LEVEL OF SERVICE AND MINIMUM COST : ILLUSTRATION WITH A CYCLICAL TRANSPORTATION SYSTEM.

Sergio R. Jara Díaz
Departamento de Ingeniería Civil
Universidad de Chile
Casilla 5373, Santiago
Chile.

1. INTRODUCTION

The estimation of cost functions for the different transportation industries hss played an important role in the analysis of industry structure and technology from a microeconomic perspective. Such an analysis can be performed on the basis of the so called dual properties of the cost function, which are in turn a result from the theoretical observation that cost minimization takes place on the technical boundary.[1] However, the fact that transportation demand is also affected by the level of service has not been dealth with within the context of transportation cost functions. The relevance of this point is guaranteed since this effect, which is more the rule than the exception, makes transportation output endogenous, i.e. can be affected by firm's action.

In this paper we provide a frsmework to understand the role of the level of service in the behavior of a firm facing exogenous input and output. prices. We show that the firm does not necessarily operate on the technical frontier. The role of the operating options is highlighted in an example using a cyclical transportation system which illustrates the concepts.

2. LEVFL OF SERVICE AND THE TRANSPORTATION FIRM.

We understand transportation output as the real flow level of different commodities carried by a firm among different origin-destination pairs during different periods (Jara Díaz, 1982 a,b). Here we will work with its scalar version Y. We will denote the input vector by X, the input price vector by ω, and output price (fare) by P. The transformation function

$$F(X,Y) \geq 0 \qquad (1)$$

describes all the technically feasible pairs (X,Y), with equality representing the technical boundary.

In the scalar output version, F(X,Y) is given by f(X)-Y, where f(X) is the traditional production function which gives the maximum level of Y that can be technically produced from X. The usual analysis indicates that cost minimization always take place on the boundary, with the input price ratio equal to the technical rate of input substitution. This is the foundation of duality between the cost function and F(X,Y).

For a given activity system, transportation demand is usually a function of both the fare charged and other factors like security or travel time, which we will summarize under the concept of level of service, ℓ. If Y_d is the number of units (goods or passengers) per unit time willing to be relocated, and P is exogenously given and fixed, then

$$Y_d = Y_d(\ell) \qquad (2)$$

where demand increases with the level of service.

The level of service generated by the firm is, in general, a function of the level of input usage, the actual flow produced and the operating rule (r) followed, i.e.

$$\ell = \ell_r(X,Y) \quad . \qquad 2 \qquad (3)$$

We expect ℓ to be increasing with X_i and decreasing with Y. Available research (Gálvez and Gibson, 1983) suggests that technical optimality is associated with a particular operating rule r_o. In other words, if $\psi(r,x)$ is the function that describes the flow produced from a technical viewpoint, then

$$f(X) = \psi(r_o,X) = \underset{r}{\text{Max}}\ \psi(r,X) \qquad (4)$$

r_o will be momentarily assumed to be the prevailing rule. Combining equations 2 and 3 we get

$$Y_d = Y_d\left[\ell_{r_o}(X,Y)\right] = h(X,Y) \qquad (5)$$

Thus, an actual flow Y can be generated from an input set X provided both capacity and the generated demand are sufficient, i.e.

$$f(X) \geq Y \qquad (6)$$

$$Y \leq h(X,Y) \qquad (7)$$

The level of service constraint (7) can be shown to correspond to an implicit constraint

$$Y \leq g(X) \qquad (8)$$

such that g(X) increases with X_i. We will name g(X) the iso-demand function. Its representation in the input space is decreasing and convex (see appendix).

The relevant cost minimizing problem for a transportation firm willing to generate an actual flow Y (parametrically given) is thus

$$\underset{X}{\text{Min}}\ \omega X^T \qquad (9)$$

subject to

$$Y - f(X) \leq 0$$
$$Y - g(X) \leq 0$$
$$X_i \geq 0$$

The feasible set is represented in figure 1, assuming f(X) = Y and g(X) = Y intersect at some point. In this context, the solution to problem (9) will take place on the boundary of the shaded area. There are three possible outcomes, depending on the input price ratio :

i) technical constraint active, level of service constraint inactive (point 1); such a solution implies that flow Y is produced at capacity, but the associated level of service generates a demand Y_d such that Y_d-Y is not satisfied.

ii) level of service constraint active, technical constraint inactive (point 2); this solution is characterized by excess capacity and complete satisfaction of demand.

iii) both constraints active (point 3); flow, demand and capacity coincide.

