EMPIRICAL METHODS FOR IMPROVING TRANSIT SCHEDULING

by
Mark Abkowitz
Israel Engelstein
Department of Civil Engineering
Rensselaer Polytechnic Institute
Troy, New York 12181
USA

1. INTRODUCTION

It is generally agreed that precise knowledge of transit vehicle run times is an important operational problem that impacts both travelers and operators. Operators rely on run times for both setting schedules and allocating vehicles to routes. Travelers are affected in terms of reliability of service, which stems directly from the predictability of vehicle run times. The proper amount of run time and slack to build into a schedule, and how to control real-time reliability problems, can have a profound effect on service regularity and thus on the productivity and efficiency of transit operations.

Many researchers have studied factors affecting bus running time and what corrective actions should be taken when reliability deteriorates (Turnquist and Bowman, 1979; Barnett, 1974; Jackson and Stone, 1976, Koffman, 1978). In the absence of empirical data due to the high costs associated with direct observation, most studies have been restricted to the use of models which are theoretically-based and those developed from simulation. A major limitation of this work has been the assumptions made in order to represent actual transit operations.

Recently, extensive empirical data has been collected on bus routes in several U.S. cities. This data consists of several days of observations of many transit vehicles operating on each day, at various points along the route. In some cases, information on physical and dynamic characteristics of transit route segments, headways and passenger wait time was also collected.

The availability of this data creates the opportunity to conduct empirical analyses of the factors affecting transit vehicle running time and their effect on bus bunching and passenger wait time. These analyses could then serve as representative inputs to the schedule and real-time control decisions cited earlier.

The primary objective of this research is to make available to both planning departments and street supervisors methods for improving service regularity through proper scheduling and real-time control, based on models developed and validated from empirical data. An interest in developing transferable models and methods which are simple in nature and do not require extensive data from the property utilizing the methods is an important motivating factor in the research. The availability of empirically-based methods of this kind would alleviate the need to analyze individual problems by manually collecting extensive amounts of data at each property, yet permits the identification of planned and real-time schedule modifications that can be implemented to simultaneously improve efficiency and productivity.

2. RESEARCH METHODOLOGY

The research design consists of analyzing six sequential steps:
(1) mean running time
(2) running time variation
(3) headway variation
(4) passenger wait time
(5) identifying the optimal control strategy
(6) operator compatibility with the developed methodology
Steps 1-4 are interdependent issues which, once resolved, serve as inputs to the fifth step. The last step concerns transfering the research results into an environment which is compatible with the transit operator.

Mean running time is defined as part of the scheduling process because the schedule and timetable are based on the mean running time. The research emphasis is on the temporal and spatial factors affecting mean running time.

Running time variation is an important measure in defining unreliable service. The causes of running time variation and the degree to which running time variation propagates as the vehicle proceeds down the route are both issues of interest. A priori, one would expect that running time variation is correlated with mean running time and that delays tend to accumulate once a vehicle falls behind schedule.

It has been proven theoretically and demonstrated empirically that the waiting time of passengers at stops is related to the headway variation. To be able to reduce the headway variation effectively, it is important to know what causes it and how the headway variation propagates along the route. It is also important to understand the relationship between the headway variation before and after the control stop, and to what degree a control strategy causes a reduction in the headway variation.

The effectiveness of headway-based and schedule-based strategies are considered. A headway-based strategy is defined here as holding the bus to a certain amount, x_0. If the coming headway is less than x_0, the bus is held up to x_0. If the coming headway is greater than x_0, the bus is not held. Headway-based holding is most suitable for routes operating with shorter headways that are uniform. When headways are short and uniform, passengers are assumed to arrive more randomly at stops and they are primarily concerned about the headways and not the schedule. Similarly, operators are concerned about keeping vehicles evenly spaced so that vehicle availability remains stable.

