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ESTIMATING MISSING LINK VOLUMES AT THE AMSTERDAM ORBITAL MOTORWAY

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Abstract

For motorway usage, authorities keep track of link flow profiles. These are tables of one hour volumes stored for each hour, type of day and month. For part of the network these profiles can be directly observed using detection loops. The remaining profiles need to be estimated using time dependent traffic data and additional static information. This paper discusses six estimation methods, characterized by the selection of data used, the assumptions on interdependencies between OD-cells they represent and their estimation mechanisms. A novelty is the computation of the correlation between different OD-cells with common period, origin or destination. Elaborate experiments with synthetic and empirical data for the Amsterdam orbital motorway, show that methods taking into account these correlations lead to improved estimation results, in particular when applied to empirical data.

INTRODUCTION

In the Netherlands, the motorway authorities keep track of average traffic volumes on motorways. This is done in to support the planning of road maintenance, to monitor traffic performance, to identify trends and future bottlenecks, etc. Data are stored in the form of flow-profiles. These profiles represent average hourly volumes, averaged per month, day and hour of the day.

Traffic is only counted on a limited number of links of the motorway network. These links are referred to as the observed links. The estimation of the flow-profiles for the other links is the subject of the present paper. It is envisaged that such estimates can be based on a combination of time-dependent traffic counts from the observed links and static information contained in Origin-Destination (OD) tables that are available from national planning models.

Although this problem is relatively unknown in literature, it resembles the well-known problem of estimating OD-matrices from traffic counts. The estimation of an OD-matrix from traffic counts under the assumption of certain route choice proportions also implies an estimate of all link flows in the network, including the unobserved ones. Another approach is to maximize entropy under the constraints of the given flow observations. In (Sherali *et al*, 1994) an approach is described minimizing the total traveled distance on a network under the constraint of the observed flows. The minimization was performed with Linear Programming techniques.

An approach that was pursued in (Van Der Zijpp *et al*, 1997) is to isolate a subnetwork that contains the unobserved link and its direct upstream and downstream observed links. This is followed by the estimation of the flow profile for this link based on time dependent observations from its most nearby links and a static OD-table projected on the local network.

This paper further elaborates on this approach, but also presents a novel network wide approach. The latter method is based on updating the prior OD-matrix using the assumption that the deviations that occur between cells of the true OD-matrix and corresponding apriori specified matrix are positively correlated, and that this correlation becomes stronger if these cells have more factors like origin, destination, month, type of day and hour in common.

Both types of methods are compared in a series of experiments based on both synthetic and empirical data, leading to the conclusion that taking into account correlations significantly improves the estimates from the network wide approach up to the point that it outperforms the subnetwork approach.

PROBLEM DESCRIPTION

The objective is to estimate flow profiles for links of a network for which no observations are available. To this end the following data are available:

- *Observed flow profiles:* Observations of flows, averaged per hour, type of day, month and year. Typically for 5% of the links such flow profiles are known.
- *Prior OD table.* This table is imported from the Dutch National Mobility Model and was last updated in 1990. Since then traffic has grown substantially and a number of changes of the

network have taken place. Only the twenty-four-hour weekday matrix was used, although separate matrices exist for peak and off-peak periods and for weekdays and non-weekdays.

- *Road networks.* A network containing the motorways and the major roads in the underlying network. No information on route choice proportions is available. Instead, routes are computed on the basis of link lengths assuming All-or-Nothing (AON) route choice behavior.

The following notation is used:

T_{ij}^{0}	cell in a priori matrix, containing daily traffic volumes from origin i to destination j
y_{kt}	observed flow on link k in hour t
τ_{ijk}	assignment map: $\tau_{ijk}=1$ if the route from i to j traverses link k; $\tau_{ijk}=0$ otherwise

The notation does not distinguish between types of day (Sun., Mon., etc.) or month. Day type and month are assumed constant in the subsequent derivations.

