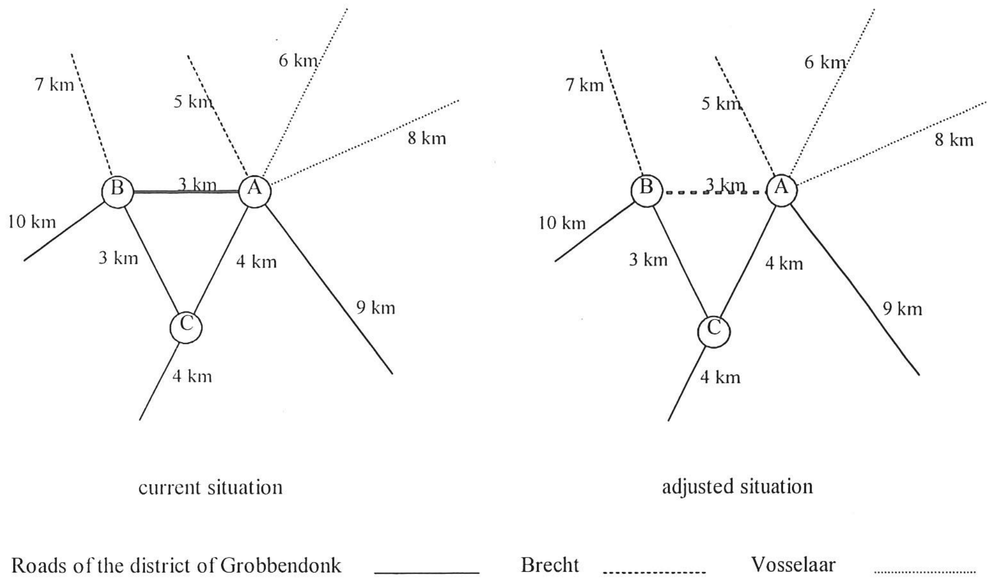

Figure 2 - an example

Table 1 includes the following data for four districts: total distance to be serviced, number of tours that are currently being performed at the district, average tour length, and at the last column, a lower bound on the number of tours calculated by dividing the total mileage by 'effective' tour length of 35 km (which is calculated by standard road width and truck capacity). The data in the Table does not include the highways which are serviced separately.

It is obvious from the Table that a lot of rounding up occurs for calculating the lower bounds. It is also apparent that average tour lengths are far below their optimal potential length of 35 km. Hence, even without improving the partition into districts, it is potentially possible to eliminate four tours (for a total of 25 tours instead of 29), by making tours longer and closer to their capacity limits.

Table 1 - Districts and tours in the province of Antwerp

District	total distance (km)	# of tours	avg. tour length (km)	lower bound
Brecht	201.57	9	22.4	$\lceil 5.76 \rceil=6$
Vosselaar	180.78	7	25.83	$\lceil 5.17 \rceil=6$
Grobbendonk	177.27	6	29.54	$\lceil 5.06 \rceil=6$
Geel	238.33	7	34.05	$\lceil 6.81 \rceil=7$
Total	797.95	29		$\lceil 22.8 \rceil=23$

Furthermore, if we ignore the restrictions imposed by the partitions into districts, then we achieve (at the last row of the Table), a lower (realistic) bound of 23 tours. Clearly, a more careful partition should take into consideration: node degree, total distances, capacity, and possibly other factors. Hence it is clear that it is possible to improve the most important objective of minimizing the number of trucks (and tours), while making sure that the other criteria (minimize dead-mileage and total mileage) would not be worse off. At the same time, other considerations, such as road maintenance are being taken into account.

