# ON THE ROLE OF OPERATING POLICY IN TRANSPORT PRODUCTION

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## 1. INTRODUCTION

The fact that operating policies and rules influence the quantity and quality of transportation production is widely recognised. In the main theoretical developments on transport supply recently published (Florian and Gaudry, 1980, Manheim, 1980, and Morlok, 1980) they enter as a variable in performance and supply functions or procedures. However, what is understood by the concept of operating policy varies among authors and its specific role in transport processes has been suggested but not systematically explored.

The precise concept definition has some unavoidable consequences in the set-up of a general framework to approach transportation production analysis. For this reason, we begin by introducing the main concepts through a basic model for an individual operator carrying only one commodity, presented in Section 2. Section 3 is devoted to operating policy variables identification while the range of options open to the operator is examined in Section 4. The meaning and uniqueness of technical optimality regarding operating policy are discussed in Section 5. Finally, some considerations are made on extensions to the model and further research directions.

The subject is so broad that it is impossible to be exhaustive and although we have intented to provide a detailed coverage, some issues are hardly sketched.

## 2. A BASIC MODEL OF TRANSPORT PRODUCTION

Transport as a production process involves the transformation of a set of inputs into a set of products. Product is defined as a quantity of a given commodity (freight or passengers) carried between two points defined in time and space. Then, its measure is time and O - D specific. Inputs required are numerous but they can be grouped into three categories: means of transportation (facilities, including right-of-way, terminal sites and equipment, and vehicles), labour and auxiliary materials (such as energy source). But it should be noted that if nothing is shipped, there is no transport production regardless of vehicle trips being actually accomplished. In this sense, the commodity to be transported is also an input (Gálvez , 1978).

The operator's task is to combine all these inputs in order to produce transport outputs. This implies a transformation technology, the art of performing that combination , that we call operating policy. It must be distinguished from the technology of means of transportation, which is the art of producing some of the inputs needed by a transport process. Then, when we refer to transportation technology we are alluding operator's methods and decisions. Similarly, when talking about inputs technological characteristics, only those directly relevant to transport production should be borne in mind. For instance, pavement thickness is not such a characteristic, while its roughness or support capacity clearly are.

Let us now turn to look at a single operator activity. We will first assume that there is only one O-D pair and one commodity to be transported between them. Also, that every vehicle of his fleet operates under identical conditions, in a closed circuit embracing the origin and the destination including an empty return journey (Gálvez, 1982). The output produced (in physical units (P.U.) per unit time (U.T.)) is given by:

$$V = k\, f \qquad (1)$$

where:

V = mean intensity of commodity transported [P.U./U.T.]
k = shipment size for one vehicle [P.U./vehicle]
f = fleet frequency [vehicles/U.T. ]

Of course, this amount is limited by the available resources. Shipment size k cannot be greater than vehicle capacity K. Frequency cannot exceed right-of-way capacity Q nor a maximum given by fleet size and vehicle cycle time. Finally, terminal facilities capacity also restrict V. We will put it in the form $\mu S$, being S the number of sites and $\mu$ the individual capacity of them, and using a superscript + or - to denote loading facilities, respectively. In summary, we have a multidimensional capacity restriction that leads to a system capacity $\lambda$ |P.U./U.T.| , given by:

$$\lambda = \min \{KQ;\ \mu^+ S^+\ ;\ \mu^- S^-\ ;\ \max kf\} \qquad (2)$$

The first three elements cover the influence of infrastructure while the last introduces the effects of mobile resources availability. Under our assumptions:

$$f = \frac{\eta\, B}{t_c} \qquad (3)$$

where:

B = fleet size |veh|
$\eta$ = portion of time that a vehicle is in service
$t_c$ = vehicle cycle time |U.T.|

As long as $t_c$ may depend upon k (at least through time spent at terminals) the product kf does not necessarily increase with k. Then, it is not guaranteed that $f_{max}$ is associated to vehicle capacity K and, accordingly, that max kf = $Kf_{max}$. It will only happen if $\frac{\partial (kf)}{\partial k} > 0$ for $0 < k \leq K$.

Now, obviously $V < \lambda$. But at this point it should be remembered that there is a quite special input: the commodity to be transported. It is well known that, in general, its availability depends upon some characteristics of the transportation process itself. As they are a function of transport technology, we conclude that the latter influences the amount of output produced not only through resources consumed (to which a price, an output characteristic, can be associated) but also through level of service (a process characteristic). On the other hand, level of service may vary with the amount of output at the individual process level, because of factors like the influence of shipment size on cycle time. This variation is reinforced, and in some cases governed, by congestion effects but conceptually exists even in its absence. From this discussion it is clear that interaction with demand is at the heart of transport production. Therefore, an equilibrium framework is needed to model it and is not a further stage where production and demand are matched.

Two functions are then added to our model: a demand and a level of service functions. The first will be expressed as:

$$V_D = D(A,\ L,\ P) \tag{4}$$

where:

$V_D$ = demand volume |P.U./U.T.|
$A$ = activity system variables
$L$ = level of service
$P$ = user out-of-pocket costs (fares, or appropriate costs if car user)

Demand volume represents the availability of the input "transported commodity". It follows that the latter is measured as a flow.

