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A VERIFICATION OF A CELL-TYPE TRANSMISSION MODEL

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Abstract

Motorway traffic has been modelled as a fluid, using equations from hydrodynamic theory. These models have been well accepted but Daganzo claimed that some of the solutions of such models were unrealistic and allowed vehicles to travel backwards (1994). His alternative, the Cell Transmission Model, incorporates queuing and has been shown to be theoretically robust. The extended Statistical Traffic Model (STQ) was developed in response to this work. It is based on a macroscopic traffic model formulated as a dynamic state-space model, STM, developed at Lancaster. STQ is applied to motorway networks in Holland to provide short term predictions of the variables flow, density and velocity and compared to the original STM.

INTRODUCTION

As conditions on motorway traffic networks worsen, methods of alleviating the recurrent congestion are sought. Currently, various methods are in place, notable Variable Message Signs, toll-gates and such like; their sole aim being the control of the traffic flow. Most research at this time is concentrated on accurately modelling the traffic flow on these motorways; the results to be used in a control process. Models for traffic are usually either microscopic, where the behaviour of individual vehicles is modelled, or macroscopic looking at the aggregate behaviour of vehicles. For our purposes, we use macroscopic models, mainly based on fluid dynamics and queuing theories, to model the motorway traffic networks in Holland.

Our ultimate aim is to produce accurate travel times for motorists, thus enabling them to make some form of route choice if necessary and also providing them with useful information during their journey. In general, functions for calculating travel times require flow and velocity measurements. Therefore, if we are to forecast such travel times, accurate forecasts of flow and velocity are required. In this paper our primary aim is pursued, that is, to accurately model and predict motorway traffic.

Many authors have discussed modelling motorway traffic, some deriving highly theoretical models, but there has been little validation of these in some cases. We shall consider two models; a minimum velocities model (STM) derived by Garside and Whittaker (1994) and a queuing model based on a cell transmission model of Daganzo (1994, 1995). To date, the first model has been tested extensively with empirical data. However, the second model has only been tested with simulated networks and we intend to validate STQ using a real life network. We require that both models in question reproduce the traffic state to a suitable level of accuracy both for estimation and prediction.

Thus, there are two main goals we wish to achieve in the course of this paper. The first is the accurate modelling of the traffic state and the second is the accurate prediction over a particular time horizon, currently 20 minutes, of the future traffic state. The paper is divided as follows. The first section introduces the modelling and prediction framework employed in this paper, namely the state space model with Kalman filtering. We also outline the features of motorway traffic that we feel our model should reproduce. We describe the physical equations we believe represent traffic dynamics sufficiently accurately. These equations are extended and we suggest possible relationships between the variables that we are modelling. We introduce the two models we focus upon in this paper and discuss the application of the models using empirical data from two networks each with different characteristics. Finally we compare the performance of each model.

THE MATHEMATICAL TRAFFIC MODEL

This section outlines the features we require from our traffic model, the framework for building our model and the equations fundamental to such models.

We formulate our models in terms of the three commonly used traffic variables, flow, q, density, r (or occupancy, o) and velocity, v, within a state-space framework which allows for the forecasting smoothing and interpolation of data such as we are dealing with here. We are able to employ recursive techniques such as Kalman filtering easily to the state-space model.

Ultimately we require our model for traffic to accurately represent the traffic behaviour. Thus is should have the following features,

- the ability to model and predict free flow during uncongested periods
- the ability to model and predict the occurrence of congestion
- the ability to model and predict the propogation of congestion.

These should be captured in the equations modelling traffic dynamics which can be incorporated into the state-space framework.

The state-space framework

Our state-space framework holds the equations describing the average traffic behaviour of a set of vehicles on a section of road by representing the current state of the traffic in terms of flows, velocities and densities at some previous time. We can formulate a model in terms of state vectors, x_t where the components of these vectors are the aforementioned variables. The state-space framework also incorporates a measurement equation which describes the relationship between an observed state vector, y_t, and our state vector which is generally unobserved. We present the general state space formulation in eqn (1)

$$\begin{aligned} x^*_{t+1} &= Ax_t + Bu_t + q_t, \\ y_t &= Cx_t + r_t. \end{aligned} \tag{1}$$

The quantities q_t and r_t represent the process and measurement noise respectively and u_t represents the exogenous variables. We consider the variances of these measurements and denote the covariance matrices by Q and R respectively. In the application R = I and Q is block diagonal.