Given the low number of observed links the problem of estimating the unobserved link volumes from the available traffic counts only is seriously underspecified. Therefore additional information, like an external OD table, is needed. However the OD-table that is available may only be used as an indication because it is outdated and applies to the average traffic level rather than to the flow level a specific hour and day. In addition to that, only little information is available on route choice.

DEVELOPMENT OF ESTIMATION PROCEDURES

In (Van Der Zijpp *et al*, 1997) an algorithm based on the identification of subnetworks was proposed. The idea behind this was that the volumes on the unobserved links may be expressed as a weighted sum of volumes on observed nearby links. In addition to this the present paper also proposes a number of network-wide approaches and compares the performance of these with the subnetwork approach.

Network-wide approaches

A network-wide method is a method that does not apply a selection to the traffic counts that are used.

Method 1: assigning the prior matrix (PRIOR)

The most simple network-wide approach is simply to assign the prior matrix to the network. This solution is used as a reference for more advanced methods. Estimates for the flow profile on link q are hence obtained by:

$$\hat{y}_{qt} = \sum_{i,j} T_{ij}^{0} \tau_{ijq} \qquad (1)$$

were the subscript t identifies the period/hour.

Method 2: assigning the updated prior matrix (WLS)

The second most simple method is to update the prior matrix before assigning it to the network. The method used for this is the weighted least square (WLS) method. The updated matrix is determined by:

$$T_{ijt} = \underset{T_{ij}}{\arg\min} \left[\sum_{i,j} \frac{(T_{ij}^0 - T_{ijt})^2}{T_{ij}^0} + \sum_{k \in K_{obs}} \frac{(y_{kt} - \sum_{i,j} T_{ijt} \tau_{ijk})^2}{w_k} \right] \quad (2)$$

where the deviations from the prior matrix are weighted with the value T_{ij}^0 and the difference between traffic count and model-predicted value is weighted with the value w_k. In the experiments the weights w_k were set to the small value 1, expressing a high confidence in the traffic counts accuracy. Note that applying WLS only affects OD-cells that contribute to at least one traffic count.

Solving the minimization problem by computer does not require much CPU time or memory usage: it requires the inversion of a matrix which height corresponds with the number of observations.

Method 3: assigning the updated prior matrix, using correlations between OD-cells (GLS)

The difference between the actual value of an OD-cell on a specific day and in a specific hour, T_{ijt}, and its prespecified prior value, T_{ij}^0, is likely to be positively correlated among OD-cells. This is because different OD-cells have their origin, destination or time of day in common. Suppose estimates of the covariances between OD-cells are available and stored in a matrix Ω, then the generalized least square method (GLS) may be used to update the prior matrix. Parallel to the WLS procedure above the GLS estimate for the updated matrix is given by:

$$\begin{aligned} &\overline{T}_t = \underset{\overline{T}_t}{\arg\min} \left[(\overline{T}_t - \overline{T}^0)' \Omega^{-1} (\overline{T}_t - \overline{T}^0) + (\hat{\overline{y}}_t - \overline{y}_t)' . (\hat{\overline{y}}_t - \overline{y}_t) \right] \\ &\text{with} : \overline{T}_t = \{..T_{ijt}..\}, \overline{T}_t^0 = \{..T_{ijt}{}^0..\}, \overline{y}_t = \{..y_t..\}, \hat{\overline{y}}_t = \{..\hat{y}_t..\}, \hat{y}_t = \sum_{i,j} T_{ijt} \tau_{ijk} \end{aligned} \quad (3)$$

Note that storing the covariance matrix Ω may require a large amount of computer memory. Computational demands of the GLS method are equal to the requirements for the WLS method. It is assumed that the entries in the covariance matrix meet the following model:

$$T_{ijt} = (1 + \alpha_t / \sqrt{T_{ij}^0}).(1 + \beta_i / \sqrt{T_{ij}^0}).(1 + \gamma_j / \sqrt{T_{ij}^0}).(1 + \varepsilon_{ij} / \sqrt{T_{ij}^0}).T_{ij}^0 \quad (4)$$

were α_t, β_i, γ_j and ε_{ij} are independent zero mean error terms, with variance σ_α^2, σ_β^2, σ_γ^2, and σ_ε^2 respectively. Consequently, the entries in the covariance matrix can be computed as follows:

$$\begin{aligned} &\text{cov}[T_{ijt} T_{rst}] = \mathrm{E}[(T_{ijt} - \mathrm{E}[T_{ijt}])(T_{rst} - \mathrm{E}[T_{rst}])] \\ &= \sqrt{\mathrm{T}_{ij}^0} \sqrt{\mathrm{T}_{rs}^0} \mathrm{E}[(1+\alpha_t)^2 (1+\beta_i)(1+\beta_r)(1+\gamma_j)(1+\gamma_s)(1+\varepsilon_{ij})(1+\varepsilon_{rs}) - 1] \\ &= \sqrt{\mathrm{T}_{ij}^0} \sqrt{\mathrm{T}_{rs}^0} ((1+\sigma_\alpha^2)(1+\sigma_\beta^2)^{\delta(i,r)} (1+\sigma_\gamma^2)^{\delta(j,s)} (1+\sigma_\varepsilon^2)^{\delta(i,r)\delta(j,s)} - 1) \end{aligned} \quad (5)$$

where $\delta(i,r)$ represents element (i,r) of the identity matrix.

The approach described above can be extended to allow for an arbitrary number of factors. In the experiments the following values were used:

$$\sigma_\alpha^2=0.7,\ \sigma_\beta^2=0.1,\ \sigma_\gamma^2=0.1,\ \text{and}\ \sigma_\varepsilon^2=0.1 \tag{6}$$

These values express that variations in OD-demand are largely due to a common time of day factor. The effect of applying GLS assuming such a dominant common factor is that not only cells that contribute to traffic counts are updated but also the other cells.

Subnetwork approach

Methods that involve updating a prior matrix and subsequent assignment have two shortcomings:

- Only OD-cells that contribute to a traffic count are updated (method 3 excepted). As the fraction of links for which flows are observed is small, a large part of the OD-cells is likely to be left unaffected, implying that the estimate of the unknown fraction for a large part depends on the prior matrix.

- The result of the update from traffic counts heavily depends on the assumptions on route choice.

The subnetwork approach presented in (Van Der Zijpp *et al*, 1997) was designed to alleviate these problems. In this approach the data used in the estimation is confined to observations in the direct vicinity of the link for which a flow profile is to be estimated. It is expected that this reduces the sensitivity of the method for assumptions on route choice.

Method 4: Updating the prior matrix using subnetwork traffic counts (WLS-SUB)

This method is identical to the method described under 'method 2', except for the fact that it only uses traffic counts that are directly upstream or directly downstream of the unobserved link.

For this purpose we define a subnetwork in such a way that all OD paths traversing the subnetwork, cross at most one observed link *upstream* of the unobserved link q and at most one observed link *downstream* of link q. Note that 'upstream' and 'downstream' refer to routes. It is possible to construct a network such that the upstream links of one route overlap with the downstream links of another route. However, for the networks considered for practical applications, such instances do not occur. Consequently, we may think of upstream links and downstream links as mutually exclusive sets.

Method 5: Updating the prior matrix using subnetwork traffic counts and taking correlations between OD-cells into account (GLS-SUB)

This method is identical to the method described under 'method 3', except for the fact that it only uses traffic counts that relate to links in the subnetwork defined around the unobserved link (see above).

Method 6: Expressing the unknown link flow as a weighted sum of surrounding flows (FACTORS)

It may be argued that only updating cells contributing to observations, is not sufficient. It is likely that cells in OD matrices display a substantial positive correlation because the number of trips in different cells is influenced by many common factors, such as a common origin, destination or hour of departure. When adjustments need to be made to one group of cells in order to match a traffic count, it is likely that other cells need to be adjusted as well. Taking into account positive correlations between OD-cells like as done in methods 3 and 5, is one way to deal with this problem.

In (Van Der Zijpp *et al*, 1997) an alternative approach is described. The main points of this approach are repeated below.

