Freight transportation, holding cost and environment: a simple microeconomic model*

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April 30, 2007

Abstract

We build a theoretical model that is meant to explore the choice of both mode and frequency of freight transportation. Our goal is to understand how the public intervention could limit the greenhouse gas emissions that are released in the transportation industry. We show how the demand for freight transportation is established at the trade-off between holding and transportation costs. The public intervention impacts the demand for freight in case the announced trade-off is sensitive in the tax applied on the fuel. We find that both market constraints (i.e. needs of immediate delivery) and transportation technology constraints reduce the effectiveness of the public intervention.

*This paper is part of a research program financed by ADEME (French Environment and Energy Conservation Agency), Agreement no. 0303c0013. We thank Louis Alligier and Charles Lopez, our colleagues in the project, for very useful discussions. We also thank Claude Pellegrin who provided us very useful guidelines about inventory management.

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1 Introduction

The issue of climate change and greenhouse gases (GHG) emissions is acknowledged today as acute and stimulates political debates for finding ways that limit this negative externality. A reply that has been given till now is that of establishing global limits of emissions of CO2 in various polluting industries, so that the Kyoto objectives can be achieved.

Generally, the pollution level is seen as an effect of production activities. Firms in polluting industries are required to limit their productions to reasonable levels. However, much of the GHG emissions is due to transportation (mainly CO2 emitted when burning fuel), not production activities, though a clear link between the two exists. Indeed, it is intuitive that firms that need to transport their goods can limit the emission of CO2 in the organization of production and distribution flows, by properly deciding both the frequency of distribution and the mode of transportation.

This issue links the inventories literature to the transportation literature. Firstly, the inventory literature has its basis established by Harris (1913). The method relied to his work is named economic order quantity. Subsequently, Lippman (1971) and Aucamp (1982) explore the demand for inputs of a firm (the downstream market), and implicitly its choice of transportation mode and frequency. Burns, Hall, Blumenfeld et Daganzo (1985) and Daganzo (1998) analyze the problem of a producer on the upstream market, who has to ship its goods to the consumption market.

Our work is mainly related to the second kind of literature that we have just mentioned. Contrary to these similar attempts, our model and its results "measure" the effectiveness of the regulatory regime "against" pollution in transportation industry.

We first model the individual demand for freight transportation. We show how the optimization of inventories holding costs endogenizes the cost of transportation. As a result, the demand for transportation is at the trade-off between the holding cost and the transportation cost. Subsequently, we examine the impact of making firms who transport goods pay for the emissions of CO2 in their transport decisions.

We believe that the results achieved by this work will be robust with the policy needs in that it illustrates how the individual needs for freight transportation can be corrected for social goals, by characterizing the entire framework of optimal inventories and transportation activities. Firms maybe induced to shift to less polluting modes and immobilize for a longer time their inventories whenever the regulator determines them to internalize the cost of pollution. Indeed, the understanding of the particular characteristics of freight transportation decisions with respect to production activities, puts in evidence the "instruments" that are "in the hands" of the regulator, at the aim of limiting the GHG emissions through transportation, without reducing the overall production of the goods that need to be distributed for consumption.

The paper is structured as follows. In Section 2 we present the general set-up of the inventory organization of the firm, to which our issue belongs. In Section 3 we find the holding cost of one producer, related to both warehousing and transporting final products. The transportation cost is expressed in Section 4. We describe subsequently in Section 5 the trade-off between the costs previously mentioned. In Section 6 we present a concrete example with data that has been available to the authors. In Section 7 we discuss the policy implication derived from our result. Section 8 concludes.

2 The context of the analysis

We want to characterize the optimal decisions of a firm that firstly produces some good and secondly needs to transport it to its clients. Hence, our problem makes sense if the consumers of the good are located in a different area with respect to the one of the production utility.

We denote by t the period of analysis (i.e., one year). For an easy treatment, we assume that the overall demand D over the period t, is uniformly distributed. The market clears at the quantity level Q that satisfies $Q = D$. The market is competitive, which means that the firm cannot strategically influence it while deciding its production and transportation activities. The consequence that is relevant for our analysis in this assumption is that it is infinitely costly for the firm to produce less than Q during the period t.

The inventory management is generally treated as a problem of choosing optimally the stock holding of inputs, that are to be used in production. The practice distinguishes between two different methods in the organization of the inventory flows. The first one is named the economic order quantity (EOQ). In this kind of organization, the firm organizes a planning of the number of orders and the size of each order of necessary inputs, such that a constant consumption rate is satisfied in the activity. The second method is that of the period order quantity (POQ). In this setup, the firm organizes its activity in such a way that it covers "sudden" consumption of inputs. This is due to uncertainty about the consumption needs. Rather than planning the number and the zise of orders over a long time interval, the firm simply waits that the state of consumption needs is revealed, before making an order for the necessary inputs.