For our purposes, the model must perform two functions, namely to track the current state accurately and then predict future states based upon the estimates obtained for the current state.

Tracking the current state is achieved in two stages. Firstly, x_{t+dt} must be estimated using previous estimated stated via the transition equation in eqn (1). Secondly, the estimation, x^*_{t+dt} must be error corrected on the basis of the expected observation and the actual observation. The error-correction equation is

$$x'_{t+dt} = x^*_{t+dt} + K_{t+dt}(y_{t+dt} - C\,x^*_{t+dt}), \tag{2}$$

where K_{t+dt} is the Kalman filter matrix. The Kalman filter, Kalman, 1960, is used at this stage to error correct the state vector because it has been shown to be the optimal linear Bayes estimator of x_t, Ghosh and Knapp, 1978, Harvey, 1989. The filter depends on the observations only through the previous estimate of the state and the current prediction error and so allows a recursive computation.

For a prediction horizon *k*, future states x_{t+k} are predicted by repeatedly applying the transition equation to the most recent version of x_t. The Kalman filter only has an effect on the estimate of the current state so that we start the prediction stage as close as possible to the current observation. To guarantee accuracy of the prediction we must ensure that the model is stable, that is, the answers do not `blow up' as k increases. This requires that the eigenvalues of the matrix A all have modulus less than 1. This can be defined in physical terms, vf = dt/ds where vf is the free flow velocity and ds and dt are spatial and temporal intervals respectively.

Equations describing traffic dynamics

Our reference point for deriving our modelling equations is fluid dynamics theory, Lighthill and Whitham (1955), Richards (1956), where we can think of our traffic as a continuum and apply an equation analogous to the conservation of mass equation for fluids. We assume conservation of vehicles,

$$\frac{\partial r}{\partial t} + \frac{\partial q}{\partial s} = S \tag{3}$$

where S is the net inflow, t and s are spatial intervals and q and r are flow and density.

However, traffic is a discrete entity so we treat it as such. We use the discretised form of the conservation equation where, if we take a particular section s of motorway we say that, during some time interval dt, the number of vehicles entering the section from the nearest upstream neighbour and on-ramps if present, is the same as the number of vehicles leaving it (approximately). Letting the discrete variables $r_{s,t}$, $v_{s,t}$ and $q_{s,t}$ denote density, velocity and flow on a section s at time t, the discrete conservation of vehicles equation is,

$$r_{s,t+dt} = r_{s,t} + k_s^{-1}(q_{s-ds,t} - q_{s,t}) - S \tag{4}$$

Eqn (4) describes the behaviour of density on the section over time, and is dependent on the current values of density and flow there as well as the flow upstream. The parameter u is dependent on the section length and the temporal interval as well as the number of lanes on the section. This equation readily lends itself to the state-space formulation.

Flow can be written in terms of the product of velocity and density, q = vr, where we now use the discretised version,

$$q_{s,t+dt} = \alpha_s v_{s,t} r_{s,t} \tag{5}$$

The parameter α_s is used to ensure that the units are the same on both sides and is calibrated from the data. We use a dynamic flow equation rather than a stationary one since we believe that this provides a better description of the traffic behaviour. It also means that flow is included in the state vector. There are many alternative forms of this equation, for example, Hilleges and Wiedlich (1996) and Papageorgiou *et al.* (1989).

We now review two models proposed in the literature, in particular the models of Whittaker *et al.* (1994) and Daganzo (1995). All models incorporate the conservation equation (4) but differ in the flow equations used.

The simplest model for describing traffic flow uses the flow equation (5) along with a velocity-density relationship derived by Smulders and referenced in Papageorgiou (1990). However, this model does not truly represent the real-life scenario since there is no interaction between adjoining sections modelled as one would expect.

The minimum velocities model (STM)

This model, proposed by Garside and Whittaker (1994), uses the conservation equation (4), a dynamic flow equation and a stationary velocity-density relationship proposed by Smulders (1984).