Level of service is a vector of attributes and for simplicity, we will use a function of the form (Gibson, 1981):

$$L = L_{T''}(T',\ T'',\ k, f) \tag{5}$$

where:

$L$ = vector of level of service components
$T'$ = vector of transportation inputs (other than transported commodity) technical characteristics.
$T''$ = operating policy

It should be noted that transported commodity acts through variable k in this function. Frequency f is included as an argument of the function because of its influence on waiting times. Although the assumption of vehicle homogeneity avoids congestion effects, these could be reflected - in the single operator context - also through the frequency. Of course, in this case the functional form will be different. Variables k and/or f (see Section 3) belong to T" but have been separately identified just to emphasize their role.

As previously discussed, output produced cannot exceed demand volume. Then, we can impose, in an equilibrium perspective:

$$V = \min \{\lambda \ ; \ V_D\} \tag{6}$$

Equations (1) to (6) constitute a basic model of transport production.Let us now examine some of its features. To begin, equilibration is not confined to eqns. (4) and (5) (the typical demand - performance approach). Eqns. (1) to (3) that represent the transportation function (Gálvez, 1982), play also an important role.

There is a mix of generic variables (like T'' or T') and specific ones (e.g., $\eta$ or K) and the variable $t_c$ requires to be broken down into elementary components. It has been preferred to clarify these aspects in Section 3, being confident in that they do not obscure the fundamental logic that underlies the model.

Transport production is not completely described by these equations. In fact, P appears as an exogenous variable and there is no explicit provision for dependance of T'' upon resources consumed. Moreover, a resources consumption function has deliberately not been included.

The set of inputs T' can be divided into those that can be used in many transport processes and those that are wholly consumed in one process. The first group embraces what we have called means of transportation while the second is composed by labour (not workers) and auxiliary materials. They differ in that the former type and amount are, strictly speaking , a datum for the operating policy, which in turn can determine the amount of the latter (not its type). In other words, there are fixed and variable inputs. We will denote them by $T'_F$ and $T'_V$ respectively.

The selection of inputs type (often referred to as "technology choice") may be based on operational considerations but is not an issue in operating policy, as previously defined. Also the amount of means of transportation that the operator allocates to a particular service will obviously depend upon the equilibrium volume and this is a function of, among other variables, the operating policy T''. Nevertheless, it does not conceptually mean that such a decision is a component of this policy. It is very important for the correct understanding of its role in transport production to distinguish between what T'' is and what are its consequences.

Up to this point, we have identified five issues not explicitly incorporated in the model: fares determination, T'' selection, resources consumption, "technology choice" and decision on the amount of fixed inputs. Jointly they are the result of operator's behaviour and therefore cannot be analysed without modeling it (or establishing a set of assumptions). Doing this would require the introduction of economic considerations that lie beyond the scope of this paper. Our purpose is to investigate what the operator <u>can</u> do and not what he <u>will</u> do. That is, to deal with T'' as a technology. However, it is to mention that Jara-Díaz (1982, 1983) has developed a similar framework in an economic context. He shows how consistent cost functions can be derived and also that level of service effects may lead a carrier to operate under capacity to maximize profits, even under no time variability of demand.

In equation (6) it is implicit that if there is enough demand (or it is exogenous and then it is provided a capacity identical to the predetermined volume) the system will be operated at capacity and, by eqn. (2), this implies that $\mu$ rates and the product kf are maximized. Previous discussion on fixed and variable inputs makes clear that these maximum values are subject to an operating policy decision: the amount of variable inputs available . It exists a relationship between both types of inputs such that there is a limit beyond which the addition of variable resources can not increase the capacity offered. It is apparent that the operator should not choose an inputs combination that exceeds the limit but it is not obvious whether he should select one exactly at the limit. This is a question of optimality that can be relevant at least because of the discrete nature of some fixed inputs and will be examined in more detail in Section 5.

To summarize, the operating policy - the art of combining inputs to produce outputs - has a widespread influence on transport production. Capacity and level of service supplied, resources consumed (and through them, all the economic decisions of the operator) and ultimately, the output level achieved, are dependent upon it. In the following sections, the basic model developed will be used to study the technology embodied by T'.

## 3. OPERATING POLICY VARIABLES

Let us examine the variables and functions included in our model. Demand function D, A variables and, by assumption, variable P are exogenously determined. The level of service function is technological in nature but, for simplicity, we will not give it an explicit form here. Focusing on operating policy, fixed inputs $T_F'$ will also be given as well as the type of inputs $T_V'$. Now it is necessary to assess which variables contained in the model lie in this category.