The solution of EOQ is then the size of one lot ordered for production. The solution trades off a higher number of production cycles, which reduces the holding costs of inventories, against the economies created by the fixed costs that are related to the number of production cycles. Such fixed costs are generally "bureaucratic" expenses in ordering new inputs in the activity. The optimal size of inventories that are to be produced after each cycle is named in the literature "the quantity of Wilson" (see Giard, 2003).

We remark then that the second method is "inflexible" in that the seize of the inventories is not really a choice. By contrast, as we show in this study, the first method is quite insightful for our analysis, in which we want to understand the way the inventory management interacts with transportation decisions.

We build our model on the EOQ method. The pecularity of our model is that we characterize the size of inventories of final products (rather than inputs), that are to be transported to the consumption market. We shift then the problem from the choice of input orders, to the one of frequency of outputs. We agree that "bureaucratic" expenses, similar to the ones related to input orders, still hold in our context. Then, the solution to our problem is similar to the quantity of Wilson of EOQ. We "enrich" the solution by adding the cost of transportation in the analysis, which allows us finally to understand how the GHG emission could be internalized in the problem of the producer.

3 Quantity of Wilson, transportation and holding cost

The model of Wilson establishes the optimal holding of the inputs that are used in production. In this setup, the consumption of inputs in the production activity is continuous. By contrast, the entry of inputs in the warehouse is discrete at optimum, as due to the fixed costs associated to each entry.

An additional dimension becomes relevant, once we consider the real world situation that the consumers are generally located in different areas than the one of the production utility. This dimension is the transportation cost, which differs between different sizes of finite product to be sent to destination. We need in this sense the link between production and transportation activities, rather than the one between input orders and production.

We assume that the consumption of the good on the market of consumption is continuous (in the same vein as the EOQ method, that assumes that the consumption of inputs is continue). This hypothesis well accommodates for the real world situations in which enterprises that have large scale productions serve a quite stable demand during the year[1]. By contrast, the technology of transportation, that will be presented in the next Subsection, creates economies from lowering the transportation frequency.

Figure 1 illustrates the timing of production and transportation activities, over

[1]With stochastic demand, EOQ and POQ interact. The model we present in what follows can be complexified in order to consider unpredictable shocks of the demand.

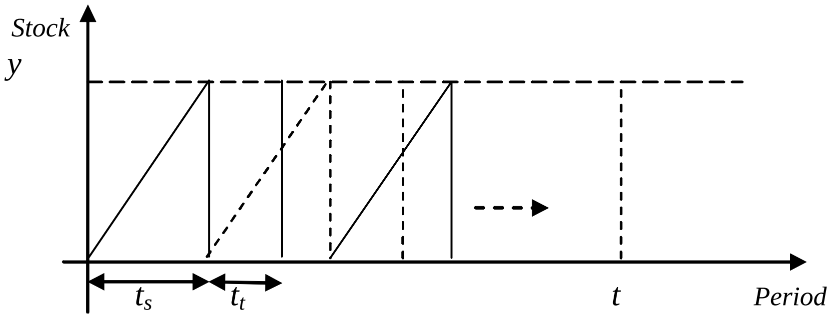

Figure 1: Production cycles and transportation

the period t. t_s is the duration of one production cycle. By t_s we basically define any production period that lies between two shipments of the freight from the place of production utility to the destination. During t_s, the cumulated inventory of finite products is y, where $y \geq 0$. $t_t(K, d)$ is the time required for transporting the batch $y \leq K$, with a transport mode of capacity K, at a distance d. The following equalities are satisfied

$$\begin{aligned} y &= \frac{Q}{n}, \\ t_s &= \frac{t}{n}, \end{aligned} \tag{1}$$

where n is the number of production cycles during the period t.

Our problem is to determine the transportation flow of the goods produced by the firm during the period t. Therefore, we need to find the optimal value of n that minimizes the overall cost of the producer. The equations above show that we can equally search for the unknown value of y, or that of t_s, since Q and t are given.

One additional remark need to be made here. The number n of shippings of the item of quantity y must satisfy two boundary conditions. Firstly, $n \geq 1$, so that $y \leq Q$. We assume in what follows, for practical considerations, that this inequality is strictly satisfied, if not otherwise stated. Secondly, the time t_s must be positive, so that $t_s > 0$. We assume for simplicity that $t_s \geq 1$ (i.e., the production of the quantity y must take at least one day). From a combination of conditions (1) and the inequality just mentioned, the same condition can be written as $y \geq Q/t$. We will call this last inequality in what follows the consumption constraint, as it is determined by the slope of production/consumption of the good. The space of values y is then restricted to the interval $[Q/t, Q]$.

We can now express the similarity between our setup and EOQ. Firstly, any triangle in Figure 1 has the perpendicular segments t_s and y, while the slope of the vertical segment is the slope of production (consumption) of the good. This triangle is equivalent to the one of input orders and production in EOQ method. The average quantity that is warehoused between production and its transportation is $y/2$. This is the quantity of Wilson. Similarly, the average quantity that is warehoused between the moments of arrival at the destination and its consumption is $y/2$. Overall, the average batch is y.