The flow equation incorporates a minimum velocity term where driver anticipation of the vehicles on the block ahead is modelled. Assuming that there is homogenous traffic flow on two adjacent sections s and s+ds, a possible rule for the movement of traffic from s to s+1 in a time interval would state that traffic can only move forward providing there is sufficient room on the next block downstream. To model whether there is sufficient room Garside and Whittaker consider the minimum of the two velocities on the blocks,

$$q_{s,t+dt} = \alpha_s \min(v_{s,t}, v_{s+ds,t}) r_{s,t} \tag{6}$$

The STM (Statistical Traffic Model) is applied to a network of blocks (not necessarily homogenous) and for each block there are two state variables, flow and density, and one other, velocity. The network is defined such that each block corresponds to the location of a monitoring station from which the traffic observations are obtained. Stability of the model is an important issue and partly affected by the length of the blocks. For short links, where the block length is of the order of 100 metres, adjustments need to be made to the time intervals used in the model.

The extended STM with queuing (STQ)

This model is similar to the STM but uses a four case scenario for the flow equation based on a cell transmission model devised by Daganzo (1994). It is a discrete model describing the dynamic behaviour of variables such as velocity and flow on a network divided into cells and assumes that all cells are homogenous and that in free flow, all cells completely discharge in one time interval. Daganzo considers the flow from a cell as a function of the capacity on that cell and that on the nearest one downstream. He claims that this approach accounts for shock waves, a phenomena observed in the traffic flow and poorly represented in the standard hydrodynamic models.

The extended STM, known as the STQ, was developed in response to his work. Like Daganzo, it has an explicit flow relationship. This flow equation aims to model the supply and demand of adjacent blocks and thus model congestion more closely. Modelling the supply and demand in this way strives to ensure that the output from one block does not exceed the available capacity on the block ahead.

$$q_{s,t+dt} = \begin{cases} Q(r_{s,t}) & r_{s,t}, r_{s+ds,t} < rcrit \\ \min(Q(r_{s,t}), q^*_{s+ds,t}) & r_{s,t} < rcrit, r_{s+ds,t} > rcrit \\ qcong_s & r_{s,t} > rcrit, r_{s+ds,t} < rcrit \\ q^*_{s+ds,t} & r_{s,t}, r_{s+ds,t} > rcrit \end{cases}$$

For free flow traffic, the flow density relationship follows a quadratic,

$$Q(r_{s,t}) = \beta r_{s,t} + \chi r_{s,t}^2,$$

and a constant qcong(s) during congestion.

We also define

$$q^*_{s,t+dt} = q_{s+ds,t} - q^{off}_{s+ds,t} + q^{on}_{s+ds,t}.$$

Unlike Daganzo's model, STQ considers a four case scenario for flows where the flow out of a block is dependent on the traffic density on that block and the nearest one downstream.

The link performance function

The final equation used is called the link performance function which describes the relationship between density, flow and velocity. There are many different relationships proposed for the link performance function dependent on the nature of the traffic and so on, authors include Papageorgiou (1989a, 1989b, 1990), Hilleges and Wiedlich (1996), Pipes (1964). We mention here the necessity for accurate link performance functions. In a model where only two of the variables are in the state-space we must obtain values for the other one from the corresponding link performance function.

The link performance function is an important behavioural characteristic of traffic flow. We now consider the velocity density relationship for which several formulae have been proposed. These are based on the observation that, under homogenous condition, increasing density in a given freeway stretch leads to shorter headways and reduced mean velocity. For the purpose of this paper, we describe the function used to model the relationship between velocity and density proposed by Smulders which is discontinuous at the point of transition from free flow to congestion,

$$V(r) = \begin{cases} \alpha r + \beta & r < rcrit \\ d\left(\dfrac{1}{r} - \dfrac{1}{rjam}\right) & r > rcrit \end{cases}$$

It is well known that this point of discontinuity corresponds to traffic's natural instability at the point where free flow transforms to congestion. However, the identification of rcrit for individual blocks of the network is a difficult task. It is of vital importance in both the functional representations of the velocity-density and flow-density relationships for the STM and STQ respectively. It acts as both a change point for different modelling regimes (for v(s,t) in the STM and q(s,t+dt) in the STQ) and will indicate for which range of densities (r < rcrit) the linear relationships need fitting. Figure 1 shows that empirical velocity-density and flow-density relationships for a particular station on the A12 network for one particular day along with the parameter estimates for alpha and beta obtained by using successively larger values of rcrit.

