Differentiation of Taxi Spot Markets and Social Welfare

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Abstract

There exist thick market externalities in taxi spot markets that larger market will make service transactions more efficient. When heterogeneous customers visit a market, a mismatching externality also arises that taxicabs could be mismatched with the types of customers not to be preferred. If spot markets are differentiated based upon efficient matching pairs of taxicabs and customers, information asymmetry between customers and taxis could be partly resolved, eventually increasing matching efficiency between taxicabs and customers. In this paper, equilibrium models of spot markets are presented incorporating both thick market economies and mismatching diseconomies. The impacts of fare regulation of taxicabs and market differentiation policies upon social welfare are also investigated by simple numerical examples.

1. Introduction

In terminal facilities, such as large airports where a lot of unspecified passengers use taxi services, taxi stands (hereafter, call spot markets) only for a specific class of passengers, e.g., those customers that make long distance trips, are often settled. If heterogeneous passengers use only one spot market, there possibly exists a mismatching in service transactions between them, the type which taxis expect passengers and one which passengers expect taxis are not necessarily the same. In such a case, differentiation in taxi spot markets depending upon both types of passengers and taxis, such as difference in fare system and destinations of passengers, may make transaction between them more efficient.

Both passengers and taxis have to pay transaction costs to move to markets and wait for partners, in order to transact services in taxi spot markets. Therefore, a thick market externality works where the transaction of services becomes more efficient with decreasing in waiting time for both agents when both arrival rates increase (Matsushima *et al.*, 2006a). Service transaction in taxi spot markets becomes more efficient as scale of market becomes large without congestion in the markets. This suggests that differentiation in spot markets is not effective if we focus upon scale economy related with thick market externality.

On the other hand, information asymmetricity exists in the transactions between passengers and taxis in such markets. The taxis cannot recognize the service which passengers need until when they meet their customers. If market transaction is made on the first-come-first-serve basis, passengers cannot know the service type of taxi they are about to ride, until they actually ride it. Differentiation in taxi spot markets can make both passengers and taxis select a desirable spot market, to make each partner partially know private information, such as type of taxis and information about passengers. Such a self selection mechanism makes matching between taxis and passengers efficient.

Taxi markets are regulated by the government in many countries. In Japan for example, the entry to the taxi markets was regulated thorough a licensing system by the road transportation law in Japan. The principal of a homogenous fare was applied by the authorization system of taxis' fare. However, deregulations of the fare system and the entry to the market were introduced in the 1970's in various cities in US starting with Atlanta. The entry and fare systems were widely deregulated in Sweden in 1991. In Japan, the coordination of demand and supply and the fare system are being deregulated gradually.

Although not a few studies about taxi market come to existance, there are some former researches to analyze the structure of taxi market and analyze the inpact of the regulation policies to the market (Orr, 1969, Douglas 1972, DeVany 1975, Schreiber 1977, Willams, 1981). Both theoretical and empirical studies about the outcome of deregulation in taxi markets are published (Teal *et al.*, 1995, Laitila *et al.*, 1995). These studies mainly focus on demand and supply in a whole city area reporting skeptical results about the effect of deregulation, depending upon pioneering case studies in US and Sweden (Teal *et al.*, 1995, Laitila *et al.*, 1995). Re-regulation of taxi markets is now investigated in order to solve the harmful effects caused by deregulation. For example, the fixed fare system is introduced again for the taxi service between the airport and city center of Stockholm.

Taxi services are transacted at the local spot markets in a city. Only a few studies are published about the service transaction in spot markets and the necessity of public regulation. Even though there exist some reports about parking behavior, those reports fail to analyze market equilibrium. Matsushima *et al.* (2006a) proposed a market equilibrium model with a thick market externality in spot markets to analyze the congestion effects and spatial equilibrium of spot markets (Matsushima *et al.*, 2006b). This paper expands their model to formulate a market equilibrium model with an externality caused by the mismatching between heterogeneous taxis and passengers. This paper also tries to analyze the effects of market differentiation polices and fare regulation upon market efficiency in services in spot markets.

Both taxis and passengers have to visit a spot market in order to transact a service. However, both agents cannot perfectly access the market condition until they arrive at the market. Both taxis and passengers decide whether they visit the market or not based on imperfect guess about the market condition. All agents have to pay transaction costs in order to visit the market. If a queue at the market already exists, they have to queue and wait to be matched with a partner. Both passengers and taxis have to bear transaction costs in order to realize the transaction. Because of 1) imperfect guess and 2) transaction, there is a pecuniary externality in a transaction at spot markets (Howitt, 1987, Howitt, 1990, Kobayashi *et al.*, 1998, Matsushima *et al.*, 2006). Thick market externality, where the expectation of increase in demand and supply for both agents is realized if both expect an increase respectively, works at the market. On the other hand, the expectation of a decrease in demand and supply is also self-fulfilling (Farmer, 1993). As strategic externality (Bulow *et al.*, 1985, Cooper *et al.*, 1988) works in matching market with imperfect information and transactions, a multiplier effect thorough a positive feedback mechanism works at spot markets. Especially, the service transaction is more efficient as the market scale becomes larger due to such an externality, as long as both agents are homogenous.

Taxis and passengers with various needs, transact services at spot markets. Taxis do not recognize which type of passenger is to be matched with until the service transaction is realized. The type of passengers is private information for the taxis, which is not known to them in advance. If the service is transacted on a first-come-first-served basis, passengers cannot choose type of taxi (e.g. fare system) to be matched with. As the information asymmetricity works mutually between taxis and passengers, market inefficiency where both taxis and passengers are not matched with desirable partners, may occur at spot markets (hereafter, called ‘mismatching externality’.) If the effect of this externality is large, adverse selection may occur where specific types of passengers and taxis leave the market. This kind of externality becomes large if agents are more heterogeneous.

Mismatching externality occurs because of mutual information asymmetricity between taxis and passengers. Differentiation of markets is one of the possible policies to conquer such information asymmetricity, where several stands are set in one market depend upon the type of passengers. Several taxi stands, such as that only for passengers with short distance trip or that for passengers with long distance trip, are set to allocate specific taxis and passengers. That makes passengers and taxis recognize the type of partners in advance and to make the matching between taxis and passengers more efficient. Though this scheme makes service transaction efficient, it is necessary to investigate carefully how it affects the efficiency in the whole market. Firstly, thin market externality may occur as this scheme physically divides the market based upon the type of agents. Secondly, it is not clear whether the differentiation policy is compatible with the passengers' and the taxis' incentives. Passengers have to get higher utility when they visit the specified stand in order for this scheme to work effectively. Taxis also have to have an incentive to use the stand. That is, the differentiation policy should be incentive-compatible with the behaviors of both agents. If it is incentive-incompatible, the behaviors of both agents has to be controlled by fare regulation policy. Fare regulation policy also affects the welfare level of both passengers and taxis. If both agents are heterogeneous, it is important to decide effective policies to regulate the fares or differentiate the markets by considering both the thick market and the mismatching externalities.

This paper proposes a spot market equilibrium model with both externalities to investigate a desirable fare regulation and differentiation policy. In the next section, a market equilibrium model with heterogeneous passengers and taxis is formulated. In section 3, market equilibrium with differentiation policy is derived. Section 4 analyzes the structure of market equilibrium. The effect of introducing fare regulation and market differentiation on social welfare is analyzed in section 5.

2. Double-queuing Model

(1) Assumptions

The analysis of this paper is based upon taxi spot markets which are located at large terminals, e.g., airports. Only one taxi stand is set at the spot market to form a queue either by taxis or passengers, respectively. Though spot markets with several stands are treated later on, the analysis in this section is considering a taxi stand market. Passengers who use the terminal can choose several modes to and from the terminal. They have information about average waiting time at the market based upon past experiences. It is not until they arrive at the market that they recognize the situation of spot markets. It is assumed that passengers will not leave from the market once they arrive at the market. Taxis also know the average waiting time from past experiences. taxis can decide whether to join the queue or not, after observing the condition of the market. A taxi leaves the market if it finds a very long queue. As taxis enter the market, as long as they can get positive profit, equilibrium with 0 profit will occur in the long run. The maximum length of taxis' queue is endogenously determined as the result of long term equilibrium. When either a taxi or a passenger arrives, service transaction is instantaneously completed with a positive queue length. A taxi has to wait for a customer when no agent is in the market. By following Matsushima *et al.*, service transaction at the market between taxis and passengers is formulated using the double-queuing model (Kendall, 1951, Sansieni *et al.*, 1961).