Given a subnetwork such as described above, OD flows that contribute to the flow on the unobserved link q may be subdivided as follows:

- flows that contribute to observations *upstream*, but not *downstream* of q,
- flows that contribute to observations *downstream*, but not *upstream* of q,
- flows that contribute to observations both *upstream* and *downstream* of q,
- flows that contribute to neither observations *upstream*, nor *downstream* of q.

Given the categorization of flows given above we may express the unknown flow profile as follows:

$$y_{qt} = \sum_{r \in R_q} y_{rt} a_{rq} + \sum_{s \in S_q} y_{st} b_{sq} - \sum_{r \in R_q} \sum_{s \in S_q} y_{rt} c_{rsq} + v_{qt} \tag{7}$$

with:

y_{rt}	the flow on upstream link r during hour t ($t = 0,1,\ldots,23$),
y_{qt}	the flow on link q during hour t,
y_{st}	the flow on downstream link s during hour t,
R_q , S_q	the sets of upstream and downstream observed links of q,
a_{rq}	the proportion of the upstream observed volume y_{rt} that contributes to y_{qt} ,
b_{sq}	the proportion of the downstream observed volume y_{st} that contributes to y_{qt} ,
c_{rsq}	the proportion of the upstream observed volume y_{rt} that contributes to both y_{qt} and y_{st} ,
v_{qt}	the number of vehicles that traverse link q, but do not contribute to any observation; this number is referred to as the *unobserved* flow.

The local multipliers a_{rq} , b_{sq} and c_{rsq} are unknown, but can be estimated using the prior matrix, leading to the following approximations:

$$\hat{a}_{rq} = \frac{\sum_{i,j} T_{ij}^0 \tau_{ijr} \tau_{ijq}}{\sum_{i,j} T_{ij}^0 \tau_{ijr}} , \quad \hat{b}_{sq} = \frac{\sum_{i,j} T_{ij}^0 \tau_{ijs} \tau_{ijq}}{\sum_{i,j} T_{ij}^0 \tau_{ijs}} , \text{ and } \hat{c}_{rsq} = \frac{\sum_{i,j} T_{ij}^0 \tau_{ijr} \tau_{ijq} \tau_{ijs}}{\sum_{i,j} T_{ij}^0 \tau_{ijr}} \tag{8}$$

Substituting these values and re-examining the first term of (7) yields:

$$\sum_{r \in R_q} y_{rt} \hat{a}_{rq} = \sum_{r \in R_q} y_{rt} \frac{\sum_{i,j} T_{ij}^0 \tau_{ijr} \tau_{ijq}}{\sum_{i,j} T_{ij}^0 \tau_{ijr}} = \sum_{r \in R_q} y_{rt} \frac{\sum_{i,j} T_{ij}^0 \tau_{ijr} \tau_{ijq}}{y_r^0} = \sum_{i,j} T_{ij}^0 \tau_{ijq} \sum_{r \in R_q} \frac{y_{rt}}{y_r^0} \tau_{ijr} \tag{9}$$

$$\text{with}: y_r^0 = \sum_{i,j} T_{ij}^0 \tau_{ijr}$$

This can be interpreted as weighing the apriori OD-cells with the ratio of observed flow y_{rt} and apriori assumed flow y^0_r. Rearranging the other terms in (7) in a similar way, and weighing the OD-flows that are observed both upstream and downstream with the square root of the factors that apply to upstream-only and downstream-only observed cells, we get:

$$\hat{y}_{qt} = \sum_{i,j} T_{ij}\tau_{ijq} \sum_{r\in R_q} \sum_{s\in S_q} \left(\tau_{ijr}(1-\tau_{ijs})\frac{y_{rt}}{y^0_{rt}} + (1-\tau_{ijr})\tau_{ijs}\frac{y_{st}}{y^0_{st}} + \tau_{ijr}\tau_{ijs}\sqrt{\frac{y_{rt}}{y^0_{rt}}}\sqrt{\frac{y_{st}}{y^0_{st}}} \right) + v_{qt} \qquad (10)$$

This formula, which is an extension of the work presented in (Van Der Zijpp *et al*, 1997) can be implemented on the basis of the assignment map τ. This facilitates the implementation of the method, relative to equation (7); in practice standard procedures to compute assignment maps are available, while dedicated software should be produced to implement (7).