The nature of the function used in an application is dependent on the network in question and also is subject to day to day variation due to factors such as the prevailing weather conditions.

APPLICATION OF TRAFFIC MODELS

We now go on to evaluate and apply the models outlined. We firstly simulate the alternative models and evaluate their performance through an application to a segment of the A2 Amsterdam network. We then proceed to implement the model on a stretch of the A12 between Gouda and Den Haag, incorporating the available measurements to make 15 step ahead forecasts of the traffic variables.

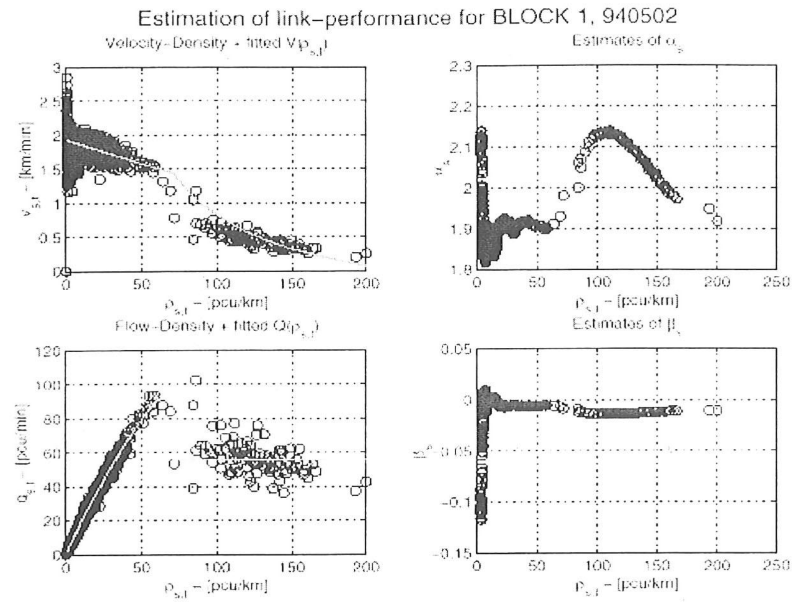

Figure 1. Empirical link performance functions for data from A12 Den Haag-Gouda

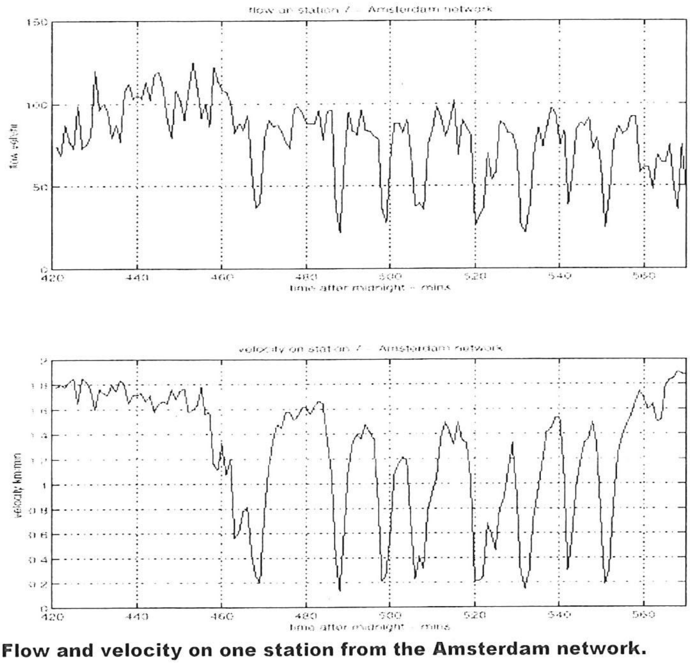

Figure 2. Flow and velocity on one station from the Amsterdam network.

Before proceeding to implement the models we firstly outline a number of important issues which must be considered. In order to evaluate model performance, that is, to assess the accuracy of the model when trying to reproduce current situations, we must have some form of error measurement.

