METHODOLOGICAL ISSUES IN MODELING TIME-OF-TRAVEL PREFERENCES

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ABSTRACT

The purpose of this paper is to address three methodological issues that arise when modeling time-of-travel preferences: unequal period lengths, schedule delay in the absence of desired time-of-travel data, and the 24-hour cycle. Varying period length is addressed by using size variables. Schedule delay is treated by assuming either arrival or departure time sensitivity and using market segment specific utility functions of time-of-travel, or using distributions of desired times-of-travel. The 24-hour cycle is modeled by using trigonometric or constrained piecewise linear utility functional forms. The methodologies developed in this paper are applied to the modeling of time-of-travel choices in the context of tour-based models using the 2000 Bay Area travel survey, and selected model estimation results are presented.

Keywords: time-of-travel modeling, schedule delay, cyclicality, tour-based model, revealed preference data

1. INTRODUCTION

1.1 Motivation and Objective

The topic of time-of-travel preferences is of relevance for virtually all types of transportation services due to its importance in evaluating demand management policies such as congestion pricing and predicting transportation system performance. The purpose of this paper is to discuss methodological issues related to modeling time-of-travel preferences. The paper specifically addresses three methodological issues: (1) modeling time periods of unequal length, (2) accounting for schedule delay when data on the desired times-of-travel are unavailable, and (3) modeling the cyclical properties of time-of-travel preferences.

The first issue arises because of the discretization of continuous time into time intervals or periods. These time intervals are often of varying length, for example, due to the small volume of trips for certain time intervals. We wish to address how to account for time periods of unequal length in the specification of the time-of-travel model.

The second issue is related to schedule delay, which is a fundamental concept in modeling time-of-travel choice (Vickrey, 1969; Cosslett, 1977; Abkowitz, 1980; Hendrickson and Kocur, 1981; Small, 1982). It postulates that travelers have desired arrival or departure times, and that travel at other times incurs disutility. If the desired times-of-travel were known, it would be relatively straightforward to include schedule delay in a time-of-travel choice model. However, these data are often not collected especially in revealed preference surveys and are difficult to forecast, and therefore it is important to find methods that account for schedule delay.

The third issue arises because of the 24-hour cycle and its implications on time-of-travel preferences. That is, for a model designed for a one-day time frame, times t and $t+24$ hours correspond to the same time instance. Therefore, these two times exhibit the same arrival (or departure) time preference and should have the same utility of arrival (or departure). We wish to address how to account for these cyclical properties in the utility specification of the time-of-travel model.

1.2 Literature Review

Time-of-travel choice has been studied using different methodological approaches which vary by the level of temporal analysis, model structure, and type of data collected. These various approaches include the use of discrete (see for example Small, 1982) vs. continuous time models (Wang, 1996; van Vuren et al., 1999); model structures ranging from logit to other more general models such as nested logit (Brownstone and Small, 1989), Ordered Generalized Extreme Value (Small, 1987), multinomial probit (Liu and Mahmassani, 1998), and error components logit (de Jong et al., 2003; RAND Europe, 2005; Hess et al., 2007); and presence of schedule delay terms, generally available from stated preference but not revealed preference surveys (see for example de Jong et al., 2003). A review of the earlier literature on time-of-travel procedures can be found in Alfa (1986), Cambridge Systematics (1999), and Bates (2002). More recent work can be found in de Jong et al. (2003), Hess et al. (2005), RAND Europe (2005), Cambridge Systematics (2005), Abou Zeid et al. (2006), and Hess et al. (2007).

Next we discuss how the literature has generally treated the methodological issues raised in this paper. First, the number and length of the time periods used have varied, with earlier efforts using a few number of coarse time periods and more recent work using more detailed time periods. For example, in RAND Europe (2005) and Hess et al. (2007), periods as short as 1 hour or 15 minutes are used in the model. Generally, time period-specific constants for arrival time, departure time, and/or duration are included in the utility equations of the time periods. If the periods are of unequal length, these constants will capture the effect of unequal lengths but will mask the pattern of time-of-travel preferences, and therefore the use of size variables is preferred, as discussed later.