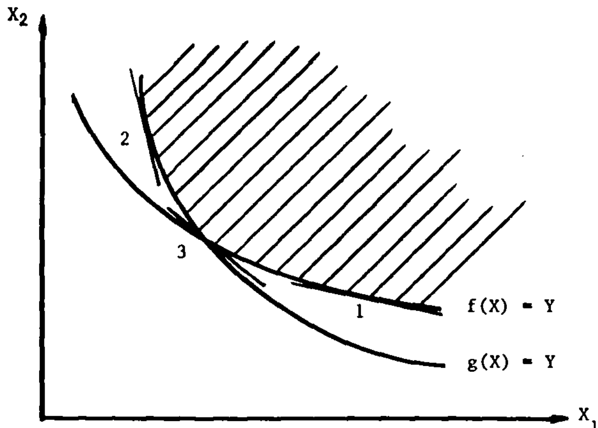

Figure 1. Cost minimization with technical and level of service constraints.

However, a solution like 2 implies that $f(X_1^2, X_2^2)$ is strictly greater than Y and the assumed operating rule can be changed. It would be in the firm's interest to find an operating rule r_1 such that

$$\ell_{r_1}(X,Y) > \ell_{r_0}(X,Y) \qquad (10)$$

Then the production of Y should fulfill two conditions :

$$Y = Y_d\left[\ell_{r_1}(X,Y)\right] \qquad (11)$$

$$Y = \psi(r_1, X) \qquad (12)$$

Equations 11 and 12 form a system that can be solved as $Y=\lambda(X)$, which is a function in the input space that passes through point 3 and belongs to the sub-space determined by $g(X) < Y$ and $f(X) > Y$, by construction. The $\lambda(X)$ locus is, undoubtly, a set of feasible input combinations that implies an expenditure reduction with respect to a type 2 solution. As the operating rule can not be changed in either a type 1 or type 3 solutions($f(X)=Y$), then cost minimization will take place either on $f(X)=Y$ or on $\lambda(X)=Y$. This is represented in solid lines in figure 2. It should be noted that the firm may deliberately choose to operate below capacity in the production of a given flow Y, depending upon the input price ratio.

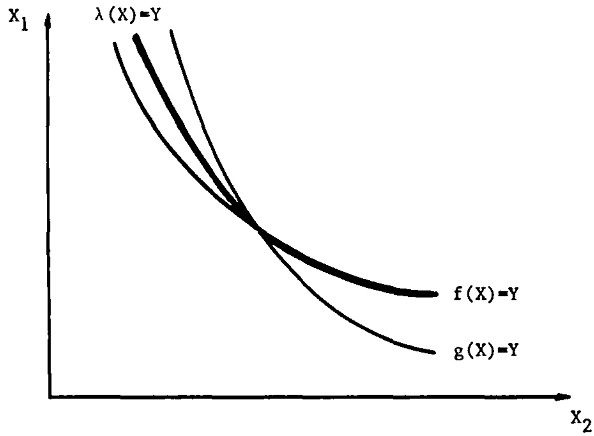

Figure 2. Locus of potential solutions considering changes in the operating rule.

In summary, the introduction of the level of service modifies the feasible set in a cost minimizing context. Only a type 1 solution preserves the traditional equality amoung input price ratios and technical rates of substituttion, which is the basis of the duality hetween the transformation function and the cost function. Cost minimization on $\lambda(X)=Y$ implies that input price ratios are equal to "subjective" substitution rates among factors; such a solution generates a cost level which is higher than the one associated to the corresponding capacity. This makes the actual cost function $C(Y)$ to be different to the "dual" cost function $C^d(Y)$, such that

$$C(Y) \geq C^d(Y) \quad , \tag{13}$$

with equality representing type 1 solutions.

