STRATEGIC AIRLINE ALLIANCES: COMPLEMENTARY VS. PARALLEL ALLIANCES

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Abstract

This paper presents a model that examines the effects on market outcome and welfare of two types of strategic airline alliances: complementary vs. parallel alliances. It is identified that the two alliances have different effects on total output and consumer surplus. The complementary alliance is likely to increase total output, while the parallel alliance is likely to decrease it. Consequently, the former increases consumer surplus, while the latter is likely to decrease it. We find sufficient conditions under which each type of alliance improves total welfare.

INTRODUCTION

In order to attract more passengers in an increasingly competitive environment, major international airlines have been seeking to extend the range of their network and access new markets. Most international carriers have focused on integrating two or more existing networks through international airline alliances.

Strategic alliances may provide opportunities for partners involved to reduce costs by coordinating activities in some fields: joint use of ground facilities such as lounges, gates and check-in counters; codesharing[1] or joint operation; block space sales;[2] joint advertising and promotion; exchange of flight attendants; and so on. Alliances also produce several benefits for consumers. Alliance partners can better coordinate flight schedules to minimize travellers' waiting time between flights while providing sufficient time for connections.

Although alliances generate benefits for both partners involved and consumers, it may reduce the number of competitors and thus increase the combined market power of alliance partners. As a result, the partners may increase air fares if they behave collusively and abuse their strengthened market power. On the other hand, it is also possible for air fares to decrease since alliances between non-market-leaders can increase their competitiveness against the market leader. By focusing on "complementary" alliances in the trans-Pacific markets, Oum, Park and Zhang (1996) empirically show that the alliances between non-leaders reduce the leader's equilibrium price.

Despite the growing importance of international airline alliances, few researchers have devoted effort to constructing formal models of the alliances. This paper constructs a formal model to examine the effects on market outcome and economic welfare of different types of alliances: "complementary" and "parallel" alliances. The "complementary" alliance refers to the case where two firms link up their existing networks and build a new complementary network in order to feed traffic to each other. The "parallel" alliance refers to collaboration between two firms competing on the same routes. Two types of parallel alliances are considered: "no-shut-down" and "shut-down" parallel alliances. The difference between the two is that each partner continues to individually provide services on the route in the first type, while two partners integrate their services in the second type.

THE BASIC MODEL

Pre-alliance situation

We begin by constructing a pre-alliance situation first where none of airlines have yet to make any type of alliance. As depicted in Figure 1, a network is considered, consisting of three gateway cities located in different countries: A, B and H. There are three origin and destination markets, AH, BH and AB, and three firms (or carriers) are operating in the network. Firm 1 is assumed to serve all three markets (AH, BH and AB) using its hub-and-spoke network. Firms 2 and 3 are assumed to serve AH and BH markets, respectively.

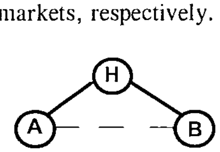

Figure 1. A simple air transport network

It is furthermore assumed that in the pre-alliance situation, travellers do not use multiple carriers' interline connecting services because of poor connections between firms 2 and 3.

Complementary alliance situation

Consider a situation where firms 2 and 3 make a "complementary" alliance. Both firms jointly provide connecting services for passengers travelling between cities A and B, while continuing to provide local services as before. In order to compete with firm 1's connecting services, the partners enhance quality of their connecting services. They agree to share revenues and costs from the connecting services.

The "full" price demand model is considered from the viewpoint that each firm's demand in each market depends not only on its air fare, but also its service quality (De Vany, 1974; Panzar, 1979). Each firm's demand in each market in the complementary alliance situation may be written as

$$Q_{AH}^{i} = D_{AH}^{i}(\rho_{AH}^{i}, \rho_{AH}^{j}) \quad for\ i=1,2,\ i \neq j$$

$$Q_{BH}^{i} = D_{BH}^{i}(\rho_{BH}^{i}, \rho_{BH}^{j}) \quad for\ i=1,3,\ i \neq j$$

$$Q_{AB}^{i} = D_{AB}^{i}(\rho_{AB}^{i}, \rho_{AB}^{j}) \quad for\ i=1,2+3,\ i \neq j$$

where ρ_k^i is the full price of using carrier i's service in market k, which is the sum of air fare, denoted by p_k^i, and value of service quality. Solving the demand functions for ρ_k^i may yield

$$\rho_k^i = d_k^i(Q_k^i, Q_k^j) \quad for\ k=AH, BH, AB,\ i \neq j.$$

We assume that outputs of rival carriers are substitutes in each city-pair market:

$$\frac{\partial d_k^i}{\partial Q_k^j} < 0, \quad for\ k=AH, BH, AB,\ i \neq j. \tag{1}$$

The value of service quality can be regarded as cost of service quality from the viewpoint of carriers. Two different costs of service quality are considered: (i) schedule delay cost on each route, and (ii) inconvenient connecting cost at the connecting airport.

The schedule delay cost is a passenger's schedule delay time arising from the difference between the passenger's desired departure and actual departure time. Research has found that the schedule delay cost depends largely on the carrier's flight frequency, which in turn depends on its *total traffic* (e.g., Douglas and Miller, 1974). Thus, if Q is the total passengers carried by carrier i on route k, then the schedule delay cost may be written as $g_k^i(Q)$. It is assumed that $g^{/}(\cdot)<0$, that is, the schedule delay cost of an airline declines with its traffic on the route. The schedule delay cost for the non-stop services is $g_k^i(Q_k^i + Q_{AB}^i)$ for k = AH and BH, while the schedule delay cost for the connecting service is the sum of the schedule delay cost on each of two local routes, $g_{AH}^i(Q_{AH}^i + Q_{AB}^i) + g_{BH}^i(Q_{BH}^i + Q_{AB}^i)$.