Second, schedule delay information has generally been included in time-of-travel models estimated from stated preference data where information about preferred times of travel are likely to be collected. However, when these models are used in application or when models are estimated from revealed preference data, the schedule delay terms are normally excluded since information about scheduling preferences are unavailable or difficult to forecast (see for example Hess et al., 2007). The inherent assumption is that the alternative-specific constants will capture these schedule delay effects, among other things. However, as we show later in the paper, the constants capture the effects of schedule delay only if additional assumptions are employed.

Third, functional forms have been developed to approximate alternative-specific constants so as to avoid overfitting and identification problems as the number of these constants increases with the number of time periods. For example, in Hess et al. (2005), exponential, power, and empirical functions are used to approximate these constants. However, to the best of our knowledge, the cyclical properties of time-of-travel preferences have not been dealt with in developing time-of-travel choice models.

1.3 Contributions and Organization

The contributions of this research are (1) the use of size variables to account for unequal period lengths, (2) the development of methods which obviate the need for explicitly incorporating schedule delay in the utilities of the time period alternatives, (3) the use of continuous cyclic functions of time which ensure that the utility at a time t is equal to the utility at time $t+24$ hours, and (4) the demonstration of the developed methods empirically using a tour-based traffic modeling approach for the San Francisco Bay Area. These methodological issues and their solutions have been developed by the authors of this paper for a project whose results are documented in Cambridge Systematics (2005) and Abou Zeid et al. (2006). The purpose of this paper is to provide the detailed derivations and analyses.

The remainder of this paper is organized as follows. Section 2 discusses the issue of unequal period lengths and proposes a method to account for it. Section 3 develops methods for incorporating schedule delay even though related data might be unavailable. Section 4 derives the continuous functions of time that satisfy the cyclicality property of time-of-travel. Section 5 presents a case study describing the application of these methodological issues to the San Francisco Bay Area and presents selected model estimation results to illustrate the concepts. Section 6 concludes the paper.

2. MODELING TIME PERIODS OF UNEQUAL LENGTH

For discrete choice modeling of time-of-travel, a number of alternatives are defined which could be of varying lengths. In this section, we discuss how to account for this issue in the utility specification of the model.

Let t be an index for continuous time, where $t \in [0, 24]$. Let $v(t)$ denote the systematic utility of time-of-travel t and $f(t)$ be the probability density function of time-of-travel choice. For a continuous logit model (Ben-Akiva and Watanatada, 1981; de Palma et al., 1983), $f(t)$ is given by:

$$f(t) = \frac{e^{v(t)}}{\int_0^{24} e^{v(t')} dt'} \tag{1}$$

We discretize the 24-hour time horizon into H periods. For a time period h where $h = 1, \ldots, H$, let $t_s(h)$ denote its start time (with respect to an arbitrary reference point), Δ_h its length, and $P(h)$ the choice probability of time period h. $P(h)$ can be expressed as follows:

$$P(h) = \int_{t_s(h)}^{t_s(h)+\Delta_h} \frac{e^{v(t)}}{\int_0^{24} e^{v(t')} dt'} dt = \frac{\int_{t_s(h)}^{t_s(h)+\Delta_h} e^{v(t)} dt}{\int_0^{24} e^{v(t')} dt'} \tag{2}$$

Applying the mean-value theorem for integrals, define for the interval $\left[t_s(h), t_s(h) + \Delta_h\right]$ the systematic utility $V(h)$ of period h, equal to the value of $v(t)$ at a "mid-point" of time interval h, and express (2) as follows:

$$P(h)=\frac{e^{V(h)}\Delta_h}{\int_0^{24} e^{v(t')}dt'}=\frac{e^{V(h)}\Delta_h}{\sum_{h'=1}^{H} e^{V(h')}\Delta_{h'}}=\frac{e^{V(h)+\ln\Delta_h}}{\sum_{h'=1}^{H} e^{V(h')+\ln\Delta_{h'}}} \tag{3}$$

Thus, time periods of unequal length can be accounted for by adding the natural logarithm of the length of the period (size variable) to its systematic utility and constraining the coefficient of the size variable to 1.

3. ACCOUNTING FOR SCHEDULE DELAY

Schedule delay is a fundamental concept in modeling time-of-travel choice which captures the disutility caused by traveling at times other than the desired times-of-travel. In this section, we discuss two approaches that can be used to account for schedule delay when data on the desired times-of-travel are unavailable.