In equations (7) and (10) v_{qt} denotes the unobserved part of the flow on link q: this flow equals the sum of all OD-flows that traverse link q, but not any observed link. In absence of any other information, it is assumed that these flows show the same variations with relative to their apriori predicted values as exhibited by the observed flows (see Van Der Zijpp *et al*, 1997).

PRIOR MATRICES

Each of the six methods described in the previous link is applied in combination with a prior matrix. Experiments were done with three types of matrices.

Priori matrix 1: Unmodified matrix

This matrix was retrieved from the Dutch National Mobility Model (see Rijkswaterstaat & Hague Consultancy Group, 1990). This matrix consisting of 1349 by 1349 cells was assigned to a representation of the national road network using all-or-nothing assumptions and afterwards a matrix for the study area was obtained (see Van Der Zijpp *et al*, 1997 for details). The matrix available applies to a 24-hour average for weekdays. This matrix was scaled linearly to match the length of the period of analysis.

Prior matrix 2: Updated matrix

The updated matrix is obtained by adapting the unmodified matrix using a constrained weighted least square method and the *average* observed values. The implication of this is that the apriori assigned value (method 1) is an unbiased estimate for the unknown link volume. Applying this matrix in practice is only possible after collecting data on the average flow level.

Prior matrix 3: Matrix of ones

As an alternative to above two matrices a (linearly scaled) matrix consisting of ones is used. The idea behind using this matrix is that in general a motorway network is designed in such a way that the importance of different on- and off-ramps is comparable.

TRAFFIC COUNTS

Traffic counts were collected at the Amsterdam orbital motorway at 34 locations. Records of data include the years 1990 to 1995. After inspection of the data it was decided only to use data from 1995 to avoid problems with missing data. Data are collected per hour, type of day and month. In addition to this a yearly average of the 24 hour flow is stored. For the experiments three sets of empirical data are prepared. These sets are coded with '1DM' for the data aggregated at 1 hour and stored per type of day and month,'24DM' for data aggregated 24 hours and stored per type of day and month and '24DY' for data aggregated 24 hours and stored per type of day but averaged over one year.

In addition to these empirical data sets also a synthetic data set was generated. The synthetic data set was obtained by randomly generating an OD-matrix using the unmodified prior matrix as a mean. The traffic counts were obtained by assigning the matrix. This was repeated 100 times resulting in artificial flow profiles of length 100.

EXPERIMENTS

Experiments were performed using data from the Amsterdam orbital motorway. This is a network consisting of 493 nodes, 495 links, 72 on-ramps and 73 off ramps. For 34 links traffic counts are available (see Figure 1).

Combining the type of method, prior matrix and used traffic count data (see previous links) results in a large number of test configurations. Each configuration corresponds with one experiment and is characterized by the following items (see also Figure 2) :

- *The traffic count data used*. Traffic counts are available for 34 links and were collected in 1995. A distinction is made between the following datasets:
  - 1 hour aggregated, distinguished per type of day and per month (1DM),
  - 1 day aggregated, distinguished per type of day and per month (24DM),
  - 1 day aggregated, distinguished per type of day and averaged over 1995 (24DY).
- *The prior OD-matrix used*. This can either be:
  - the unmodified 'normal' matrix derived from the national mobility model (N),
  - an updated version of this matrix (U), or
  - a uniform matrix, each cell value set equal to one (O).
- *The estimation method used*:
  - assigning the prior OD table (PRIOR),
  - updating the prior matrix using all traffic counts available, assuming uncorrelated OD-cells (WLS),
  - updating the prior matrix using all traffic counts available, assuming correlated OD-cells (GLS),
  - updating the prior matrix using only counts from the subnetwork assuming uncorrelated OD-cells (WLS-SUB),
  - updating the prior matrix using only counts from the subnetwork assuming correlated OD-cells (GLS-SUB), or
  - expressing the unknown link flow as a weighted sum of surrounding link flows, using factors computed using the prior matrix (FACTORS).