The second component of the cost of service quality is a passenger's inconvenience cost due to connections. Carlton, Landes and Posner (1980) estimate that travellers place an extra cost of $13-17 (in 1978 dollars) for a single carrier's one-stop connecting services, as compared to its non-stop services. This extra cost for alliance partners' connecting services will be even larger, if the partners' connecting service is inferior to the single carrier's connecting service. For convenience of analysis, without loss of generality, we assume that the inconvenient connecting cost for the single carrier's connections is zero, but that for the partners' connections, denoted by γ, is positive. However, the partners' connecting cost will decrease as the level of their coordination increases at the airport H.

Carrier i's production cost function on route k may be expressed as $C_k^i(Q)$, implying its round-trip cost of carrying Q passengers on the route. This production cost function reflects economies of traffic density, satisfying $C_k^{i\prime}(Q)>0$ and $C_k^{i\prime\prime}(Q)<0$. [3]

Given these demand and cost specifications, profit function for the non-aligned carrier and aligned partners can be expressed as:

$$\begin{aligned}\Pi^{1c} \equiv\ & Q_{AH}^1\left[d_{AH}^1(Q_{AH}^1, Q_{AH}^2) - g_{AH}^1(Q_{AH}^1 + Q_{AB}^1)\right] + Q_{BH}^1\left[d_{BH}^1(Q_{BH}^1, Q_{BH}^3) - g_{BH}^1(Q_{BH}^1 + Q_{AB}^1)\right] \\ & + Q_{AB}^1\left[d_{AB}^1(Q_{AB}^1, Q_{AB}^{(2+3)}) - g_{AH}^1(Q_{AH}^1 + Q_{AB}^1) - g_{BH}^1(Q_{BH}^1 + Q_{AB}^1)\right] \\ & - C_{AH}^1(Q_{AH}^1 + Q_{AB}^1) - C_{BH}^1(Q_{BH}^1 + Q_{AB}^1)\end{aligned} \tag{2}$$

$$\begin{aligned}\Pi^{(2+3)c} \equiv\ & Q_{AH}^2\left[d_{AH}^2(Q_{AH}^1, Q_{AH}^2) - g_{AH}^2(Q_{AH}^2 + Q_{AB}^{(2+3)})\right] + Q_{BH}^3\left[d_{BH}^3(Q_{BH}^1, Q_{BH}^3) - g_{BH}^3(Q_{BH}^3 + Q_{AB}^{(2+3)})\right] \\ & + Q_{AB}^{(2+3)}\left[d_{AB}^{(2+3)}(Q_{AB}^1, Q_{AB}^{(2+3)}) - g_{AH}^2(Q_{AH}^2 + Q_{AB}^{(2+3)}) - g_{BH}^3(Q_{BH}^3 + Q_{AB}^{(2+3)}) - \gamma\right] \\ & - C_{AH}^2(Q_{AH}^2 + Q_{AB}^{(2+3)}) - C_{BH}^3(Q_{BH}^3 + Q_{AB}^{(2+3)})\end{aligned} \tag{3}$$

where superscript c stands for complementary alliance.

It can be shown that $\partial^2\pi^{ic}/\partial Q_{AH}^i \partial Q_{BH}^i = 0$. We can also show that

$$\frac{\partial^2\pi^{ic}}{\partial Q_k^i \partial Q_{AB}^i} = -2g_k^{i\prime}(\cdot) - g_k^{i\prime\prime}(\cdot)\cdot(Q_k^i + Q_{AB}^i) - C_k^{i\prime\prime}(\cdot), \quad k = AH, BH. \tag{4}$$

In (4), the first term is positive because an airline's schedule delay cost decreases with its traffic. The second term is positive if g is linear or concave. The third term is also positive because of economies of traffic density. (4) can be positive even if g is convex. More generally, we assume that (4) is positive, implying that there exist network complementarities between local and connecting services.

In (1), outputs of rival carriers are assumed to be substitutes in each city-pair market. We further assume

$$\frac{\partial^2\pi^{ic}}{\partial Q_k^i \partial Q_k^j} < 0, \quad k = AH, BH, AB, \quad i \neq j. \tag{5}$$

Parallel alliance situation

Next, consider another post-alliance situation where firms 1 and 2 make a "parallel" alliance. Two types of parallel alliances are considered. The first is that each partner continues to provide local services in the AH segment and choose their quantities to maximize their joint profits. Another type is that the partners integrate services in the AH segment in a way that the hub partner continues to provide local services, but the non-hub partner stops producing local services. For simplicity of analysis, it is assumed that the partners equally share revenues and costs arising from the joint services.

For consistency, we consider the same demand and cost specifications as used in the complementary alliance. In particular, the inverse demand functions for the parallel alliance may be written as

$$\rho^i_{AH} = d^i_{AH}(Q^i_{AH}, Q^j_{AH}) \ \ for\ i=1,2\ ,\ i \neq j$$

$$\rho^i_{BH} = d^i_{BH}(Q^i_{BH}, Q^j_{BH}) \ \ for\ i=1,3\ ,\ i \neq j$$

$$\rho^1_{AB} = d^1_{AB}(Q^1_{AB})$$

where Q^2_{AH} is positive for the "no-shut-down" case; Q^2_{AH} is zero for the "shut-down" case.