Schedule-based holding is considered most suitable for long headway routes where the schedule is not as tight and allows for an easy implementation of this procedure. It can also be suitable in the case where headways are uneven and the schedule is designed to meet certain demand requirements. In both cases, the passenger concern is "not to miss" a certain bus, so the buses should adhere to schedule. To implement a schedule-based policy, there is a need to construct a reasonable schedule and enforce adherence to it, such as proper incentives for drivers and a mechanism for accurate monitoring of their performance.

Within this framework, the choice of where to locate a holding point is extremely important. The criteria often used to address that issue is to minimize passenger wait time. For this reason, the relationship between scheduled headway, headway variation and wait time is examined. In this study, empirical wait time models covering a range of 3-12 minute headways are estimated and the results are compared with theoretical wait time models and other researchers' findings.

After completing the first four phases of the research, an optimization routine is developed to determine: 1) if control is effective on the route, 2) the appropriate holding strategy to implement given schedule characteristics and 3) where to locate the control stop and the optimal holding time given route and schedule characteristics. For the headway-based strategy, the objective is to minimize the total waiting time of passengers, including those delayed onboard the vehicle at the holding point. For the schedule-based strategy, the objective is to maximize the effectiveness of control, where effectiveness is defined in the subsequent discussion. Different control strategies (schedule vs. headway-based) are considered for different scheduled headways, as passenger arrival patterns are hypothesized to vary according to scheduled headway.

An important issue to consider is the eventual implementation of the models and methods by the transit operator. This can be accommodated by developing computer software so that the decision methodology can be utilized. The software is programmed for a microcomputer system, since many transit operators are presently using or considering microcomputers in managing their operations and the program could be used by them without incurring additional cost.

3. RESEARCH RESULTS

3.1 Mean Running Time

The data used in this analysis was collected in 1978 from Queen City Metro in Cincinnati, Ohio by General Motors using automated vehicle monitoring (AVM) equipment. The data consisted of observations on two bus routes, each roughly ten miles in length, which travel over city streets, extending radially from the CBD along a traffic corridor. The routes run into the CBD and return to the suburban origin. Outside of layovers (time spent between the end of the previous run and beginning of the next run) at the CBD and suburban terminal, no other holding points are used on these routes. Peak period headways are 12 minutes, dropping off to 15-20 minutes during the off-peak. These routes have qualities which are common to transit routes in many metropolitan areas (Bevilacqua et al., 1979). Additional information on the routes included physical characteristics (length between observation points, number of traffic signals, parking restrictions, stop signs, yields and unsignalized intersections) as well as dynamic characteristics (average boarding and alighting, average number of stops made, time of travel, direction of travel). This data was segmented by observation point and operating period (Abkowitz and Engelstein, forthcoming).

The analysis focused on determining the physical and dynamic factors affecting mean running time. This was accomplished using linear regression with mean running time as the dependent variable and the route characteristics as the independent variables. Model specification in all phases of this research was guided by a criteria which included consideration of variables that could be justified _a priori_ as explanatory variables of the dependent variable, had the expected coefficient signs and had statistically significant coefficient estimates (t-statistics). The overall statistical fit of the model (corrected R^2) and potential dependencies among the independent variables were also considered.

The final mean running time model appears in Table 1. It was found that mean running time is highly influenced by trip distance, people boarding and alighting and signalized intersections, and to a lesser degree by parking restrictions on the route, time of day and direction of travel. The model results tend to confirm prior views. The order of importance of the explanatory variables also seems reasonable. The finding that running time is positively related to the number of signalized intersections is consistent with observations made by Welding (1957).

It is interesting to note that the value of the constant implies a maximum average running speed of 21 miles per hour. Adding the average numbers of boarding, alighting, signals and typical parking restrictions, the average running speed decreases to 14 miles per hour. These values are quite reasonable for bus movement in an urban corridor.