Let h denote a time-of-travel period, h^* denote a desired time-of-travel period, a denote an arrival time period, a^* denote a desired arrival time period, d denote a departure time period, d^* denote a desired departure time period, $TT(h)$ denote the travel time in period h, and $SD(h,h^*)$ denote the schedule delay for travel period h given a desired time-of-travel period h^*. Let t denote a time-of-travel, t^* denote a desired time-of-travel, $tt(t)$ denote the travel time corresponding to time-of-travel t, and $sd(t,t^*)$ denote the schedule delay for time-of-travel t given a desired time-of-travel t^*.

3.1 Approach 1: Assume Constant Desired Times-of-Travel by Market Segment

We specify the utility of a time-of-travel period as a function of the travel time, schedule delay, and size of the period. We also include an alternative-specific constant and allow the specification to include other explanatory variables.

For trips with a desired arrival time (e.g. the trip from home to work), the systematic utility of an arrival time period a can be expressed as follows:

$$V(a) = \alpha_1(a) + \beta_1 TT(a) + \gamma_1 SD(a, a^*) + \ln \Delta_a + \cdots \tag{4}$$

And for trips with a desired departure time (e.g. the trip from work to home), the systematic utility of a departure time period d can be expressed as follows:

$$V(d) = \alpha_2(d) + \beta_2 TT(d) + \gamma_2 SD(d, d^*) + \ln \Delta_d + \cdots \tag{5}$$

In the above equations, $\alpha_1(a)$ and $\alpha_2(d)$ are alternative-specific constants, β_1, β_2, γ_1, and γ_2 are coefficients to be estimated, and $\ln \Delta_a$ and $\ln \Delta_d$ are the size variables described in the previous section.

For a desired arrival time period a^*, modeling arrival time choice means that $\gamma_1 SD(a, a^*)$, which is a function of the difference between a and a^*, can be expressed as a function $g_1(a)$ for a given market segment if a^* is assumed to be constant for individuals in that market segment. $g_1(a)$ is then an attribute of period a (whose value does not vary across individuals in a market segment) and is absorbed by the alternative-specific constant (for the respective market segment) of the systematic utility of period a; in this case, there is no need to explicitly include a schedule delay term in the systematic utility.

If on the other hand departure time choice were modeled for a trip with desired arrival time, schedule delay would depend on travel time and cannot be a constant. This is seen by noting that for a given trip, $t_a = t_d + tt(t_d)$, where t_a is the arrival time which corresponds to a departure time t_d. Therefore, the schedule delay $sd(t_a, t_{a^*})$ for a departure at time t_d and a desired arrival at time t_{a^*} will be a function of t_d, travel time $tt(t_d)$, and t_{a^*}. Even if t_{a^*} is assumed to be constant for individuals in a market segment, the travel time will assume a different value for individuals in that market segment who travel between different origins and destinations.

By a similar argument, for a desired departure time, modeling departure time choice reduces the schedule delay to a constant if desired departure time is assumed to be constant for individuals in a market segment.

To sum up, we model arrival time choice if the trip is arrival sensitive (i.e. with a desired arrival time) and model departure time choice if the trip is departure sensitive (i.e. with a desired departure time). Schedule delay functions become arrival and departure specific constants by market segment.

3.2 Approach 2: Latent Desired Times-of-Travel

An alternative approach to the one described above is to assume a probability density function $f(t^*)$ for the latent (unobserved) desired time-of-travel t^* such that:

$$\int_0^{24} f(t^*)\,dt^* = 1 \tag{6}$$

and

$$f(0) = f(24) \tag{7}$$

Let $P\left(h|t^*\right)$ denote a time-of-travel choice model with an explicit schedule delay term that depends on t^*. Then, the time-of-travel choice probability $P(h)$ can be computed by integrating the conditional choice probability $P\left(h|t^*\right)$ over the density of the desired time-of-travel, as follows:

$$P(h) = \int_0^{24} P\left(h|t^*\right) f\left(t^*\right) dt^* \tag{8}$$

The two techniques discussed above thus account for schedule delay in time-of-travel choice models when desired time-of-travel data are unavailable, which is typically the case especially with revealed preference surveys.

4. MODELING THE 24-HOUR CYCLE

In this section, we discuss the specification of the alternative specific constants of the model. Two points are worth noting. The first point is that instead of using dummy variables for the time periods, we specify these constants as continuous functions of time. The advantages of this approach are (1) the reduction in the number of unknown parameters which need to be estimated, especially if the data do not contain observations for all arrival and departure time periods, and (2) the smoothing of discontinuities in the utility function that would result if dummy variables for the periods were used instead.

