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IMPLICIT AVAILABILITY/PERCEPTION LOGIT MODELS FOR ROUTE CHOICE IN TRANSPORTATION NETWORKS

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Abstract

The main problems of path choice modeling are, in general, the very large dimension of the choice set and the high overlapping among paths connecting the same OD pair.

In this paper a general framework dealing with both these problems is proposed. This framework consists of a path choice model implicitly accounting for intermediate degrees of availability/perception of alternative paths, where overlapping among paths is simulated as a factor influencing perception of paths.

The model was calibrated and compared numerically on available RP route choice data with good results.

INTRODUCTION

Random utility (r.u.) models are undoubtedly the most used tools for the simulation of path choice in transport networks. These models simulate the choice of a decision maker i among a set of feasible alternative paths (choice set) C_i and their operational use requires that the analyst is able to correctly specify this choice set for each individual.

This assumption may be unrealistic in many practical cases and in particular in modeling path choice when hundreds of paths are potentially available; the result of ignoring this aspect may cause significant mis-specification problems [Stopher 1980, Williams and Ortuzar 1982]. The problem of simulating availability/perception of alternative paths in transport networks has been ignored until recently; lately some papers addressed the issue of modeling route perception with heuristic [Cascetta E., Russo F., Vitetta A 1997] or explicit [Antonisse et al 1985], [Cascetta E., Russo F., Viola 1997] models.

In this paper a model of path choice implicitly accounting for intermediate degrees of availability/perception of alternative paths is presented.

The first part of the paper discusses the general problem of modeling the choice set in r.u. theory and a recently proposed model of implicit availability/perception (IAP) [Cascetta, Papola 1997] (section 2). Successively the IAP model is applied to route choice and different Logit specifications are proposed (section 3). Finally IAP Logit model with first and second order approximations are calibrated and compared numerically on available RP route choice data.

2 CHOICE SET MODELING

The problem of choice set simulation has been dealt with in the literature following two basically different approaches:

- simulating the perception/availability of an alternative implicitly in the choice model of the same alternative (e.g. by introducing some perception/availability attributes in the utility function of the alternatives such as car availability or "label" variables)
- simulating the choice-set generation explicitly in a separate model

The implicit approach is more convenient from the operational point of view as it doesn't require explicit information about the individual's choice set and allows the use of standard specification/calibration routines. It has been adopted more or less consciously in most specifications of r.u. models proposed in the literature. On the other hand it lacks of theoretical consistency as "utility" attributes are confused with availability ones and mis-specification errors may arise if the same attribute plays a dual role.

In the second approach the probability of choosing an alternative j is generally formulated as a two stage choice model [Mansky 1977]:

$$p^i(j) = \sum_{C \in G^i} p^i(j/C) P^i(C) \tag{1}$$

where:

i = the generic individual

j = the generic alternative

C = the generic choice set

G^i = the set of all possible non empty choice set for the individual i.

$p^i(j/C)$ = the probability of individual i choosing alternative j given that the choice set is C

$P^i(C)$ = the probability that individual i considers choice set C.

Different models have been proposed for the specification and calibration of $P^i(C)$, such as captivity models [Gaudry and Dagenais 1979], random constraints models [Swait and Ben-Akiva 1987], and latent choice set models [Ben-Akiva and Boccara 1995]. For a recent review see Ben-Akiva and Boccara (1995).

Explicit choice-set approaches assume that an alternative is either available/perceived by the decision maker or not and have some shortcomings due to computational complexity (the number of feasible choice-set belonging to G_i increases exponentially with the number of alternatives and heuristic rules to reduce the dimensionality of G are often used).

Recently a different approach to choice set modeling was proposed based on the assumption of intermediate degrees of availability/perception and the inclusion of an availability/perception model in the systematic utility of standard r.u. models.