3.2 Running Time Variation

Running time deviation models were also estimated using data collected in Cincinnati. Similar route characteristics were considered as potential independent variables in the model specification and an additional variable representing running time deviation at the previous observation point was defined to measure propagation effects and to counteract the autocorrelation which was expected between adjacent segments of the route.

The final model appears below:

$$s_t = 3.73 + 1.00\ s_{t-1} + 11.95\ d_t \qquad (1)$$

corrected $R^2 = 0.89$

where s_t = standard deviation of running time at observation point t (seconds)

d_t = distance between points t and t-1 (miles)

It strongly suggests that running time deviation at early points on the route propagates as the vehicle proceeds further downstream. This result is consistent with observations made by Doras (1979) on bus routes in Paris and by Loo (1981) in a study of a Minneapolis bus route. Additional deviation is introduced on each subsequent route segment, primarily as a function of the length of the segment. This finding agrees with observations made by Welding (1957) and Sterman and Schofer (1974), who both concluded that route length was positively correlated with their reliability measures.

The results imply that immediate corrective actions to service instability warrant serious consideration. It also suggests that structuring shorter routes may lead to service improvement, provided that the improvements outweigh the additional delays incurred by passengers who must transfer. Finally, the lack of significance of other explanatory variables indicates that road problems influence delay and only indirectly the variation, implying that it might be better to improve reliability by investing in better control than to change route characteristics (e.g., signals).

TABLE 1. MEAN RUNNING TIME MODEL

$$\begin{aligned}\text{Mean Running Time (seconds)} = {} & \beta_1 + \beta_2 * (\text{link length}) + \beta_3 * (\text{pax. boarding}) + \beta_4 * (\text{pax. alighting}) \\ & + \beta_5 * (\%\ \text{on-street parking}) + \beta_6 * (\text{signalized intersections}) \\ & + \beta_7 \quad (\text{if daytime off-peak}) + \beta_8 \quad (\text{if afternoon peak}) \\ & + \beta_9 \quad (\text{if outbound travel})\end{aligned}$$

VARIABLE	COEFFICIENT VALUE	T-STATISTIC	VARIABLE MEAN	AVG. CONTRIBUTION (coef. value * variable mean)
Constant	-122.04		1.0	-122.04
link length	216.54	10.89	2.05	443.91
pax. boarding	6.03	5.74	9.37	56.5
pax. alighting	3.83	3.83	11.57	44.3
% on-street parking	114.59	3.15	0.09	10.31
signalized intersections	8.16	5.13	10.64	86.82
if daytime off-peak	30.43	2.16	0.25	7.61
if afternoon peak	41.73	2.78	0.25	10.43
if outbound travel	25.80	1.82	0.5	12.9

No. of Observations = 56

Corrected R^2 = 0.92

$F(8,46)$ = 76.5

Standard Error = 41.9

Durbin-Watson Statistic = 1.92

While the running time deviation model is very reasonable, there was some concern over the ease with which it could be applied, since the running time deviation at the previous point would have to be known. For this reason, a second set of models were estimated using just the mean and variance of running time, segmented by time-of-day according to the following three operating periods:
(1) morning peak - 6:00 a.m. - 9:00 a.m.
(2) daytime off-peak - 9:00 a.m. - 3:00 p.m.
(3) afternoon peak - 3:00 p.m. - 6:00 p.m.

The estimation results were:

Period 1: $s_t = 1.399 + .0454\,\bar{t}$ corrected $R^2 = 0.82$ (2)

Period 2: $s_t = 0.977 + .0529\,\bar{t}$ corrected $R^2 = 0.90$ (3)

Period 3: $s_t = 0.707 + .082\,\bar{t}$ corrected $R^2 = 0.90$ (4)

where: s_t = standard deviation of running time (minutes)
$\bar{t}$ = mean running time (minutes)

These models also show that running variation increases along the route together with the mean running time.