3. ILLUSTRATION WITH A CYCLICAL SYSTFM.

Let us define a simplified version of Gálvez' (1978) cyclical system, which operates moving a flow Y between a given origin-destination pair with one loading site at the origin O and one unloading site at the destination D. We define

- B = fleet size
- K = vehicle capacity in physical units [P.U.]
- k = load size in [P.U]
- t_1 = travel time from O to D, loaded, in time units [T.U.]
- t_2 = travel time from D to O, unloaded, in [T.U.]
- u = loading-unloading capacity of any site, in [P.U./T.U.].

The flow moved from O to D can be easily shown to he descrihed by

$$\psi(r,X) = \frac{Bk}{t_1 + t_2 + \frac{2k}{\mu}} \tag{14}$$

where load size is taken as the operating rule. As $\psi(r,X)$ is increasing in k, and k has an upper bound K, we get

$$f(X) = \underset{r}{\mathrm{Max}}\ \psi(r,X) = \frac{BK}{t_1 + t_2 + \frac{2K}{\mu}} , \qquad (15)$$

such that the operating rule associated to the technical frontier is full capacity operation.

We will assume a linear in ℓ demand function

$$Y_d = a + b\ \ell \qquad a, b \in R^+ . \qquad (16)$$

The level of service will be taken as time in the system (or total travel time) with a minus sign, i.e.

$$\ell = - \left(\frac{1}{2q} + t_1 + \frac{k}{\mu} \right) \qquad (17)$$

where q is frequency, given by

$$q = \frac{B}{t_1 + t_2 + \frac{2k}{\mu}} \qquad \text{(Gálvez, 1978)} . \qquad (18)$$

Taking the operating rule associated to the technical frontier, we get

$$Y_d = a - b \left[\frac{1}{2B} \left(t_1 + t_2 + \frac{2K}{\mu}\right) + t_1 + \frac{K}{\mu} \right] \qquad (19)$$

which is directly our g(X) because ℓ in equation 17 does not depend on the flow level Y.

Assuming only vehicles of a given capacity are available in the market, the transportation firm's decision variables are B and μ. From Eq. 15, the curve f(X) = Y in the (B, μ) space has the form

$$B = \frac{Y}{K} \left(t_1 + t_2 + \frac{2K}{\mu}\right) , \qquad (20)$$

which is represented in figure 3.

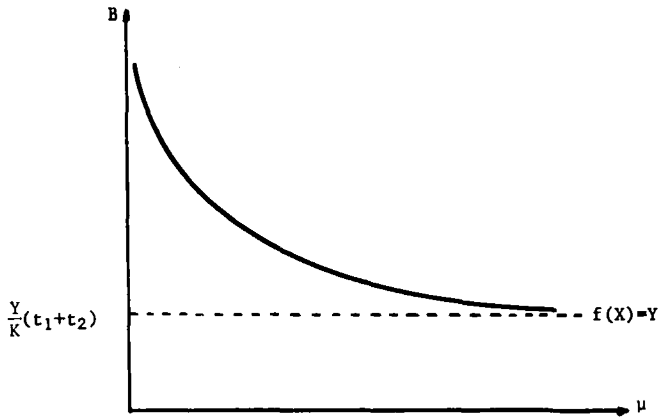

Figure 3. Technical frontier in a cyclical system.

Equation 19 is the basis to obtain g(X) = Y, given by

$$B = \frac{t_1 + t_2 + \frac{2K}{\mu}}{2\left[\frac{a-Y}{b} - t_1 - \frac{K}{\mu}\right]} \tag{21}$$

and represented in figure 4.

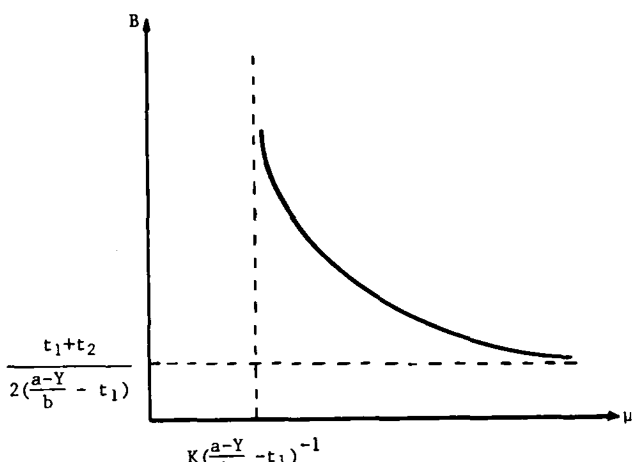

Figure 4. Iso-demand function.