The second point is based on the fact that time-of-travel is cyclic. The cycle length for weekday urban trips is 24 hours. The implication of this observation is that the utility of arrival (departure) at a time t should be equal to the utility of arrival (departure) at time $t+24$ hours. Therefore, in addition to using continuous functions of time as discussed above, these functions need to satisfy the cyclicality property. We discuss below two types of functions which can be used for that purpose.

4.1 Trigonometric Function

We make use of the property that for any trigonometric function $y(.)$, we have $y(0)=y(2k\pi)$, where $k \in Z^{+}$. Since for our application we require that $v(0)=v(24)$, we define a mapping function $z_k(t)$ that maps $t=0$ to 0 and $t=24$ to $2k\pi$ as follows:

$$z_k(t)=\frac{2k\pi t}{24},\ 0 \le t \le 24, k \in Z^{+} \tag{9}$$

with $z_k(0)=0$ and $z_k(24)=2k\pi$

Therefore, a utility function which is a trigonometric function of the mapped arguments will then guarantee that $v(0)=v(24)$. Consider for example the following utility function, which is based on the idea of the Fourier series (Fourier, 1822):

$$\begin{aligned} v(t) = {} & \beta_1 \sin\left(\frac{2\pi t}{24}\right)+\beta_2 \sin\left(\frac{4\pi t}{24}\right)+\cdots+\beta_K \sin\left(\frac{2K\pi t}{24}\right) \\ & +\gamma_1 \cos\left(\frac{2\pi t}{24}\right)+\gamma_2 \cos\left(\frac{4\pi t}{24}\right)+\cdots+\gamma_K \cos\left(\frac{2K\pi t}{24}\right) \end{aligned} \tag{10}$$

For sufficiently large K this series can be used to approximate any cyclical function. The coefficients β's and γ's need to be estimated from data.

Letting $t_m(h)$ denote the mid-point of time period h (measured from some arbitrary reference point), the utility of arrival or departure in period h can be expressed as follows, where the mid-point of a time period is used to represent the period:

$$\begin{aligned} V(h) &= v\left(t_m(h)\right) \\ &= \beta_1 \sin\left(\frac{2\pi t_m(h)}{24}\right) + \beta_2 \sin\left(\frac{4\pi t_m(h)}{24}\right) + \cdots + \beta_K \sin\left(\frac{2K\pi t_m(h)}{24}\right) \\ &\quad + \gamma_1 \cos\left(\frac{2\pi t_m(h)}{24}\right) + \gamma_2 \cos\left(\frac{4\pi t_m(h)}{24}\right) + \cdots + \gamma_K \cos\left(\frac{2K\pi t_m(h)}{24}\right) \end{aligned} \tag{11}$$

This utility function satisfies the cyclicality property since $v(t) = v(t+24)$. Note that this trigonometric function is specified as a combination of sines and cosines of angles with different frequencies. Having both sines and cosines in the formulation (as opposed to having only sines or cosines) is needed to ensure that every time t between 0 and 24 will have a unique utility value. Moreover, the use of angles with different frequencies is needed to get a better model fit compared to using only one frequency. The truncation point K could be determined empirically based on the resulting profile of the utility function and the statistical significance of the terms comprising the function.

4.2 Piecewise Linear Function

An alternative to the use of the trigonometric function described above is the piecewise linear function with additional constraints. For a piecewise linear function of time with K breakpoints $b_1, ..., b_K$ between 0 and 24, we define the variables:

$$t_k(t) = \max\left[0, \min\left(t - b_{k-1}, b_k - b_{k-1}\right)\right], \quad k = 1, ..., K+1, \quad t \in [0, 24] \tag{12}$$

where $b_0 = 0$ and $b_{K+1} = 24$. The utility function component corresponding to the piecewise linear function of time can be expressed as follows:

$$v(t) = \beta_1 t_1(t) + \beta_2 t_2(t) + \beta_3 t_3(t) + \cdots + \beta_K t_K(t) + \beta_{K+1} t_{K+1}(t), \qquad (13),$$

where β_1, β_2, β_3, …, β_K, and β_{K+1} are unknown parameters to be estimated. Figure 1 shows a schematic diagram of the utility function given by expression (13) with three breakpoints.