(2) Specification of a Double-queuing Model

Let us assume that a state variable m shows the queuing setting of either the taxis or the passengers in a spot market. When $m > 0$, m taxis make a queue, while when $m < 0$ m passengers make a queue. As service between passengers and taxis are transacted instantaneously, it is not possible that both passengers and taxis make queues. This paper takes no consideration of the congestion in spot markets in order to narrow our focus of the effect of market differentiation upon mismatching externality. The passengers do not leave the market once they arrive. The maximum length of passengers' queue is assumed as $-\infty \leq m \leq M$. Let us indicate $P^t(m)$ as the probability where a system is in state m at time t. The state equations can then be written as follows.

$$P^{t+\Delta t}(M) = (1 - \lambda\Delta t)P^t(M) + (1 - \lambda\Delta t)\mu\Delta t P^t(M - 1) + o(\Delta t)! \tag{1a}$$

$$P^{t+\Delta t}(m) = (1 - \lambda\Delta t)(1 - \mu\Delta t)P^t(m) + (1 - \lambda\Delta t)\mu\Delta t P^t(m - 1) + \lambda\Delta t(1 - \mu\Delta t)P^t(m + 1) + o(\Delta t)! \quad (m = -\infty, \cdots, M - 1), \tag{1b}$$

where $o(\Delta t)!$ is the higher order term and $o(\Delta t)!/\Delta t \to 0$ is $\Delta t \to 0$. Considering $\Delta t \to 0$ limit, in the long-run steady states, we can see that

$$-\lambda P(M) + \mu P(M - 1) = 0 \tag{2a}$$

$$-(\lambda + \mu)P(m) + \mu P(m - 1) + \lambda P(m + 1) = 0 \ (m = -\infty, \cdots, M - 1). \tag{2b}$$

$P(m)$ $(m = -\infty, \cdots, M)$ is the stationary probability when the queue of either taxis or passengers is m. When the stability of the steady state exists, it should hold that $\mu > \lambda$. As $\sum_{m=-\infty}^{M} P(m) = 1$ holds, $P(m)$ can be written as follows.

$$P(m) = (1 - \rho)\rho^{M-m} \quad (-\infty \leq m \leq M), \tag{3}$$

where $\rho = \lambda/\mu$. Given the average arrival rates of the customers and the taxis (λ, μ), the average lengths of the suppliers' queue and the consumers' queue are given by

$$E(m > 0 : \lambda, \mu) = M - \frac{\rho}{1 - \rho}(1 - \rho^M) \tag{4a}$$

$$E(m < 0 : \lambda, \mu) = \frac{\rho^{M+1}}{1 - \rho}, \tag{4b}$$

respectively (Matsushima *et al.*, 2006). The average waiting time of the consumers and that of the suppliers, denoted by $T(\lambda, \mu, M)$ and $S(\lambda, \mu, M)$ respectively, with the arrival rates (λ, μ), are given by

$$S(\lambda, \mu) = E(m > 0 : \lambda, \mu)/\mu \tag{5a}$$

$$T(\lambda, \mu) = E(m < 0 : \lambda, \mu)/\lambda. \tag{5b}$$

The probability ξ that a newly arrived supplier leaves the market without joining the suppliers' queue is defined by $P(M) = 1 - \rho$.

(3) Maximum Length of the Taxis' Queue

Suppose that there is no physical limit on the length of the suppliers' queues. Each supplier who arrives at the spot market observes the current length of the suppliers' queue and decides to join the queue or to leave. The average waiting time of the mth suppliers in the suppliers' queues, denoted by $W(m)$, is given by

$$W(m) = \frac{m}{\lambda}. \tag{6}$$

The expected profit of taxis that join the queue is

$$EU(m, p) = p - \frac{m}{\lambda}, \tag{7}$$

where p is the fare for the taxi services. Suppliers will join the queue as long as they can expect positive average profits from the spot market. The transaction cost for visiting the market has already been sunk by the time the supplier arrives at the spot market. In a competitive market, the maximum length of the suppliers' queues is determined in such a way that the maximum number of the suppliers waiting in the queues is a number that can guarantee nonnegative expected profits. From the non-negativity condition of the profits, the maximum length of the suppliers' queues, $M(\lambda, p)$, is defined by

$$M(\lambda, p) = [p\lambda], \tag{8}$$

where the notation $[\cdot]$ means the maximum natural number that does not exceed $p\lambda$.

3. Heterogeneity in Both Taxis and Passengers and Market Equilibrium

(1) Assumptions

The average arrival rate μ, λ may change in the long run as taxis and/or passengers enter or leave the market. taxis try to enter the market as long as the expected profit from entering the market is positive. Passengers may enter the market as long as the expected utility from consuming the service of the taxies is larger than the reserved utility from consuming other services. The average arrival rate of passengers and taxis in spot markets may simultaneously converge to an equilibrium level in a long period from entering and leaving the market. In this subsection, a model of passengers' and taxis' behavior to enter or leave a market with one spot is formulated to define the market equilibrium in the long run.

Heterogeneous passengers and taxis make transaction in the spot market. As matching in the market is transacted on a first-come-first-served basis, neither passengers nor taxis can recognize the partners' type in advance. In this setting, pooling equilibrium occurs under imperfect information where heterogeneous agents do not know the partners' type. To make our discussion simple, passengers who visit the spot markets can be classified into passengers who make a long distance trip (type 1) and those who make a short distance trip (type 2). It is assumed also that two types of taxis arrive at the market. Either type of taxis sets a different fare system, or transaction cost to access the market and is also different for each type. Let us indicate a fare by type k $(k = 1, 2)$ taxis for type i $(i = 1, 2)$ passengers p_{ki}. The Fare is set in a temporal term, and given exogenously at an instant. This assumption for the fare system reflects that type 1 taxis are based upon a place far away from the terminal, while type 2 taxis have a base close to the market, for example. Both types of taxis have to go back to their base after the transaction is completed. Assume also that the taxis visit the market without passengers.

Let us indicate the one way transaction cost of type k $(k = 1, 2)$ taxis who visit the spot market from their base $d_k/2$, in a temporal term. When a type 1 taxi is matched with a type 1 passenger, it pays a transaction cost d_1 to arrive at the objective place, and comes back to its base without payment. The taxi also has to pay a transaction cost in the case where the transaction is not realized. However, it has to bear both the transaction cost d_2 to move between the spot markets and its partner's objective place, and d_1 to move between the market and its base. The same process can be applied to type 2 taxis. Therefore, the transaction cost for type k $(k = 1, 2)$ taxis that are matched with type i $(i = 1, 2)$ passengers can be written as follows.

$$d_{ki} = \begin{cases} d_k & \text{when matched with type } \; i = k \; \text{ passengers} \\ d_k + d_i & \text{when matched with type } \; i \neq k \; \text{ passenger} \\ d_k & \text{with no transaction} \end{cases} \tag{9}$$

The following equation holds for either type of taxis and passengers.

$$d_{11} < d_{12}, \quad d_{21} > d_{22}. \tag{10}$$

This shows that the total transaction cost is minimized when type 1 taxis are matched with type 1 passengers, while type 2 taxis are matched with type 2 passengers. This type of matching sets is defined as efficient matching.

(2) The Behavior of the Taxies

Assume that the type k $(k = 1, 2)$ taxis' arrival rate is μ_k and that the type i $(i = 1, 2)$ passengers' arrival rate is λ_i. Both arrival rates are independent. Assume also that enough taxis are available in the market. As 2 types of passengers and taxis make a queue at the same stand, the service transaction in the market can be expressed by a double queuing model with arrival rates $\lambda = \lambda_1 + \lambda_2, and \mu = \mu_1 + \mu_2$, respectively. Either type of taxis can enter the market with the probability $\rho = \lambda/\mu$. They are matched with type 1 passengers with a probability λ_1/λ, and type 2 passengers with the probability λ_2/λ. A taxi may leave the market with the probability $1 - \rho$.

The expected transaction cost which type k $(k = 1, 2)$ haa to bare is

$$\overline{d_k} = d_k + \rho \frac{\lambda_i}{\lambda} d_i \ (k, i = 1, 2, \ i \neq k). \tag{11}$$

From Eq.(11), the relations between $\overline{d_1}$ and $\overline{d_2}$ are expressed as

$$\begin{aligned} &\overline{d_1} \geq \overline{d_2} \quad \text{when} \left(\tfrac{\mu - \lambda_2}{\mu - \lambda_1} \leq \tfrac{d_1}{d_2} \right) \\ &\overline{d_1} < \overline{d_2} \quad \text{when} \left(\tfrac{\mu - \lambda_2}{\mu - \lambda_1} > \tfrac{d_1}{d_2} \right). \end{aligned} \tag{12}$$

The expected transaction cost for either type of taxis depends upon the ratio of the passengers' type in the market. Taxis cannot recognize the partner's type until they are matched, as there is only one stand in the market. The expected fare revenue of type k taxis $\overline{p_k}$ is

$$\overline{p_k} = \frac{\sum_{i=1}^{2} \lambda_i p_{ki}}{\lambda}. \tag{13}$$

As it is shown later, both types of taxis cannot always enter the market in a market equilibrium. Here assume that only type k taxis can enter the market. The maximum queue length of type k taxis can be shown as follows from Ea.(8).

$$M(\lambda_1, \lambda_2, p_{k1}, p_{k2}) = [\overline{p_k} \lambda]. \tag{14}$$

taxis may enter the market when the queuing length is shorter than the maximum length of queuing, otherwise they leave the market. Type k taxis that visit the market can get an expected profit $\overline{p_k} - S'(\lambda, \mu)$ with a probability $\rho = \lambda/\mu$, while acquire 0 profit with a probability $1 - \rho$. $S'(\lambda, \mu) = S(\lambda, \mu)/\rho$ shows the taxis' average waiting time, and $S(\lambda, \mu)$ is expressed in Eq.(5a). Considering that type k taxis bear a transaction cost $\overline{d_k}$ in order to visit the market, the expected net profit of type k taxis that visit the market is

$$EU_k(\lambda_1, \lambda_2, \mu_k, p_{k1}, p_{k2}) = \rho \overline{p_k} - S(\lambda, \mu) - \overline{d_k}. \tag{15}$$

