TRAVELLERS' RESPONSE TO TIME-RELATED QUALITY OF TRANSPORTATION SERVICE

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1. INTRODUCTION

The purpose of travel is to arrive at one's intended destination within the intended time. Accordingly, the traveller's choice of transport mode or route depends upon the time when he can arrive at his intended place by whichever means. When the travelling cost, comfort and other factors are not overriding, the traveller selects the mode or route that requires the shortest time to travel.

The travel time is the time interval from the departure time from the traveller's home to the arrival time at his destination. It has more or less a probabilistic fluctuation in general. Therefore, the traveller behaves so that the probability of being late for the intended time will assume a sufficiently small value when he is appointed the time for arrival. In the following, the appointed time for arrival is expressed as A.T.A.

When the traveller decides his departure time to reduce the probability of his being late, the time interval from the departure time to the A.T.A. is different from the travel time itself. This time interval is virtually spent for travel, so it is called virtual time-consumption, V.T.C., in the following. The difference between the V.T.C. and the travel time is remarkable when the fluctuation of the travel time is large. The author has mentioned in the above that the means requiring the shortest time is selected. It is clear that the shortest 'time' means the shortest 'V.T.C.' more precisely. So consideration on the human decision-making process up to departure is the first step of our study.

The time-related quality of the transportation service is considered as the joint evaluation of the swiftness and the reliability of the mode of travel. It is well represented by the traveller's decision of the departure time and V.T.C. as described later. In the following, the travellers' decision-making process in response to the probabilistic fluctuation of travel time is dealt with in the manner of system identification.

2. GENERALIZED DEPARTURE TIME

(1) System identification

'System identification' is a procedure to find the response function of a black-box system by comparing the input and the output of the system. This procedure requires that the input and the output have a causal relation. When the system has plural inputs, the causal relation is assured by the measurement of the output change produced by the change of one input that is taken into account while the other inputs are held constant.

This is a stricter requirement than those for forecasting because the forecast means the estimation of hardly detectable facts from more easily detectable facts, so the procedure for forecasting does not automatically guarantee the presence of a causal relation between the predictor variables and the criteria variables although analysts may often expect it.

Engineers often attempt not only to estimate the present or future transportation trends but also to vary and to control them by various technical means. Therefore, engineers will not be satisfied by mere forecast modeling, and system identification will be required.

There are two possible ways to apply the above mentioned procedure to the human beings. The first is to select an individual and to give him a change of input to measure the change of his response in the condition controlled by the observer. (The 'controlled condition' means that the controlling factors ,i.e. the inputs are held constant except the one that is taken into account .) The second is to compare the differences of the input-output pairs among an identical group of human beings in the controlled conditions. The latter method is used in this paper because it is practical in the application.

(2) Fundamental assumptions

The identical group mentioned above is the group that consists of individuals whose criteria for judgment are identical. note1) The identical criteria are involved by satistying the following conditions.

1) The evaluation of being late is identical among the group.
2) The evaluation of time consumption for travelling is identical.
3) The given information of the probabilistic fluctuation of travel time is identical.

For the convenience of theoretical calculation, the following assumptions are added to the above.

4) Each individual is able to adjust his departure time but can not control the travel time.
5) Each individual decides his departure when he has judged that the probability of being late is sufficiently small while the V.T.C. is reasonably small.
6) The probability of being late at the departure fluctuates with individual- and occasional- differences among the group. The distribution of these differences is unique to the group.

(3) Definition of generalized departure time

Suppose that an individual belongs to a group and he travels length ℓ . The travel time is denoted by t_n and the P.D.F. (Probability Density Function) of t_n by $\phi_{tn}(t|\ell)$,where $\phi_{tn}(t|\ell)$

is the conditional P.D.F. on the trip length ℓ. He possesses full information on $\phi_{tn}(t|\ell)$, while he can not control it according to the assumptions 3) and 4). Let the origin of the time axis coincide with the A.T.A., and use t_s for the departure time at his home, t_a for the arrival time at his intended destination, respectively.