In other words each alternative j may belong to the choice set with a certain degree of membership $\mu_C(j)$; the choice set is then represented by a fuzzy set of continuous variables in the interval (0,1). This intermediate degree of membership tries to represent various degrees of availability/perceptions of a given alternative by a given decision maker in a given choice context that may correspond to an alternative which in principle is available but isn't "fully" considered for a particular trip under particular circumstances. Coherently with latent choice set interpretation in Ben Akiva et al [1995] and latent variable models in Ben Akiva et al [1997], the $\mu_C(j)$, representing the availability/perception of alternative j, can be seen as a latent variable.

Furthermore the proposed model suggest an integration of the choice model with the availability/perception model by introducing a suitable functional transformation of $\mu_C(j)$ directly in the utility function of the same alternative:

$$U_j^i = V_j^i + \ln \mu_C^i(j) + \varepsilon_j^i \tag{2}$$

where:

U^i_j = perceived utility of alternative j for decision maker i

V^i_j = systematic utility of alternative j for decision maker i

ε^i_j = random residual of alternative j for decision maker i

$\mu^i_C(j)$ = degree of membership of alternative j to the fuzzy choice set C for decision maker i ($0 \le \mu \le 1$) representing availability/perception of alternative j for decision maker i

The logarithmic transformation is such that extreme cases (full availability/no availability) are correctly represented.

The latent variable $\mu^i_C(j)$ for individual i, can be seen as a random variable unknown to the analyst and expressed as the sum of his expected value and a random residual:

$$\mu_C^i(j) = E[\mu_C^i(j)] + \eta_j^i = \overline{\mu}_C(j)[\mathbf{Y}] + \eta_j^i \tag{3}$$

where the expected value $\overline{\mu}_C(j)$ is modeled as a function of a vector Y of availability/perception attributes. By expanding in Taylor series up to the second order term $\ln \mu_C^i(j)$, the perceived utility (2) becomes [see Cascetta Papola 1997]:

$$U_j^i \approx V_j^i[\mathbf{X}] + \ln \overline{\mu}_C(j)[\mathbf{Y}] - \frac{\left(1 - \overline{\mu}_C(j)[\mathbf{Y}]\right)}{2\overline{\mu}_C(j)[\mathbf{Y}]} + \sigma_j^i \qquad \text{with } \sigma_j^i = \varepsilon_j^i + \eta_j^i \tag{4}$$

where X is a vector of attributes connected with the utility of each alternative (e.g. l.o.s. or socio-economics variables in a modal choice model).

Obviously it is possible to use the first order approximation of $\ln \mu_C^i(j)$; in such a case we have:

$$U_j^i \approx V_j^i[\mathbf{X}] + \ln \overline{\mu}_C(j)[\mathbf{Y}] + \sigma_j^i \qquad \text{with } \sigma_j^i = \varepsilon_j^i + \eta_j^i \tag{5}$$

Under the above assumptions the probability of choosing alternative j can be expressed as:

$$p^i(j)=\Pr\left(U_j^i \geq U_k^i \quad \forall k \neq j \in C\right)=$$
$$=\Pr\left(\sigma_k^i-\sigma_j^i \leq V_j^i+\ln\bar{\mu}_C(j)-\frac{\left(1-\bar{\mu}_C(j)\right)}{2\bar{\mu}_C(j)}-V_k^i-\ln\bar{\mu}_C(k)+\frac{\left(1-\bar{\mu}_C(k)\right)}{2\bar{\mu}_C(k)} \quad \forall k \neq j \in C\right) \tag{6}$$

or alternatively, by using the first order approximation of $ln\mu^i{}_C(j)$:

$$p^i(j)=\Pr\left(U_j^i \geq U_k^i \quad \forall k \neq j \in C\right)=$$
$$=\Pr\left(\sigma_k^i-\sigma_j^i \leq V_j^i+\ln\bar{\mu}_C(j)-V_k^i-\ln\bar{\mu}_C(k) \quad \forall k \neq j \in C\right) \tag{7}$$

where C is the overall set of all feasible alternatives.