Beyond the interest of this research effort, the running time deviation models can be used to improve schedules by allowing for the appropriate amount of slack so that succeeding runs are unlikely to be impacted by delays on earlier runs. Using an assumption of the distributional form for running time and the running time models, the appropriate slack time can be determined for a given confidence level.

For example, for a normal distribution of running time, if mean running time from terminus to terminus is 30 minutes and the standard deviation of running time is 3 minutes, the operator can be 95% sure of having buses begin the next run on time by allowing just under 6 minutes of slack in the schedule (union work rules are a separate consideration). This analysis can be extended rather easily to determine the vehicle requirements to operate a route given the desired headway, mean running time, running time deviation and confidence level.

3.3 Headway Variation

Headway variation analyses focused on two issues: 1) the relationship between scheduled headways, running time variation and headway variation, and 2) the impact of control on headway variation beyond the control point. The discussion in Sections 3.3 and 3.4 apply only to the headway-based strategy, as the schedule-based strategy does not address regulating headways or the impact of headway variation on system wait time.

The data used in the first analysis was collected in Minneapolis, Minnesota on a 24-mile route running through the downtown area, with the CBD located approximately midway on the route. This route has many branches and a trunk portion of 10 miles where the average headway is 5 minutes, but not uniform. The available data consisted of ride checks on 12 consecutive buses, where the arrival times, departure times and number of passengers boarding and alighting were recorded at each stop. The data was collected for eight weekdays in the afternoon peak period during 1981. The route was split into three segments (northbound, downtown, southbound) and the running time and headway variation were computed at each observation point. The observed headways were modified to separate planned headway variation (because of non-uniform schedule) from actual headway variation due to unreliability.

A linear regression model was estimated, which yielded the following result:

$$s_h = -34.4 + 1.698\ s_t \qquad (5)$$

corrected $R^2 = 0.96$

where: s_h = standard deviation of headway (seconds)

s_t = standard deviation of running time (seconds)

The model represents an extremely good statistical fit, and indicates that the standard deviation of running time and headway are strongly correlated, as expected. However, the data is extremely limited by the previously described characteristics of this route and the fact that one route does not allow for study of the expected effect of scheduled headway on headway variation. This led to a decision to also consider simulation as an analysis tool.

Several researchers have previously used simulation to study real-time control (Bly and Jackson, 1974; Koffman, 1978; Turnquist and Blume, 1980). However, these studies did not explicitly examine factors affecting headway variation, but rather had a more general focus on the overall benefits to passengers.

The inputs to our simulation program included: 1) scheduled headway, 2) average running time to each stop, and 3) variation of running time. The scheduled headway was set at 3, 6 and 9 minutes, and running times were assumed to come from a beta distribution. Stop locations ranged from average running times of 5-90 minutes from the route origin and the coefficient of variation ranged from 0.05 to 0.17. The output of the simulation consisted of headway variation for each combination of scheduled headway, running time and running time variation. These results were used as inputs to model estimation using headway variation as the dependent variable.

The simulation results indicated that the headway variation increases rather quickly near the beginning of the route and then reaches an upper bound. The time it takes to reach the upper bound depends on the scheduled headway and variation in running time. This was borne out by the following model estimation result:

$$v_h = (-12.2 + 6.948\bar{h})(1 - e^{-0.0447\ v_t}) \qquad (6)$$

where: v_h = headway variation (minutes)

$\bar{h}$ = scheduled headway (minutes)

v_t = running time variation (minutes)

The residual mean square for this model is 4.08, indicating a good statistical fit.

The impact of headway-based control on headway variation was also examined using simulation, as no empirical data on control is presently available for model development. Recall, the headway-based approach is to hold to a threshold value, x_0, if the coming headway is less than x_0. The simulation design was to introduce control at stops located 10, 20, 30 and 40 minutes from the route origin, varying the threshold value in 0.5 minute increments from 0 to the scheduled headway. Headways of 3, 6 and 9 minutes were considered. The output measures included headway variation before and after the control stop.