It can be easily shown that, if both curves intersect, it will happen at only one point and the relative position is given in figure 5.

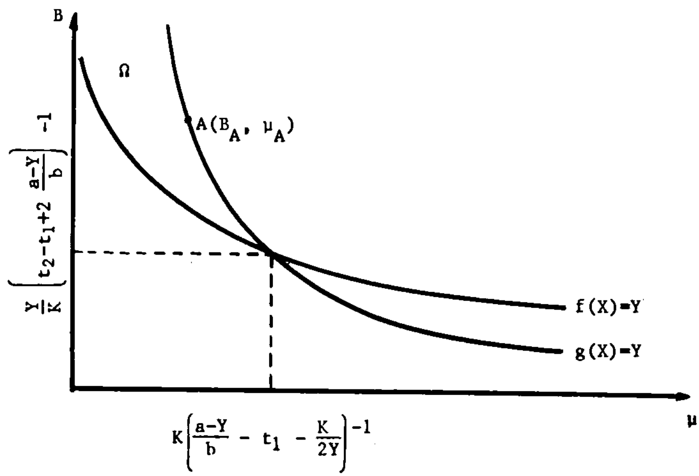

Figure 5. Technical and level of service constraints in the input space.

As suggested in the previous section, a point like A implies that using B_A vehicles and a loading-unloading capacity μ_A, the cyclical system can generate a flow greater than Y if operated at full vehicle capacity. Simultaneously, the associated service level (travel time) generates a demand exactly equal to Y. Therefore, from a technical viewpoint the same flow could be produced with a load size less than K; this would simultaneously improve the level of service by virtue of equations 17 and 18. Thus, changing the operating rule, the feasible set can be enlarged including part of the sub-space Ω indicated in figure 5. The boundary must fulfil the following conditions

$$Y = \frac{Bk}{t_1 + t_2 + \frac{2k}{\mu}} \tag{22}$$

$$Y = a - b\left[\frac{1}{2B}\left(t_1 + t_2 + \frac{2k}{\mu}\right) + t_1 + \frac{k}{\mu}\right] \tag{23}$$

Equation 22 represents the $Y_d[\ell_{r_1}(X,Y)]$ function (Eq. 11) and Eq. 23 corresponds to the $\psi(r_1, X)$ function (Eq. 12). Both equations form a system that can be solved in Y or in k. The solution in Y is our $\lambda(X)$ function, which in this case happens to be given by

$$B = \frac{(t_1 + t_2)\left(1 + \frac{2Y}{\mu}\right)}{2\left(\frac{a-Y}{b} - t_1\right)} + \frac{2Y}{\mu} \tag{24}$$

It can be easily checked that Eq. 24 (which is valid only in the Ω space), passes through the intersection between $f(X) = Y$ and $g(X) = Y$.

On the other hand, the solution in $k = k(B, \mu)$ gives the load size associated to each point on $\lambda(X) = Y$. However, directly from eq. 22.

$$k = \frac{Y(t_1 + t_2)}{B - \frac{2Y}{\mu}} \qquad (25)$$

which increases with μ (and decreases with B). As frequency is given by Y/k, it can be also interpreted as q increasing with B (or decreasing with μ). Figure 6 summarizes all this properties.

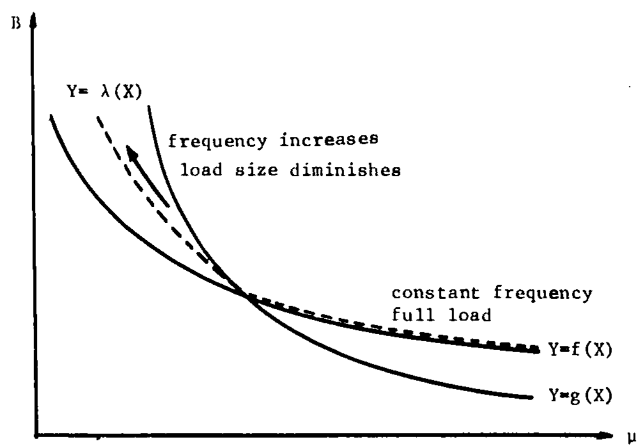

Figure 6. Boundary of feasible input combinations and associated operating rules.