The relation between t_a and t_s is

$$t_a = t_s + t_n \quad . \tag{1}$$

Because t_n is a random variable, t_a has a probabilistic quantity even if t_s is strictly given as t_{s1}. The conditional P.D.F. of t_a for given ℓ and t_{s1}, $\phi_{ta}(t|\ell, t_{s1})$ is

$$\phi_{ta}(t|\ell, t_{s1}) = \phi_{tn}(t - t_{s1}|\ell) \quad . \tag{2}$$

The probability of being late when he decides to depart at time t_{s1} is

$$\begin{aligned} \alpha_1 &= \int_0^\infty \phi_{ta}(t|\ell, t_{s1})dt \\ &= \int_0^\infty \phi_{tn}(t - t_{s1}|\ell)dt \\ &= \int_{-t_{s1}}^\infty \phi_{tn}(t|\ell)dt \end{aligned} \tag{3}$$

Note that the time origin coincides with the A.T.A. and t_{s1} has a negative value.

Eq.(3) represents the probability of being late, but from another point of view, it means a variable-transform, from t_{s1} to α_1. Because eq.(3) is monotonous, α_1 has one-to-one correspondence to t_{s1}. Let the transform be expressed as follows.

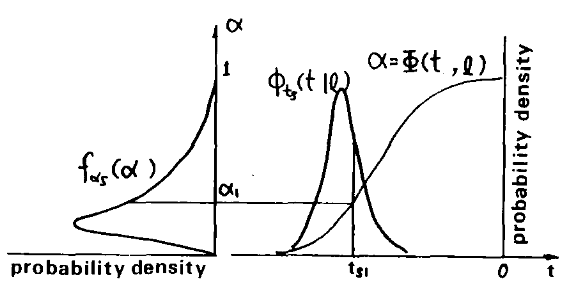
$$\alpha = \Phi(t, \ell) \tag{4}$$

The more detailed explanation on this relation is as follows. Fig.1 (a) shows the correspondence of point t_{s1} on t-axis to point α_1 on α-axis. This correspondence means the mapping of the region of $(-\infty, 0)$ on t-axis to the region of $(0,1)$ on α-axis. The assumption 5) insists that each individual decides his departure time measuring the time in the probability of being late. In this sense, α is the generalized time scale in the probability of being late. The individual decides his departure at $\alpha=\alpha_s$ that he has judged as optimal to departure, i.e. α_s is the departure time selected on α-axis. Let α_s be called as 'Generalized Departure Time (G.D.T.)'. The actual departure time is obtained by the inverse transform of α_s to the actual time axis t. Although the members of the group have different distances to travel and different fluctuations of travel time, their departure times are uniformly evaluated on α-axis.

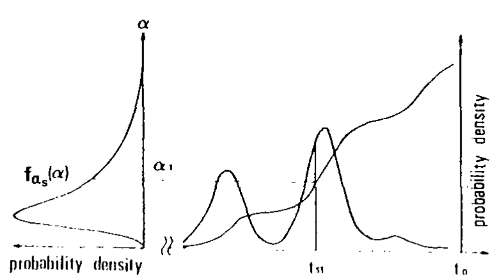
According to the assumption 6), the fluctuation of α_s is represented by the P.D.F., $f_{\alpha s}(\alpha)$. Then the P.D.F. of t_s, is obtained by the inverse transform of $f_{\alpha s}(\alpha)$.

$$\phi_{ts}(t \mid \ell) = f_{\alpha s}(\alpha)\frac{d\alpha}{dt} \tag{5}$$

For example, consider a traveller who uses a scheduled transit system such as a railway. If he misses a train at the station, he must wait till the next train departs. Therefore the transfer function of eq.(4) produces a step-like relation between t and α. The relation is illustrated in Fig.1 (b). Accordingly, the P.D.F. of the departure time given by eq.(5) has few peaks as also shown in the figure when the time interval between the train departures is relatively small. note 6)



(a) personal transit systems



(b) scheduled transit systems

Fig.1 Relationship between G.D.T. and actual departure time

$f_{\alpha_s}(\alpha)$ represents the criterion for decision-making that is unique to the group. The purpose of this paper is to find the quantitative expression of $f_{\alpha_s}(\alpha)$.