EFFECTS OF COMPLEMENTARY ALLIANCE

Effects on firms' outputs and profits

Let us first analyze the effects of the complementary alliance. We consider an equilibrium that arises when the non-aligned carrier (i.e., firm 1) and the aligned partners (i.e., firms 2+3) play a Cournot game in each market of the network.[4] By using vectors Q^1 and Q^2, (2) and (3) can be simplified as

$$\max_{Q^1} \ \Pi^{1c} = \Pi^{1c}(Q^1, Q^2) \tag{6}$$

$$\max_{Q^2} \ \Pi^{2c} = \Pi^{2c}(Q^1, Q^2; \gamma) \tag{7}$$

where $Q^i \equiv \left(Q^i_{AH}, Q^i_{BH}, Q^i_{AB}\right)$ for i = 1, 2. For convenience of notation, superscript 2+3 is replaced by 2. Assume that there exists a "stable" Cournot-Nash equilibrium $\left(Q^1(\gamma), Q^2(\gamma)\right)$ which satisfies the following first-order conditions for maximization of (6) and (7):

$$\Pi^{1c}_1(Q^1(\gamma), Q^2(\gamma)) \equiv 0 \tag{8}$$

$$\Pi^{2c}_2(Q^1(\gamma), Q^2(\gamma); \gamma) \equiv 0\ . \tag{9}$$

Assume that the second-order conditions are also satisfied, i.e., the following Hessian matrices are negative definite for i = 1, 2:

$$\Pi_{ii}^{ic} \equiv \begin{bmatrix} \Pi_{AH_iAH_i}^{ic} & \Pi_{AH_iBH_i}^{ic} & \Pi_{AH_iAB_i}^{ic} \\ \Pi_{BH_iAH_i}^{ic} & \Pi_{BH_iBH_i}^{ic} & \Pi_{AB_iAB_i}^{ic} \\ \Pi_{AB_iAH_i}^{ic} & \Pi_{AB_iBH_i}^{ic} & \Pi_{AB_iAB_i}^{ic} \end{bmatrix}.$$

Proposition C-1. Under the complementary alliance conditions, firm 1 produces less output in markets AH, BH and AB, but the alliance partners produce more output in both their local market and the AB market than under the pre-alliance conditions.

Proof. Differentiating (8) and (9) with respect to γ yields

$$\Pi_{11}^{1c}\frac{dQ^1}{d\gamma} + \Pi_{12}^{1c}\frac{dQ^2}{d\gamma} = 0, \tag{10}$$

$$\Pi_{21}^{1c}\frac{dQ^1}{d\gamma} + \Pi_{22}^{1c}\frac{dQ^2}{d\gamma} + \Pi_{2\gamma}^{2c} = 0 \tag{11}$$

where $\Pi_{2\gamma}^{2c} \equiv [0, \ 0, \ -1]^T$. Solving (10) and (11) for $(dQ^1/d\gamma, dQ^2/d\gamma)$, we have

$$\frac{dQ^1}{d\gamma} = \left[I - \left(\Pi_{11}^{1c}\right)^{-1}\Pi_{12}^{1c}\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{21}^{2c}\right]^{-1}\left(\Pi_{11}^{1c}\right)^{-1}\Pi_{12}^{1c}\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{2\gamma}^{2c} \tag{12}$$

$$\frac{dQ^2}{d\gamma} = -\left[I - \left(\Pi_{22}^{2c}\right)^{-1}\Pi_{21}^{2c}\left(\Pi_{11}^{1c}\right)^{-1}\Pi_{12}^{1c}\right]^{-1}\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{2\gamma}^{2c}. \tag{13}$$

Differentiating (8) with respect to Q^2 yields the following 3-by-3 "derivative" matrix of carrier 1's reaction functions: $R_2^{1c} \equiv \partial R^{1c}(Q^2)/\partial Q^2 \equiv -\left(\Pi_{11}^{1c}\right)^{-1}\Pi_{12}^{1c}$ where $R^{1c}(Q^2(\cdot))$ is carrier 1's reaction function for the aligned partners' outputs. Similarly, a "derivative" matrix of the partners' reaction functions for firm 1's outputs can be defined as $R_1^{2c} \equiv \partial R^{2c}(Q^1)/\partial Q^1 \equiv -\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{21}^{2c}$.

In what follows, we show that every element of R_2^{1c}, R_1^{2c} matrices is negative: First, it turns out that both Hessian inverse matrices are negative matrices. $\left(\Pi_{11}^{1c}\right)^{-1}$ can be expressed as

$$\frac{1}{|\Pi_{11}^{1c}|}\begin{bmatrix} \Pi_{BH_1BH_1}^{1c}\Pi_{AB_1AB_1}^{1c} - \left(\Pi_{BH_1AB_1}^{1c}\right)^2 & \Pi_{AH_1AB_1}^{1c}\Pi_{BH_1AB_1}^{1c} & -\Pi_{AH_1AB_1}^{1c}\Pi_{BH_1BH_1}^{1c} \\ \Pi_{AB_1AH_1}^{1c}\Pi_{BH_1AB_1}^{1c} & \Pi_{AH_1AH_1}^{1c}\Pi_{AB_1AB_1}^{1c} - \left(\Pi_{AH_1AB_1}^{1c}\right)^2 & -\Pi_{AH_1AH_1}^{1c}\Pi_{BH_1AB_1}^{1c} \\ -\Pi_{AB_1AH_1}^{1c}\Pi_{BH_1BH_1}^{1c} & -\Pi_{AH_1AH_1}^{1c}\Pi_{AB_1BH_1}^{1c} & \Pi_{AH_1AH_1}^{1c}\Pi_{BH_1BH_1}^{1c} \end{bmatrix}.$$

By the second-order conditions and the network complementarities condition (4), every element of $\left(\Pi_{11}^{1c}\right)^{-1}$ is negative. Similarly, $\left(\Pi_{22}^{2c}\right)^{-1}$ is also negative matrix. Secondly, Π_{12}^{1c} and Π_{21}^{2c} are negative diagonal matrices because of condition (5). Thus, both R_2^{1c} and R_1^{2c} are negative matrices.