The simulation output provided data for model estimation, which yielded the following result:

$$v_a = 0.5448\, v_b^{0.713} (\bar{h} - x_0)^{0.734} \quad (7)$$

corrected $R^2 = 0.94$

where: v_a = headway variation after control at control stop (minutes)

v_b = headway variation before control at control stop (minutes)

$\bar{h}$ = scheduled headway (minutes)

x_0 = threshold value (minutes)

An important implication of this model is that the headway variation reduces to zero when x_0 is equal to the scheduled headway, independent of the level of variation before control. This finding does not suggest that it is always better to hold according to a threshold of the scheduled headway, since the optimal strategy also depends on the number of people onboard at the control stop and those waiting downstream. The model, when combined with equation 6, implies that the benefits of control are not distributed uniformly to all stops after the control point. Instead, it appears that the maximum benefits are felt near the control point, with the headway variation beginning to increase again downstream until it reaches an upper bound.

3.4 Passenger Wait Time

The wait time analysis was conducted using data collected in Los Angeles as part of the evaluation of AVM equipment implemented at SCRTD. The data was collected on four routes with headways varying from 3-12 minutes. Checkers were located at specific stops on the routes and noted passenger and vehicle arrival times to stops and the weather conditions at the time of observation. Three days of data were collected on each route in both directions, with the exception of one route where only one direction was observed. Separate analyses were performed on the 3-minute headway route and the other routes (8-12 minute headways), since there was reason to expect passenger arrival patterns might be related to the scheduled headway.

The regression estimate for the 3-minute route was:

$$\bar{w} = 77.34 + 0.0028\, v_h \quad (8)$$

corrected $R^2 = 0.66$

where: $\bar{w}$ = average passenger waiting time (seconds)

v_h = headway variation (seconds)

It is interesting to note that the observed wait times were 7% lower than would be predicted using a theoretical model which assumes random passenger arrivals (Holroyd and Scraggs, 1966). Perhaps this can be explained by the many people who might catch a bus by running to meet it, thereby incurring no wait time.

The wait time model for the 8-12 minute headway routes was:

$$\bar{w} = -47.02 + 0.497\,\bar{h} + 0.00121\,v_h \qquad (9)$$

corrected $R^2 = 0.69$

where $\bar{h}$ = scheduled headway (seconds)

The negative constant, and the coefficient for the mean headway which is less than 0.5, results in lower wait time than predicted by the theoretical model.

The wait time analysis results are not unusual and are consistent with findings reported by Holroyd and Scraggs (1966), O'Flaherty and Mangan (1970), Seddon and Day (1974) and Joliffe and Hutchinson (1975). If anything, they suggest that the accepted assumption of random arrivals for headways lower than 10-12 minutes should be reconsidered.

3.5 Identifying The Optimal Control Strategy

The results of the steps described in Sections 3.1-3.4 are used as inputs to this decision process in resolving the following questions:

(1) should a holding strategy be implemented?
(2) which kind of control is appropriate?
(3) where should the control point be located?
(4) for headway-based control, what is the optimal threshold value?

Question 2 is determined outside of the decision algorithm and depends on the length and uniformity of scheduled headways for reasons previously described. The remainder of the questions are addressed within the decision algorithm.

The algorithm for headway-based control is to minimize the following objective function:

$$TW = \sum_{i=1}^{j-1} n_i \times \bar{w}_i + b_j \times d_j(x_o) + \sum_{i=j}^{N} n_i \times \bar{w}_i \qquad (10)$$

where: TW = expected total wait time on route
j = the control stop
x_0 = threshold value
n_i = number of people boarding at stop i
b_j = number of people onboard at stop j
$\bar{w}_i$ = average wait time at stop i
N = total number of stops on route
$d_j(x_o)$ = expected delay at the control stop for a threshold of x_o

The minimum expected total wait time will occur at a specific j and x_0, which will result in the identification of the optimal control point and threshold value. The minimum expected total wait time is then compared to the expected total wait time without control to determine if control represents an improvement and the magnitude of the benefit provided.