The V.T.C. can be obtained for any members of the group, for any modes or routes, by giving their places of departure and arrival, the P.D.F. of travel time and $f_{\alpha_s}(\alpha)$. The P.D.F. of travel time is more or less controllable by choosing the type of transit system, by adjusting the method of its operation. Consequently, the planners can control the V.T.C. of the users.

(4) Estimation of generalized departure time

$f_{\alpha_s}(\alpha)$ in eq.(5) is estimated as follows.
When the P.D.F. of actual departure time $\phi_{t_s}(t|\ell)$, and the P.D.F. of travel time, $\phi_{t_n}(t|\ell)$, are given, the P.D.F. of the arrival time, $\phi_{t_a}(t_a|\ell)$, is obtained from the eq.(1).

$$\phi_{t_a}(t_a|\ell) = \int_{-\infty}^{0}\phi_{t_n}(t_a-t|\ell)\ \phi_{t_s}(t|\ell)\,dt \tag{6}$$

Giving the distribution of the trip length among the group as the P.D.F. of ℓ, $g_\ell(\ell)$, the P.D.F. of the arrival time of all members is,

$$\phi_{t_a}(t_a) = \int_0^{\ell_0}\phi_{t_a}(t_a|\ell)\,g_\ell(\ell)\,d\ell \tag{7}$$

where ℓ_0 is the upper limit of the trip length.
Substituting eqs.(5) and (6) in to eq.(7), we obtain the following equation.

$$\begin{aligned}\phi_{t_a}(t_a) &= \int_0^{\ell_0}\int_{-\infty}^{0}\phi_{t_n}(t_a-t|\ell)\,f_{\alpha_s}(\Phi(t,\ell))\frac{d\Phi}{dt}\,dt\ \ g_\ell(\ell)\,d\ell \\ &= \int_0^{\ell_0}\int_0^{1}\phi_{t_n}(t_a-t|\ell)\,f_{\alpha_s}(\alpha)\,d\alpha\ \ g_\ell(\ell)\,d\ell \end{aligned}\tag{8}$$

Note that the relation between t and α is given by

$$t = \Phi^{-1}(\alpha,\ell) \quad .$$

Because ℓ and α are independent of each other, the order of the integrations is exchanged. Then we obtain the following equations.

$$K_2(t_a,\alpha) = \int_0^{\ell_0}\phi_{t_n}(t_a-t\ |\ell)\,g_\ell(\ell)\,d\ell \tag{9}$$

$$\phi_{t_a}(t_a) = \int_0^{1}K_2(t_a,\alpha)\,f_{\alpha_s}(\alpha)\,d\alpha \tag{10}$$

Eq.(10) is an integral equation of the first kind of Fredholm type and it can be solved by substituting the observed data in to $g_\ell(\ell)$, $\phi_{t_n}(t\ |\ell)$ and $\phi_{t_a}(t)$.

Generally speaking, the travel time consists of plural components, i.e. walking time, in-vehicle time, waiting time and so on. But in this paper, the fluctuation of travelling speed is only taken into account for the sake of simplification, as described later. Denoting the travel speed as v, and the P.D.F. of v as $q_v(v)$, the travel time t_n is

$$t_n = \ell / v \tag{11}$$

then $\phi_{t_n}(t \mid \ell)$ is obtained by the transform of $q_v(v)$.

$$\phi_{t_n}(t \mid \ell) = q_v(\ell/t)\left|\frac{dv}{dt}\right| = \ell/t^2 \; q_v(\ell/t) \tag{12}$$

3. G.D.T. OF COMMUTERS

(1) Observation of commuters' behavior

Commuters in cities have their own homes, places of employment and modes of travel which are different to one another, so it is difficult to obtain the simple expression of the probabilistic distributions of t_n, ℓ and t_a. However, if the following conditions are satisfied, the commuters' behavior gathering at a railway station is equivalent to the behavior in travelling to their places of employment.