By using R_2^{1c} and R_1^{2c}, (12) and (13) can be rewritten as

$$\frac{dQ^1}{d\gamma} = -\left[I - R_2^{1c} R_1^{2c}\right]^{-1} R_2^{1c} \left(\Pi_{22}^{2c}\right)^{-1} \Pi_{2\gamma}^{2c} \tag{14}$$

$$\frac{dQ^2}{d\gamma} = -\left[I - R_1^{2c} R_2^{1c}\right]^{-1} \left(\Pi_{22}^{2c}\right)^{-1} \Pi_{2\gamma}^{2c} . \tag{15}$$

The stability of Cournot-Nash equilibrium implies that the magnitude of the eigenvalues of matrices $R_2^1 R_1^2$ and $R_1^2 R_2^1$, must be less than one (Zhang and Zhang, 1996). Hence, by the Neumann lemma,[5] $\left(I - R_2^{1c} R_1^{2c}\right)^{-1}$ and $\left(I - R_1^{2c} R_2^{1c}\right)^{-1}$ exists and

$$\left(I - R_i^{ic} R_i^{jc}\right)^{-1} = I + \left(R_i^{ic} R_i^{jc}\right) + \left(R_i^{ic} R_i^{jc}\right)^2 + \cdots + \left(R_i^{ic} R_i^{jc}\right)^n + \cdots \qquad for\ i = 1, 2,\ i \neq j.$$

Since $R_j^{ic} R_i^{jc}$ is a positive matrix, then $\left(I - R_j^{ic} R_i^{jc}\right)^{-1}$ is also a positive matrix. Therefore, $dQ^1/d\gamma > 0$ and $dQ^2/d\gamma < 0$ since R_2^{1c} is a negative matrix and $\Pi_{2\gamma}^{2c}$ is a negative vector. *Q.E.D.*

The intuitive explanations for Proposition C-1 are as follows: If the partners provide better quality of connecting services in market AB, inconvenience cost (γ) will decrease, which in turn increases connecting traffic for the partners. This connecting traffic increase implies that the partners can feed more traffic to each other. As a result, schedule delay cost for local non-stop services will decrease and average operating costs on the non-stop routes will decrease. Consequently, increases in Q_{AB}^{2+3} lead to decreases in the partners' air fares in the AH and BH markets, which in turn increases AH and BH traffic as well. On the other hand, increases in Q_{AB}^{2+3} decrease Q_{AB}^1, resulting in increased carrier 1's unit cost on the AH and BH routes and increased schedule delay cost for its local services.

Proposition C-2. Under the complementary alliance conditions, firm 1 earns less profit, but the alliance partners earn more profit, as compared to the pre-alliance conditions.
Proof. Substituting the Cournot-Nash equilibrium $\left(Q^1(\gamma), Q^2(\gamma)\right)$ into (6) and (7), and differentiating these with respect to γ, we have

$$\frac{\partial \Pi^{1c}}{\partial \gamma} = \sum_{k=AH}^{AB} \frac{\partial \Pi^{1c}}{\partial Q_k^1} \frac{dQ_k^1}{d\gamma} + \sum_{k=AH}^{AB} \frac{\partial \Pi^{1c}}{\partial Q_k^2} \frac{dQ_k^2}{d\gamma} = \sum_{k=AH}^{AB} \frac{\partial d_k^1}{\partial Q_k^2} \frac{dQ_k^2}{d\gamma} Q_k^1 \tag{16}$$

$$\frac{\partial \Pi^{2c}}{\partial \gamma} = \sum_{k=AH}^{AB} \frac{\partial \Pi^{2c}}{\partial Q_k^2} \frac{dQ_k^2}{d\gamma} + \sum_{k=AH}^{AB} \frac{\partial \Pi^{2c}}{\partial Q_k^1} \frac{dQ_k^1}{d\gamma} + \frac{d\Pi^{2c}}{d\gamma} = \sum_{k=AH}^{AB} \frac{\partial d_k^2}{\partial Q_k^1} \frac{dQ_k^1}{d\gamma} Q_k^2 - Q_{AB}^2 . \tag{17}$$

By the first-order conditions, the first term of the right-hand side of the first eqns of (16) and (17) disappears. By condition (1), $\partial \Pi^{1c}/\partial \gamma > 0$ and $\partial \Pi^{2c}/\partial \gamma < 0$. *Q.E.D.*

Effects on market outcome and economic welfare

To examine changes in *total output* due to the complementary alliance, we further assume that the aligned partners and non-aligned competitors are symmetric and the partners can provide connecting services at the same quality as the non-partner's (i.e., $\gamma = 0$).

Proposition C-3. For the symmetric case, the complementary alliance results in (i) increased total output and (ii) decreased "full" price in markets AH, BH, and AB. Therefore, consumers in these markets are better off due to the complementary alliance.
Proof. Let Q be total output vector and $\rho(Q)$ be corresponding "full" price vector. By definition of Q,

$$\frac{dQ}{d\gamma} = \frac{dQ^1}{d\gamma} + \frac{dQ^2}{d\gamma}. \tag{18}$$

Rearranging (10) and using $R_2^{1c} \equiv -\left(\Pi_{11}^{1c}\right)^{-1}\Pi_{12}^{1c}$, we can have

$$\frac{dQ^1}{d\gamma} = R_2^{1c}\frac{dQ^2}{d\gamma}. \tag{19}$$

Substituting (15) and (19) into (18) yields

$$\frac{dQ}{d\gamma} = -\left[I + R_2^{1c}\right]\left[I - R_1^{2c}R_2^{1c}\right]^{-1}\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{2\gamma}^{2c}. \tag{20}$$