Vehicle capacity K is determined by the "technology choice". The same happens with characteristics like power that will (partially) influence the cycle time $t_c$, maintenance requirements that have an impact on $\eta$, or the ease of loading and unloading that will be reflected in $\mu^+$ and $\mu^-$. The vehicle fleet size is a decision on fixed inputs amount as it is the number of sites $S^+$ and $S^-$ and the number of units of right-of-way (upon which Q depends). "Technology choice" also defines the type of terminal sites and equipment and the right-of-way standard, affecting $\mu^+$ $\mu^-$ and Q, and the type of labour W and auxiliary materials M. Then, we can redefine our generic variable T' decomposing it into a vector of inputs and a vector of characteristics, what makes easier the linkage of this technical formulation with the traditional microeconomic notation. Using the symbol X for inputs, we have [1]:

$$X = \{B, S^+, S^-\} \qquad (7)$$

We will denote the second by TCH, whose components are inputs characteristics, as K and the others that - resulting from "technology choice"-influence but do not fully determine variables $t_c$, $\eta$, $\mu^+$, $\mu^-$, Q, W and M. Before entering to operating policy variables identification and description, it is useful to analyse vehicle cycle time. It has three basic components:

$$t_c = t_T + t_r + t_D \qquad (8)$$

where:

$t_T$ = loading and unloading times spent at terminals |U.T.|

$t_r$ = running time |U.T.|

$t_D$ = unavoidable "dead" time |U.T.|

Component $t_T$ is always given by:

$$t_T = k\left(\frac{1}{\mu^+} + \frac{1}{\mu^-}\right) \qquad (9)$$

Running time $t_r$ is more complex. For a given route it is a function of the operating speed v, which in turn depends upon vehicle and right-of-way characteristics, traffic regulations, energy consumption (denoted by $E_r$) and the shipment size (in some cases its influence can be negligible but usually this does not happen). Energy consumption and running time are cumulative totals that result from the mix of resistances to movement, vehicle motor power and efficiency and driving style. The latter is a true operational variable. Anyway, $E_r$ level will pose a limit to the travel time that can be achieved. We can then write:

$$t_r = t_r\,(T',\ T'',\ k,\ E_r) \qquad (10)$$

It should be noted that the operator can specify a driving style in terms of target cruising speeds for each route section but it is just a practical issue and not a conceptual one. What is important is that the specification of an operating speed is to a large extent the same as establishing a running time, then little understanding can be gained from doing it in theoretical formulations.

"Dead" time $t_D$ covers the durations of some operations that must be performed in every cycle but do not contribute to the transport process itself. Examples are fuel tank filling, vehicle inspection (or cleaning) preceding every round-trip (they usually take place in air transport) or time spent at control offices (police stations, customs, etc.). Sometimes these operations can be accomplished simultaneously with loading or unloading and therefore do not increase cycle time. In short:

$$t_D = t_D\,(T',\ T'') \qquad (11)$$

If congestion effects are introduced, waiting times at terminals and additional delays will arise. They are also dead times in terms of the production process but cannot be regarded, in general, as unavoidable in our single operator model.

Now, the list of variables affected by the operating policy is: Q, $\mu^+$, $\mu^-$, n, $t_r$, $t_D$, |W,M| (their amount), k and f, for which an individual analysis will be conducted to overview the role of operational decisions:

- $Q$ : the right-of-way regulations, the traffic control systems (if they exist) and the maintenance [2] policy exert a strong influence on capacity. Also, even assuming an absolute vehicle homogeneity, capacity will depend on operating speed because the vehicles have a finite length.

- $\mu^+$, $\mu^-$ : the rates at which the transported commodity can be loaded or unloaded are dependent upon labour and energy supplied, the equipment maintenance policy and the stowage procedures. Operator's decisions on the use of available sites give rise to another variable: $\varepsilon$, the proportion of sites in service, that is a $S$ multiplier.

$\eta$ : it is affected by labour amount attached, vehicles maintenance policy and operator's decisions on the use of available vehicles.

$t_r$, $t_D$: corresponding comments have been already made. However, note that the operator has a degree of freedom to allocate some components of $t_D$ to a particular stage in vehicle cycle.

$|V,M|$ : the operator possesses full control on its amount. Variable E belongs to this category through M.

k : the operator can determine its value, in the range $0 < k \leq K$.

f : the operator can also fix its value, in the range $0 < f \leq f_{max}$. In our model it is a constant throughout the reference period.

But it must be realised that k and f cannot be simultaneously predetermined by the operator. Recalling the model specified in Section 1, and if we assume that A,P,D and L functions, T' and T" variables (except k and f) are given, we would have as dependent variables the following: $\lambda$,L,$V_D$,V,k and f. The system of equations allows to determine exactly five of them. Then it flows that k or f result from equilibration. And it should be remembered that they are mutually related by eqn. (2), which can be rewritten, introducing eqns. (8) to (11), as:

$$f = \frac{\eta\ E}{t_r(T',\ T'',k,E) + t_D(T',T'') + k\left(\frac{1}{\mu^+} + \frac{1}{\mu^-}\right)} \qquad (12)$$

Even if demand is exogenous, what may lead to $V_D = V = kf$ and to drop level of service function, k or f will be an outcome. Otherwise, one of the remaining variables already assumed as given should become an unknown. In such a case, we have a design problem where, for instance, required fleet size B can be calculated.

The preceding analysis shows that in spite of existing several variables affected by the operating policy, there is a reduced number of decision fields associated with it that are responsible for its impacts. These will be named control variables and the following classification is proposed:

a) Operations regulations (OR). It embraces traffic regulations and control systems, driving style, stowage procedures and complementary operations (those who originate "dead" times other than delays) type, frequency, duration and allocation within the vehicle cycle.

b) Maintenance policy (MP). There is one for each fixed input.

c) Discretional use of available fixed inputs (DUF). As it has been seen, a vehicle, a site, etc may be not used because of an operator's decision, in spite of being in good condition. It is different from the case of temporary withdrawal from service for maintenance purposes.

d) Amount of variable inputs (AVI). It entails the determination of the quantity of W,E and any other resource of this type.

e) Shipment size or frequency (k or f).