Since $v(0) = 0$, the cyclicality property $v(0) = v(24)$ implies that we need to have $v(24) = 0$. Since $b_K < 24$, we have:

$$v(24) = \beta_1 b_1 + \beta_2(b_2 - b_1) + \beta_3(b_3 - b_2) + \cdots + \beta_K(b_K - b_{K-1}) + \beta_{K+1}(24 - b_K) = 0 \qquad (14)$$

Therefore,

$$\beta_{K+1} = \frac{-\beta_1 b_1 - \beta_2(b_2 - b_1) - \beta_3(b_3 - b_2) - \cdots - \beta_K(b_K - b_{K-1})}{24 - b_K} \qquad (15)$$

Substituting expression (15) for β_{K+1} in the utility expression (13), we obtain:

$$\begin{aligned} v(t) = {} & \beta_1 t_1(t) + \beta_2 t_2(t) + \beta_3 t_3(t) + \ldots + \beta_K t_K(t) \\ & + \left[\frac{-\beta_1 b_1 - \beta_2(b_2 - b_1) - \beta_3(b_3 - b_2) - \cdots - \beta_K(b_K - b_{K-1})}{24 - b_K}\right] t_{K+1}(t) \end{aligned} \qquad (16)$$

Rearranging terms, we obtain:

$$\begin{aligned} v(t) = {} & \beta_1\left(t_1(t) - \frac{b_1}{24 - b_K} t_{K+1}(t)\right) + \beta_2\left(t_2(t) - \frac{b_2 - b_1}{24 - b_K} t_{K+1}(t)\right) \\ & + \beta_3\left(t_3(t) - \frac{b_3 - b_2}{24 - b_K} t_{K+1}(t)\right) + \cdots + \beta_K\left(t_K(t) - \frac{b_K - b_{K-1}}{24 - b_K} t_{K+1}(t)\right) \end{aligned} \qquad (17)$$

The corresponding utility expression for the time period is:

$$
\begin{aligned}
V(h) &= v\left(t_m(h)\right) \\
&= \beta_1 \left(t_1\left(t_m(h)\right) - \frac{b_1}{24-b_K} t_{K+1}\left(t_m(h)\right) \right) \\
&\quad + \beta_2 \left(t_2\left(t_m(h)\right) - \frac{b_2-b_1}{24-b_K} t_{K+1}\left(t_m(h)\right) \right) \\
&\quad + \beta_3 \left(t_3\left(t_m(h)\right) - \frac{b_3-b_2}{24-b_K} t_{K+1}\left(t_m(h)\right) \right) \\
&\quad + \cdots + \beta_K \left(t_K\left(t_m(h)\right) - \frac{b_K-b_{K-1}}{24-b_K} t_{K+1}\left(t_m(h)\right) \right)
\end{aligned}
\tag{18}
$$

Thus, a piecewise linear function with K breakpoints and one of the unknown parameters (β_{K+1}) expressed as a function of the other parameters (β_1 to β_K) requires the estimation of K parameters and satisfies the cyclicality property of time. Note that the choice of breakpoints could be based on aggregate arrival time (or departure time) profiles and apriori knowledge of time-of-travel preferences. They can be adjusted according to the estimated profiles and the statistical significance of the terms comprising the function.

5. EMPIRICAL RESULTS

In this section, we describe a case study that shows the application of the above modeling methods to the San Francisco Bay Area using the 2000 survey. We first present an overview of the approach used and then show selected model estimation results. Additional details on the methodology, model estimation, and application can be found in Cambridge Systematics (2005) and Abou Zeid et al. (2006).

5.1 Overview of the Modeling Approach

We use the 2000 Bay Area Travel Survey to estimate logit time-of-travel choice models using a tour-based approach. The models are estimated for auto tours/trips of different purposes: work, school, shopping, eat-out, personal business, pick-up/drop-off, discretionary, and work-based (subtours). The explanatory variables used include level of service variables (such as travel time), demographic variables, mode (drive-alone vs. carpool), etc. A total of 35 time periods are used, all of which are half-hours except for the first and last periods (early morning and late evening hours) which are of longer duration.