1) The train service has an extremely low frequency so that being late for one train means inevitably being late for the A.T.A. at their places of employment.
2) No alternative transit system is available when they miss a train.
3) The regularity of the train service is sufficient so that they can arrive before the A.T.A. when they get on the train.

These conditions mean a case where the step-like relation shown in Fig.1 (b) has only one step.

The observation of commuters' behavior was carried out at Komuro station of the Hokuso Railway and in the residential area of its environs, Funabashi city, Chiba prefecture. In this case, most of the commuters travel to Tokyo and the following conditions are satisfied in addition to the above-mentioned conditions.

4) Almost all of the commuters have access to the railway only on foot. note 2)
5) Almost all of the commuters have no alternative means to travel to their places of employment in the peak hours of travelling. note3)

The time intervals of train operation, counted numbers of trains and passengers are shown in Table-1. note4)

interval	number of trains	number of passengers
19 min.	2	492
23	2	616
32	6	803
38	2	282

Table-1 Counted numbers of trains and passengers

The following items were observed and measured.

(i) Arrival time of commuters at the station.
(ii) Distribution of ℓ measured from the detailed map indicating the location of inhabitants.
(iii) Commuters' walking speed.

The black dots in Fig.2 show an example of the arrival time of passengers at the platform of the station. The arrival time was obtained by addition of walking time from the wicket to the platform to the measured arrival time at the wicket.

Fig.3 shows the distribution of trip length ℓ obtained from the map.

Fig.4 shows the distribution of the walking speed. note[5] The average and standard deviation are 1.34 m/s and 0.39 m/s, respectively. The distribution is well approximated by a logarithmic normal distribution shown as a line in the figure.

(2) Numerical calculation

Let $\{t_i\}$ and $\{\alpha_j\}$ be the representive coordinates of t and α respectively. The number of t_i and α_j are both N. Denote the values of $\phi_{t_a}(t_i)$, $f\alpha_s(\alpha_j)$ and $K_2(t_i, \alpha_j)$ as P_i, f_j and K_{ij}, respectively. In addition, let F_j be $f_j \times \Delta\alpha_j$, where $\Delta\alpha_j$ is the weighting for the numerical integration.

Using the above notations, eq.(10) is expressed as follows.

$$\{P_i\} = [K_{ij}]\{F_j\} \qquad (13)$$

$\{F_j\}$ is the vector of cell frequencies and it must satisfy the restraints of $F_j \geq 0$ and $\Sigma F_j = 1$. But when eq.(13) is solved as a simultaneous equation, $\{F_j\}$ does not satisfy the restraints,

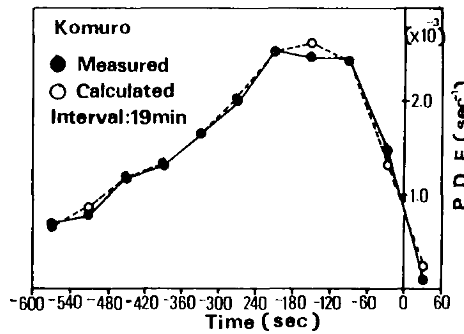


Fig.2 Distribution of arrival

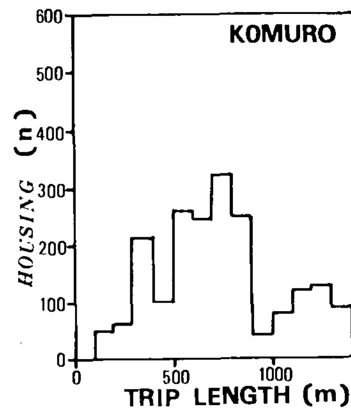


Fig.3 Distribution of Trip Length

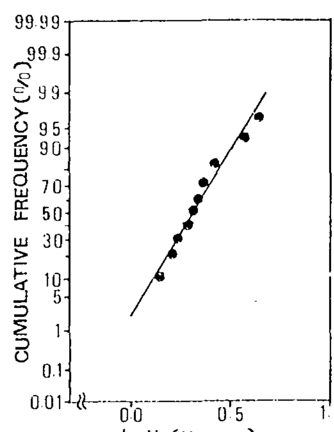


Fig.4 Distribution of Walking Speed

because $\{P_i\}$ includes the random fluctuation involved from the observation. Thus, the minimum chi-square procedure is used.[8] This procedure estimates $\{F_i\}$ to obtain the minimum χ^2 value calculated with the observed $\{P_i\}$ and that derived by the eq.(13), satisfying the above restraints.