We add now the "immobilization" of the inventory during its transportation. The average inventory holding is changed accordingly, as follows

$$\frac{ynt_s + ynt_t\left(K, d\right)}{t}. \tag{2}$$

The first part, $ynt_s/t = y$ is the average quantity warehoused during one production-consumption cycle. The second, $ynt_t/t = Qt_t/t$, is the average quantity immobilised during the transportation. The duration $t_t(.)$ of one transport is a function of the transport capacity chosen and the distance between origin and destination. (2) is rewritten as

$$y + Q\frac{t_t\left(K, d\right)}{t}.$$

Let us denote now by c_f the opportunity cost associated to the possession of one unit of the good during the period t, instead of having it sold to the client. Also, we denote by c_w the unit cost of the warehouse in which the good is stocked. Therefore, we can write the cost of possession of the good during the period t, namely the holding cost, as

$$C_s\left(K, y, d, Q\right) = c_f\left(y + Q\frac{t_t\left(K, d\right)}{t}\right) + c_w y. \tag{3}$$

The opportunity cost c_f is somehow arbitrarily determined, in that it is defined by the financing service of each enterprise. However, as such costs are generally covered with short term bank credits, c_f can be assimilated to the interest rate required by the bank for such credits. Also, we remark that the warehousing varies between 0 and y, so that the producer needs in her activity a space of size y, that costs $c_w y$.

From the expression (3) and *Assumption 1* stated below, it is straightforward that $C_s\left(y\right)$ is overall increasing in y. Indeed, if the firm could transfer its goods directly to the consumers at no cost, then it is not economically justified to warehouse finite products. This is the key issue that drives our analysis in what follows. The goods need to be transported to the destination, in which case there are economies of larger size of inventory y that is to be transported to the destination.

4 The shipping cost

We characterize now the choice of the transport mode, for any given flow of quantities y that must be transported to destination. In the next Section we derive the optimal quantity y that internalizes both the holding and the transport cost of the firm.

We assume that each vehicle of transportation is defined by its capacity K, where $K > 0$. We denote by $p(K, y)$ the function of the price of each capacity unit and one unit of distance between origin and destination. Therefore, for a vehicle of capacity K, the cost of transporting a quantity y over one unit of distance is $yp(K, y)$. We make now two hypotheses, which are essential for the rest of our analysis and reflect in fact the technology of the transportation industry.

Assumption 1 *The duration of transportation is such that* $\partial t_t(K, d)/\partial d > 0$.

Assumption 2 *2.1. The price* $p(K, y)K$ *of one transport mode increases in* K. *2.2 The unitary price* $p(K, y)$ *is such that* $\partial p(K, y)/\partial y < 0$ *and* $\partial^2 p(K, y)/\partial y > 0$, *for any* $y \leq K$. *Moreover* $sp(K, K) > p(sK, sK)$, *for any integer* $s > 1$.

Assumption 1 states that the period of transport increases in the distance to be made, at any given capacity. On the other hand, $t_t(K, d)$ is not necessarly monotonic in K. For instance, the duration of river transportation is higher than the one of the road, over the same distance. On the other hand, transporting freight by airplane is faster than transporting it by motorway. *Assumption 2.1* states that transporting a quantity y with a mode of size $K > y$ is not optimal. By *Assumption 2.2*, any transport of a batch y with several vehicles of capacity $K < y$, rather than one vehicle of capacity $K = y$, is suboptimal. Overall, the optimal capacity to be chosen is the smallest capacity k that satisfies the condition $y \leq k$. Also, as we show later on with real data, $p(K, y)$ is a convex function of y.

We denote by $\overline{k}$ the maximal capacity of transportation, that is technologically possible. The cost of transporting the batch y with capacity k over a distance d is then

$$P(k, y, d) = dyp(k, y),$$

where

$$k = \begin{cases} \{\min K/K \geq y\}, & if\ y \leq \overline{k} \\ \overline{k}, & otherwise \end{cases} \tag{4}$$

We remember that the number of transports realized during the period t is $n = Q/y$. Therefore, the transportation cost of the period is

$$C_t(k, y, d, Q) = nP(k, y, d) = Qdp(k, y)$$

5 The trade-off between holding cost versus transportation cost

We show in this Section the way the demand for transportation of a vertically integrated firm is formed.