Time-of-travel choice modeling is done at two levels: primary activity and secondary activity. A primary activity of the tour can be defined to be the activity of longest duration on the tour, the activity with highest priority, etc; all other activities are considered secondary.

The primary activity divides the tour into two half-tours. Since scheduling decisions on a tour are interrelated, the two half-tours comprising a tour are scheduled simultaneously. We assume that the half-tour from home to the primary activity is arrival sensitive, while the half-tour from the primary activity to home is departure sensitive. Therefore, we model the joint choice of arrival time and departure time at the primary activity. Since there are 35 time periods, scheduling the tour at this level involves a choice among 630 alternatives (equal to $35 \times (35+1)/2$). An alternative (a, d) is thus characterized by an arrival time period a, a departure time period d, and a duration $t_m(d) - t_m(a)$, and its systematic utility can be expressed as follows:

$$
\begin{aligned}
V(a,d) = {} & \alpha_1(a) + \beta_1 TT(a) + \ln\Delta_a \\
& + \alpha_2(d) + \beta_2 TT(d) + \ln\Delta_d \\
& + \alpha_3\left(t_m(d) - t_m(a)\right) + \cdots
\end{aligned}
\tag{19}
$$

where we have included size variables for the arrival and departure time periods, defined as the number of half-hour periods within a given time period, since not all periods are of equal duration. The schedule delay terms for both half-tours are accounted for by alternative-specific constants by market segment, assuming that desired times-of-travel are constant by market segment.

For secondary activities before the primary activity, we can compute (in model application) the departure time t_d^s from the secondary activity given the modeled arrival time t_a (corresponding to period a) at the primary activity, as follows: $t_d^s + tt\left(t_d^s\right) = t_a$, where $tt\left(t_d^s\right)$ is the travel time corresponding to departure time t_d^s. Time-of-travel choice for secondary activities before the primary activity is then a choice of arrival time from a choice set of at most 35 time periods (periods corresponding to arrival times larger than t_d^s will be unavailable).

Similarly, for secondary activities after the primary activity, we can compute (in model application) the arrival time t_a^s at the secondary activity given the modeled departure time t_d (corresponding to period d) at the primary activity and the travel time $tt\left(t_d\right)$ corresponding to departure time t_d, as follows: $t_a^s = t_d + tt\left(t_d\right)$. Time-of-travel choice for secondary activities after the primary activity is then a choice of departure time from a choice set of at most 35 time periods (periods corresponding to departure times smaller than t_a^s will be unavailable).

5.2 Selected Model Estimation Results

Figures 2 and 3 show the utility function values corresponding to the estimated arrival time functions for a work tour and a school tour model, respectively. In each of these figures, the "base" plot represents the arrival time function net of all interactions (with other variables), and every other plot represents the sum of the "base" plot and the interaction of the arrival time function with the variable that is referenced. Every plot is further divided by the function value at 8 AM, so that for a given plot one can compare the relative rather than absolute utilities across arrival time periods.

For the work tour model, trigonometric arrival time functions with truncation point $K = 4$ are used. The results shown in Figure 2 can be interpreted as follows. Compared to a full-time worker, the utility of arrival to work for a part-time worker increases after 8 AM because many part-time jobs occur during night shifts for example. For a worker without work time flexibility, the utility of arrival before 8 AM increases and the utility after 8 AM (and up to 2 PM) decreases relative to a worker with full or partial work time flexibility; this is expected since workers without work time flexibility prefer to arrive early than late. For a female with kids in the household, the utility of arrival before 8 AM decreases and the utility for most time periods after 8 AM increases relative to a female without kids in the household or to a male; the presence of kids in the household causes later arrivals to be more favorable because of the need to take care of the kids (e.g. dropping kids at school or day care centers, etc.).

For the school tour model, piecewise linear arrival time functions are used with three breakpoints chosen in the following periods: 6:45 – 7:15 AM (b_1 = 4), 7:45 – 8:15 AM (b_2 = 5), and 9:15 – 9:45 AM (b_3 = 6.5), where $t = 0$ represents 3 AM. The results shown in Figure 3 can be interpreted as follows. Relative to a kid of age 5-15, the utility of arrival to school for a kid of age 16-17 increases before 8 AM and decreases after 8 AM; high school students generally arrive at school earlier than elementary school students. For kids where all adults in the household work full time, the utility of arrival earlier than 8 AM increases more than the utility of arrival after 8 AM; when all adults in the household work, the kid is more likely to be dropped off earlier because of the work arrival time constraints of the adult workers. Finally, note that in the time periods close to 8 AM, the effect of the distance variable is to decrease the utility of arrival after 8 AM more than it decreases the utility of arrival before 8 AM; long distance travel does not favor late arrivals.