Let us fix the passengers' arrival rate λ_1, λ_2 and the taxis' fare p_{ki} $(i = 1, 2)$. From the result of free entries to the market, the conditional equilibrium arrival rate μ_k^* with passengers' arrival rate λ_1, λ_2 given exogenously can be derived. Suppose a conditional market equilibrium, where λ_1, λ_2

are exogenously given, in which only either type 1 or type 2 taxis enter the market. Suppose also that $EU_1(\lambda_1, \lambda_2, \mu_1^*, p_{11}, p_{12}) > EU_2(\lambda_1, \lambda_2, \mu_2^*, p_{21}, p_{22})$ is satisfied in such a conditional market equilibrium. taxis may enter the market in the long run until the expected net profit becomes 0 when they visit the market. $EU_1(\lambda_1, \lambda_2, \mu_1^*, p_{11}, p_{12}) = 0$ is satisfied in the conditional market equilibrium. As a result, type 2 taxis cannot enter the market as $EU_2(\lambda_1, \lambda_2, \mu_2^*, p_{21}, p_{22}) < 0$ holds. On the other hand, type 1 taxis cannot enter the market when $EU_1(\lambda_1, \lambda_2, \mu_1^*, p_{11}, p_{12}) < EU_2(\lambda_1, \lambda_2, \mu_2^*, p_{21}, p_{22})$ is satisfied. Therefore, taxis with a higher expected net profit occupy the market in the long term. The equilibrium arrival rate of taxis that occupy the market can be expressed as μ^* which satisfies the following condition.

$$\max_k \left\{ \frac{\lambda}{\mu^*}\overline{p_k} - S(\lambda, \mu^*) - \overline{d_k} \right\} = 0 \tag{16}$$

Let us indicate a type of taxis that gives the maximum value k^* for the equation above. Both $\mu_{k^*}^* = \mu^*$ and $\mu_l^* = 0$ are satisfied when the following condition holds for l with $k^* \neq l$.

$$\frac{\lambda}{\mu^*}\overline{p_{k^*}} - \overline{d_{k^*}} > \frac{\lambda}{\mu^*}\overline{p_l} - \overline{d_l} \tag{17}$$

When the above equation becomes an equlity, the equilibrium arrival rate of taxis μ^* can be calculated, even though the ratio of arrival rate is unsettled.

(3) The Behavior of the Passengers

Let us assume that type k taxis arrive at the market with an arrival rate μ_k. Passengers cannot select the partner's type as taxis are allocated on a first-come-first-served basis. The expected fare which type i passengers have to bear is

$$\overline{P_i} = \frac{\sum_{k=1}^{2} \mu_k p_{ki}}{\mu}. \tag{18}$$

The subjective expected utility $EV_i(\lambda_1, \lambda_2, \mu_1, \mu_2, \overline{P_i})$ of type i passengers visiting the market with an arrival rate λ_i is

$$EV_i(\lambda_1, \lambda_2, \mu_1, \mu_2, \overline{P_i}) = v_i - \overline{P_i} - T(\lambda, \mu), \tag{19}$$

where v_i shows the utility of type i passengers who consume taxis' services' $\overline{P_i}$ is the fare type i passengers pay in a temporal dimension. Type i passengers decide whether to visit the market by considering their expected utility $EV_i(\lambda_1, \lambda_2, \mu_1, \mu_2, \overline{P_i})$. Assume that the probabilistic term of the passengers' utility v_i is distributed along the probabilistic distribution function $F_i(v_i)$ (the probabilistic density function $f_i(v_i)$) with a range $[0, \overline{v_i}]$ independently from the passengers' type. $\overline{v_i}$ is the upper limit of the passengers' utility who consume the taxi services. Let us normalize the reserved utility level 0 when the passengers consume any other service. That means passengers who can acquire a positive utility from consuming taxi services may use the taxi service. The following condition have to be satisfied in order that type i passengers visit the market.

$$T(\lambda, \mu) + \overline{P_i} \leq v_i \tag{20}$$

The number of passengers h_i who consume the taxi services can be represented as follows when the potential number of type i passengers is $\overline{H_i}$.

$$h_i = \overline{H_i}\{1 - F_i(T(\lambda, \mu) + \overline{P_i})\} \tag{21}$$

If we assume that the arrival interval of each passenger to the market is distributed based upon a Poisson arrival with mean $1/\nu_i$, the average arrival rate of h_i passengers is expressed as $\lambda_i = h_i \nu_i$. The arrival rates of type i passengers in the long term equilibrium are λ_i^* $(i = 1, 2)$, which satisfies

$$\lambda_i^* = \sigma_i\{1 - F_i(T(\lambda^*, \mu) + \overline{P_i})\} \ (i = 1, 2), \tag{22}$$

where $\sigma_i = \nu_i \overline{H_i}$. The market equilibrium where fare p_{ki} $(k, i = 1, 2)$ is exogenously given is expressed as $\lambda_1^*, \lambda_2^*, \mu_{k^*}^*$ which satisfies

$$\lambda_1^* = \sigma_1\{1 - F_1(T(\lambda^*, \mu^*) + \overline{P_1})\} \tag{23a}$$

$$\lambda_2^* = \sigma_2\{1 - F_2(T(\lambda^*, \mu^*) + \overline{P_2})\} \tag{23b}$$

$$\min_k \left\{ \frac{\lambda^*}{\mu^*}\overline{p_k} - S(\lambda^*, \mu^*) - \overline{d_k} \right\} = 0, \tag{23c}$$

where k^* is the k tha minimizes Eq. (23c).

4. Fare Regulation and Differentiation in the Market

(1) The Settings

There exists information asymmetricity in spot markets where the taxis and the passengers cannot recognize the partners' type in advance and where unspecific taxis and passengers make service transaction. Efficient matching between taxis and passengers is not guaranteed with heterogeneous agents. When the heterogeneity of both agents is large enough, mismatching externality may occuur where a specific type of taxis occupy the market or the taxis' transaction cost increases. In order to reduce this type of externality, it is necessary to introduce a differentiation policy to separate the markets depending on the types of both agents. In the next subsection, a case where taxis and passengers transact services at one stand with fare regulation is considered. A pooling equilibrium can be derived with one stand in the market. In the following section, the case where several stands are set for each type of passengers is considerd. A separating equilibrium is derived with multiple stands. It is also investigated whether separating equilibrium is incentive compatible with the taxis' and passengers' behavior.

(2) Pooling Equilibrium and Fare Regulation

The taxis' fare is exogenously given in the previous section. However, the fare is derived endogenously as a result of the market competition in a long tperiod when taxis can freely set their fare. Assume that taxis can freely decide their fare under the condition where a public agent sets the price cap $p_1,\ p_2$ for each type of passengers. Let us suppose that both types of taxis set their fare at p_{1i}, p_{2i} $(i = 1, 2)$. However, this is not sustainable. Only type k^{**} taxis, which pay a lower transaction cost, occupy the market because eq.(17) is satisfied in the market equilibrium as taxis have to join the queue in order to make a transaction in the spot market. On the other hand, taxis who leave the market have to set a higher fare than that of type k^{**} taxis in order to enter the market because passengers cannot select their desirable type of taxis. Type k^{**} taxis can also increase the fare in such a situation. They set an upper limit p_1, p_2 to their fare from the result of the long term entry competition of taxis. Type k^{**} taxis can stop the other types of taxies to enter the market. This result shows that the price cap regulation works as a fare regulation. The market may disappear as a result of long term competition without a price cap regulation. Price cap regulation is necessary to keep the spot market going. Such pooling equilibrium (PE) can be expressed by $\lambda_1^{**}, \lambda_2^{**}, \mu^{**}$ which satisfies the following conditions.

$$\lambda_1^{**} = \sigma_1\{1 - F_1(T(\lambda^{**}, \mu^{**}) + p_1)\} \tag{24a}$$

$$\lambda_2^{**} = \sigma_2\{1 - F_2(T(\lambda^{**}, \mu^{**}) + p_2)\} \tag{24b}$$

$$\frac{\lambda^{**}}{\mu^{**}}\overline{p} - S(\lambda^{**}, \mu^{**}) = \min_k\{\overline{d_k}\} \tag{24c}$$

$\overline{p} = \sum_{i=1}^{2} p_i\lambda_i/\lambda$. When Eq.(17) is satisfied, $\mu_{k^{**}}^{**} = \mu^{**}, \mu_l^{**} = 0$. When Eq.(17) is satisfied as an equality, both types of taxis can enter the market with an unspecified ratio of both types. Only type k^{**} taxis having a lower transaction cost can enter the market in the long term equilibrium, while the other type of taxis cannot enter the market. As a result, the taxis' transaction cost $d_{k^{**},i}$ which type k^{**} taxis have to pay when they are matched with type i passengers satisfies the following conditions.

$$d_{k^{**}i} \geq d_i \ (i = 1, 2). \tag{25}$$

Because of the information asymmetricity between taxis and passengers, taxis are matched with passengers who impose on them a higher transaction cost. The transaction cost that taxis bear increases as compared with the case of efficient matching. This means that a mismatching externality occurs where both taxis and passengers are not matched with their desirable partners in a pooling equilibrium with heterogeneous agents.

(3) Separating Equilibrium and Fare Regulation

Suppose that 2 taxi stands are set in the market and that type 1 taxis and passengers use stand 1 while type 2 taxis and passengers use stand 2. For the moment, assume that every agent is obligated to use a specific stand for its type for efficient matching. The incentive compatibility of usage regulation is analyzed later in this paper. There is also price cap regulation with p_1 for transaction between type 1 passengers and taxis and p_2 for that between type 2 passengers and taxis, respectively. The price cap regulation has an essential meaning same as the fare regulation through market entry competition in the long run as discussed ealier in the paper.