Only for completeness, let us take a rapid look at what happens if the assumption of uniqueness of O - D pair, transported commodity and route is relaxed. First, the operator will have to decide how to serve different O - D combinations (route structure) and how to use the vehicle fleet, terminals, personnel and so on: allocating portions of them to specific commodities and/or O-D pairs or making some kind of combinations. Secondly, given all this it may still happen that there exist alternative trajectories through the network among which a choice is mandatory (route selection). In summary, two new control variables appear: inputs allocation and routeing. In what follows, we return to our simplified model.

According to the stated notation, the elements of the set T are:

$$T'' = \{OR;MP;DUF;AVI;k \text{ or } f\} \quad (13)$$

Solely k and f are specific variables while the remaining four are generic ones. As such, they cover a range of detailed actions, that are similar in nature but whose efficiency in terms of their impact on the variables incorporated in the model may significantly differ in each particular situation. The previous analysis has identified some of those actions and impacts but is in no case exhaustive. Anyway, establishing an operating policy implies the specification of all the vector T'' components in terms of well defined variables from which those included in the model can be determined.

## 4. OPTIONS IN OPERATING POLICY

Options open to the operator have been stated in Section 3 in terms of generic control variables. We will say that a strategy is a vector containing values for a set of independent specific control variables, from which the whole set of operating parameters can be calculated. In what follows, we will refer to the cyclical model detailed in Sections 2 and 3.

There are two types of control variables: those that fix the amount of a variable input (AVI) and those that fix the value of a parameter (not an input). The control variables are related through the model equations already stated. We will analyse first some technological options related to problem-specific matters, that can be explored in a subsystem context. Then, we will deal with the options for the whole system, considering two cases: exogenous and endogenous demand.

Let us consider first eqn.(10). For given values of k and $E_r$ [3], there are many driving styles compatible with them, each leading to a different travel time $t_r$. For instance, it is possible to disaggregate in many ways the total energy spent between the loaded trip from O to D and the empty return trip. A driving style specification must be a detailed description that can be understood by the driver. Target velocities are often used, for each right-of-way condition, that involve a specific throttle setting (or equivalent) instructions, and consequently a set of values for the load factor of the motor, resulting in a rate of energy spending. Such a model has been discussed by Schwarzkopf and Leipnik (1977) for the special case of a car, in an optimization context, and by Gálvez and De la Carrera (1981), in a simulation context. The selection of a cruising speed for ships and aircrafts is another example of driving style determination.

The right-of-way maintenance policy (when existing), a MP parameter, also conditions the amount of energy necessary for obtaining a travel time $t_r$ with a load size k.

Similarly, for known k, $F_r$ and TCP parameters, traffic regulations will influence the attainable $t_r$. Traffic theory tools allow to establish an explicit relation between $t_r$ and regulations, as well as traffic engineering supplies tools for optimisation in this field, already well established (see, for example, Robertson, Lucas and Baker, 1980, or Gibson, Saavedra and Spoerer, 1982). Thus, once driving style, right-of-way maintenance policy, and traffic regulations have been specified, we can find the value of $t_r$ and $E_r$ for any value of k.

We can define a family of driving styles so that we can find a value of $t_r$ for any combination of k and $E_r$. We would expect that $t_r$ increases with k ($t_r$ constant), and decreases with $E_r$ (k constant), and also that $E_r$ increases with k ($t_r$ constant).

We will consider now eqn. (9). Variables $\mu^+$ and $\mu^-$ are not inputs. They only summarize the terminal technology, including stowage procedures. A value of $\mu^+$ or $\mu^-$ is associated with TCP parameters, and AVI values. Focusing on energy consumption, we can write:

$$\mu^+ = \mu^+_{T'} \ (T', E^+_T)$$

$$\mu^- = \mu^-_{T'} \ (T', E^-_T) \qquad (14)$$

Once selected a terminal technology, we can expect that an increase in $E^+_T$ (or $E^-_T$) should imply an increase in $\mu^+$ (or $\mu^-$).

We will discuss now the $\eta$ parameter, that is related to two different policies. The first is the vehicle maintenance policy (a MP parameter), that determines a mean proportion of time $\eta_1$ that a vehicle is in service, or in other words, the mean proportion of vehicle fleet that is actually circulating. The second is a deliberate schedule slack $\eta_2$ or its equivalent, the withdrawal of a number of vehicles from service. These two policies are different in nature. The first is strongly conditional on the TCH parameters, and usually decisions are constrained to a narrow range.

Inputs consumption in maintenance operations is highly inelastic. Nevertheless, there is a chance of increasing or decreasing the labour used in maintenance choosing between vehicles waiting to be repaired or workers waiting for a vehicle. We have:

$$\eta = \eta_1(T') \; \eta_2 \qquad (15)$$

The $\eta_2$ parameter can be considererd as a pure policy variable, because it is not conditioned by TCH, varying in the range $(0,1]$.