6. CONCLUSION

This paper has addressed three methodological issues related to time-of-travel modeling: unequal period lengths, schedule delay, and the 24-hour cycle.

We deal with the first issue by using size variables to account for time intervals of different lengths.

Two methods are proposed for modeling schedule delay when desired times-of travel are unobserved. The first one is to use market segment specific utility functions of time-of-travel and model arrival time choice for arrival sensitive trips and departure time choice for departure sensitive trips. The second one is to use a probability density function of the latent desired time-of-travel.

The third issue is that the utility function of time-of-travel needs to be cyclical and can be modeled using either trigonometric or constrained piecewise linear functions.

A case study was presented to demonstrate the time-of-travel modeling methodologies developed in this paper. Tour-based time-of-travel models were estimated using the 2000 Bay Area Travel Survey, and selected model estimation results for work and school tours were presented. 35 time periods, all consisting of 30-minute intervals except for two periods of longer duration, were used in the model. Both the trigonometric and the constrained piecewise linear utility functions were demonstrated. The approach developed here has also been used to estimate time-of-travel choice models for tours of other purposes both at the primary and secondary activity levels. The estimated models have then been applied to the San Francisco County Transportation Authority model, a microsimulation activity-based model, and used to test various scenarios such as highway and transit improvements and congestion pricing. The tests showed that the time-of-travel distributions were reasonable and peak spreading was observed when congestion levels increased. Furthermore, the time-of-travel distributions predicted by the model for a baseline scenario compared favorably with observed patterns. The detailed estimation and application results can be found in Cambridge Systematics (2005).