We denote by c_e the fixed cost associated to any shipping of the batch y. Over the period t, the cumulated fixed cost for preparing the expedition is

$$nc_e = \frac{Q}{y}c_e$$

The overall cost borne by the producer in holding the production y and transporting it with transport capacity k is

$$\begin{aligned} C(k, y, d, Q) &= C_s(k, y, d, Q) + C_t(k, y, d, Q) + \frac{Q}{y}c_e \\ &= c_f\left(y + Q\frac{t_t(k, y, d)}{t}\right) + c_w y + Qdp(k, y) + c_e\frac{Q}{y} \end{aligned} \tag{5}$$

The problem of the producer is then the following

$$\left\{ \begin{array}{c} \underset{y,k}{Min} C(k, y, d, Q) = C_s(k, y, d, Q) + C_t(k, y, d, Q) + \frac{Q}{y}c_e \\ s.t. \\ k \leq \overline{k} \\ y \geq Q/t,\; y \leq Q \end{array} \right. \tag{P}$$

The first constraint derives from the transportation technology (Section 4) while the second and the third ones derive from the equality between the slope of the consumption and that of production of the good (Section 3). The function $C(k, y, d, Q)$ is convex in y and the constraints are linear. Therefore, a solution y^* exists and it is unique.

We remark in the formulation of the problem (P) the relevance of the transportation decision of the producer for environmental policy. Whenever the slope of the demand and the transport technology do not constrain the objective function, the transport mode is chosen such that $y = k$, where k solves the trade-off between the holding cost, increasing in k, and the transportation cost, that is decreasing in k. However, the market and the transport technology limit the choice of the transportation mode. We deal with this issue in the next Subsection.

5.1 The solution

We denote by y^* the general solution of (P) and by y^u the solution of the unconstrained problem. We find

$$y^u = \sqrt{\frac{Qc_e}{c_f \left(1 + Q\frac{\partial t_t(k,y^u,d)/\partial y^u}{t}\right) + c_w + Qd\frac{\partial}{\partial y^u} p\left(k, y^u\right))}}. \tag{6}$$

Let us distinguish now the regions in which the solution y^u lies, once we consider the impact of the production technology constraint and the transportation technology constraints in the problem.

Case 1 $y^u < Q/t$

A corner solution arises in this case, which is $y^* = Q/t$. This is the situation in which the slope of production (consumption) of the good is such that the consumption constraint (see Section 3) is binding. The firm has, in this case, no flexibility in its transportation decisions. The batch that has to be transported to destination cannot be in the region $y > Q/t$, without rationing the consumers of the destination market[2]. Moreover, if $Q/t \leq \overline{k}$, then, by *Assumption 2*, the producer chooses the smallest transport capacity k that satisfies this inequality. Otherwise, $\overline{k}$ is to be chosen.

The number of vehicles that are necessary for one sending of the batch y is $s = 1$, whenever $Q/t \leq \overline{k}$. Otherwise, s is the upper integer approximation of $Q/\left(t\overline{k}\right)$.

Case 2 $Q/t < y^u \leq Q$.

In this case the constraints of production (consumption) frequency are both slack. It remains to find the impact of the transportation technology.

Case2.1 $y^u < \overline{k}$

The solution is $y^* = y^u$. Moreover, the smallest transport capacity k that satisfies $y^u \leq k$ is chosen. The solution is interior in this situation and it satisfies the desirable trade-off.

Case 2.2 $y^u > \overline{k}$

Then, $\frac{\partial}{\partial y^u} p\left(k, y^u\right) = 0$. Therefore, the solution of the problem is $y^* = \max\left(y^c, \overline{k}\right)$, where y^c is a reduced form of the expression of y^u (see (6)), as follows

$$y^c = \sqrt{\frac{Qc_e}{c_f\left(1 + Q\frac{\partial t_t(k,y^c,d)/\partial y^c}{t}\right) + c_w}}$$

[2]We note that under our initial assumption of continuous consumption of the good, rationing the consumers is infinitely costly for the firm. A more complete analysis would be to investigate the strategic decision of the firm, whether to rationize or not the consumers, according to the market competition. This is left for further research.

The number of vehicles required for one transport is $s = 1$, whenever the solution is $y^* = y^u$. Otherwise, s is the upper integer approximation of $y^*/\overline{k}$.

These are the two relevant cases that define the solution y^* in the problem and, consequently, the choice of transport capacity. We show in the next Section (where we present a reduced form of the unconstrained solution y^u) that there exists a certain threshold $\underline{Q}$ that separates the regions of total production of the period t for which either one of the two cases is satisfied.

Finally, we can establish the demand for transportation of one producer, and its potential impact on the policy implementation for reducing the traffic in freight transportation. The firm requires vehicles of capacity $k = y^*$ (or $\overline{k}$, such that $\overline{k} < y^*$), for the transportation of the quantity Q, during the time interval t. The total number of vehicles that form the demand of the firm during the period under scrutiny, denoted m, is the upper approximation of Q/k, where $k = y^* < \overline{k}$, or $k = \overline{k}$.