The preferred alternative is the root mean square error proportional (RMSEP),

$$RMSEP = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}}{\frac{1}{n}\sum_{i=1}^{n} y_i}$$

One of the fundamental aspects of modelling any physical process is the estimation of parameters involved in the equations used. We can estimate the parameters in several ways,

- where possible we use our data and regression techniques to estimate parameters in the link performance function and the dynamic flow equation
- we can use a grid search method using an algorithm to search for the other parameter values which minimise some cost function. Methods to do this include Nelder - Mead and Powell's method. We do this for all the parameters left in the model, even those with theoretical values.
- some parameters we cannot identify easily, for example, rcrit, and this is identified by eye.

Once we have estimated parameters for our model is it sensible to perform a sensitivity analysis on the values found for completeness. However, since we are more interested with the comparison of the models using the same parameter values where appropriate we do not discuss this issue in more detail.

Data from the motorways is collected using pairs of inductance loops embedded in the surface of the road. Each time a vehicle passes over such loops, several measurements, such as vehicle passage, speed and type are recorded. These are passed to a monitoring station where the measurements are converted to our traffic variables, usually flow and occupancy, the latter converted to density. The data is generally aggregated across lanes so we only have single values for flow, density and velocity. Furthermore, the data is usually collected at one minute intervals. Recent improvements in the data collection methods assure that the data collected is synchronised between stations. Other problems include missing data from malfunctioning loops.

Examples of flow and velocity from the networks that we model are shown in Figures 2 and 3. Figure 2 shows that both the flow and velocity behaviour on this network exhibits cyclic behaviour, a feature of stop-and-go traffic. This type of behaviour is more demanding to reproduce and so is a real test of the capabilities of a model. From Figure 3 we can see serial correlation between stations.

For each application site we have a large section of motorway divided into blocks. We like to divide a section such that the end of each block coincides with the location of a pair of inductance loops and hence we have no choice over the spatial intervals used in the model equations. This is in contrast to Hilleges and Wiedlich who simulate traffic on a network and choose their block lengths to satisfy particular conditions. Generally the length of a block is of the order of 1 kilometre. There are three main block types used to build a network, namely sections, forks and joins. They can be defined in terms of their up- and down- stream neighbours; sections having only one of each, forks have one upstream and two downstream and joins have two upstream and one downstream.

We now go on to apply the models to the two test sites. Table 1 summarises the locations used in our testing.

Table 1. Summary of site data information

Site	No. of lanes	No. of blocks	Block length	traffic behaviour
A2 Amsterdam	2/3	16	0.5km	stop-and-go
Gouda-Den Haag	3	10	1km	recurrent congestion

Application to A2 Amsterdam

In this section we discuss the testing and development of STQ and compare it to the STM. Garside (1996) and Whittaker, Garside and Lindveld (1994) describe the development and implementation of the STM.

The models STM and STQ were used to model flows, densities and velocities from various sections of the Amsterdam test site. We model a section length of 0.5km by one block where boundary variables are from blocks s-ds and s+ds. The parameters for each model and the parameter values implemented are given in Table 2.

Table 2. Summary of model parameters

parameter	a(s)	u	vf	beta	rjam	d	qcong	rcrit
unit		min/km	km/min		veh/km		veh/min	veh/km
model								
STM	1	0.06	1.95	0.006	250	91.33	-	80
STQ	1	0.06	1.95	0.006	250	-	85	80

We use simulink (TM) to build our network. Simulink enables the simulation of non-linear dynamic equations using icons to represent mathematical operations, sources, delays and such like. Figure 4 shows an example of the simulink iconographic representation of equations used to model a block from a network.

Figures 5 and 6 show the results obtained by the STQ and STM models. Table 3 presents the errors.

Table 3. Results for RMSEP for flow, velocity and density for STQ and STM

RMSEP	STQ	STM
flow	0.26	0.38
velocity	0.29	0.23
density	0.37	0.26

All 3 quantities are estimated well by both models. STM performs better for modelling density and velocity but STQ models flow the better of the two. One further point to be noted is the performance of the models when estimating velocity at the end of the time period. The STM still estimates velocity to drop a long way whilst STQ estimates the velocity to be closer to observed values at this point. This difference in behaviour can be attributed to the reliance of the STM on the velocity ahead as well as that on the block in question. Therefore, the velocity ahead may still drop at this time although the congestion has cleared from the block on which we are looking.