The algorithm for schedule-based control is to identify the stop which maximizes:

$$\frac{sdv_t}{on/down} \qquad (11)$$

where: sdv_t = standard deviation of running time

on = number of passengers onboard

down = number of passengers waiting downstream

This measure was selected to identify stops with high unreliability and many passengers affected downstream. For this application, if either terminus is identified as the optimal point, the control strategy is assumed to be ineffective relative to no control. Further research should be conducted to compare the results of this approach with other methods of control point selection for schedule-based holding.

3.6 Operator Compatibility With The Developed Methodology

The decision algorithm has been coded in Pascal for the Apple II microcomputer. For each stop, the user defines the number of boardings and alightings, distance and number of intersections from the previous stop, direction and time period of travel, and if available, percentage of on-street parking allowed from the previous stop. Most of this data is available or can be easily collected by the transit property. This data file serves as an input to the decision algorithm.

The user is prompted to describe the scheduled headway, which determines whether headway-based or schedule-based control is being considered. The input file of stop information is combined with the models previously described to form the inputs to the objective function.

The model output includes a statement of whether control is effective, a priority listing of the most effective control stops, and for headway-based control, corresponding threshold values and absolute and relative benefits of control over the no control case. The priority listing is useful in situations where it is impractical to implement control at a particular stop (e.g., traffic conditions) and near-optimal alternatives are worthy of consideration. The absolute and relative benefits provide for a comparison across routes (e.g., because of constraints on the number of available street supervisors).

Sample output for headway-based control appears at the top of page 11. To assist transit operators in utilizing the developed methodology, a user's manual has been written which accompanies the software.

LIST OF EFFECTIVE CONTROL STOPS BY ORDER

STOP 13,	THRESHOLD	3.75	MIN,	REDUCTION	49.21	MIN,	%REDUCTION	4.98%
STOP 21,	THRESHOLD	3.00	MIN,	REDUCTION	47.15	MIN,	%REDUCTION	4.77%
STOP 20,	THRESHOLD	3.00	MIN,	REDUCTION	46.81	MIN,	%REDUCTION	4.74%
STOP 22,	THRESHOLD	2.75	MIN,	REDUCTION	46.70	MIN,	%REDUCTION	4.73%
STOP 12,	THRESHOLD	4.00	MIN,	REDUCTION	46.36	MIN,	%REDUCTION	4.69%
STOP 11,	THRESHOLD	4.25	MIN,	REDUCTION	42.04	MIN,	%REDUCTION	4.26%

3.7 Further Discussion

The models reported in Section 3 represent an attempt to use empirical data to establish factors which affect transit route performance and passenger level of service. They should not be interpreted as "cause and effect" models because the collinearity between variables and lack of information on other potentially significant explanatory variables make it difficult to understand the individual contributions of each factor. Thus, the models should be considered primarily for their value in providing reasonable estimates of performance and service given the availability of information on route characteristics.

The analyses were also restricted by the range of empirical data available for study. Although the routes which were evaluated appear to be representative in terms of route and service characteristics, nevertheless the number of routes are limited and the issue of transferability is unresolved. Present research activities include efforts to validate the developed models. Other assumptions which were made in conducting this research include independence of routes within the network and on-time vehicle departures from the route origin.

4. CONCLUSIONS

Several findings can be reported from this research activity. Mean running time is strongly influenced by trip distance, passengers boarding and alighting and signalized intersections with other route characteristics having a lesser effect on this measure. Running time deviation magnifies and propagates as vehicles proceed downstream. Headway variation is very highly correlated with running time variation, with scheduled headway also impacting this measure. Headway-based control decreases headway variation, the magnitude of the change being dependent on the threshold level. Finally, models of passenger waiting time which assume random passenger arrivals overestimate observed waiting times, even for short headway routes.