This example is illustrative of the theory developed in the previous section. Its shows how the feasible set of input combinations is modified by the fact that the form those inputs are combined has a correspondence with the level of service offered by the firm. In particular, we have shown that a firm serving a cyclical system may deliberately choose to operate its vehicles in a less-than-full-load fashion in order to increase frequency, so disminishing travel time and positively influencing demand. As this was done for a (parametrically) given flow, we can conclude that variability of demand is not the sole responsible of operations below capacity, usually observed in practice.

The example also confirms the idea of cost optimality (minimum) outside the technical boundary, weakening the foundations of duality.

4. SYNTHESIS AND CONCLUSIONS.

By introducing level of service in the analysis of transportation production, we have been able to show the modification of the feasible input set in the provision of a given flow. This results from the observation that input usage not only generates a transportation capacity, but also an associated level of service that, in general, also depends on the operating rule and on the actual flow level produced. Thus, a given flow can be generated if two conditions are fulfilled : sufficient capacity and sufficient demand. For a given operating rule, each point in the input space has an associated capacity and an associated demand. The set of points fulfilling both conditions has a boundary that only partially coincides with the technical boundary, which weakens the foundations of duality due to eventual cost minimizing points strictly within the technically feasible set.

All this concepts have been illustrated within the context of a transportation firm operating a cyclical system. We have concluded that the firm may deliberately operate below capacity in order to minimize the cost of producing a given actual flow Y. In particular, the firm may choose to disminish load size in order to increase frequency, so offering a better level of service.[3]

We have then designed a framework that provides a solid basis for the transportation analysts' claim on the inadequacy of the minimum cost criteria (associated to capacity) to decide among transportation alternatives. We have also reinforced the idea that microeconomic analysis of transportation activities requires a theoretical reformulation. We postulate the content of this paper as part of that task.

NOTES.

1. For a review of the applied work, see Jara Díaz (1982, a)
2. An antecedent of this kind of relation is Manheim's performance function ϕ_E (R,S,T,V) (Manheim, 1980).
3. This may well explain the behaviour of airline shuttle services.

APPENDIX. SHAPE OF THE ISO-DEMAND FUNCTION.

We have

$$Y_d = Y_d(\ell) \qquad \text{(a)}$$

$$\ell = \ell_r(X,Y) \qquad \text{(b)}$$

where

$$\frac{\partial Y_d}{\partial \ell} > 0 \, , \quad \frac{\partial \ell}{\partial X_i} \geq 0 \, , \quad \frac{\partial \ell}{\partial Y} \leq 0 \, . \qquad \text{(c)}$$

In addition

$$Y \leq Y_d \left[\ell_r(X,Y) \right] = h(X,Y) \, . \qquad \text{(d)}$$

We define

$$H(X,Y) = Y - h(X,Y) = 0 \qquad \text{(e)}$$

which represents an input function $g(X) = Y$: the iso-demand function.

Then

$$\frac{\partial Y}{\partial X_i} = -\frac{\partial H/\partial X_i}{\partial H/\partial Y} = \frac{\partial h/\partial X_i}{1-\partial h/\partial Y} = \frac{\partial Y_d/\partial \ell \quad \partial \ell/\partial X_i}{1-\partial Y_d/\partial \ell \quad \partial \ell/\partial Y} > 0 \quad \text{and} \qquad \text{(f)}$$

$$\frac{\partial X_i}{\partial X_j} = -\frac{\partial H/\partial X_j}{\partial H/\partial X_i} = -\frac{\partial Y_d/\partial \ell \;\; \partial \ell/\partial X_j}{\partial Y_d/\partial \ell \;\; \partial \ell/\partial X_i} = -\frac{\partial \ell/\partial X_j}{\partial \ell/\partial X_i} < 0 \quad . \qquad \text{(g)}$$

Besides, as one input approaches a zero level, the other input (2 inputs case) is likely to tend to infinity in order to keep the service level constant.
Inequality (f) indicates that $g(X)$ increases from the origin; inequality (g) shows that $g(X) = Y$ is decreasing, and the last observation suggests that it is likely to be convex.

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