Application to Start 4 Network: Gouda-Den Haag

We now apply the models outlined in the state-space framework to make predictions of the traffic on the Start 4 network. This network consists of two carriageways of the A12 between Gouda and Den Haag, one in each direction, consisting of 5 blocks. Figure 7 contains a schematic

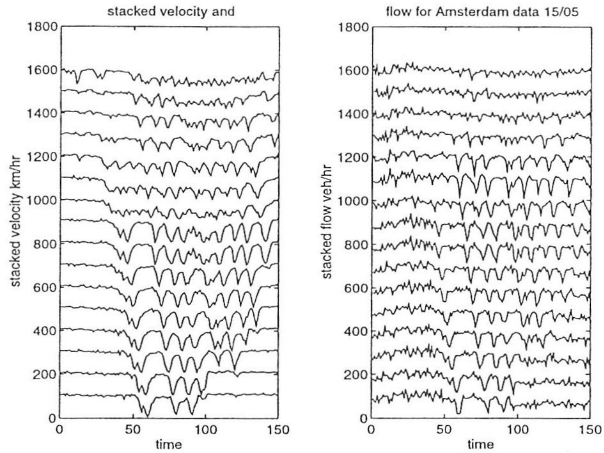

Figure 3. Stacked flows and velocities showing stop and go traffic conditions on the Amsterdam network.

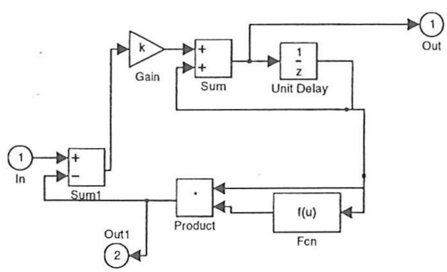

Figure 4. Simulink model of the model equations on a section block.

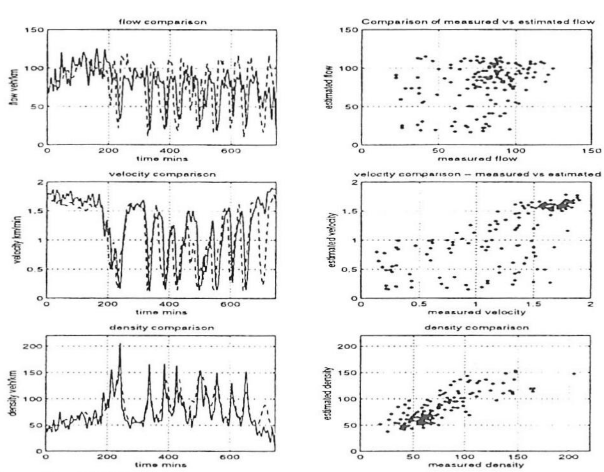

Figure 5. Results from the STM

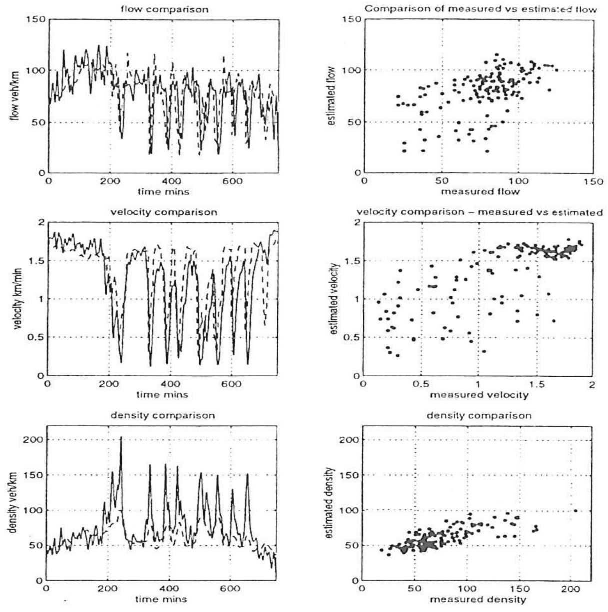

Figure 6. Results from the STQ model for estimating traffic variables on station 7 of the Amsterdam network.