5.2 Discussion

The trade-off between holding cost and transport cost is easier reflected by the reduced form unconstrained solution, expressed by y_r^u below. Indeed, let us consider the case where $c_e = 0$ and $\partial t_t/\partial y = 0$ at any y. These simplifications are without loss of generality as in reality they have little impact on transportation decisions. For instance, the duration of the transport is not really different between different transport capacities of the same mode (as for instance, two tracks of different dimensions). Then, from (5), y_r^u solves

$$-\frac{\partial}{\partial y_r^u} p\left(k, y_r^u\right) = \frac{1}{Qd}\left(c_w + c_f\right) \tag{7}$$

We can then establish from (7) and *Assumption 2.2* that the optimal inventory y_r^u

1) increases in Q

2) increases in d

3) decreases in c_f and c_w

4) increases in $-\frac{\partial}{\partial y^u} p\left(k, y^u\right)$

Indeed, the sensibility of y with respect to its determinants in quite intuitive. For instance, 1) states that firms that have very high production capabilities and serve large sizes of the demand are very likely to use big transport capacities, like railways or shipment. The small producers are more likely to use roads. 2) is the immediate consequence of the analysis of Section 3, as c_f and c_w are the parameters that define the holding cost. 3) derives from the technology of the transportation industry, that we stated in *Assumption 2*. Basically 2) and 3) show how the holding and shipping costs are traded-off at optimum, while 1) shows that the trade-off is different between enterprises that have different production levels.

Let us add the consumption constraint in the problem. There exists a threshold of quantity $\underline{Q}$, for which the first order condition (7) is satisfied exactly at the inferior boundary of the feasible set $[Q/t, Q]$. Therefore, if $Q = \underline{Q}$ then the solution is $y^* = y_r^u = \underline{Q}/t$ and $\underline{Q}$ solves

$$-\frac{\partial}{\partial\left(\underline{Q}/t\right)} p\left(k, \underline{Q}/t\right) = \frac{1}{\underline{Q}d}\left(c_w + c_f\right) \tag{8}$$

By 1) we can distinguish three relevant regions of the quantity Q in the determination of the solution. At any Q such that $Q < \underline{Q}$ the solution is interior (*Case 2* of the previous Subsection). In the region $Q > \underline{Q}$ a corner solution (*Case 1*) should arise, according to (8). However, by 1), there exists a third region of Q for which the solution is again interior. We will see in the next Subsection how the boundary between the three regions is determined, in a concrete example.

The model we have developed in this study shows that the producers should be sensitive to the cost of shipping while deciding the transport capacity and, consequently, the frequency at which they choose to transport their products. From the social perspective, it would be desirable that the producers reduce as much as possible the frequency, by choosing vehicles of high capacity. Moreover, the modes of large capacity are also emitting less GHG per ton-km then the small ones. For instance, the rail transportation is emitting less GHG per ton-km than the motorway transports (see Section 7).

The policy implementation that could used in this sector at the aim of limiting the GHG emissions is that of a fee added to the normal price of the oil. This policy would be effective if the producers were sensitive to the price of the transport, so that the trade-off we have mentioned were established at lower frequencies.

However, we show in what follows that the trade-off expressed above is not really sensitive in the shipping cost, in which case the producers are not likely to internalize the GHG emission in their inventory management. We use for this illustration real data from the French market.

6 Example

Let us assume first that the road transportation is the only mode available on the market. We present first the shipping parameters, for which we use the values

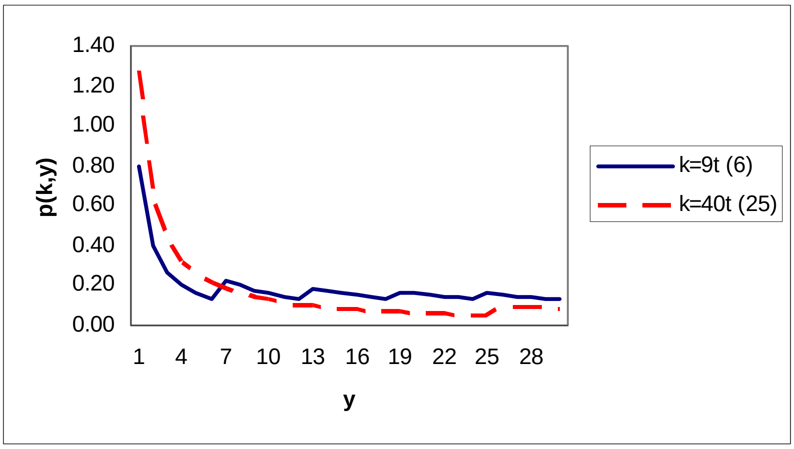

Figure 2: $p(k, y)$ and y for road transportation

indicated by the official website of CNR[3]. We add to this information the data about the payload of each category of vehicle. These values are presented in Table 1.

We calculate then $p(k, y)$ by using these values. For instance, consider a vehicle that has a grossweight of 9 (tons). Because its payload is 6, $p(6, 4) = 0.8/4$ ($euros$), while $p(6, 6) = 0.8/6$. The cost of transportation with this vehicle is minimized if $k = 6$. Figure 2 illustrates comparatively $p(k, y)$ for $k = 9$ and $k = 40$. We observe that the curve of $p(9, y)$ is below (over) the one of $p(40, y)$, whenever $y \leq 6$ ($y > 6$).