A NOTE ON SOLUTION PROCEDURES

The strong connection between districting and routing can be utilized to generate efficient districting procedures, which take into account considerations related to routing as well. Such an approach is demonstrated in Cattrysse *et al* (1998), and is briefly described here. The cycle approach was introduced by Male *et al* (1978) and implemented for routing waste collection vehicles. The approach is based on creating an Eulerian graph and partitioning it into cycles. The partition is characterized by having a large number of small cycles where each arc belongs to exactly one cycle. The cycles approach was later used by Eglese (1994) for performing routing for salt spreading operations. Both Male *et al*, and later Eglese use the cycles approach for determining routings in the context of capacitated arc-routing problems. Their approaches rely on constructing routes by aggregating cycles, and benefit from a reduced size of the problem (due to aggregation of cycles instead of arcs), and from the flexibility in constructing the routes (due to the relative small size of the cycles and the existence of many of them).

Why is the cycle approach not always good for routing?

Routings based on the cycle approach are not necessarily optimal, furthermore, there is no good reason to assume that optimal or good routings are composed of cycles. Eglese himself has criticized the approach in Eglese *et al* (1994), and provided examples in which it is far from optimal. The main drawback of the cycle approach for arc-routing problems emanates from the fact that a cycle has to be either included or excluded. Moreover, the construction of cycles is independent of capacity considerations, and hence by combining cycles additional tours may be required and excessive dead-mileage can occur, as was demonstrated in Eglese *et al* (1994). Construction of routes based on cycles is often in contrast to the behavior of the more efficient capacitated arc-routing algorithms (see for example, Pearn, 1991), which construct tours around 'expensive' arcs, that is, arcs that are far away from the depot, and hence create 'large' tours which are typically very different from the routes constructed using the cycle approach. Clearly routing based on the cycle approach tends to serve the inner zones first, and thus are likely to incur more dead-mileage.

Why is the cycle approach good for districting?

Earlier we have touched upon the difficult question of what is a good districting scheme. Here are some guidelines that should be kept in mind when designing good districts:

- Districts should be balanced and compact.
- Districts should facilitate good routings.
- Districts should be robust with respect to variations in the operations that would be performed in them.
- Neighbouring districts should have minimal overlap.

Some of the inherent characteristics of the cycle approach can lead to good districting according to the measures mentioned above. The existence of many, relatively small cycles allows the aggregation into balanced and compact districts. A potential for good routings (within the given district borders) is maintained by ensuring that districts constitute an Eulerian graph, moreover districts are separable (by cycles) and each collection of cycles is also unicursal. The construction of cycles does not take explicitly into consideration capacity constraints, and hence the districts are not sensitive to changes in capacity, which are the most binding constraints of the associated routing problems. Furthermore, the separability and additivity of cycles, provides robustness with respect to small variations in the parameters of the problem, and easy adjustment of the final partition to topological and geographical constraints and considerations. A detailed description of the cycle approach for districting and its implementation appear in Cattrysse *et al* (1998).

DISCUSSION

In this article we have stressed the importance of treating the problem of districting explicitly, rather than automatically associating it with either location or routing operations. However, it is important to mention that the problem of districting does not exist for all types of services, even if the covered area is large. Sometimes, especially in the private sector, there is no need for a tactical or strategical bridging step between depot location and vehicle routing. This is typically the case when a single type of activity is organized from a depot, for instance the delivery of soft drinks. In that case, there is perhaps only a need for an operational division of the area. The required partitioning into sub-units can change frequently, without much difficulty. This operational partitioning should not be viewed as districting in the sense discussed earlier.

The problem of districting is very much related to a traditional task of geographers: dividing a large area into smaller sub-areas that have a large degree of homogeneity. This division is usually based on static characteristics such as demographic density, type of habitation, soil characteristics, etc. However, in the context of this article, districting is intrinsically based on a series of movements from one place to another in order to render a service. Usually several types of transit and services need to be considered. Nevertheless, the various services should be related for obvious reasons. Consequently, there exist distinct districts for waste collection, other districts for emergency medical care, and still other districts for road maintenance. Since good districting should take into account the different tasks performed, districting requires a multi-criteria approach. Due to the similarity of the services performed within each district, the different objectives are often not conflicting. Hence, for many instances it is expected to result in an 'easy' multi-objective problems and often, as in the case for road gritting in the Province of Antwerp, only one or two objectives have to be considered explicitly.

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