By using the symmetric condition and $R_1^{2c} \equiv -\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{21}^{2c}$, (20) can be rewritten as

$$\frac{dQ}{d\gamma}\Big|_{\gamma=0} = -\left[I + \left(\Pi_{22}^{2c}\right)^{-1}\Pi_{21}^{2c}\right]^{-1}\left(\Pi_{22}^{2c}\right)^{-1}\Pi_{2\gamma}^{2c}. \tag{21}$$

Using the result $(AB)^{-1} = B^{-1}A^{-1}$, we can further simplify (21) as follows:

$$\frac{dQ}{d\gamma}\Big|_{\gamma=0} = -\left[\Pi_{22}^{2c} + \Pi_{21}^{2c}\right]^{-1}\Pi_{2\gamma}^{2c}. \tag{22}$$

Note $\Pi_{22}^{2c} + \Pi_{21}^{2c}$ is a negative definite matrix. Its inverse matrix, $\left(\Pi_{22}^{2c} + \Pi_{21}^{2c}\right)^{-1}$ can be expressed as

$$\frac{1}{\left|\Pi_{22}^{2c} + \Pi_{21}^{2}\right|}\begin{bmatrix} \left(\Pi_{B_2B_2}^{2c} + \Pi_{B_2B_1}^{2c}\right)\left(\Pi_{C_2C_2}^{2c} + \Pi_{C_2C_1}^{2c}\right) - \left(\Pi_{B_2C_2}^{2c}\right)^2 & \Pi_{A_2C_2}^{2c}\Pi_{C_2B_2}^{2c} & -\Pi_{A_2C_2}^{2c}\left(\Pi_{B_2B_2}^{2c} + \Pi_{B_2B_1}^{2c}\right) \\ \Pi_{C_2A_2}^{2c}\Pi_{B_2C_2}^{2c} & \left(\Pi_{A_2A_2}^{2c} + \Pi_{A_2A_1}^{2c}\right)\left(\Pi_{C_2C_2}^{2c} + \Pi_{C_2C_1}^{2c}\right) - \left(\Pi_{A_2C_2}^{2c}\right)^2 & -\Pi_{B_2C_2}^{2c}\left(\Pi_{A_2A_2}^{2c} + \Pi_{A_2A_1}^{2c}\right) \\ -\Pi_{C_2A_2}^{2c}\left(\Pi_{B_2B_2}^{2c} + \Pi_{B_2B_1}^{2}\right) & -\Pi_{C_2B_2}^{2c}\left(\Pi_{A_2A_2}^{2c} + \Pi_{A_2A_1}^{2c}\right) & \left(\Pi_{A_2A_2}^{2c} + \Pi_{A_2A_1}^{2c}\right)\left(\Pi_{B_2B_2}^{2c} + \Pi_{B_2B_1}^{2c}\right) \end{bmatrix}$$

where subscripts A, B and, C represent AH, BH, and AB, respectively. Since every element of $\left(\Pi_{22}^{2c} + \Pi_{21}^{2c}\right)^{-1}$ is strictly negative, the inverse matrix is a negative matrix. Combining it with $\Pi_{2\gamma}^{2c}$ vector, we have $dQ/d\gamma|_{\gamma=0} < 0$. Thus, $d\rho(Q)/d\gamma|_{\gamma=0} > 0$. Consequently, consumer surplus in each market increases due to the complementary alliance. *Q.E.D.*

In order to analyze changes in total welfare due to the complementary alliance, we assume a partial equilibrium framework in which consumer demand for air travel in each market is derived from a utility function which can be approximated by the form

$$\sum_{k=AH}^{AB} U_k(Q_k^1, Q_k^2) + Z$$

where Z is expenditure on a competitively supplied *numeraire* good, and $\partial U_k/\partial Q_k^i = \rho_k^i$. Recall that ρ_k^i is the full price of using carrier i's service in market k, i.e., $\rho_k^i = p_k^i + g_k^i(\cdot)$.

Then consumer surplus in each market can be written as

$$CS_k = U_k(Q_k^1, Q_k^2) - \rho_k^1 Q_k^1 - \rho_k^2 Q_k^2, \tag{23}$$

and total surplus can be written as

$$W = \sum_{k=AH}^{AB} CS_k + \left(\Pi^1 + \Pi^2\right). \tag{24}$$

Substitution of (2) and (3) into (24) can yield the following expression for W:

$$\begin{aligned} W = \sum_{k=AH}^{AB} U_k(Q_k^1, Q_k^2) &- \sum_{i=1}^{2}\left[g_{AH}^i(Q_{AH}^i + Q_{AB}^i)\cdot(Q_{AH}^i + Q_{AB}^i) + g_{BH}^i(Q_{BH}^i + Q_{AB}^i)\cdot(Q_{BH}^i + Q_{AB}^i)\right] \\ &- \sum_{i=1}^{2}\left[C_{AH}^i(Q_{AH}^i + Q_{AB}^i) + C_{BH}^i(Q_{BH}^i + Q_{AB}^i)\right] - \gamma Q_{AB}^2 \end{aligned} \tag{25}$$

where again, for simplicity, superscript 2+3 is replaced by 2.