In case the batch to be transported is in the region $y \leq 6$, then the smallest vehicle that is technologically possible has the lowest cost. Indeed, a rational producer would not choose to transport its goods with a vehicle of higher capacity if it does not expect to fill it with products, as stated by *Assumption 2.1*. In the region $y > 6$, as already told, the vehicle of capacity 40 has a lower shipping cost. This comparison shows that there are economies in using larger transportation modes. Absent any holding cost and absent the consumption constraint, the rational producer would prefer to transport its goods with lower number of vehicles, of capacity 40, rather than more vehicles, of capacity 9, whenever y is in the region $y \geq 6$.

Let us introduce now the holding cost. We assume that the firm produces sugar and it has to transport its product on a distance of 500 km. Each transport from the warehouse to the destination takes generally one day. The annual cost of warehousing goods in France, according to Terrier (2006), has an average of 80 $euros/ton$. The cash flow brought by 1 ton of sugar is around 630 $euros$. We calculate the unit cost c_f of holding the sugar as $c_f = r * 630$ $euros/year$, where $r = 0.15$ is an annual rate which represents the opportunity cost of the good that is produced, but not sold.

[3]Comité National Routier, website www.cnr.fr; this is a French institution whose activity is to evaluate and do research on the road transportation market in France.

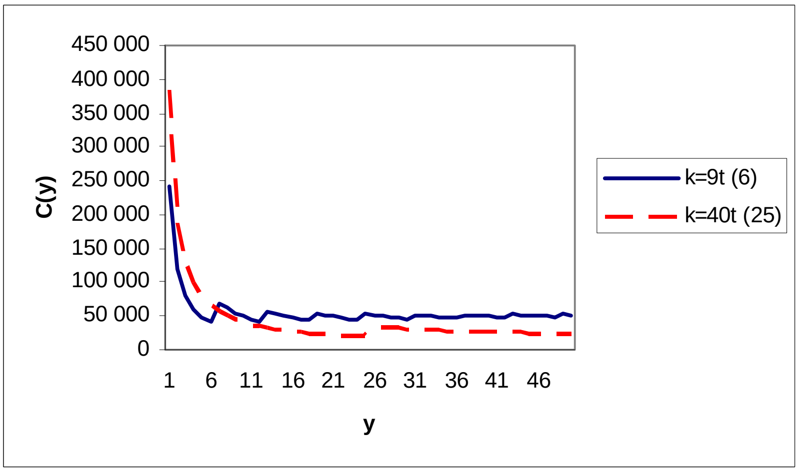

Figure 3: $C(y)$ and y for road transport when $Q = 3000$ (in *tones*)

We observe in the theoretical model that the three cases of Subsection 5.1 can be illustrated at different production scales Q, while keeping the other parameters constant.

For instance, if we take $t = 250$ days, as the average number of working days during the year, then (8) is solved at $\underline{Q} = 4000$. From our previous analysis, an interior solution is realized for quantities that are in the region $Q \leq 4000$. Take for instance $Q = 3000$ (*tons*). The relevant set of y that satisfies the consumption constraint is $[12, 3000]$.

Table 3 presents some values of $C_s(y)$, $C_t(y)$ and $C(y)$, while Figure 3 illustrates $C(y)$ between the two different transport capacities. This example reveals that for small producers, the cost of transportation is much larger than the holding cost. We are then in Case 2.2, in which the solution is $y^* = 25$. No trade-off exists, but the social goal is achieved, in that the biggest vehicle available on the market is chosen by the producer.

We assume now that $\underline{Q} = 4000 < Q < 6200$. A corner solution arises, so that $y^* = Q/t$ (Case 1). The firm cannot "freely" choose between the transport capacities that are available on the market, in which case the frequency of transport during the year is given to the firm. If $Q > 6200$, the solution is interior (Case 2.1). The biggest capacity is chosen, both in Case 1 and 2.1, but the vehicules are charged entirely only in the second case.

We can now "extend" the transportation market to other modes that have higher capacities than the road. We take as example the rail transportation. The average cost of a train of 500 tons (20 wagons) according to the data available about the French market is of 15.3 *euros* per each *km* (SNCF, 2002). Then, $p(k, y)$ can be calculated in a similar way as we did for the road transportation case. In Germany, the average cost is said to be very different, of 25 *euros* (Doll, 2005). Figure 4