Model specifications (6) and (7) will be referred to in the following as Implicit Availability/Perception random utility (IAP r.u.) respectively with first and second order approximation. Such models may be specified in turn in different ways depending on assumptions made on the joint distribution of random residual $\sigma^i{}_j$ and the way in which the average degree of availability/perception of an alternative j, $\bar{\mu}_C(j)$, is modeled. Different models can thus be generated corresponding to different hypotheses. In the next section we will see some examples of possible specifications.

IMPLICIT AVAILABILITY/PERCEPTION LOGIT MODELS FOR ROUTE CHOICE

The problem of alternatives perception is particularly acute in path choice where hundreds of alternatives are potentially available for large networks. On the other hand, available empirical evidence shows that only a limited number of paths are actually perceived by travelers [Cascetta E., Russo F., Vitetta A. 97], [Golledge R.G. 1997].

Recently a modification of the Logit route choice model, named C-Logit, was proposed [Cascetta et al 96] introducing a commonality factor CF_k reducing the systematic utility for heavily overlapping paths:

$$U_j^i = V_j^i - CF_j + \varepsilon_j^i \tag{8}$$

with:

$$CF_j = \beta_0 \ln\left(1+\sum_{k\neq j}\frac{C_{j,k}}{\left(C_j \cdot C_k\right)^{1/2}}\right) \tag{9}$$
$$0 \leq CF_j \leq \infty \quad \text{if } \beta_0 > 0$$

where $C_{j,k}$ is the sum of link costs shared between paths h and k, C_j and C_k are the cost of respectively paths h and k, β_0 is an unknown parameter (with an expected positive sign) and the summation is extended to all paths k available for the same OD relation. By assuming $\varepsilon^i{}_j$ i.i.d. $(0;\alpha)$ Gumbel, the C-Logit model can be derived:

$$p^i(j)=\frac{\exp\left[\alpha\left(V_j^i - CF_j\right)\right]}{\sum_n \exp\left[\alpha\left(V_n^i - CF_n\right)\right]} \tag{10}$$

In this paper the C-Logit model is redefined as a first order IAP model and extended to a second order IAP model still under the hypothesis of random residuals $\sigma^i{}_j$ i.i.d $(0,\alpha)$ Gumbel. In this case second and first order IAP r.u. models (6) and (7) becomes:

$$p^i(j)=\frac{\exp\left[\alpha\left(V_j^i+\ln\bar{\mu}_C(j)-\frac{1-\bar{\mu}_C(j)}{2\bar{\mu}_C(j)}\right)\right]}{\sum_n\exp\left[\alpha\left(V_n^i+\ln\bar{\mu}_C(n)-\frac{1-\bar{\mu}_C(n)}{2\bar{\mu}_C(n)}\right)\right]} \tag{11}$$

$$p^i(j)=\frac{\exp\left[\alpha\left(V_j^i+\ln\bar{\mu}_C(j)\right)\right]}{\sum_n\exp\left[\alpha\left(V_n^i+\ln\bar{\mu}_C(n)\right)\right]} \tag{12}$$

which are simple MNL models with a kernel in the utility function given by the average perception/availability factor of the alternative j. Models (11) and (12), in the following, will be denoted by IAPLG2 and IAPLG1 respectively.

The general expressions (11) and (12), can be specified differently depending on the way in which the average availability/perception $\bar{\mu}_C(j)$ is modeled.

A possible solution, for example is to directly simulate $\bar{\mu}_C(j)$ with a single perception attribute Y_j having the same variation range of $\bar{\mu}_C(j)$:

$$0\le Y_j\le 1 \tag{13}$$

In this case assuming for the systematic utility V_j the usual linear specification:

$$V_j^i=\sum_h\beta_h X_{hj}^i \tag{14}$$

models (11) and (12) thus become:

$$p^i(j)=\frac{\exp\left[\sum_h\beta'_h X_{hj}^i+\alpha\left(\ln Y_j^i-\frac{1-Y_j^i}{2Y_j^i}\right)\right]}{\sum_n\exp\left[\sum_h\beta'_h X_{hn}^i+\alpha\left(\ln Y_n^i-\frac{1-Y_n^i}{2Y_n^i}\right)\right]} \tag{15}$$

$$p^i(j)=\frac{\exp\left(\sum_h\beta'_h X_{hj}^i+\alpha\ln Y_j^i\right)}{\sum_n\exp\left(\sum_h\beta'_h X_{hn}^i+\alpha\ln Y_n^i\right)} \tag{16}$$

where β'_h coefficients include the α Gumbel parameter ($\beta'_h=\alpha\beta_h$).

If we use, as the perception attribute Y_j, a variable of path independence (IND_j) obtained as the inverse of the expression between the parenthesis in eqn (9):

$$Y_j=IND_j=\frac{1}{1+\sum_{k\neq j}\frac{C_{j,k}}{\left(C_j\cdot C_k\right)^{1/2}}} \tag{17}$$

having the desired variation range expressed in (13), by substituting eqn (17) in eqn (16),we easily obtain C-Logit model (10), that can be thus seen as an IAPLG1 model with $\bar{\mu}_C(j)$ directly simulated with the only attribute IND_j reported in eqn (17). This attribute can be seen as a normalized measure, in the interval [0,1], of the independence of the generic path with respect to all other paths available for the same OD pair; the underlying assumption is that the independence of a path increases the perception of the same path as a real alternative.

Alternatively more complex structures including a wider range of attributes can be used to model $\overline{\mu}_C(j)$

For example a binomial Logit specification can be adopted to model $\overline{\mu}_C(j)$:

$$\overline{\mu}_C(j) = \frac{1}{1+\exp\left(\sum_k \gamma_k Y^i_{kj}\right)} \tag{18}$$

Note that the binomial logit specification of (18) is similar to the model proposed in Swait and Ben-Akiva (1987) and in Ben-Akiva and Boccarà (1995) and can be similarly interpreted as a random constraint model with a logistic distribution of the random component.

In this case, assuming for the systematic utility V_j the same expression (14), models (11) and (12) becomes respectively:

$$p^i(j) = \frac{\exp\left[\sum_h \beta'_h X^i_{hj} - \alpha \ln\left(1+\exp\left(\sum_k \gamma_k Y^i_{kj}\right)\right) - \frac{\alpha}{2}\exp\left(\sum_k \gamma_k Y^i_{kj}\right)\right]}{\sum_n \exp\left[\sum_h \beta'_h X^i_{hn} - \alpha \ln\left(1+\exp\left(\sum_k \gamma_k Y^i_{kn}\right)\right) - \frac{\alpha}{2}\exp\left(\sum_k \gamma_k Y^i_{kn}\right)\right]} \tag{19}$$

$$p^i(j) = \frac{\exp\left[\sum_h \beta'_h X^i_{hj} - \alpha \ln\left(1+\exp\left(\sum_k \gamma_k Y^i_{kj}\right)\right)\right]}{\sum_n \exp\left[\sum_h \beta'_h X^i_{hn} - \alpha \ln\left(1+\exp\left(\sum_k \gamma_k Y^i_{kn}\right)\right)\right]} \tag{20}$$

with the same meaning of coefficient β'_h.

The estimation of unknown parameter included in IAP Logit models such as (15), (16), (19) and (20), can be generally carried out using different Maximum Likelihood estimators whose specification depends on the available information (see Cascetta Papola 1998). If the only information available are the choices of a random sample of n decision-makers, y_i, (i=1,2,...,n) where y_i is the observed (from both an RP and an SP interview) alternative chosen by decision maker i, then the likelihood function assumes the classical form:

$$L = \prod_{i=1}^{n} p(y_i / X_i, Y_i, \alpha, \beta, \gamma)$$

where $p(y_i / X_i, Y_i, \alpha, \beta, \gamma)$ expresses the general choice probability of alternative y_i as a function of the vectors of attributes X and Y and unknown parameters (α, β, γ) appearing in the used model. ML estimator of α, β, γ can then be obtained as:

$$\alpha^{ML}, \beta^{ML}, \gamma^{ML} = \arg\max_{\alpha,\beta,\gamma} L(\alpha, \beta, \gamma)$$