Let us now consider the $\varepsilon$ parameter, that is analogous to $\eta$ but applies to terminals. It also entails two different policies. The first, $\varepsilon_1$, is the mean proportion of time a site is off-service because of maintenance operations. The second, $\varepsilon_2$, is a discretional parameter, equivalent to the withdrawal from service of a number of sites. Then we have:

$$\varepsilon^+ = \varepsilon_1^+ \; (T') \; \varepsilon_2^+ \qquad (16)$$

$$\varepsilon^- = \varepsilon_1^- \; (T') \; \varepsilon_2^-$$

Another parameter identified is $t_D$, in eqn. (11), which arises from the complementary operations defined by the operator. As stated, he can define their type and characteristics, so there is a margin for arbitrary decisions. But its allocation within the vehicle cycle may affect capacity (if cycle time is reduced) or demand volume through changing the cycle time split between loaded and unloaded stages (level of service will vary). It follows that $t_D$ should not be considered as a wholly discretional parameter. Summarizing, we have detected seven problem-specific operating policies (related each to traffic regulations, driving style, terminal technology , maintenance of vehicles, sites and right-of-way and complementary operations) and three discretional parameters: $\varepsilon_2^+$, $\varepsilon_2^-$ and $\eta_2$.

We will discuss now the case of <u>exogenous demand</u>, where equations (4) and (5) are replaced by

$$V_D = V_D^* \qquad (17)$$

where $V_D^*$ is an arbitrary value. We will first pay attention to the cycle time determination. From eqns.(8), (9), (10) and (11), we obtain

$$t_c = k \left( \frac{1}{\mu^+} + \frac{1}{\mu^-} \right) + t_r \; (k, \Gamma_r) + t_D \qquad (18)$$

So, it is clear that once defined the mentioned policies, it is necessary to know the k parameter to obtain the cycle time. Then, we will set k at an arbitrary level $k^0$ , $0 < k^0 < K$.

From eqn.(3) we have:

$$\frac{f}{\eta_2} = \frac{\eta_1 B}{t_c} \tag{19}$$

All the variables, at the right-hand side of eqn.(19) are given, then only one variable at the left-hand side remains to be determined. The capacity of each subsystem is defined by:

$$\lambda^+ = \varepsilon_1^+ \mu^+ S^+ \qquad (\varepsilon_2^+ = 1) \tag{20}$$

$$\lambda^- = \varepsilon_1^- \mu^- S^- \qquad (\varepsilon_2^- = 1) \tag{21}$$

$$\lambda_M = \frac{\eta_1 B\ k^0}{t_c} \qquad (\eta_2 = 1) \tag{22}$$

$$\lambda_Q = k^0\ Q \tag{23}$$

And from eqn.(2) the system capacity is:

$$\lambda = \min\ (\lambda^+\ ,\ \lambda^-\ ,\ \lambda_M,\ \lambda_Q) \tag{24}$$

In general, any component can be the active constraint and we will explore all the possibilities.

Case 1.- $\lambda = \lambda^+$

In this case, $\varepsilon_2^+ = 1$ seems adequate, and we should correct the other discretional parameters as follows:

$$\varepsilon_2^- = \frac{\lambda^+}{\lambda^-} \text{ and } \eta_2 = \frac{\lambda^+}{\lambda_M} \tag{25}$$

Case 2.- $\lambda = \lambda^-$ , is analogous to case 1.

Case 3.- $\lambda = \lambda_M$

In this case, $\eta_2 = 1$ seems adequate, and we have:

$$\varepsilon_2^+ = \frac{\lambda_M}{\lambda^+} \text{ and } \varepsilon_2^- = \frac{\lambda_M}{\lambda^-} \tag{26}$$

Case 4.- $\lambda = \lambda_Q$

In this case it is necessary to correct all the discretional parameters as follows:

$$\varepsilon_2^+ = \frac{\lambda_Q}{\lambda^+} \quad , \varepsilon_2^- = \frac{\lambda_Q}{\lambda^-} \quad \text{and } \eta_2 = \frac{\lambda_Q}{\lambda_M} \tag{27}$$

At this point, we have completely defined the three discretional parameters for any situation, with the only assumption that k has an arbitrary value. Frequency f can be calculated from eqn. (19).

We have to check now inequation (6).

Case A.- $\lambda \leqslant V_D^*$

In this case, $V = \lambda$. Depending upon the active constraint, remaining parameters will have their values defined. However, demand is not fully satisfied. We can modify the proportion of unsatisfied demand by varying the parameter $k^o$ only if we are in Case 3 or 4. Otherwise, to acomodate more flow in the system we would be obliged to increase the corresponding $\varepsilon_1$ and/or $\mu$ parameter. To do this, additional resources (e.g. labour or energy) would be required.

Case B.- $\lambda > V_D^*$

In this case, $V = V_D^*$ and the transported commodity availability is an active constraint. There should be excess capacity in all the subsystems and we must correct the discretional parameters as follows:

$$\varepsilon_2^+ = \frac{V_D^*}{\lambda^+} \quad , \quad \varepsilon_2^- = \frac{V_D^*}{\lambda^-} \quad \text{and } \eta_2 = \frac{V_D^*}{\lambda_M} \tag{28}$$

In all cases, of course, eqn. (1) is sufficient to determine the frequency f.