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LIST OF FIGURES

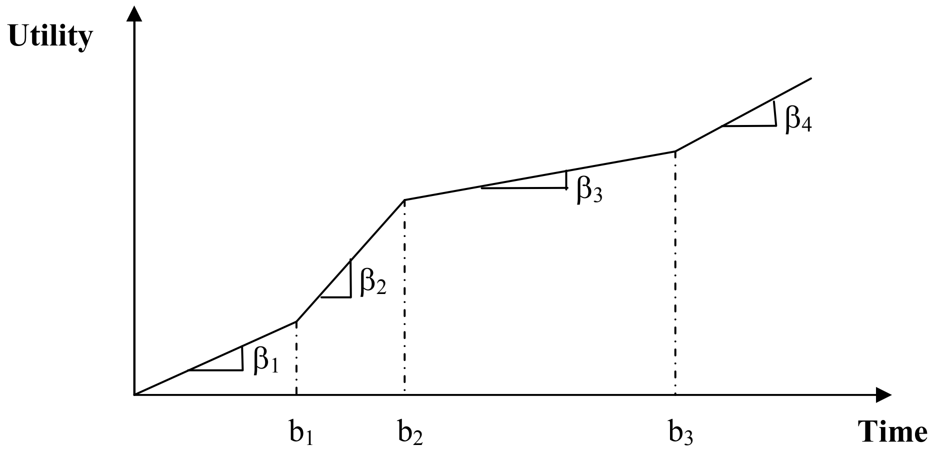
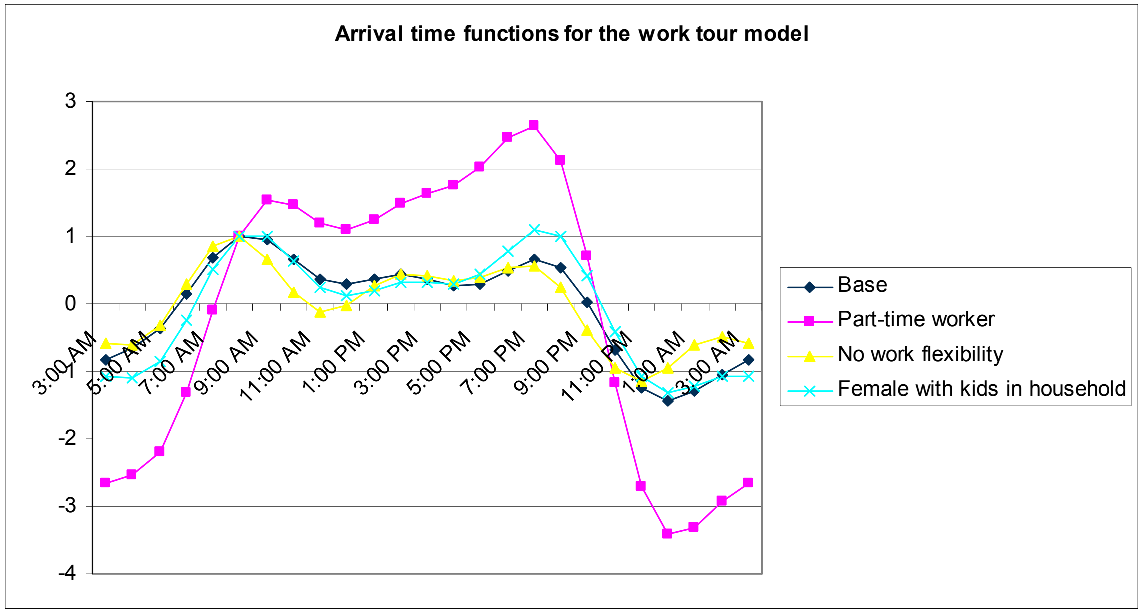
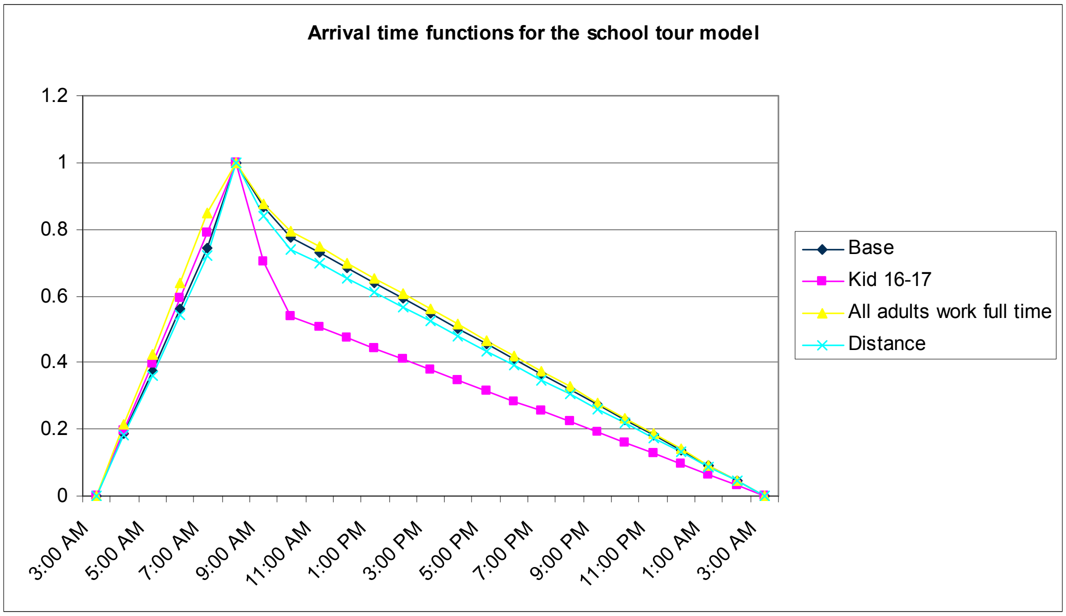


Figure 1



Arrival time functions for the work tour model
3
2
1
0
-1
-2
-3
-4
3:00 AM
5:00 AM
7:00 AM
9:00 AM
11:00 AM
1:00 PM
3:00 PM
5:00 PM
7:00 PM
9:00 PM
11:00 PM
1:00 AM
3:00 AM
Base
Part-time worker
No work flexibility
Female with kids in household

Figure 2



Arrival time functions for the school tour model
1.2
1
0.8
0.6
0.4
0.2
0
3:00 AM
5:00 AM
7:00 AM
9:00 AM
11:00 AM
1:00 PM
3:00 PM
5:00 PM
7:00 PM
9:00 PM
11:00 PM
1:00 AM
3:00 AM
Base
Kid 16-17
All adults work full time
Distance

Figure 3