The service transaction in each stand can be described by an independent double queuing model. The average waiting time at stand i of taxis and passengers are represented as $S_i(\lambda_i, \mu_i)$, $T_i(\lambda_i, \mu_i)$, assuming each arrival rate λ_i and μ_i, respectively. The maximum queue length of type i taxis is

$$M_i(\lambda_i, p_i) = [p_i \lambda_i]\,. \tag{26}$$

The expected net profit of type i taxis is

$$EU_i(\lambda_i, \mu_i, p_i) = \rho_i p_i - S_i(\lambda_i, \mu_i) - d_i, \tag{27}$$

where $\rho_i = \lambda_i/\mu_i$. The conditional equilibrium arrival rate of type i taxis with exogenous passengers' arrival rate λ_i is μ_i° which satisfies the following condition.

$$\frac{\lambda_i}{\mu_i^\circ} p_i - S_i(\lambda_i, \mu_i^\circ) - d_i = 0 \tag{28}$$

On the other hand, the subjective expected utility $EV_i(\lambda_i, \mu_i, p_i)$ of type i passengers who visit stand i is expressed as follows.

$$EV_i(\lambda_i, \mu_i, p_i) = v_i - p_i - T_i(\lambda_i, \mu_i) \tag{29}$$

The equilibrium arrival rate of type i passengers with exogenous μ_i is λ_i°, which satisfies the following equation.

$$\lambda_i^\circ = \sigma_i\{1 - F_i(T_i(\lambda_i^\circ, \mu_i) + p_i)\} \ (i = 1, 2) \tag{30}$$

From the above conditions, the separating equilibrium (SE) can be expressed with $(\lambda_1^\circ, \mu_1^\circ), (\lambda_2^\circ, \mu_2^\circ)$ which satisfies the following equations.

$$\lambda_1^\circ = \sigma_1\{1 - F_1(T_1(\lambda_1^\circ, \mu_1^\circ) + p_1)\} \tag{31a}$$

$$\lambda_2^\circ = \sigma_2\{1 - F_2(T_2(\lambda_2^\circ, \mu_2^\circ) + p_2)\} \tag{31b}$$

$$\frac{\lambda_1^\circ}{\mu_1^\circ} p_1 - S_1(\lambda_1^\circ, \mu_1^\circ) = d_1 \tag{31c}$$

$$\frac{\lambda_2^\circ}{\mu_2^\circ} p_2 - S_2(\lambda_2^\circ, \mu_2^\circ) = d_2 \tag{31d}$$

As only taxis with lower transaction cost visit each stand, there is no mismatching externality in this setting.

In the above discussion, each type of taxis and passengers are forced to visit a specific stand. It is necessary to investigate whether such regulation to their behavior is incentive compatible or not. When type k ($k = 1, 2$) taxis visit stand k, they make a transaction with type k passengers and pay a transaction cost d_k, while when they visit stand j ($j \neq k$), they are matched with type j passengers and pay a transaction cost $d_k + d_j$. In the market equilibrium, the following conditions are always satisfied.

$$EU_2(\lambda_1^\circ, \mu_1^\circ, p_1) \leq EU_1(\lambda_1^\circ, \mu_1^\circ, p_1) = 0 \tag{32a}$$

$$EU_1(\lambda_2^\circ, \mu_2^\circ, p_2) \leq EU_2(\lambda_2^\circ, \mu_2^\circ, p_2) = 0 \tag{32b}$$

In this case, type 1 taxis have an incentive to visit stand 1, while type 2 taxis have an incentive to visit stand 2. This result means that differentiation policy is incentive compatible with the taxis' behavior. In the following, let us compare the expected utilities of the passengers of each type when they visit the other stand in the market equilibrium, in order to investigate the incentive compatibility of the passengers' behavior. Assume that $T_1(\lambda_1^\circ, \mu_1^\circ) > T_2(\lambda_2^\circ, \mu_2^\circ)$ is satisfied regarding the waiting time in both stands. Then

$$EV_1(\lambda_1^\circ, \mu_1^\circ, p_1) < EV_1(\lambda_2^\circ, \mu_2^\circ, p_2) \tag{33a}$$

$$EV_2(\lambda_1^\circ, \mu_1^\circ, p_1) < EV_2(\lambda_2^\circ, \mu_2^\circ, p_2) \tag{33b}$$

are satisfied which shows that either type of passengers have an incentive to visit stand 1. This type of market differentiation policy is not incentive compatible with the passengers' behavior. In order to realize a market differentiation through the passengers' self-selection mechanism, public agent introduce a penalty fare system $\hat{p}_1, \hat{p}_2$ which satisfies the following conditions.

$$p_1 + T_1(\lambda_1^\circ, \mu_1^\circ) < \hat{p}_1 + T_2(\lambda_2^\circ, \mu_2^\circ) \tag{34a}$$

$$\hat{p}_2 + T_1(\lambda_1^\circ, \mu_1^\circ) > p_2 + T_2(\lambda_2^\circ, \mu_2^\circ) \tag{34b}$$

That is, $\hat{p}_1$ is charged when type 1 passenger visit stand 2, while $\hat{p}_2$ is charged when type 2 passenger visit stand 1. Passengers do not pay penalty fares as each type of passengers is forced to use a specific stand under this penalty fare system.

(4) Spot Markets Specifics

The mismatching externality in pooling equilibrium and the incentive compatibility in separating equilibrium are closely related with an assumption of a transaction mechanism where taxis and passengers are matched on a first-come-first-served basis. Both taxis and passengers cannot select their desirable partners in this mechanism. As a result, market entry competition of taxis brings about an increase in the taxis' fare. When an expensive fare is set, those taxis wait for passengers even if they have to wait long enough. Taxis who set a lower fare cannot enter the queue because they acquire a negative profit. That is, only taxis with a higher fare occupy the market. Because of the peculiarity in the service transaction of the spot market, fare regulation is required in order for the market competition to bring about an equilibrium fare which is socially optimum. In order to secure the passengers' incentive compatibility in a separating equilibrium, it is necessary to introduce a market differentiation policy through the passengers' self-selection behavior. There might be other fare systems to be consistent with the incentive compatibility. As this paper assumes 2 types of passengers, the fixed fare system is adequate. In general, a non-linear fare system can be applied to solve these problems.

5. Characteristics of Market Equilibrium and Multiple Equilibria

Thick Market Externality

In spot markets, there exists thick market externality where an increase in the passengers' and taxis' arrival rate brings about an increase in the others' arrival rate through an active market transaction. In a market with thick market externality, a service transaction becomes efficient when the market scale is large, unless congestion or a mismatching externality is present. Let us pay attention to the fact that the average waiting length in spot markets can be expressed with Eqs.(4a), (4b). The average waiting length $E(m > 0 : \lambda, \mu)$, $E(m < 0 : \lambda, \mu)$ was a degree of 0 for the average arrival rate μ, λ. For any $\mu > \lambda \geq 0$ and $\theta \geq 0$, the following equations are satisfied.

$$E(m < 0 : \lambda, \mu) = E(m < 0 : \theta\lambda, \theta\mu) \tag{35a}$$

$$E(m > 0 : \lambda, \mu) = E(m > 0 : \theta\lambda, \theta\mu) \tag{35b}$$

Therefore, for the average waiting times for taxis and passengers S, T, the following conditions are satisfied.

$$S(\lambda, \mu) = \theta S(\theta\lambda, \theta\mu) \tag{36a}$$

$$T(\lambda, \mu) = \theta T(\theta\lambda, \theta\mu) \tag{36b}$$

In this way, a positive feedback mechanism where an increase (decrease) in the arrival rate of one agent brings about an increase (decrease) in the arrival rate of the other agent in spot markets. As a result, average waiting time decreases when the arrival rates of both taxis and passengers increase simultaneously, which makes market transactions efficient, that is, thick market externality is at work. There possibly exist multiple equilibria in markets with scale economies. In this section, thick market externality at work in spot markets is investigated. A mechanism where multiple equilibria occur both in pooling equilibrium and separating equilibrium is also analyzed. A mechanism in pooling equilibria is analyzed first for the sake of convenience.

(1) The Structure of Separating Equilibrium

In separating equilibrium, market equilibrium is realized in each stand, independently. Without loss of generality, let us focus upon stand 1. A type 1 passenger and a type 1 taxi make a transaction at stand 1. The market equilibrium at stand 1 can be defined as the combination of equilibrium arrival rate $(\lambda_1^\circ, \mu_1^\circ)$, which satisfies the following conditions.

$$\lambda_1^\circ = \sigma_1\{1 - F_1(T_1(\lambda_1^\circ, \mu_1^\circ) + p_1)\} \tag{37a}$$

$$\frac{\lambda_1^\circ}{\mu_1^\circ} p_1 - S_1(\lambda_1^\circ, \mu_1^\circ) = d_1 \tag{37b}$$

From Eq.(20), the following equation should be satisfied for the passengers to have an incentive to visit the market.

$$T_1(\lambda_1^\circ, \mu_1^\circ) + p_1 \leq v_1 \tag{38}$$

There exist a μ_1 which satisfies $0 \geq \rho_1 \geq 1$ for any λ_1 in Eq.(37b) (Matsushima *et al.*, 2006). Let us indicate that μ as $\mu_1^\circ(\lambda_1)$. Then Eq.(37a) can be rewritten as $\lambda_1 = \Gamma_1(\lambda_1, \mu_1^\circ(\lambda_1))$. Both sides of Eq.(37a) can be divided into $y = \lambda_1$, $y = \Gamma_1(\lambda_1, \mu_1^\circ(\lambda_1))$, which are shown as 2 diagrams in **Figure-1**. The 45 degree line in this figure shows $y = \lambda_1$. Cross points in this figure show the λ_1 which satisfies Eqs.(37a), (37b) simultaneously. Let us indicate the shift dynamics of λ_1 in a market disequilibrium in the following equation.

$$\frac{d\lambda_1}{dt} = \zeta_1\{\lambda_1 - \Gamma_1(\lambda_1, \mu_1^\circ(\lambda_1))\} \tag{39}$$

where $\zeta_1 > 0$ indicates a parameter. In the initial event, let us suppose that the initial arrival rate of type 1 passengers is plotted at C which satisfies $\lambda_1^B < \lambda_1$. The term $d\lambda_1/dt \geq 0$ is satisfied from Eq.(39), where the arrival rate of type 1 passengers finally converges to the equilibrium solution A. The equilibrium solution $(\lambda_1^\circ, \mu_1^\circ(\lambda_1^\circ))$ is a stable equilibrium. On the other hand, $d\lambda_1/dt \leq 0$ holds if $0 \leq \lambda_1 < \lambda_1^B$ is satisfied, that is, the arrival rate of type 1 passengers decreases. Then it converges to a stable equilibrium $(0, 0)$. There are two stable equilibria $(0, 0)$ and $(1.76, 0.90)$ in the case shown in **Figure-1**. The authors have already proved that the equilibrium $(0, 0)$ where a spot market is not realized is always stable, and that there are two stable equilibrium including $(0, 0)$ when Eqs.(37a) and (37b) have 2 equilibria solutions which satisfy the condition (38) other than $(0, 0)$ (Matsushima *et al.*, 2006).