Proposition C-4. For the symmetric case, total welfare rises due to the complementary alliance.
Proof. Differentiating (25) with respect to γ and using $\partial U_k/\partial Q_k^i = \rho_k^i = p_k^i + g_k^i$, we can show

$$\begin{aligned} \frac{dW}{d\gamma} &= \sum_{i=1}^{2}\sum_{k=AH}^{BH}\left[p_k^i - g_k^{i'}\cdot(Q_k^i + Q_{AB}^i) - C_k^{i'}\right]\frac{dQ_k^i}{d\gamma} \\ &+ \sum_{i=1}^{2}\left[p_{AB}^i - \sum_{k=AH}^{BH}\left(g_k^{i'}\cdot(Q_k^i + Q_{AB}^i) - C_k^{i'}\right)\right]\frac{dQ_{AB}^i}{d\gamma} - \left[Q_{AB}^2 + \gamma\frac{dQ_{AB}^2}{d\gamma}\right]. \end{aligned} \tag{26}$$

Notice the first and second bracketed terms of (26) are positive by the first-order conditions. Since $dQ_k^1/d\gamma > 0$ and $dQ_k^2/d\gamma < 0$ for each market k, the overall effect of the complementary alliance on total welfare is not clear. However, under the symmetric condition and $\gamma = 0$, (26) can be reduced to

$$\frac{dW}{d\gamma}\Big|_{\gamma=0} = \sum_{k=AH}^{BH}\left[p_k^1 - g_k^{1'}\cdot(Q_k^1+Q_{AB}^1) - C_k^{1'}\right]\left(\frac{dQ_k^1}{d\gamma}+\frac{dQ_k^2}{d\gamma}\right)$$

$$+\left[p_{AB}^1 - \sum_{k=AH}^{BH}\left(g_k^{1'}\cdot(Q_k^1+Q_{AB}^1) - C_k^{1'}\right)\right]\cdot\left(\frac{dQ_{AB}^1}{d\gamma}+\frac{dQ_{AB}^2}{d\gamma}\right) - Q_{AB}^2.$$

By the first-order conditions and Proposition C-3, $dW/d\gamma|_{\gamma=0}<0$. *Q.E.D.*

EFFECTS OF PARALLEL ALLIANCE

Effects of no-shut-down parallel alliance

We first analyze the effect of the "no-shut-down" parallel alliance where two partners continue to individually provide local services after their alliance. For the convenience of analysis, we define θ as: $\theta=1$ for post-parallel alliance; $\theta=0$ for pre-alliance. We then treat θ as continuous in the range $0\le\theta\le 1$, and assume that carrier i's output in market k, $Q_k^i(\theta)$, is continuous and differentiable in θ in the entire range. By these assumptions, the overall effect of switching from the pre-alliance to the "no-shut-down" parallel alliance can be calculated as the integral of the infinitesimal effect as follows:

$$\Delta Q_k^i(\theta) \equiv Q_k^i(1) - Q_k^i(0) = \int_0^1\left[dQ_k^i(\theta)/d\theta\right]d\theta.$$

It turns out to be easy to sign the infinitesimal effect, $dQ_k^i(\theta)/d\theta$. Consequently, the overall effect, $\Delta Q_k^i(\theta)$, can be determined as well if the sign of the infinitesimal effect remains unchanged in the range, which can be verified.

Each firm's post-alliance profit function can be expressed as

$$\max_{Q^1}\ \Pi^{1p}(Q^1,Q^2,Q^3;\theta) \equiv \Pi^1 + \theta\cdot\Pi^2$$

$$\max_{Q^2}\ \Pi^{2p}(Q^1,Q^2,Q^3;\theta) \equiv \Pi^2 + \theta\cdot\Pi^1$$

$$\max_{Q^3}\ \Pi^{3p}(Q^1,Q^3) \equiv \Pi^3$$

where superscript p stands for parallel alliance; $Q^1\equiv\left(Q_{AH}^1, Q_{BH}^1, Q_{AB}^1\right)$, $Q^2\equiv Q_{AH}^2$, $Q^3\equiv Q_{BH}^3$; and

$\Pi^1\equiv Q_{AH}^1\left[d_{AH}^1(\cdot)-g_{AH}^1(\cdot)\right]+Q_{BH}^1\left[d_{BH}^1(\cdot)-g_{BH}^1(\cdot)\right]+Q_{AB}^1\left[d_{AB}^1(\cdot)-g_{AH}^1(\cdot)-g_{BH}^1(\cdot)\right]-C_{AH}^1(\cdot)-C_{BH}^1(\cdot)$,

$\Pi^2 \equiv Q_{AH}^2\left[d_{AH}^2(\cdot)-g_{AH}^2(\cdot)\right]-C_{AH}^2(\cdot)$, $\Pi^3 \equiv Q_{BH}^3\left[d_{BH}^3(\cdot)-g_{BH}^3(\cdot)\right]-C_{BH}^3(\cdot)$.

Proposition P-1. If the non-hub partner (i.e., firm 2) produces the same amount of output after the "no shut-down" parallel alliance, then the hub partner (i.e., firm 1) produces less output in all three markets, and the non-partner (i.e., firm 3) produces more output in market BH than under the pre-alliance.

Proof. Since the non-hub partner does not change its output in the parallel alliance, the first-order conditions for firms 1 and 3 may be respectively written as

$$\Pi_1^{1p} = 0\,,\ \Pi_3^{3p} = 0\,.$$

Assuming that there exists a "stable" equilibrium, $\left(Q^1(\theta), Q^3(\theta)\right)$, which satisfies the first-order conditions for firms 1 and 3, that is,

$$\Pi_1^{1p}\left(Q^1(\theta), Q^3(\theta); \theta\right) \equiv 0 \tag{27}$$

$$\Pi_3^{3p}\left(Q^1(\theta), Q^3(\theta)\right) \equiv 0\,. \tag{28}$$

Differentiating (27) and (28) with respect to θ yields

$$\Pi_{11}^{1p}\frac{dQ^1}{d\theta} + \Pi_{13}^{1p}\frac{dQ^3}{d\theta} + \Pi_{1\theta}^{1p} = 0\,, \tag{29}$$

$$\Pi_{31}^{3p}\frac{dQ^1}{d\theta} + \Pi_{33}^{3p}\frac{dQ^3}{d\theta} = 0 \tag{30}$$

where $\Pi_{1\theta}^{1p} \equiv \left[Q_{AH}^2 \cdot \left(\partial d_{AH}^2/\partial Q_{AH}^1\right), 0\,, 0\right]^T$, the first element of which is negative by condition (1).