The actual procedure is as follows.
1) By giving the arbitrary initial values $\{F_j\}$ of eq.(13) as $\{F_j^0\}$, $\{P_i\}$ is obtained as $\{P_i^0\}$.
2) χ_i^2, the cell component of χ^2 value, is calculated as follows.

$$\chi_i^2 = (P_i - P_i^0)^2 / P_i^0 \tag{14}$$

where, P_i : observed cell frequency, P_i^0: calculated cell frequency.
3) Let i_{max} be the cell number which gives the maximum χ_i^2 value, and $\{F_j^0\}$ be replaced by $\{F_j^1\}$ as follows.

$$\{F_j^1\} = \{1-\Delta F_1\}\{F_j^0\} + \{\Delta F_{imax}\} \tag{15}$$

$$\{\Delta F_{imax}\} = \Delta F_1 \{0,\ldots 0,\overset{i_{max}}{1},0\ldots 0\}^T \tag{16}$$

ΔF_1 is obtained by the following procedure.
Assume that the replacements shown by eqs.(15) and (16) are made, χ^2 value is obtained as follows

$$\chi^2 = \sum_i \frac{\{P_i - P_i^0(1-\Delta F_1) - K_{i,imax}\Delta F_1\}^2}{P_i^0(1-\Delta F_1) + K_{i,imax}\Delta F_1} \tag{17}$$

ΔF_1 is decided as follows so that the maximum decrease of χ^2 is obtained .

$$\frac{\partial(\chi^2)}{\partial(\Delta F_1)} = 0 \tag{18}$$

4) Replacing $\{F_j^0\}$ of step 1) with $\{F_j^1\}$, $\{P_i^1\}$ is obtained in place of $\{P_i^0\}$.
5) Replacing $\{P_i^0\}$ of steps 2) and 3) with $\{P_i^1\}$, ΔF_2 and $\{\Delta F_j^2\}$ are obtained in place of ΔF_1 and $\{F_j^1\}$.
6) The calculation is repeated until χ^2 value no longer decreases in the same manner.

Note that step 3) is effective only when the element of K_{ij} gives the maximum for $i = j$ among the jth row vector.

The representive coordinates $\{t_i\}$ are selected with interval of 60 sec.

cell No.	t_i	α_i
1	-570 sec	3.700×10^{-7}
2	-510	3.387×10^{-6}
3	-450	2.964×10^{-5}
4	-390	1.214×10^{-4}
5	-330	4.779×10^{-4}
6	-270	1.787×10^{-3}
7	-210	6.251×10^{-3}
8	-150	2.011×10^{-2}
9	- 90	5.814×10^{-2}
10	- 30	3.161×10^{-1}
11	+ 30	5.597×10^{-1}

Table-2 Representative coordinates

$\{\alpha_j\}$ are selected so that the peak of $\phi_{ta}(t|\alpha_j)$ coincides with t_i. $\{t_i\}$ and $\{\alpha_j\}$ are shown in Table-2. $\Delta\alpha_j$ is decided as follows.

$$\Delta\alpha_j = \sqrt{\alpha_{j+1}\,\alpha_j} - \sqrt{\alpha_j\,\alpha_{j-1}} \tag{19}$$

(3) Results of calculation

An example of $\{P_i\}$ obtained from the calculation is shown in Fig.2 by white dots. The correlation between the calculated and observed $\{P_i\}$ is close, and the significance level of goodness of fit exceeds 95% for 492 passengers. In other cases, significance levels of 75 - 95% are obtained except the time interval of 32 min. The calculated results of $\{F_j\}$ for various intervals are shown in Fig.5 as cumulative distribution curves. They are similar to one another. The averages and the standard deviations of $\{F_j\}$ are shown in Table-3.