[Insert Figure-1 here]

(2) The Structure of Pooling Equilibrium

To make the discussion simple, let us suppose that the fare of both types of taxis are set in the same level $p_{11} = p_{21} = p_1,\ p_{21} = p_{22} = p_2$ and that the transaction cost is set as d. From Eqs.(23a), (23b) and (23c), pooling equilibrium can be defined as $\lambda_i^{**}, \mu_i^{**}$ which satisfies the following conditions.

$$\lambda_1^{**} = \sigma_1\{1 - F_1(T(\lambda^{**}, \mu^{**}) + p_1)\} \tag{40a}$$

$$\lambda_2^{**} = \sigma_2\{1 - F_2(T(\lambda^{**}, \mu^{**}) + p_2)\} \tag{40b}$$

$$\frac{\lambda^{**}}{\mu^{**}}\overline{p} - S(\lambda^{**}, \mu^{**}) = \overline{d} \tag{40c}$$

As the taxis' fare and transaction cost are same for all types, condition (17) is defined from the expected transaction cost. Also $\overline{d_1} < \overline{d_2}$ is satisfied when $\lambda_1 > \lambda_2$ holds, that is, type 1 taxis dominate the market. When $\lambda_1 < \lambda_2$ is satisfied on the other hand, type 2 taxis dominate the market.

Suppose Eq.(40a) and (40b) are expressed as $\lambda_1 = \Theta_1(\lambda_1, \lambda_2, \mu), \lambda_2 = \Theta_2(\lambda_1, \lambda_2, \mu)$ in order to make the description simple. Suppose also that μ which satisfies Eq.(40c) for any λ_1, λ_2 is $\mu^{**}(\lambda_1, \lambda_2)$. Assume λ_2 is fixed as $\overline{\lambda_2}$. Eq.(40a) can be divided into two equations $y = \lambda_1$ and $y = \Theta_1(\lambda_1, \overline{\lambda_2}, \mu^{**}(\lambda_1, \overline{\lambda_2}))$. A point of intersection of the two equations shows a conditional equilibrium where $\overline{\lambda_2}$ is exogenously given. The equilibrium solution changes as $\overline{\lambda_2}$ changes. The locus of the equilibrium is expressed as $\tilde{\lambda}_1(\lambda_2)$. In the same way, the locus of the equilibrium solution $\tilde{\lambda}_2(\lambda_1)$ can be derived for any λ_2. The $\lambda_1 - \lambda_2$ plane can be divided into four territories, 1) $\lambda_1 > \tilde{\lambda}_1(\lambda_2),\ \lambda_2 > \tilde{\lambda}_2(\lambda_1)$ (domain I-1), 2)$\lambda_1 > \tilde{\lambda}_1(\lambda_2),\ \lambda_2 < \tilde{\lambda}_2(\lambda_1)$ (domain I-2), 3)$\lambda_1 < \tilde{\lambda}_1(\lambda_2)$, $\lambda_2 > \tilde{\lambda}_2(\lambda_1)$ (domain I-3), 4)$\lambda_1 < \tilde{\lambda}_1(\lambda_2),\ \lambda_2 < \tilde{\lambda}_2(\lambda_1)$ (domain II-1, II-2) as shown in **Figure-2**. The equilibrium solutions are defined as the arrival rates of both types which are consistent with the conditional equilibrium arrival rates which are realized when the arrival rate of other type passengers are exogenously given. That is, an equilibrium can be defined which satisfies

$(\lambda_1^\circ(\lambda_2^\circ), \lambda_2^\circ) = (\lambda_1^\circ, \lambda_2^\circ(\lambda_1^\circ))$. As symmetricity is assumed in taxis' fare and transaction for each type in this case, the two loci $\tilde{\lambda}_1(\lambda_2)$ and $\tilde{\lambda}_2(\lambda_1)$ are symmetric about the 45 degree line as shown in **Figure-2**. Type 1 taxis dominate the market in the domain above the 45 degree line, while type 2 taxis dominate the market below the 45 degree line.

[Insert Figure-2 here]

Move dynamics of $\lambda_1,\ \lambda_2$ in the market disequilibrium is defined as follows.

$$\frac{d\lambda_1}{dt} = \eta_1\{\lambda_1 - \Theta_1(\lambda_1, \lambda_2, \mu^{**}(\lambda_1, \lambda_2))\} \tag{41a}$$

$$\frac{d\lambda_2}{dt} = \eta_2\{\lambda_2 - \Theta_2(\lambda_2, \lambda_1, \mu^{**}(\lambda_2, \lambda_1))\}, \tag{41b}$$

where $\eta_1 > 0,\ \eta_2 > 0$ are parameters. Suppose that the initial points λ_1, λ_2 are located in the domain II-1. The terms $d\lambda_1/dt \le 0, d\lambda_2/dt \le 0$ are satisfied from Eqs.(41a),(41b). That is, the arrival rates of passengers of both types gradually decreases (moves to the direction of the arrow shown in the domain II-1 in **Figure-2**) to converge to a stable equilibrium $(\lambda_1^{**}, \lambda_2^{**}) = (0.42, 0.42)$. The directions of move of λ_1, λ_2 are shown in each domain in the **Figure-2**. As a result, the arrival rates of both types decreases and converges to a stable equilibrium $(\lambda_1^{**}, \lambda_2^{**}) = (0.0, 0.0)$ when the initial arrival rates are in the domain II-2. When the initial arrival rates are in other territories, the long run equilibrium arrival rates converge to the stable equilibrium $(\lambda_1^{**}, \lambda_2^{**}) = (0.42, 0.42)$. D in this figure shows the unstable equilibrium. From the above, market equilibria are different depending upon the initial arrival rates of passengers. Therefore, market equilibria exist in spot markets.

6. Differentiation Policy and Social Welfare

(1) Type of Market Equilibrium and Social Welfare

There are two types of market equilibrium, 1) pooling equilibrium $(\mu_i^{**}, \lambda_i^{**})$, and 2) separating equilibrium $(\mu_i^\circ, \lambda_i^\circ)$. As discussed in the former section, two stable equilibria possibly exist: one with positive arrival rates, and another without taxi and passenger. The social welfare is 0 when no agent visits the market. Social welfare in the market equilibrium is defined by focusing upon the market equilibrium with positive arrival rates as follows. The producers' surplus is 0 as taxis enter the market until the expected net profit is 0. Therefore, only the consumers' surplus is taken into consideration. The expected utility of each type of passengers is defined in Eq.(19) in the pooling equilibrium. When the probabilistic utility v_i is distributed along the probabilistic density function $f_i(v_i)$, social welfare SS^{PE} is expressed as follows.

$$SS^{PE} = \sum_{i=1}^{2} \sigma_i \Big\{ \int_{\tau_i^{PE}}^{\overline{v_i}} (v_i - \tau_i^{PE}) f_i(v_i) dv_i \Big\} \tag{42}$$

$\tau_i^{PE} = T(\lambda^{**}, \mu^{**}) + \overline{P}_i$ is satisfied. For the separating equilibrium SE, the social welfare SS^{SE} can be defined by replacing τ_i^{PE} in Eq.(42) with $\tau_i^{SE} = T_i(\lambda_i^{\circ}, \mu_i^{\circ}) + p_i$.

Let us take notice that the social welfares SS^{PE} and SS^{SE} are functions of regulated fares p_1, p_2. Suppose that the social welfare in a pooling equilibrium and a separating equilibrium $SS^{PE}(p_1, p_2)$, $SS^{SE}(p_1, p_2)$ are functions of p_1, p_2 in order to make this point clearly. The optimal regulated fare (p_1^{**}, p_2^{**}), $(p_1^{\circ}, p_2^{\circ})$ can be defined as follows.

$$(p_1^{**}, p_2^{**}) = \arg\max_{p_1, p_2} \{SS^{PE}(p_1, p_2)\} \tag{43a}$$

$$(p_1^{\circ}, p_2^{\circ}) = \arg\max_{p_1, p_2} \{SS^{SE}(p_1, p_2)\} \tag{43b}$$

'arg' denotes a fare that maximize the right-hand side of Eqs.(43a) and (43b).