SOME SPECIFICATION AND CALIBRATION RESULTS OF THE IAP LOGIT MODEL ON PATH CHOICE DATA

The data base

The data base was build up on a truck-drivers road-side survey, carried out for 150 bi-directional sections located throughout Italy as a part of the Italian DSS for transportation planning (Cascetta,

1995). The overall sample was segmented by truck dimensions (number of axles) and load percentage.
In order to specify and calibrate the path choice model, only a part of the global sample was used. Preliminary path choice models were calibrated on interviews relative to heavy trucks (4 or 5 axles) loaded at more than 80% of their capacity. This was considered a sufficiently homogeneous market segment with a reduced influence of non-level-of-service attributes.
The chosen path was indicated in the questionnaire through entrance/exit points for motorway sections, in addition to origin, destination and intermediate nodes. After discarding interviews relative to paths giving rise to identification ambiguities on the coded graph, a total of 1588 interviews were kept for path generation and path choice modeling.
Path generation and level-of service attributes computations were carried out using the national road network (Nuzzolo et al., 1995), which consists of all the motorways and the main national roads. The main elements of the road network are reported in Tab. 1.

Tab. 1 - Characteristics of the road network

Nodes		Links	
Type	Number	Type	Number
real	1604	motorway	706
ficticious	180	toll gates	138
		extra urban roads	2032
		urban roads	178
		ramp	298
		connectors	552
		others	225
Total	**1784**	Total	**4129**

In order to compute the level-of-service attributes, vehicles were classified according to the number of axles.
Evaluation of average travel time on each path was carried out by using functions reported in the literature. The functions proposed by the Italian National Research Council (1983) were used for motorways, while TRRL functions (1980) were used for extraurban roads.

Enumeration of feasible paths

In the case of path choice the number of physically available alternatives is usually very large and some models should be adopted to limit it to a manageable number of potentially perceived alternative paths.
In this paper an algorithm for generating paths which include a very large proportion of actually chosen paths was developed elaborating a minimum k-paths algorithm based on multi-criteria path generation. Previous experiments carried out on the Italian network (Russo and Vitetta, 1995) showed that a significant degree of coverage of chosen paths could be obtained by generating successively K "shortest" paths with respect to two criteria, namely minimum travel time and maximum motorway use. The procedure is iterated until K different paths (with a difference of at least 4% of the total cost in this application) are generated. The results show (Cascetta et al., 1996) that after 8 paths the increase in the coverage number is very marginal and the best combination is based on the first six "minimum time" paths plus the first two "maximum motorway" paths. Obviously for some O/D pairs the number of generated paths may be lower than eight as some of the paths generated by one criterion may be included also in the set generated by the other.

Model specifications

On this data base different specifications (attribute X and Y) were tested for all models discussed in section 3, i.e. models reported in eqn (15), (16), (19) and (20) representing, as just seen, different possible specification of IAP Logit models (11) and (12).
In particular models (15) and (16) have been specified using the attribute Y_j=IND_j (17) to directly simulate $\overline{\mu}_C(j)$ while in models (19) and (20) a logarithmic transformation of this attribute appear alone and together with another one in the perception model (18); the vector of attributes X is always the same for comparison purposes and include classic level of service attribute. The structure of model (19) and (20) imply that the γ coefficient is expected to have a negative sign for attributes correlated with a positive perception of the alternative and vice versa.
In conclusion the attributes used for the test model are the following:

$$\sum_h \beta_h X_{hj} = \beta_{time} \cdot time_j + \beta_{cost} \cdot cost_j + \beta_{motorway\_lenght} \cdot motorway\_lenght_j$$

$$Y_j^i \quad = IND_j$$

$$\sum_k \gamma_k Y_{kj} \quad = \gamma_{\ln IND_j} \cdot \ln IND_j + \gamma_{labelA} \cdot labelA_j$$

where:

$time_j$	=	travel time of path j
$cost_j$	=	travel cost of path j
$motorway\_lenght_j$=	total motorway length for path j	
IND_j	=	attribute (17)
$labelA_j$	=	label equal to one if path j was generated with both criteria (minimum time and maximum motorway length), 0 otherwise

In tab.2 several specification and relative calibration results are shown.