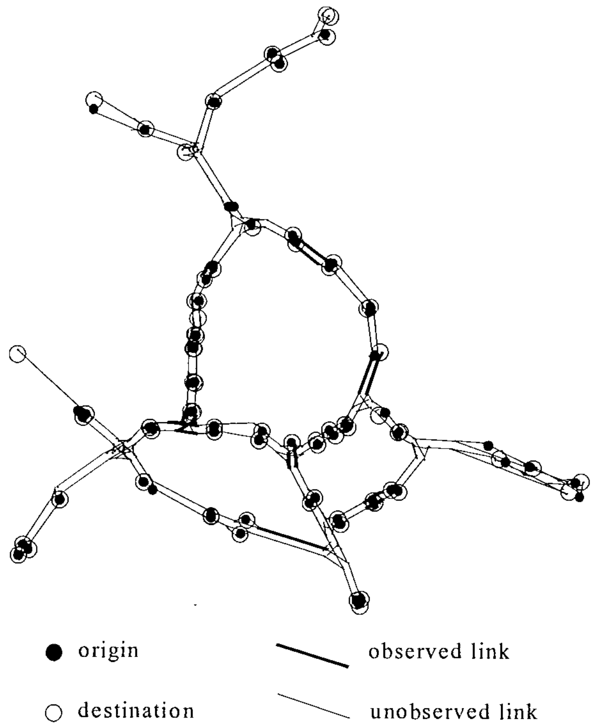

Figure 1 - The Amsterdam orbital motorway network

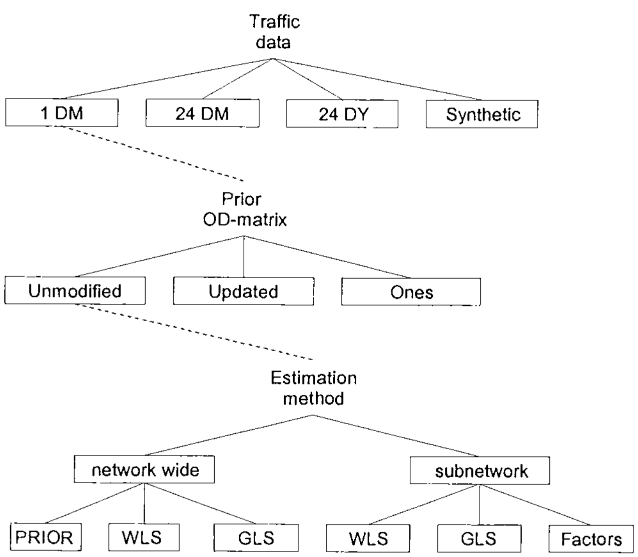

Figure 2 - Organization of the experiments

The experiments were run by sequentially estimating the flow profile for all links for which data were available, each time avoiding the use of the data of the current link (see Figure 3). The performance of the method is expressed in the mean absolute error proportional (MAEP) which was defined as follows:

$$MAEP = \frac{1}{Q}\sum_{q}\frac{\sum_{t}\left|\hat{y}_{qt} - y_{qt}\right|}{\sum_{t} y_{qt}}, \text{ with } Q \text{ the number of links involved.} \qquad (11)$$

The numerical results of the experiments are displayed in Table 1.

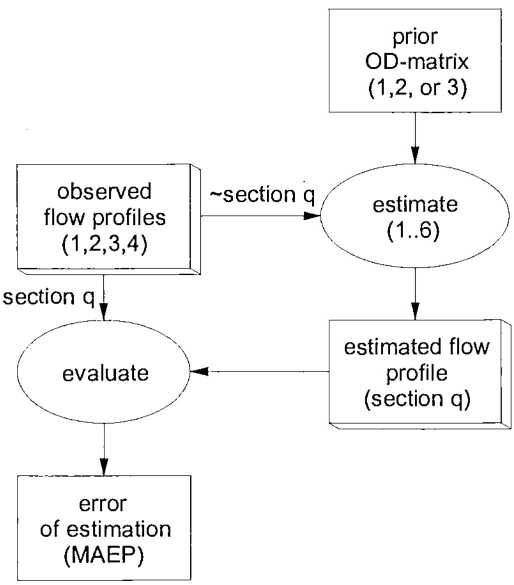

Figure 3 - Evaluation setup

Table 1 - Numerical results of experiments (MAEP)