Let us discuss now the case of endogenous demand. Here, eqn. (17) does not hold and we need an equilibration procedure to calculate the parameters of the system. Using eqns. (4) and (5), and assuming A and P are given, we can obtain:

$$V_D = V_D\ (T' \ , k, f) \tag{29}$$

The system capacity $\lambda$ can be determined in the same way as in the case of exogenous demand. Under the assumption of system operation at capacity, we can obtain a value $f_c$ for the frequency parameter. Then, using eqn. (29), we can calculate a value for $V_D(f_c)$. If $V_D(f_c) > \lambda$, the system can operate with the values of the parameters already calculated. If $V_D(f_c) < \lambda$, the demand constraint is active and we have to solve the simultaneous equations:

$$V_D = V_D\ (k^o \ , f)$$
$$V_D = f\ k^o \tag{30}$$

Solution to (30) provides values for $V_D$ and f. Then, the discretional parameters can be derived as follows:

$$\varepsilon_2^+ = \frac{V_D}{\lambda^+}, \quad \varepsilon_2^- = \frac{V_D}{\lambda^-} \quad \text{and } \eta_2 = \frac{V_D}{\lambda_M} \tag{31}$$

These three discretional parameters, in this case, will be strictly less than 1. There is excess capacity in all the subsystems.

However, as previously suggested, it is likely that when demand depends upon level of service some policies will need a more detailed specification. This may lead to additional control variables (e.g. factors conditioning comfort) that, in general we can expect to be independent. Anyway, their independence should be tested by means of a procedure like the one followed here.

It only remains to discuss the amount of variable inputs (AVI). Let us look at the total energy consumption level E, assuming that all the subsystems use the same kind of energy source. It can be calculated, per unit time, from:

$$E = fE_r + \varepsilon^+ \mu^+ S^+ E_T^+ + \varepsilon^- \mu^- S^- E_T^- \tag{32}$$

As seen before, driving style, etc. specification and a $k^o$ value will determine $E_r$. Similarly, $E_T^+$ and $E_T^-$ will come out from terminals operation technology selected. Then, all variables at the right-hand side of eqn. (32) are known. This implies that the control variables already identified suffice to establish the value of E. The same reasoning applies to the other variable inputs. We conclude that AVI is not independent. But it should be noted that if E is fixed instead of $\mu^+$, it will not lead to a particular value for $\mu^+$ because of substitution possibilities. A rule for distributing the energy input among its competitive uses is also needed.

Focusing on the proposed concept of a control strategy we have shown that not all existing control variables are independent and then a strategy contains less components than the vector T'. In our example, the seven problem-oriented policies plus k or f allow to determine the remaining parameters f or k, $\eta_2$, $\varepsilon^+$, $\varepsilon_2^-$ and AVI. The so-called problem-oriented variables are related to sub-systems operation while k or f are linked to the whole system functioning. Parameters $\eta_2$, $\varepsilon_2^+$ and $\varepsilon_2^-$ are a measure of the utilisation degree-in a physical sense-of the fixed inputs and AVI is a measure of variable inputs consumption.

Therefore, when specifying a control strategy what we are doing is to choose a way (problem-oriented parameters) and a general level (k or f) at which the resources are used. Given T', it will result in an output level (through f or k) and a degree of utilisation of resources. Obviously, different strategies may lead to different results. Moreover, it is possible that a strategy proves unfeasible as would occur if f is set to an arbitrary value that requires k > K. Also, if a given level of output is imposed, it is apparent that there are various control strategies that are able to fulfill the condition.

In summary, there are alternative strategies of combining inputs to produce outputs (this is, operating policies), which sometimes may be equivalent in output level and sometimes different, and even may be unfeasible. It is desirable then to analyse how can they be compared. To perform this comparison, distinction made above among the parameters is relevant because it makes clear that there are different levels (e.g. sub-system or system levels)at which it should be carried out. Care must be taken in providing a homogeneous basis for comparative analysis, in terms of output and resources.

## 5. OPERATING POLICY OPTIMALITY

When dealing with options in operating policy the existence of alternatives has been established. Undoubtedly, the question if operating policy, as a set of technical rules or procedures, has an optimum is quite relevant. In this section we intend to precise the scope of the problem and to outline a framework for optimal strategies generation.

In microeconomic theory a technology is said to be optimal if it allows to produce a given level of output with a non-inferior combination of inputs. We will follow this definition in our analysis.

Let us commence with the case of exogenous demand, where it is normally assumed that the transport system will be operated at capacity. Then, it is worthwhile to assess if capacity does entail a technical optimum or at least, under which conditions it will occur. The expression developed in Section 2 for capacity . introducing variable $\varepsilon$ is:

$$\lambda = \min \{kQ;\ \varepsilon^+\mu^+S^+\ ;\ \varepsilon^-\mu^-S^-\ ;\ \max kf\} \tag{33}$$

At this point, it should be apparent that-defined in this way-capacity is determined not only by input characteristics but also by operating policy. Every component at the right-hand side of(33) includes at least one variable dependent on T". Therefore, it is a straightforward conclusion that for a given vector of inputs, capacity will represent an optimal production level if, and only if, it is associated with an optimal operating policy. In other words, any T" will, jointly with T', lead to a particular value for $\lambda$ and then, unless a condition is imposed over T", there will exist a broad range of capacities. In this sense, the linkage between capacity and optimal operating policy contributes to an unambiguous definition of the former concept.