(2) Optimal Fare and Social Welfare

Let us explain the effect of differentiation policy and fare regulation upon social welfare through a simple numerical example. The results of numerical examples are shown in **Figure-3** to **Figure-7** with parameters shown in the footnotes of figures. **Figure-3** shows the relation between price cap fares p_1, p_2 and the social welfare in stable equilibrium. The optimal price cap fare in the pooling equilibrium is $(p_1^{**}, p_2^{**}) = (3.43, 1.41)$. The social welfare in pooling equilibrium is $SS^{PE} = 3.78$.

Figure-4 shows the relation between the price cap fare p_i and social welfare in a stand of pooling market i $(i = 1, 2)$. In the separating equilibrium, $p_1^\circ = 2.28$ for stand 1, and $p_2^\circ = 1.95$ for stand 2. Social welfare in separating market SS is 4.26, that makes social welfare increase by the market differentiation policy. The average waiting time of each stand under the optimal price cap regulation $(T_1(\lambda_1^\circ, \mu_1^\circ), T_2(\lambda_2^\circ, \mu_2^\circ))$ is $(0.91, 0.89)$ in this case. The penalty fare which satisfies $\hat{p}_1 > 2.31, \hat{p}_2 > 1.92$ should be set to lead passengers' behavior in order to make the differentiation policy incentive compatible with their behavior.

[Insert Figure-3 here]

[Insert Figure-4 here]

[Insert Figure-5 here]

[Insert Figure-6 here]

[Insert Figure-7 here]

Next, the relation between the market condition and the effect of market differentiation policy is discussed. **Figure-5** shows the change in social welfare in both the pooling equilibrium and the separating equilibrium under an optimal fare regulation according to the change in d with $(d_1, d_2) = (d + 0.25, d)$. As transaction of type 1 taxis is larger than that of type 2 in this case, the market is dominated by type 2 taxis. As the transaction cost of type 2 taxis which are matched with type 1 passengers becomes large, when the taxis' transaction cost d is large, the effect of mismatching externality is large in the pooling equilibrium. On the other hand, there is no mismatching externality in the separating equilibrium, while thin market externality is present. The social welfare in a separating market is large as the effect of mismatching externality is large when $d \geq 0.43$ in **Figure-5**. The pooling market is desirable when $d < 0.43$, as the effect of thick market externality is larger than that of mismatching externality. **Figure-6** shows the relation between σ and the social welfare in the market equilibrium under an optimal fare regulation with $\sigma_1 = \sigma_2 = \sigma$. The number of passengers who visit the market becomes large as the passengers' density is large. Therefore, mismatching externality works well while the effect of thin market externality in the separating equilibrium is small. A separating market which decreases mismatching externality is optimal when $\sigma \geq 2.26$ in **Figure-6**.

Figuer-7 shows the relation between the ratio of passengers and the social welfare in both types of market with $\sigma_1 + \sigma_2 = 4.5$. The horizontal axis shows the density of type 1 passengers per total density of passengers $\zeta = \sigma_1/(\sigma + \sigma_2)$. The separating market is optimal with $0.50 \leq \zeta \leq 0.68$, otherwise the pooling market. The expected gross profit for taxis is large as ζ increases, because the ratio of long distance passengers increases. As a result, more taxis arrive at the market in order to increase the social welfare. However, the effect of mismatching externality becomes large when the average transaction cost of taxis increases, as both types of passengers visit the market in a pooling equilibrium of $0.50 \leq \zeta \leq 0.68$. That is, the differentiation policy is effective in this case. The stand for type 1 passengers may disappear in a separating market when $\zeta < 0.18$, as the arrival rate of type 1 passengers is small in order to make the effect of thin market externality more effective. On the other hand, a type 2 stand cannot exist when $\zeta > 0.72$. In a pooling market, the market is alive for any ζ as the total number of passengers is fixed.

(3) Policy Implications

As two externalities, thick market externality and mismatching externality, exist in spot markets, the desirable form of the markets cannot be defined concretely. Whether the market differentiation policy is effective or not depends upon the circumstances of markets. A public agency has to investigate carefully considering both externalities when introducing a differentiation policy. The numerical examples in the paper show that the separating market gives a higher social welfare when σ and d are large. Even though general results cannot be derived from limited numerical examples, this result suggest that several stands should be set for spot markets where the transaction cost is large and the demand for taxis is large, e.g., airports.

The knowledge from this paper depends upon the assumptions that passengers are matched with taxis on a first-come-first-served basis and that there is no limit in the arrival rate of taxis. For example, passengers have to bear the waiting time in order to wait for their desirable type of taxis in a cruising market, while the price competition mechanism works as they can choose the taxis' type. Passengers cannot choose their desirable taxis when they are matched with taxis on a first-come-first-served basis. Then, taxis with higher fares can make a transaction with the passengers. Taxis have an incentive to raise the fare in the long run. When the taxis' fare is high enough, more taxis enter the market thus increasing the waiting time of taxis as a result. If there is no limit in the taxis' fare, the market is dominated only by taxis with higher fares pushing the taxis with lower fares out of the market. Taxis that are forced to leave the market may wait for passengers outside of the market, which causes congestion. From the standpoint of view of congestion relaxation in the market, queuing is the efficient way for matching. However, fare competition does not work well in spot markets, thus creating a necessity for fare regulation. When queuing is applied for transaction in spot markets, the fixed fare system for specific places may be also effective. It is necessary to develop a general equilibrium model for the whole taxi market.

This paper proposes an equilibrium model to explain thick market externality and mismatching externality by heterogeneous agents for the case where supply and demand are matched in a taxi market. This kind of mechanism can be applied for various services other than the taxi market. For example, public transportation or logistics can be explained with matching mechanism. Similar discussion may be applied for the allocation problem of the peoples'S scheduling. Expanding this model may give us useful knowledge to other areas of transportation modeling.

7. Conclusion

In taxi spot markets where taxis and passengers of all types transact taxi services, thick market externality appears where the service transaction becomes more efficient while more passengers and taxis visit the market. On the other hand, mismatching externality may also appears, where specific types of taxis dominate the market or the transaction cost of taxis increases, as both agents cannot recognize the partners' type. In this paper, a spot market equilibrium model with both thick market externality and mismatching externality is formulated. The mechanism of market equilibrium and the effect of market differentiation policy upon social welfare are analyzed for a pooling market where both types of passengers and taxis visit one stand and a separating market where several stand are set according to the type of agents. The theoretical knowledge derived from this paper is as follows.

- Specific type of taxis dominate the market in a pooling equilibrium.
- A market differentiation policy is incentive compatible with the taxis' behavior.
- When the market differentiation policy is not incentive compatible with the passengers' behavior, it is necessary to introduce fare regulations.
- Introduction of a market differentiation policy should be carefully investigated as both thick market externality and mismatching externality appears in spot markets.

Even though these results are derived under the assumption that the matching between supply and demand is transacted in a spot market, it is useful to consider transportation market differentiation policies.

Some future researchs still remains when considering a realistic market differentiation policies. Firstly, this paper assumes no limit in the arrival rate of each type of taxis. If the maximum number of taxis which can be utilized is introduced, several types of taxis can possibly coexist in a pooling equilibrium. Secondly, when the number of types of passengers is larger than the number of stands, an optimal matching pattern should be investigated. Thirdly, this paper assumes a fixed fare for each type of passengers in order to simplify the disucussion. If passengers are more heterogeneous, a discussion about an optimal nonlinear fare system which consists of fixed fare and variable fare should be investigated. Finally, this paper proposes a partial equilibrium model where taxis and passengers are matched in one spot market. A general equilibrium model with total taxi services in a city is necessary to be considered in order to check the effectiveness of fare regulation policies.

References

- Abe, M.A. and B.C. Brush (1976), "On the regulation of price and service quality: The taxicab problem", Quarterly Review of Economics and Business 16, 105-111.
- Arnott, R (1996), Taxi travel should be subsidized, Journal of Urban Economics, vol.40, pp.316-333.
- Bergantino, A.S. and E. Longobardi (2000), "Rethinking deregulation. The case of the taxi industry", Paper presented at the 6th RSAI World Congress, Lugano, Switzerland.
- Bulow, J. I., Geanakoplos, J. D., and Klemperer, P. D. (1985): Multimarket oligopoly: Strategic substitutes and complements, *Journal of Political Economy*, Vol. 93, pp. 488-511.
- Cairns, R.D. and C. Liston-Heyes (1996), "Competition and regulation in the taxi industry", Journal of Public Economics 59, 1-15.
- Cooper, R. and Andrew, J. (1988), Coordinating coordination failures in Keynesian models, *Quarterly Journal of Economics*, Vol. 103, pp. 441-463.
- DeVany, A. S. (1975), Capacity utilization under alternative regulatory constraint: An analysis of taxi markets, *Journal of Political Economy*, Vol. 83, pp.83-94.
- Douglas, G.W. (1972), Price regulation and optimal service standard: The taxicab industry, *Journal of Transport Economics and Policy*, Vol. 6, pp.116-127.
- Farmer, R. E. A. (1993), *The Macroeconomics of Self-Fulfilling Prophecies*, The MIT Press.
- Gärling, T.,Laitila, T., Marell, A., and Westin, K. (1995): A note on the short-term effects of deregulation of the Swedish taxi-cab industry, *Journal of Transport Economics and Policy*, Vol. 20, pp. 209-214.
- Häckner, J. and Nyberg, S. (1995), Deregulating taxi services: A word of caution, *Journal of Transport Economics and Policy*, Vol.24, pp.195-207.