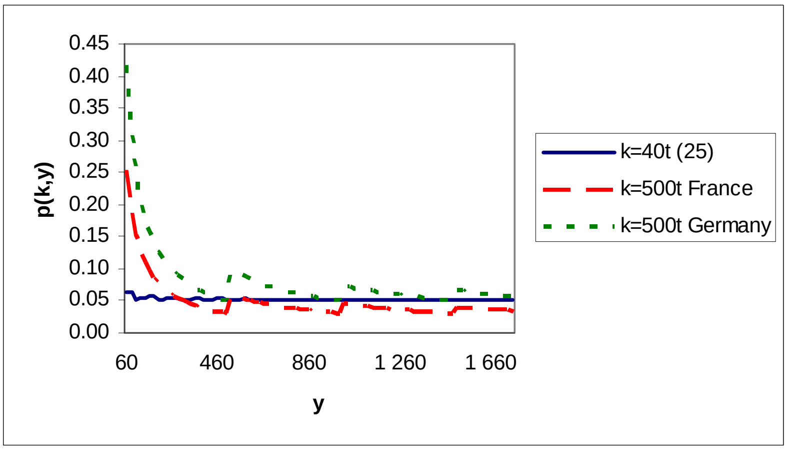

Figure 4: $p(k, y)$ and y for road ($k = 40t$) and rail ($k = 500t$) transportation

illustrates comparatively $p(k, y)$ for rail and road transportation on French and German market.

We observe in the figure that the rail freight transportation dominates the road in France, if the batch to be transported by rails is over a threshold of about 350. Consequently, the producer prefers to transport her goods by rail if Q is over a threshold that is at about $Q = 7000$. According to the price available about the German market, the rail transport is chosen only for very huge production scales $Q \geq 350000$).

7 Policy implication

We can conclude from our simple framework that the producers who need to transport their goods benefit from the economies of less frequent transportation only when Q is neither too small (Case 1), nor too high (Case 2.2). Otherwise, the producer is always constrained by the slope of the consumption at the destination market. One needs to identify then what is the range of quantities for which the rail would be a preferable option, from the social perspective. Our result, though very simple, reflects the current reality of the French market on which the railroads are very little chosen by producers that have "intermediary" production scales.

In order to understand when a transportation mode should be preferred to the other, we need to understand for which quantity of the good to be transported, one mode is socially preferable to the other. From EpE and ADEME (2005) and ECMT (1998), we can present the CO_2 emissions in kg per ton-km for road/rail transportation, as illustrated in Figure 5. The electric rail in France is much less polluting than the diesel rail transportation, since most of the electricity produced is from nuclear power plants. The curve of the average pollution per quantity of

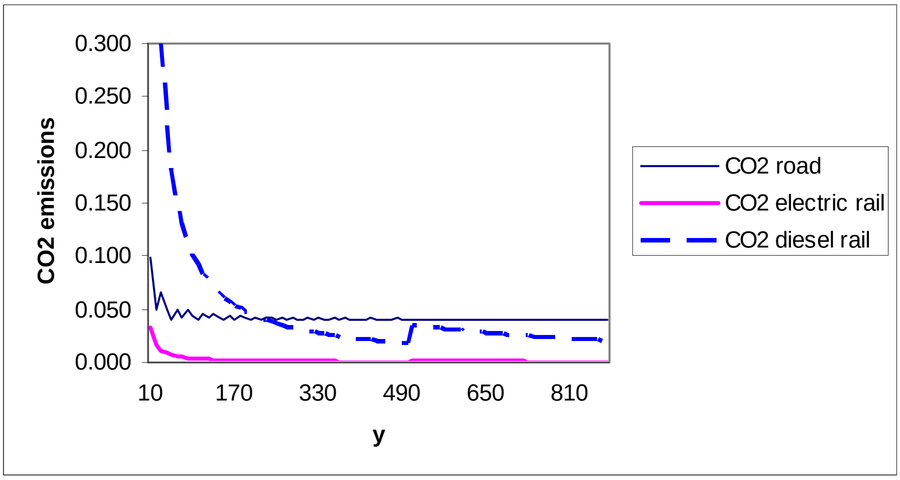

Figure 5: The emission of CO_2 in kg per tonne-km by road/rail transportation

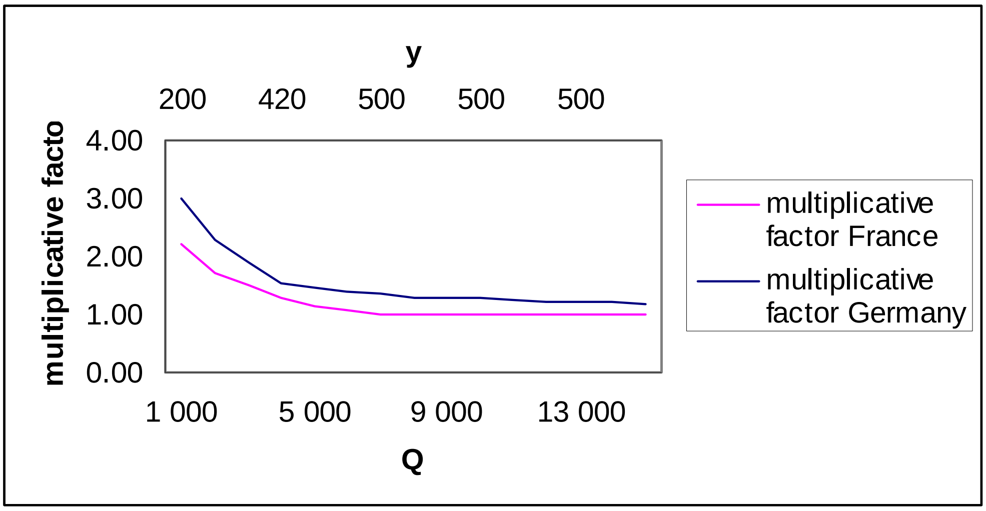

Figure 6: The threshold multiplicative factor that makes the producer indifferent between road/rail transportation