Expression (33) incorporates only partially the optimality requirement through the component max kf. Strictly speaking, each $\lambda$ component should be associated with such a requirement. Let us now examine what is an optimal T [4]. Recalling eqns. (3) and (13), max kf is equivalent to

$$\max \lambda_M = \frac{\eta_{T'}(T'')\, B\, k}{t_{r_{T'}}(T'',k) + t_{D_{T'}}(T'') + k\left(\frac{1}{\mu^+_{T'}(T'')} + \frac{1}{\mu^-_{T'}(T'')}\right)} \tag{34}$$

If we assume that there is no dependence among control variables beside eqn. (34), it follows that kf maximisation calls for maximising $\eta^-$, $\mu^+$ and $\mu^-$ and minimising $t_D$, reducing the problem to

$$\max \lambda_M = \frac{\eta^* B\, k}{t^*_r(k) + t^*_D + k\left(\frac{1}{\mu^{+*}} + \frac{1}{\mu^{-*}}\right)} \tag{35}$$

subject to $k \leq K$

where k is the only independent variable. Subscripts have been replaced by asterisks to indicate that remaining parameters have been set to their optimal values. This is a first level of optimisation that consists of finding the way in which particular subsets of inputs can be better used. For instance, $t^*_r(k)$ is a function obtained through specifying a driving style that, given the inputs, their characteristics and the rest of OR, offers the minimum $t_r$ for each k value. It should be noted that here energy consumption level E is regarded as an input [5]. The second optimisation level is provided by the solution to problem (35), an optimal k (k*) given by:

i) $$k^* = K, \text{ if } \frac{t^*_D + t^*_r(k^*)}{k^*} > \frac{d\, t^*_r(k^*)}{d\, k^*} \tag{36}$$

ii) $$k^* < K, \text{ if } \frac{t^*_D + t^*_r(k^*)}{k^*} < \frac{d\, t^*_r(k^*)}{d\, k^*} \tag{37}$$

In Section 2 we had already suggested that full load operation is not necessarily optimal. But now we can say that this is not simply an issue of "technology choice". Actually, $t^*_r(k^*)$ depends - among other variables - upon E and therefore, the same means of transportation technology and operating policy may be consistent with $k^* = K$ for a range of E values and with $k^*<K$ for another value E'. This makes clear the influence of variable inputs on optimal T'' and , consequently, on capacity $\lambda$, often not realised nor taken into account, maybe because they are not explicitly incorporated in standard capacity formulae.

It is evident that any rule such as "use full load" or "use half load" is not a general technical optimum. In fact, optimal operating policy is not associated with alike rules or with specific values but with expressions like (36) and (37). And, as suggested in the previous Section, the same will happen for other variables like $\mu^+$, $\mu^-$ or the function $t_r$. Perhaps, variables $\eta^*_I$ or $\epsilon^{+*}$ will adopt constant values.

Maximisation of $\lambda_M$ has led to some results that affect other $\lambda$ components such as $\varepsilon^+\mu^+S^+$, through $\mu^+$ maximisation. It can be seen that there is no contradiction, because a maximum value for $\mu^+$ has a positive impact on $\lambda$ in both cases. But as discussed in Section 4, not always the first level individual optimisations can be independently conducted as we have done here. In a particular problem, the number of degrees of freedom must be established first and a selection of strategy components carried out. Only these components are subject to optimisation. By definition, they can be independently optimised. Using the model relationships, the remaining T" components can be derived. Obviously, it is not guaranteed that they will also reach their particular optimal values.

Clearly, a situation like this arises when fixed inputs are not exactly proportioned and then it is not feasible that $\eta_2, \varepsilon_2^+$ and $\varepsilon_2^-$ be simultaneously equal to unity their maximum value. It would seem that in this case there are equivalent combinations of, say, k and $\eta_2$. Let us assume that the unloading terminal is restrictive and only $\lambda_o$ can be transported. Combinations would come from:

$$\frac{\eta_2 k}{t_c^*(k)} = \frac{\lambda_o}{\eta_1^* B} \qquad (38)$$

for $0 < \eta_2 \leqslant 1$ and $0 < k \leqslant K$. But, through equation (32), and taking into account that there will be an optimal rule for the energy consumption split between vehicles and terminals, each combination $(\eta_2, k)$ may lead to different total E levels. Thus, we have to impose the condition of E minimisation to be consistent with our technical optimum definition. As a consequence, k and $\eta_2$ are no longer independent. Therefore, although a universal optimal operating policy does not exist, in each case it is unique.

This discussion allows us to clarify the point made in Section 2, regarding combinations of fixed and variable inputs. Under optimal operating policies, different input vectors will produce different capacities. We have no technical rule for comparing capacities, so this is an economic problem. In a short-run context this case can be thought of as a distinct energy consumption levels comparison for the same transport system. If in the initial situation energy is not efficiently used we will have a T" optimisation problem and capacity will not be diminished. But if we start from an optimal situation, a lower E level will result in less capacity and a necessary subutilisation of fixed inputs. Whether this is desirable or not is a matter of relative prices.