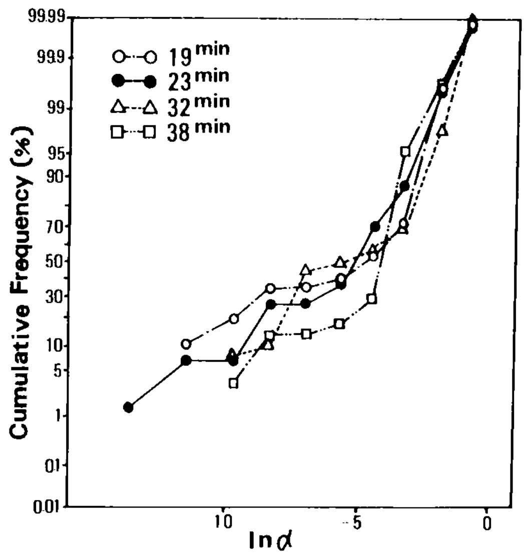


Fig.5 Calculated Results of G.D.T.

The solid line in Fig.6 shows the weighted average of the lines shown in Fig.5. Because it is convenient to use an analytical expression of $f_{\alpha s}(\alpha)$ in actual application, the solid line is approximated by the following function.

interval	average	standard deviation
19 min.	2.2 %	3.1 %
23	1.4	2.9
32	2.5	4.9
38	1.8	2.1
weighted average	2.0	3.8

Table-3 Calculated G.D.T.

$$f^{a}_{\alpha_s}(\alpha) = Af^{1}_{\alpha_s}(\alpha) + Bf^{2}_{\alpha_s}(\alpha) \tag{20}$$

where $f^{a}_{\alpha_s}(\alpha)$ is the approximation of $f_{\alpha_s}(\alpha)$,

$$f^{1}_{\alpha_s}(\alpha) = \frac{1}{\sqrt{2\pi}\,\sigma_1\,\alpha}\, e^{-(\ell n\alpha-\mu_1)^2/2\sigma_1^2} \tag{21}$$

$$f^{2}_{\alpha_s}(\alpha) = \frac{1}{\sqrt{2\pi}\,\sigma_2\,\alpha}\, e^{-(\ell n\alpha-\mu_2)^2/2\sigma_2^2} \tag{22}$$

and, $A = B = 0.5$, $\mu_1 = -4.0174$, $\sigma_1 = 1.0108$, $\mu_2 = -8.0652$, $\sigma_2 = 1.9680$, respectively.

The cumulative distribution of eq.(20) is also shown in Fig.6 by a dotted line which well agrees with the solid line. The average and standard deviation given by eq.(20) are 1.6% and 3.2%, respectively.

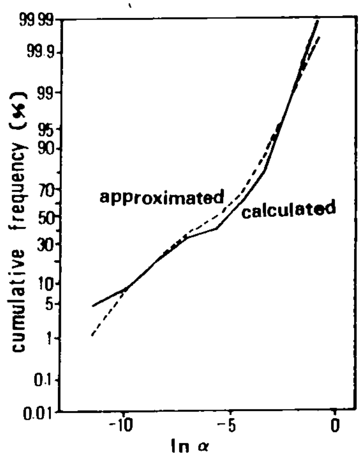


Fig.6 Approximation of G.D.T.

4. DISCUSSION

The probability distribution of the G.D.T. was obtained in a quantitative expression. This is useful to calculate the V.T.C. of individuals who are appointed the place and time of arrival. The V.T.C. represents the individual evaluation of the time-related quality of the transit system.

The P.D.F. of the commuters' G.D.T. obtained at Komuro is roughly illustrated in Fig.7. The features of the G.D.T. are as follows.

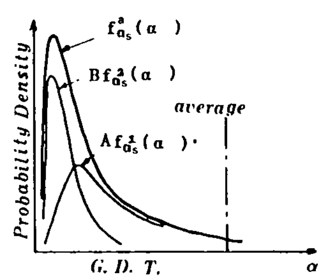


Fig.7 Illustration of G.D.T.