good transported with electric trains is always below its correspondent on the road transportation. The transportation by diesel trains shows to be also preferable to the road if y is such that $y \geq 230$.

Let us analyze now the instruments that are available to the planner. As the only social objective of our study is that of inducing the firms to internalize the GHG emission in their shipping decisions, we do need to model the social problem.

The common instrument that is used today in transportation industry in order to achieve the objective of GHG emissions reduction is that of a tax applied to the price of the fuel. In the example above, the price $p(k, y)$ of the road transportation already includes such tax. Even though the price is effective in stimulating the capacity 40 on the road instead of 9, there is some range of production scales for which it were socially optimal if the rail were chosen. This range is also very different between the French and German market.

In Figure 6 we show for different scales of activity the amount of the multiplicative factor to apply to the price of the road transportation. As the intuition suggests, the curve of such tax applied to the German market is over the one applied on the French market. Moreover, the function of the threshold is downward sloping, which means that at high production the firms internalize easier the social goal in their problem.

8 Conclusion

We have shown with a very simple model that the producers care about the economies created to them by high capacities of transportation, an essential issue of environmental problems of GHG emission. However, such decision depends on the trade-off between holding and transportation cost. We have shown that such trade-off exists only if the scale of production has some intermediary range.

From the social perspective, a relevant fact is that producers with different production scales have different sensitivity to the price of the transportation, with direct impact on the implementation of the tax policy. Indeed, in areas dominated by small producers, a lower frequency of transportation is achieved just in case the multiplicative factor is significantly high. As Figure 6 shows, a tax policy is much more effective in areas of high scale industries with respect to small scale ones.

We remark that the inventory management of the producer in our set-up is not constrained by any specific requirements of her clients. We have assumed that the firm can organize the circulation of her products such that a constant consumption of the good is realized. In reality, the clients, like supermarkets, make specific orders of goods that should be in their warehouse in certain amounts and at specific times, which means that they have bargaining power over the producer. This issue is left for future research.

We assumed all around our study that the logistics management of the producer is such that the vehicle transports only her goods. Obviously, a solution for reducing the number of shipments is that of consolidating the transportation of different production units of the same area. However, additional transaction costs would arise and limit the willingness of the producers for such consolidations.

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Table 1 (road)

Grossweight (tons)	Payload (tons)	Price/(vehicule*km) (euros)
9	6	0.8
26	16	1.15
40	25	1.28

Table 2 (river)

Grossweight (tons)	Price/(vehicule*km) (euros)
1350	19.7
4400	37.8

Table 3 (road)

	Ct(y)		Cs(y)	C(y)	
y	9t (6)	40t (25)		9t (6)	40t (25)
1	2 000 000	3 200 000	467	2 000 467	3 200 467
2	1 000 000	1 600 000	557	1 000 557	1 600 557
3	666 667	1 066 667	646	667 313	1 067 313
4	500 000	800 000	736	500 736	800 736
5	400 000	640 000	825	400 825	640 825
6	333 333	533 333	915	334 248	534 248
7	571 429	457 143	1 004	572 433	458 147
8	500 000	400 000	1 094	501 094	401 094
9	444 444	355 556	1 183	445 627	356 739
10	400 000	320 000	1 273	401 273	321 273
11	363 636	290 909	1 362	364 998	292 271
12	333 333	266 667	1 451	334 785	268 118
13	461 538	246 154	1 541	463 079	247 695
14	428 571	228 571	1 630	430 202	230 202
15	400 000	213 333	1 720	401 720	215 053
16	375 000	200 000	1 809	376 809	201 809
17	352 941	188 235	1 899	354 840	190 134
18	333 333	177 778	1 988	335 321	179 766
19	421 053	168 421	2 078	423 130	170 499
20	400 000	160 000	2 167	402 167	162 167
21	380 952	152 381	2 256	383 209	154 637
22	363 636	145 455	2 346	365 982	147 800
23	347 826	139 130	2 435	350 261	141 566
24	333 333	133 333	2 525	335 858	135 858
25	400 000	128 000	2 614	402 614	130 614
26	384 615	246 154	2 704	387 319	248 858
27	370 370	237 037	2 793	373 164	239 830
28	357 143	228 571	2 883	360 025	231 454
29	344 828	220 690	2 972	347 800	223 662
30	333 333	213 333	3 062	336 395	216 395