Since both $\left(\Pi_{33}^{3p}\right)^{-1}$ and Π_{31}^{3p} are negative matrices, $dQ^1/d\theta$ and $dQ^3/d\theta$ have opposite signs (see eqn (30)). Now, we show $dQ^1/d\theta<0$. Solving (29) and (30) for $dQ^1/d\theta$, we have

$$\frac{dQ^1}{d\theta} = -\left[I - R_3^{1p}R_1^{3p}\right]^{-1}\left(\Pi_{11}^{1p}\right)^{-1}\Pi_{1\theta}^{1p} \tag{31}$$

where $R_3^{1p} \equiv -\left(\Pi_{11}^{1p}\right)^{-1}\Pi_{13}^{1p}$ and $R_1^{3p} \equiv -\left(\Pi_{33}^{3p}\right)^{-1}\Pi_{31}^{3p}$ are derivative matrices of firm 1's (firm 3's, respectively) reaction function for firm 3's (firm 1's, respectively) output. Imposing the stability condition on the equilibrium yields that $\left[I - R_3^{1p}R_1^{3p}\right]^{-1}$ is a positive matrix. As shown in Proposition C-1, every element of $\left(\Pi_{11}^{1p}\right)^{-1}$ is negative because of the second-order conditions and the network complementarities condition (4). Therefore, $dQ^1/d\theta<0$ and $dQ^3/d\theta>0$.

Next, we show that the signs of $dQ^1/d\theta<0$ and $dQ^3/d\theta>0$ remain unchanged in the entire range of interest. In (31), the third term, $\Pi_{1\theta}^{1p}$, remains as negative in the range since the first element of $\Pi_{1\theta}^{1p}$ is always negative regardless of any value of θ in the range. By similar arguments, the signs of the first and second terms remain unchanged in the region. *Q.E.D.*

Similarly, we can show

Proposition P-2. If the hub partner produces the same amount of output after the parallel alliance, then the non-hub partner decreases its output, and the non-partner produces the same amount of output, as compared to the pre-alliance situation.

The next question naturally arises: what if both Q^1 and Q^2 are chosen endogenously? If the two partners endogenously decides their outputs, they cannot simultaneously increase output in market AH after the parallel alliance.

Proposition P-3. $dQ^1/d\theta$ and $dQ^2/d\theta$ cannot both be positive.

Proof. Denoting a "stable" equilibrium by $\left(Q^1(\theta), Q^2(\theta), Q^3(\theta)\right)$, and differentiating the first-order conditions with respect to θ, we have

$$\Pi_{11}^{1p}\frac{dQ^1}{d\theta} + \Pi_{12}^{1p}\frac{dQ^2}{d\theta} + \Pi_{13}^{1p}\frac{dQ^3}{d\theta} + \Pi_{1\theta}^{1p} = 0, \tag{32}$$

$$\Pi_{31}^{3p}\frac{dQ^1}{d\theta} + \qquad \Pi_{33}^{3p}\frac{dQ^3}{d\theta} \quad = 0 \tag{33}$$

$$\Pi_{21}^{2p}\frac{dQ^1}{d\theta} + \Pi_{22}^{2p}\frac{dQ^2}{d\theta} + \qquad \Pi_{2\theta}^{2p} = 0, \tag{34}$$

where $\Pi_{2\theta}^{2p} \equiv Q_{AH}^1 \cdot \left(\partial d_{AH}^1 / \partial Q_{AH}^2\right) < 0$.

Again, from (34), it can be easily verified that $dQ^1/d\theta$ and $dQ^3/d\theta$ have opposite signs. Eqns (32) and (33) show that $dQ^1/d\theta$ and $dQ^2/d\theta$ are interdependent with each other. Solving (32)-(34) for $dQ^1/d\theta$ and $dQ^2/d\theta$ yields

$$\frac{dQ^1}{d\theta} = -\left[I - R_3^{1p}R_1^{3p}\right]^{-1}\left(\Pi_{11}^{1p}\right)^{-1}\left(\Pi_{1\theta}^{1p} + \Pi_{12}^{1p}\frac{dQ^2}{d\theta}\right), \tag{35}$$

or

$$\frac{dQ^2}{d\theta} = -\left(\Pi_{22}^{1p}\right)^{-1}\left(\Pi_{2\theta}^{2p} + \Pi_{21}^{2p}\frac{dQ^1}{d\theta}\right). \tag{36}$$

Since $\Pi_{12}^{1p} < 0$ and $\Pi_{21}^{2p} < 0$, both $dQ^1/d\theta$ and $dQ^2/d\theta$ cannot be positive in (35) and (36). *Q.E.D.*

Although both $dQ^1/d\theta$ and $dQ^2/d\theta$ cannot simultaneously be positive in (35)-(36), it is possible that both $dQ^1/d\theta$ and $dQ^2/d\theta$ are negative in (35)-(36). This can be illustrated by the following numerical example. Assume that demand is linear as follows:

$$d_k(Q_k^i, Q_k^j) = \alpha - (Q_k^i + Q_k^j), \qquad for\ \ k = AH, BH, AB. \tag{37}$$

Assume that schedule delay cost, $g_k(\cdot)$, is also linear and that operating cost, $C_k(\cdot)$, is concave:

$$g_k^i(Q_k^i) = 1 - \delta Q_k^i, \quad C_k^i(Q_k^i) = Q_k^i - \frac{\mu}{2}\left(Q_k^i\right)^2, \quad for\ k = AH, BH, AB \tag{38}$$

where μ represents extent of increasing returns to density. Given these specifications, the explicit expressions of equilibrium output can be obtained for each firm under the pre-alliance and the post-alliance situations. In particular, when $\alpha = 4$, $\delta = .03$, and $\mu = .04$, $\Delta Q^1 \equiv \left(\Delta Q_{AH}^1, \Delta Q_{BH}^1, \Delta Q_{AB}^1\right)$ = (-.2142, -.0009, -.0119), $\Delta Q^2 \equiv \Delta Q_{AH}^2$ = -.1404, and $\Delta Q^3 \equiv \Delta Q_{BH}^3$ = .0009.