Tab.2 - Calibration results

	IAP Logit							
Coefficients	□IC(j)=INDj		□IC(j)=>binomiale					
(t_students)	1st ord.	2ndord.	1st order			2nd order		
□time	-1.937 (-10.21)	-1.939 (-10.15)	-1.940 (-10.15)	-1.114 (-5.435)	-1.371 (-6.671)	-1.937 (-10.09)	-1.114 (-5.435)	-1.368 (--6.654)
□cost	-0.027 (-10.29)	-0.027 (-10.29)	-0.027 (-10.24)	-0.021 (-7.814)	-0.022 (-8.209)	-0.027 (-10.22)	-0.021 (-7.814)	-0.022 (-8.223)
□motorway lenght	0.013 (6.183)	0.013 (6.160)	0.013 (6.126)	0.011 (4.882)	0.012 (5.35)	0.013 (6.107)	0.011 (4.882)	0.012 (5.374)
□	0.869 (4.219)	0.300 (3.728)	73.349 (3.716)	22.291 (2.837)	108.11 (7.837)	51.875 (4.044)	6.851 (2.150)	22.318 (5.288)
□lnINDj			-0.023 (-3.569)		-0.023 (-5.256)	-0.016 (-3.794)		-0.055 (-4.39)
□labelA				-0.456 (-2.60)	-0.086 (-9.235)		-0.919 (-1.499)	-0.210 (-5.471)
LogLIn(0)	-1696.8	-1696.8	-1696.8	-1696.8	-1696.8	-1696.8	-1696.8	-1696.8
LogL(□ML)	-1279.0	-1281.0	-1279.0	-1022.0	-1005.0	-1279.0	-1022.0	-1005.0
Rho2 bar	0.246	0.245	0.246	0.397	0.407	0.246	0.397	0.407

As it can be seen coefficient signs are as expected for both level of service and perception attributes; the total motorway length in a path, play a positive role in the utility of the same path for question of comforts and both IND_j and $labelA$ increase the perception of the alternative (negative coefficients). Furthermore the calibration results shows a general high significance of the perception attribute IND_j in all different functional forms and a high significance of the perception attributes in general. There is virtually no difference between first and second order approximation models in terms of goodness of fit while there are differences in the value of the perception attributes coefficients and the α Gumbel parameter that systematically decrease in the second order model.

CONCLUSIONS

Route choice models are a fundamental building block of all traffic assignment models. Traditional route choice models are MN Logit and Probit with the implicit assumption that all feasible paths are available choice alternatives.

Recently the problem of new functional specifications of route choice models and explicit modeling of path choice set formation has been addressed. This paper extends and generalizes the results of previous research on this topic proposing an Implicit Availability Perception random utility choice model which incorporates perception attributes in a consistent way and, on the other hand, is suitable for operational applications to path enumeration-based assignment model.

This IAP r.u. model offers many possibility of specifications according to different initial hypotheses. In this paper some of these possible specifications have been proposed and tested with positive results in terms of coefficient signs and significance.

In general the hypothesis that some attributes play a fundamental role in alternative's availability/perception together with the hypothesis to simulate this availability/perception through the IAP functional form seems to be supported from the results.

However further empirical validations and/or other functional specifications have to be tested for a full assessment of the IAP model performances in the context of path choice. It should however be recognized that, in any case, the proposed IAP model gives a general consistent and simple framework to model path choice taking into account the problems of choice set individuation.

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