- Howitt, P. M. and McAfee, R. P. (1987), Costly search and recruiting, *International Economic Review*, Vol. 28, pp. 89-107.
- Howitt, P. M. (1990), *The Keynesian Recovery*, Prentice Hall.
- Kendall, D. G. (1951), Some problems in the theory of queues, *Journal of the Royal Statistical Society*, Vol 8, pp.151-185.
- Kobayashi K., K. Fukuyama, and K. Matsushima (1998), Face-to-face contacts and matching equilibrium, in Lennart Andersson and Thomas Blom (eds.): *Sustainability and Development*, pp.115-126, Proceedings, Karlstad, Sweden.
- Manski, C. and J.D. Wright (1976), "Nature of equilibrium in the market for taxi services", Transportation Research Record 619, 11-15.
- Matsushima, K. and K. Kobayashi (2006a), Endogenous Market Formation with Matching Externality, in Kobayashi et al. eds, Structural Change in Transportation and Communications in the Knowledge Economy: Implications for Theory, Modeling and Data, Edward Elgar.
- Matsushima K. and K. Kobayashi (2006b), 'Spatial Equilibrium of taxi spot markets and social welfare', Paper presented at the 53rd Annual North American Meetings of the Regional Science Association International, Toronto, November 2006.
- Moore, A.T. and T. Balaker (2006), "Do economists reach a conclusion on taxi deregulation? " , Econ Journal Watch 3(1), January, 109-132.
- Orr, D. (1969), The 'Taxicab problem': A proposed solution, *Journal of Political Economy*, Vol. 77, pp.141-147.
- Sasieni, M. W. (1961), Double queues and impatient customers with an application to inventory theory, *Operations Research*, pp.771-781.

- Schreiber, C. (1977), The economic reasons for price and entry regulation of taxicabs, *Journal of Transport Economics and Policy*, Vol. 11, pp. 298-304.
- Teal, R. F. and Berglund, M. (1987), The impacts of taxicab deregulation in the USA, *Journal of Transport Economics and Policy*, Vol.21, pp.37-56.
- Willams, D. J. (1981), The economic reasons for price and entry regulation of taxicabs, A comment, *Journal of Transport Economics and Policy*, Vol. 14, pp. 81-83.
- Yang, H. and S.C. Wong (1998), "A network model of urban taxi services", Transportation Research B 32B(4), 235-246.
- Yang, H. and S.C. Wong (2002), "Demand-supply equilibrium of taxi services under competition and regulation", Transportation Research B 36B(9), 799-819.
- Yang, H., Y.W. Lau, S-C Wong and H.K. Lo (2000), "A macroscopic taxi model for passenger demand, taxi utilization and level of services", Transportation 27, 317-340.
- Yang, H., M. Ye, W. H. Tang and S.C. Wong (2003), "Modeling urban taxi services: A literature survey and an analytical example" . In W.H.K. Lam and M.G.H. Bell eds. Advanced modeling for Transit Operations and Service Planning, Pergamon, 257-286.

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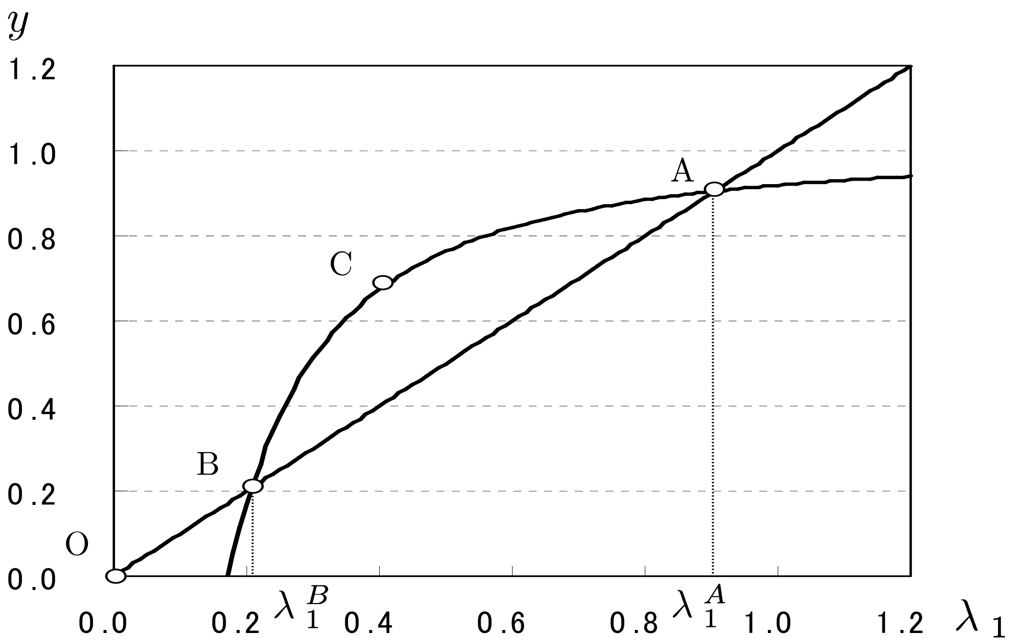
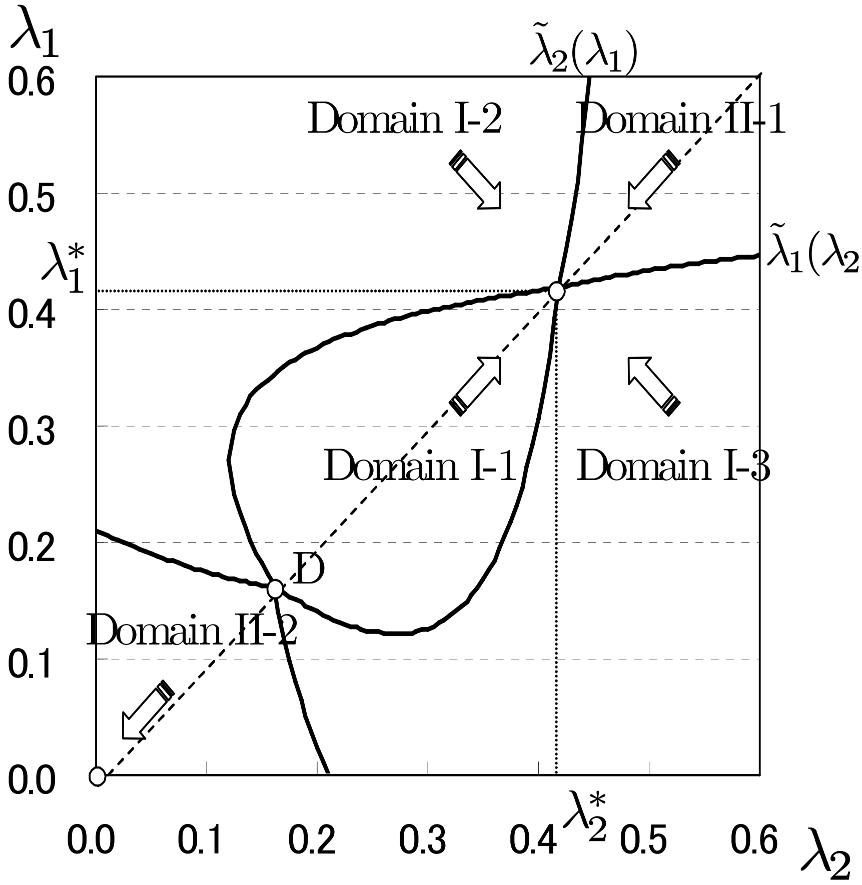
Figure-6 Passengers' density and optimal regulation policy

Figure-7 The ratio of passengers and optimal regulation policy



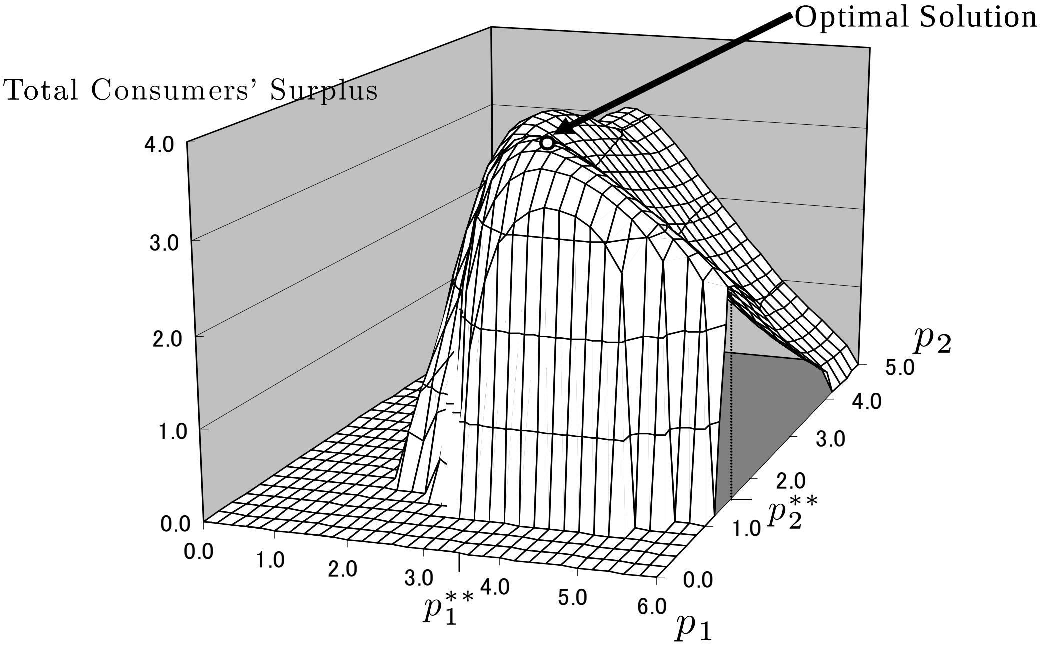
This example shows a case where v_1 is uniformly distributed in the range $[0, 5.0]$, where $\sigma_1 = 2.5$, $x_1 = 0.5$ and $p_1 = 3.0$. In this case, there are two equilibria with $(\mu_1^\circ, \lambda_1^\circ) = (1.76, 0.90)$ and $(0, 0)$.