1) The P.D.F. has a sharp peak in the region of extremely small value of α that is far less than the average value.
2) The value of the P.D.F. decreases rapidly as the value of α increases, but it does not disappears, within a range of very small value of α.
3) The P.D.F. is approximated by a combination of two log-normal distribution functions. The possibility that the commuters' group consists of two sub-groups has not been ignored.

The above-mentioned features indicate that the commuters reduce their probability of being late drastically, and the fluctuation of their judgment is small. Howevor the fluctuation of commuters V.T.C. is not always small because the decision making for departure depends on the transfer function from t to α. The commuters response to the time-related quality of transportation service is sensitive.

This result provides a fundamental viewpoint in dealing with the choice behavior. When the other effects are negligible, it is valid to accept the following principles. These are possibly the starting points for the theoretical consideration of travellers behavior. [note7)]

1) Each individual chooses one means that requires least V.T.C. among all alternatives.
2) The fluctuation of choice is derived from the fluctuation of individual judgment.

Although the travel from home to place of employment only was dealt with in this paper, the concept of G.D.T. can be applied to various modes of travels that have restraints in the place and time of arrival.

5. CONCLUSIONS

The purpose of this paper is to clarify the decision making process of travellers for departure in order to find the human evaluation of the time-related quality of transportation service. The following items were revealed.
1) The departure time of travellers who are appointed the place and the time of arrival is dealt with uniformly using the Generalized Departure Time. The G.D.T. is defined on the generalized time axis that is scaled in terms of the probability of being late for the appointed time of arrival.
2) The estimated probability density function of commuters' G.D.T. exhibits a sharp peak in the region of extremely small values of the generalized time. The P.D.F. is approximated by the combination of two log-normal distribution functions. Therefore, the Virtual Time-Consumption defined as the time interval from the departure time to the appointed time sensitively varies according to the fluetuation of travel time.
3) The time-related quality of transportation service that means the joint evaluation of swiftness and regularity of travel is well represented by the V.T.C.

6. ACKNOWLEDGEMENT

This study was carried out with the cooperation of Prof. Yoshiji Matsumoto, Dept. of Civil Eng., the Univ. of Tokyo and Mr. Toshiro Tanabe, Min. of Transp., the Japanese Government.
We are grateful to Mr. Yasuyuki Miyaki, graduate student, the Univ. of Tokyo, for his assistance, and Prof. Takashi Nishioka, the Univ. of Tsukuba, for his valuable advice on this study.

NOTES

1) The word 'identical' does not means 'strictly same' here, but means that the differences among the members can be dealt as a probabilistic fluctuation.

2) The walking speed is controllable among the travellers to some degree , so it does not agree with the assumption 4) of 2. (2), strictly. But the Relative Metabolic Rate (R.M.R.) of human beings sensitively increases as the walking speed increases.[1)] The increase of R.M.R. involves an increase of travellers' discomfort , so they reduce the discomfort by adjusting their departure time to avoid having to increase their walking speed on the basis of daily experience. In fact, the number of commuters who rushed to the station immediately before the train departure was small.

3) The percentage of commuters who use personal transit systems, i.e. cars and motorcycles from Funabashi to Tokyo is only 6% . In addition, half of them travel to a few areas of Tokyo not served by the Hokuso Railway. [2)]

4) Investigations were carried out in other residential areas which were similar to Komuro but the time intervals of the train operation were shorter, i.e. 4 - 14 min. As the time interval became large, the calculated f_{α_s} (α) approached that obtained in Komuro, and it was stable in Komuro though the time interval changed. Therefore, the time intervals observed in Komuro can be considered sufficiently large.

5) The walking speed was calculated from the time required for 97 commuters to walk a length of 50 m in Naruse-Dai, Machida city, Tokyo. The conditions for walking in Naruse-Dai were similar to those in Komuro.

6) This deduction regarding the behavior of train passengers was confirmed by observation of commuters behavior in the suburbs of Hiroshima city. The observation also revealed that commuters' G.D.T. in Hiroshima had almost the same distribution as that in Tokyo.[4)]

7) This is also confirmed by observation in Hiroshima.[4)]

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