Effects of shut-down parallel alliance

We now analyze the effects of the second type of parallel alliance. For tractability of analysis, we impose more structures on the model. First, demands and schedule delay costs for all three markets are assumed to be symmetric. Secondly, in order to use a common cost function, we assume that the distances between cities A and H, and between B and H are the same. Thirdly, we use special functions (37)-(38) for demand, schedule delay cost, and operating cost.[6]

Proposition P-4. Under the "shut-down" parallel alliance conditions, the partners produce less output in market AH, but produce more output in markets BH and AB, and firm 3 produces less output in its local market BH than under the pre-alliance conditions.

See Park (1997) for the proofs of the "shut-down" parallel alliance. The intuitive reasons for Proposition P-4 are as follows: First of all, since the AH market is now serviced only by the name of the hub partner, this market becomes a monopoly market. The hub-partner produces more than its pre-alliance output in this market, but less than total pre-alliance output, i.e., $Q_{AH}^{1b} < Q_{AH}^{1p}\left(\equiv Q_{AH}^{(1+2)p}\right) < Q_{AH}^{1b} + Q_{AH}^{2b}$. Secondly, the hub partner increases its BH and AB traffic due to the network complementarities. Thirdly, the non-partner will decrease its BH traffic since its reaction function to the hub partner's output in market BH is downward sloping.

Proposition P-5. Under the "shut-down" parallel alliance conditions, the hub partner earns more profit than under the pre-alliance conditions. Given the economies of density, the non-hub partner earns more (less, respectively) profit when the size of markets is sufficiently large (small, respectively) than under the pre-alliance situations. Firm 3 earns less profit, as compared to the pre-alliance conditions.

Proposition P-6. The "shut-down" parallel alliance results in (i) increased (decreased, respectively) total output and (ii) decreased (increased, respectively) "full" price in markets BH and AB (market AH, respectively). Therefore, consumers in these markets (this market, respectively) are better off (worse off, respectively) due to the parallel alliance.

It can be verified that decreases in consumer surplus in market AH dominate the increases in market BH and AB.

CONCLUDING REMARKS

This study analyzes the effects on market outcome and welfare of two types of alliances: complementary vs. parallel alliances. To recapitulate major findings of this study, First, the complementary alliance in a specific market has indirect positive effects on the partners' outputs in the other markets. Coordination in connecting markets allows the partners to increase service quality and decrease average operating costs in local markets.

Second, the two types of alliances have different effects on total output and consumer surplus. Given the symmetry, the complementary alliance increases total output, and decreases "full" price. Thus, consumer surplus increases as a result of the complementary alliance. On the other hand, both the "no-shut-down" and "shut-down" parallel alliances are likely to decrease total output on the alliance route. Consequently, consumer surplus is likely to decrease due to the parallel alliance.

Finally, we find sufficient conditions under which complementary alliance improves total welfare. Total welfare can rise if the partners and non-partners are symmetric and if the partners can coordinate to the extent that they are able to provide the same level of connecting services as firm 1's.

Government agents should be very careful to allow would-be parallel alliance partners to have antitrust immunity. Since the partners are significant competitors in the same markets, competition may be reduced if they are able to integrate operation with the protection of antitrust immunity. As a result, the parallel alliance reduces consumer surplus and is more likely to decrease total welfare.

ENDNOTES

[1] A codesharing agreement is a marketing arrangement between two airlines whereby one airline's designator code is shown on flights operated by its partner airline. For example, Lufthansa has been codesharing on United Airlines' flight between Frankfurt and 25 U.S. interior cities via two of United's hubs (Chicago O'Hare and Washington Dulles).

[2] If two carriers make a block space sale agreement, each carrier can buy a block of seats in the other carrier's flights and resell them to passengers. For example, Air Canada and Korean Air have signed on such an agreement on the Seoul-Vancouver-Toronto route, under which each buys 48 seats from the other's flights on the route.

[3] Caves, Christensen and Tretheway (1984) distinguish between economies of traffic density and economies of firm size. Economies of traffic density mean that output is expanded by increasing flight frequency within a given network. Economies of firm size imply that output is expanded by adding points to the network. Many studies reach a common conclusion: roughly constant returns to firm size exist, while sizeable economies of traffic density exist up to fairly large volumes of traffic.

[4] The Cournot assumption is not crucial in the duopoly market. Brander and Zhang (1990) and Oum, Zhang and Zhang (1993), using conjectural variations, find some evidence that airlines in duopoly markets behave like Cournot competitors.

[5] Neumann lemma is that if R is a real square matrix and the magnitude of eigenvalues of R is less than one, then $(I-R)^{-1}$ exists and $(I-R)^{-1} = \sum_{i=0}^{\infty} R^i$. See, for example, Ortega and Rheinboldt (1970, p.45).

[6] The linear demand and concave operating cost functions are also used in Brueckner and Spiller (1991), Brueckner, Dyer and Spiller (1992), and Nero (1996).

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