Data:	**1DM**			**24DM**			**24DY**			**Synthetic**		
Matrix:	**N**	**U**	**O**	**N**	**U**	**O**	**N**	**U**	**O**	**N**	**U**	**O**
PRIOR	0.84	0.63	0.88	0.59	0.18	0.64	0.59	0.16	0.59	0.20	0.20	0.67
WLS	0.77	0.43	0.78	0.62	0.12	0.66	0.58	0.10	0.58	0.19	0.19	0.68
GLS	0.65	0.22	0.63	0.59	0.06	0.61	0.58	0.05	0.59	0.21	0.21	0.58
WLS-SUB	0.75	0.41	0.67	0.61	0.11	0.58	0.59	0.08	0.51	0.19	0.19	0.69
GLS-SUB	0.76	0.28	0.63	0.69	0.08	0.60	0.64	0.06	0.53	0.20	0.20	0.71
FACTORS	1.00	0.25	0.62	0.94	0.07	0.59	0.81	0.05	0.51	0.20	0.20	0.76

INTERPRETATION OF RESULTS AND CONCLUSIONS

The experiments with the synthetic data can be used to check on a number of theoretical properties. Assigning the prior matrix leads to an average error of 20% (first row, 10th data column). Applying weighted least squares, either using all data (WLS) or using part of the data (WLS-SUB) slightly improves this estimate to an error of 19%. This is to be expected because the way the synthetic data are generated is in line with the underlying assumptions of the WLS method. In theory one would expect WLS to do better than WLS-SUB as the first method uses all data available. However, this effect is only visible in the fourth digit and is hence not displayed in the table. Likewise it can be explained that WLS outperforms the GLS methods as GLS methods assume a positive correlation between OD-cells that is not present in the synthetic data.

The 12th data column of the table shows an unexpected performance of the WLS methods (second and fourth rows). An increased estimation error relative to the prior matrix assignment is indicated. The probable explanation for this is that the error structure of the prior matrix (in this case a matrix of ones) is not in line with the assumptions underlying the WLS method.

From the results obtained with empirical data we can draw a number of clear conclusions:

- If no additional data are collected the best achievable error of estimation ranges between errors around 50-60%. For practical purposes this is too high of an error.
- This particular accuracy is reached when using a matrix consisting of ones as a prior matrix. This casts some serious doubts about the suitability of the prior OD-matrix or about the way it was assigned to the network for the problem at hand.
- Updating the prior matrix with static data prior to estimating the flow profiles greatly improves the estimates. After using the updated prior matrix the errors range between 5% and 20%, depending on the required level of aggregation.
- The method based on factors does not perform as well as might have been expected. In some cases it does not even lead to a reduction of the estimation error relative to the prior matrix assignment. Probably this is due to the low fraction of links that is observed. In many cases the subnetwork contains no more than one or two observed links.
- Taking into account positive correlations between OD-cells in general reduces the error of estimation significantly relative to the case where OD-cells are assumed to be uncorrelated. The effect is at its largest when applying the method to an updated prior matrix (row 3, column 2, 5 and 8).
- The greatly improved accuracy of the GLS-method compared to the WLS-method suggests that GLS could also be successfully applied to related problems such as the estimation of OD-matrices from traffic counts.
- No evidence was found that confining the use of data to a subnetwork around the unobserved link leads to a better estimate.

In view of the results, the best way to proceed in practice seems to collect traffic counts at as many locations as possible and update a matrix consisting of ones using these static data. The resulting matrix can be used as an input to a generalized least squares method that also uses dynamic data collected at the regular data collection sites. Experimenting with the covariances used may further improve the estimates obtained with this method.

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