Figure-1 Separating equilibria



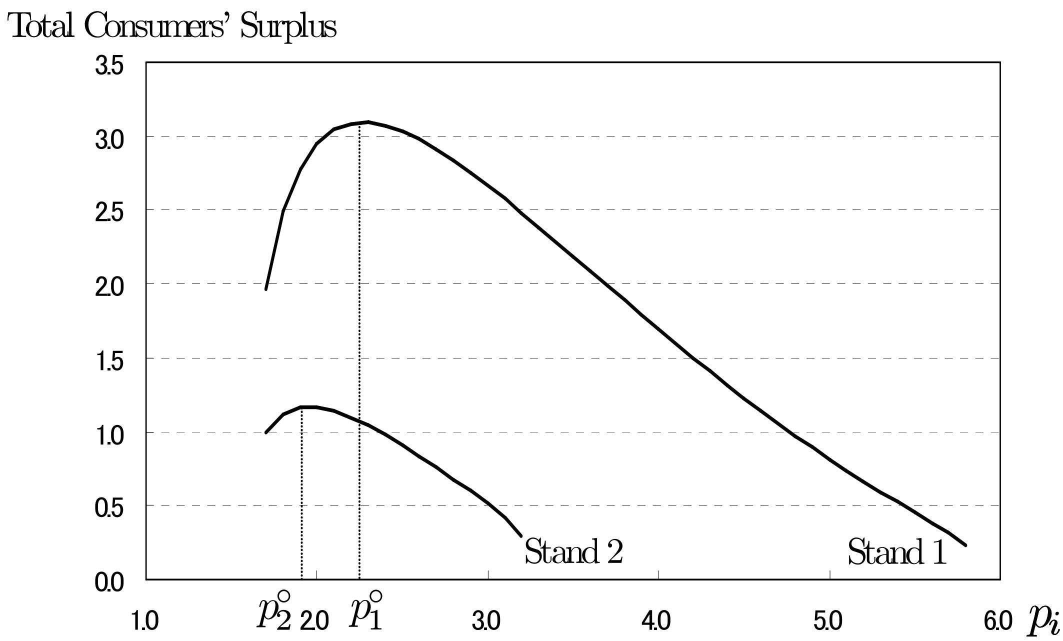
Probabilistic utility term of passengers v_i ($i = 1, 2$) are uniformly distributed in the area $[0.0, 5.0]$. This figure shows a case with $\sigma_1 = \sigma_2 = 2.5$. $d_1 = d_2 = 0.5$ and $p = p_1 = p_2 = 4.0$. The ratio of arrival rate of both types are undecided on the line $\lambda_1 = \lambda_2$ as $d_1 = d_2, p_1 = p_2$.

Figure-2 Pooling equilibria



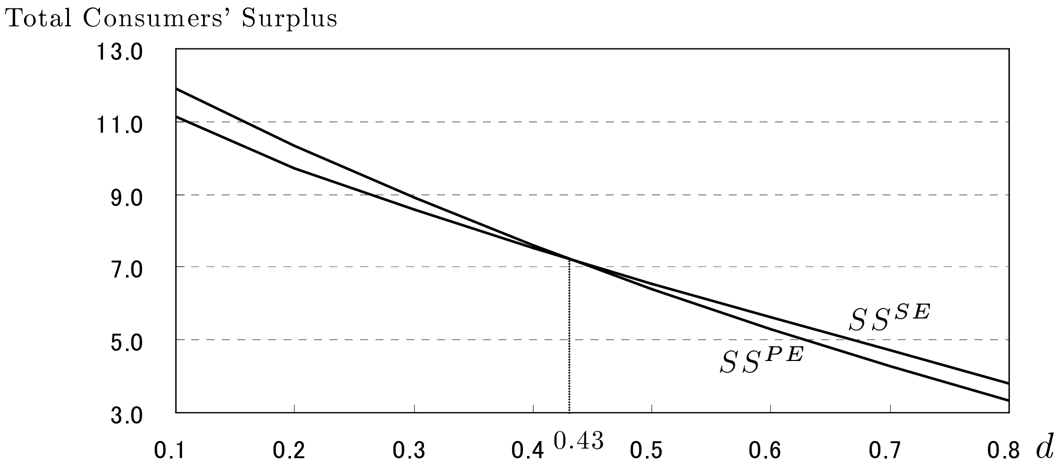
v_1 and v_2 are uniformly distributed in the area $[0.0, 7.5]$ and $[0.0, 5.0]$, respectively. This example shows the result from calculation with $\sigma_1 = \sigma_2 = 2.5$, $d_1 = 1.0$, $d_2 = 0.75$.

Figure-3 Fare and social welfare (a case with pooling equilibria)



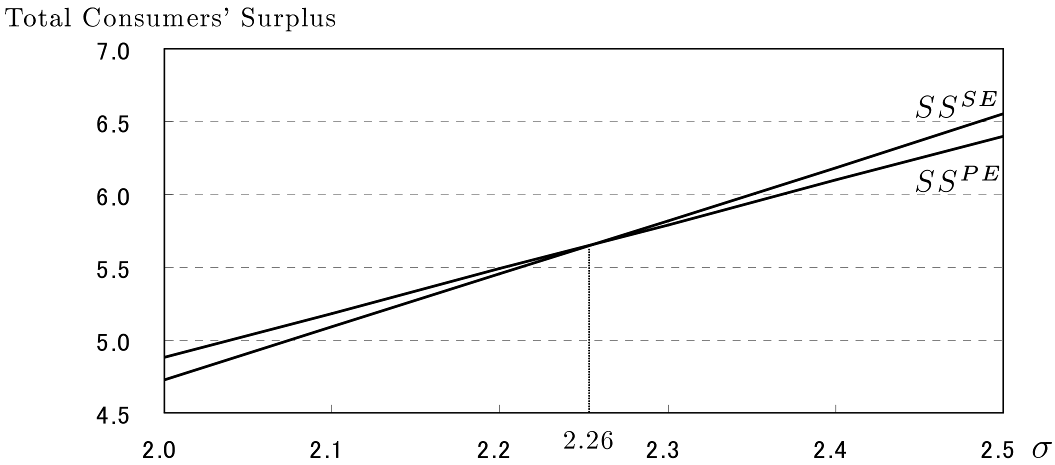
v_1 and v_2 are uniformly distributed in the area $[0.0, 7.5]$ and $[0.0, 5.0]$, respectively. This example shows the result from calculation with $\sigma_1 = \sigma_2 = 2.5$, $d_1 = 1.0$, $d_2 = 0.75$.

Figure-4 Fare and social welfare (a case with separating equilibria)



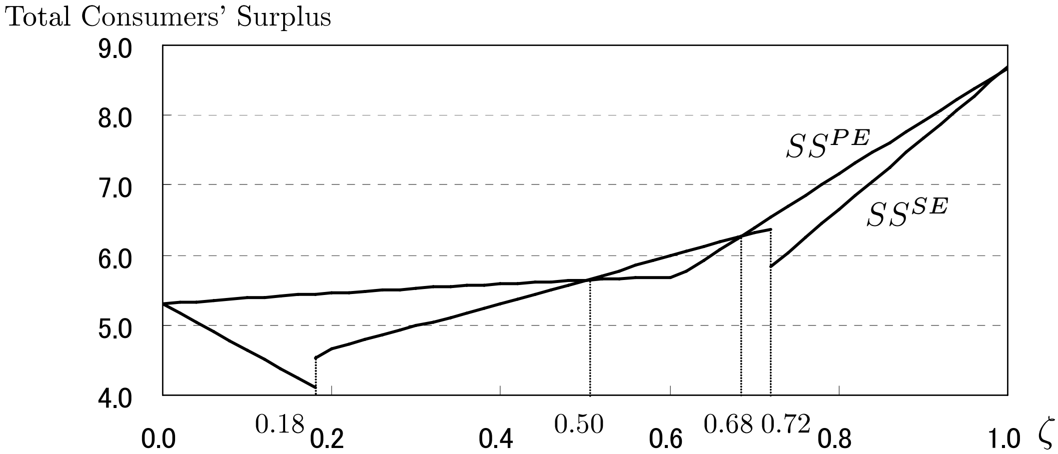
v_1 and v_2 are uniformly distributed in the area $[0.0, 7.5]$ and $[0.0, 5.0]$, respectively. This example shows the result from calculation with $\sigma_1 = \sigma_2 = 2.5$, $d_1 = d + 0.25$, $d_2 = d$.

Figure-5 Taxis' transaction cost and optimal regulation policy



v_1 and v_2 are uniformly distributed in the area $[0.0, 7.5]$ and $[0.0, 5.0]$, respectively. This example shows the result from calculation with $\sigma_1 = \sigma_2 = \sigma$, $d_1 = 0.75$, $d_2 = 0.5$.

Figure-6 Passengers' density and optimal regulation policy



v_1 and v_2 are uniformly distributed in the area $[0.0, 7.5]$ and $[0.0, 5.0]$, respectively. This example shows the relation between ζ and social welfare with $\sigma_1 + \sigma_2 = 4.5$, $\zeta = \frac{\sigma_1}{\sigma_1+\sigma_2}$, $d_1 = 0.75$, $d_2 = 0.5$.

Figure-7 The ratio of passengers and optimal regulation policy