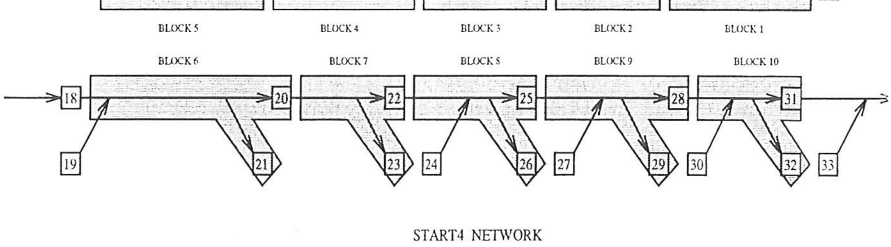

Figure 7. Block representation of START4 network consisting of the A12 Gouda – Den Haag separating exogenous and endogenous monitoring stations.

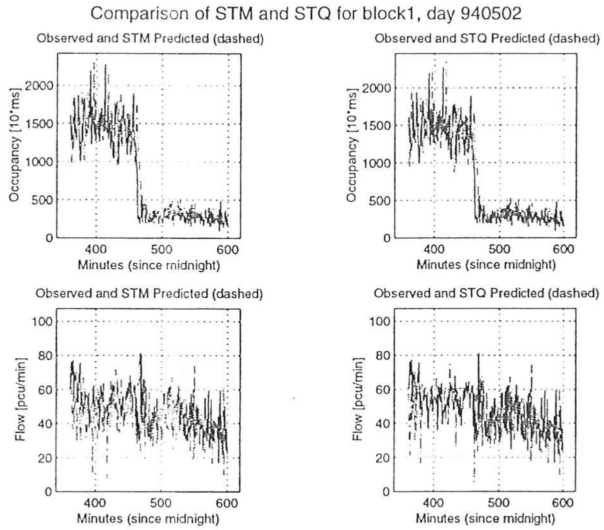

Figure 8. Evaluation of the STM and the STQ in congestion through comparison of the 15-step predictor with the observations for the occupancy on block 1 on 940502.

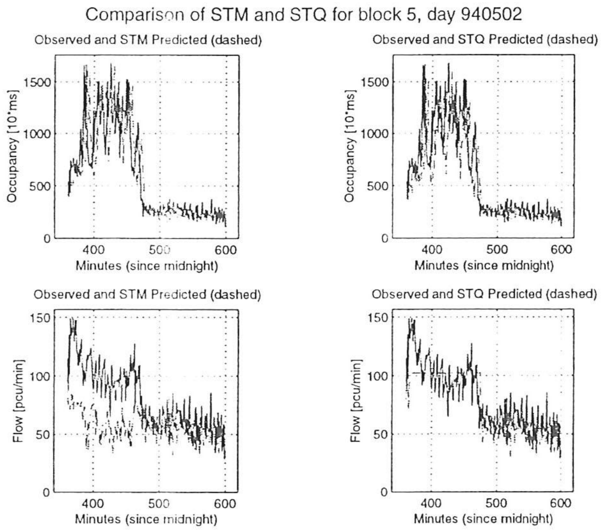

Figure 9. Evaluation of the STM and the STQ in congestion through comparison of the 15-step predictor with the observations for the occupancy on block 5 on 940502.

representation of the network in which Gouda lies to the right-hand end of the network with Den Haag at the other end.

Figures 8 and 9 show the output from the two blocks on the START4 network. In each figure the left hand pair of plots contains the 15 step ahead predictions from the STM for the occupancy and flow variables along with their associated observed values, and the right hand pair of plots show the equivalent STQ output.

For both blocks, the occupancy predictions differ only slightly between the STM and STQ. However, inspections of the output for the flow variables clearly demonstrates the advantage of the STQ over the STM. In Figure 8 it can be seen that the flow prediction from the STM shows a high degree of variability, much more than that exhibited by the STQ, indicating the instability evident in the STM model at higher values of density. The output in Figure 9 shows more clearly how the STM can fail to even predict the correct level of flow in congestion, again the STQ appears to perform much better.

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