Beyond the individual model implications, many general contributions can be attributed to this research effort. The research has addressed individually and collectively the issues which impact service reliability, resulting in pertinent information on setting timetables and allocating vehicles to routes. These impacts are then represented mathematically and utilized in the development of a decision process which can be used to improve service regularity through the implementation of real-time holding strategies. Finally, a mechanism is provided by which the operator can utilize the methodology directly to address current reliability problems. The research product is based heavily on empirical analysis, which is likely to be more representative of actual operations.

The research results have direct practical application in metropolitan areas where conventional transit service is operated. It is seen as being particularly important in these times of fiscal conservation, with its emphasis on cost reductions and increased productivity for public services.

5. ACKNOWLEDGEMENT

This research is being sponsored by the Office of Service and Management Demonstrations, Urban Mass Transportation Administration (UMTA) of the U.S. Department of Transportation. The support provided by Mr. Joseph Goodman of UMTA is particularly appreciated.

6. REFERENCES

Abkowitz, M.D. and I. Engelstein. "Factors Affecting Running Time on Transit Routes." Transportation Research. Vol. 17A, No. 2, pp. 107-113, March 1983.

Barnett, Arnold. "On Controlling Randomness in Transit Operations." Transportation Science, pp. 102-116, May 1974.

Bevilacqua, O. et al. Evaluation of the Concinnati Transit Information System (TIS). U.S. DOT, Report UMTA-MA-06-0060-79-1, 1979.

Bly, P.H. and R.L. Jackson. Evaluation of Bus Control Strategies By Simulation. Transport and Road Research Laboratory, Report No. 637, 1974.

Doras, Jean Leo. "Irregularite des Autobus et Temps D'Attente des Voyageurs." TEC, 33, pp. 13-21, 1979.

Holroyd, E.M. and D.A. Scraggs. "Waiting Times for Buses in Central London." Traffic Engineering and Control, 8(3), pp. 158-160, 1966.

Jackson, R.L. and D. Stone. Experiments in Bus Service Control Using and Interactive Model of a Typical Urban Bus Route. University of Newcastle Upon Tyne, 1976.

Joliffe, J.K. and T.P. Hutchinson. "A Behavioral Explanation of the Association Between Bus and Passenger Arrivals at a Bus Stop." Transportation Science, 94, pp. 248-282, 1975.

Koffman, David. "A Simulation Study of Alternative Real-Time Bus Headway Control Strategies". Transportation Research Record 663, pp. 41-46, 1978.

Loo, Debra F. Evaluation of Schedule-Based Holding for Transit Vehicles: A Case Study of Bus Route 5 in Minneapolis, Minnesota. U.S. DOT, Transportation Systems Center, Report No. SS-24-U.3-201, 1981.

O'Flaherty, C.A. and D.O. Mangan. "Bus Passenger Waiting Times in Central Areas." Traffic Engineering and Control, 11(9), pp. 419-421, 1970.

Seddon, P.A. and M.P. Day. "Bus Passenger Waiting Times in Greater Manchester." Traffic Engineering and Control, 15(9), pp. 442-445, 1974.

Sterman, Brian P. and Joseph L. Schofer. "Factors Affecting Reliability of Urban Bus Services." Transportation Engineering Journal, TEL, pp. 147-159, 1976.

Turnquist, Mark A. and Steven W. Blume. Evaluating Potential Effectiveness of Headway Control Strategies for Transit Systems, January 1980.

Turnquist, Mark A. and Larry A. Bowman. Control of Service Reliability in Transit Networks. U.S. DOT, Report No. DOT/RSPA/DPB-50/79/5, 1979.

Welding, P.I. "The Instability of Close-Interval Service." Operational Research Quarterly, 8, pp. 133-142, 1957.