The task of finding optimal operating policies is analytically more difficult when there are several interrelations among its variables. To illustrate this, let us extend our previous example only allowing the operator to substitute energy consumption of vehicles ($E_T$) and at terminals ($E_T^+$ and $E_T^-$). A new constraint appears: total energy consumption must be equal to the available amount E. Suppose, for simplicity, that $\mu^+(E_T^+) = \mu^-(E_T^-) = \mu(E_T)$ and $\varepsilon^+S^+ = \varepsilon^-S^- = \varepsilon S$. Thus, recalling eqn. (32), the problem is:

$$\max \lambda_M = \frac{\eta^* Bk}{t_c} \tag{39}$$

$$\text{s.t.} k \leq K$$

$$\frac{\eta^* B}{t_c} E_r + 2\mu\varepsilon^* S E_T = E$$

Optimisation has to be conducted over k, $E_r$ and $E_T$. Solution to problem (39) leads to:

$$\left(\frac{\partial\lambda_M}{\partial E_r}\right)^* / \left[f^* + \left(\frac{\partial f}{\partial E_r}\right)^* E_r^*\right] = \left(\frac{\partial\lambda_M}{\partial E_T}\right)^* / \left[E_r^* \left(\frac{\partial f}{\partial E_T}\right)^* + 2\mu^*\varepsilon^* S\right] \tag{40}$$

and $$\frac{\eta^* B}{t_c^{*2}} \left[t_D^* + t_r^*(k^*, E_r^*) - k^* A\right] + \delta_1 = 0 \tag{41}$$

where $$A = \left[t_c^* \left(\frac{\partial t_r^*}{\partial k}\right)^* + \frac{2E_r^*}{\mu^*} \left(\frac{\partial t_r^*}{\partial E_r}\right)^*\right] / \left[t_c^* - E_r^* \left(\frac{\partial t_r^*}{\partial E_r}\right)^*\right]$$

where the notation ( )* indicates derivative evaluation at the optimum. $t_c$ is the denominator of $\lambda_M$ expression and $\delta_1$ is a multiplier. From eqn. (40) it can be derived that:

$$\left(\frac{\partial t_r^*(k,E_r)}{\partial E_r}\right)^* = -\frac{\eta^* Bk}{\varepsilon^* S\left[\mu^*(E_T^*)\right]^3 t_c^*(k,E_r^*,E_T^*)} \left(\frac{d\mu^*(E_T)}{d E_T}\right)^* \tag{42}$$

Combining (42) and the total energy constraint we obtain,

$$E_r^* = E_r^*(k,E) \tag{43}$$

and $$E_T^* = E_T^*(k,E) \tag{44}$$

Introducing (43) and (44) in (41), a value for k* is attained, taking into account that it will be equal to K if $\delta_1 < 0$. Note that we now get simultaneusly a value for k* and an optimal distribution of energy consumption among vehicles and terminals. This is, two components of an optimal T'' strategy are jointly determined. In this way, more interrelations can be recognised in order to increase model's ability to represent real situations.

The case of endogenous demand is far more complex. First , some operator's decisions may affect demand volume through level of service changes. For instance, allocation of dead time to stages of the vehicle cycle in which it is unloaded is preferable because users travel time will be lower. The optimality can be found by means of a similar process as the one developed for the exogenous demand case but it will be more complicated as long as additional functions and control variables are involved. Obviously, demand sensitivity to level of service will be crucial in determining the best T" in this case.

## 6. FINAL REMARKS

The preceding analysis shows how strategies in operating policy that involve technical optimality [6] can be derived. In the examples developed we have used the shipment size k as an option but equally frequency could have been chosen. As previously stated, some basic operating rules cannot be formulated in simple ways. Nevertheless, more research is needed to appraise if any practical and easy-to follow suggestions, that lead to at least nearly optimal operation, can be readily obtained. Off-peak periods make also relevant the research on sub-optimal operation.

We have assumed that there is only one commodity and one route. Of course, broader spatial contexts are usual but in this paper we have not dealt with them. Another strong assumption made is that one operator possesses full control on all means of transportation. In practice, it often happens that there exist many operators, each one with a partial control. We have shown that optimal policies are mutually related, thus even if all of them seek for optimality in their respective fields the global result may not correspond to a consistent strategy and then sub-optimal operation will arise. This is a strong argument for planning in transport and it seems promising the fact that a control strategy does not comprise all operating decisions, because a planning authority could select a convenient subset in order to influence the whole system, minimising bureaucratic and political costs.

The framework developed in this paper for transportation technology analysis, although specified for simple cases, is flexible enough to admit extensions to cope with more complex systems. It provides the basis for transport production understanding by describing how an operator might use the available resources, and showing how he should do it to be efficient. Of course, to explain the means of transportation technology choice or the level of output produced, economic considerations are to be introduced, as pointed out in Section 2.

## NOTES

1) As we have specified the model, the right-of-way does not appear in the X vector. If capacity Q is made explicitly dependent upon the number of right-of-way units, this will be another X component.

2) Along the paper, the term maintenance policy embraces routine maintenance operations and repairs.

3) We will center discussion on energy consumption. For the other inputs a similar treatment can be done.

4) When there is only one product, output maximisation for given inputs will generate non-inferior combinations.

5) The distinction made in the paper between fixed and variable inputs is not relevant in an ergodic context, as assumed by the microeconomic theory that is being used in this section.

6) Optimal $T^*$ gives rise to transportation transformation functions (F(X,Y) = 0) as defined in microeconomic analysis.

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