THE EFFECTS OF TRAVEL TIME TO WORK ON RESIDENTIAL LOCATION DECISIONS

by
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1. INTRODUCTION

In most urban areas of Japan, the problems of transportation, housing and land value are a difficult problem. These problems, especially in large cities, are so serious that people are obliged to waste a great deal of their time, money and human energy on their trips to work, and they must live in limited and high-priced (or high-rent) housing.

In order to solve these problems, a variety of countermeasures have been taken to meet each situation. These measures have been the induction of rapid transit, improvement of the transportation network, the supply of housing by the official housing agency, the lending of funds for the purchase of housing, and so forth. These measures have not been irrelevant, but it is difficult to say that they have satisfied the people's demand. There are two major reasons why the people have not been satisfied with the past policies. One is that the housing size and price, as well as residential locations have differed from people's demand. The second reason is that the housing supply has consistently not been enough to meet the demands, because the measures were not sufficient in terms of the rapid growth of urban population.

Studies on urban spatial structure have been made in the fields of economics, sociology, geography and engineering for a long time. Most of the theories which were previously developed concerned the concentric circle zone theory which has only one market or work place in a central district of an urban area[1,2,3]. And then, the bid rent is thought of as a difference of travel cost. In previous studies, the theories of residential distribution have been developed under the condition that people would pursue the maximum utility, according to restrictions of their household equilibrium, that involves expenditure of transport costs. However, it must be considered that the previous theories were not able to reproduce real residential distributions and real rental gradients of urban land, because the utility function of the theories is only an ideological concept.

Recently, Glene Weisbrod, et al, have published a noticeable result of their research which analyzed the consummers trade-offs in a decision to move and the selection among alternative residential locations[4]. Their study focuses on the role of transportation level-of-service changes, relative to various aspects of neighborhood quality, including crime, taxes, school quality and demographic factors. The analysis of the study is based on the multinomial logit model. And the measure of accessibility in their model is regarded as a weighted function of in-vehicle-travel-time, out-of-vehicle-time and out of pocket cost. This measure is included in the model as a statistical measure.

A trip is not a natural phenomenon but one of economic behavior. So, the mechanism of a trip generation should be considered in conformity with economic principles. When people intend to move from one place to another place, they expect some utility of their trips. We can broadly divide these trips into two categories. One is a trip which provides us with some utility on the way, such as a leisure trip. The other is a trip which produces some utility at the destination only. Most of the trips in our daily life come under the latter category. And these trips should be looked upon as dependent phenomena which are generated to accomplish their primary purposes at their destinations.

Applying this method of looking at trips to work trips between usual places of residence and the place of work, the work trips are thought of as the secondary phenomena which are derived from the primary demands of getting dwellings. So, for the purpose of solving the problems of transportation and housing in urban areas, we must reveal the mechanism of housing demand which is a primary demand and the role of how travel time to work plays in the mechanism.

The purpose of this paper is to find the basic factors in the mechanism of housing demand which is a primary demand, to research the mutual relationships among these factors and to disclose the role which the travel time to work plays in the mechanism. The consideration in this study bases its premise on the consumer choice theory and the analyzed results of a survey on residential location decisions which was enforced in the Tokyo metropolitan area in 1970.

2. SOME RESULTS OF THE SURVEY

2.1 THE WAY OF SURVEY

In the survey, we observed the workers employed in Suginami Ward which is about 13 Km distant from the center of Tokyo metropolis[5]. The reason for choosing this ward as an objective zone was because we intended to observe workers who came from the suburban residential zones and workers who came from the central residential zones.

The items of the survey were, (1) on work trip: travel time and cost to work (2) the existing state of housing: housing cost, floor area, residential lot size and type of housing, (3) attributes of household: income, number of household members and number of workers, (4) reasons for residing at present address, (5) Yes or No about having a moving plan. The total number of valid samples in the survey was 1243. The number of households by tenure of dwelling in the survey is shown in Table 1.

Table 1 The number of households by tenure of dwelling.

Tenure	Owned	Rented, Owned privately	Rented, Owned publicly	Issued	Total
Number of households	651	252	177	163	1,243
Percentage	52.4	20.3	14.2	13.1	100

2.2 THE EFFECTS OF TRAVEL TIME TO WORK ON OTHER FACTORS

When the quality of dwellings can be bypassed, it is supposed that people demand mainly the housing size and the residential location as the objects for their housing demands. The floor area and the lot size are enumerated as attributes of housing size. The travel distance to work and the residential environment are enumerated as attributes for residential location. And it is supposed that the factors generating housing demand are the number of household members, household income and the proportion of household expenditure for housing. The relationships between these factors and the travel time to work are observed as follows. The residential environment is put aside as it is difficult to express the environment quantitatively.

RESIDENTIAL LOT SIZE

From the data of the survey, the percentage of detached dwellings, which had their own lot, was 59 %. Comparing the two, the average lot size in each residential zone by the survey and that of a housing survey taken in 1968 in the Tokyo metropolitan area[6], there was a correlation between them. It is shown as Fig.1. In this figure, the ordinate shows the average lot size of the usual residential places of the worker who worked in Suginami ward, and the abscissa shows the total average lot size in each residential zone.

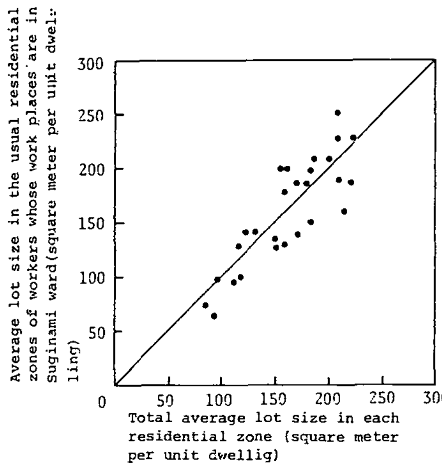

Fig.1
A comparison between the average lot size in the usual residential places of workers who worked in a certain place of work and the total average lot size in each residential zone.

The correlation may be fairly good because the correlation coefficient is 0.863. This good correlation means that there was scarcely any effect of travel time to work on the residential lot size and that the average lot size monotonously increases as the residential zones are away from the central zone. From these facts, it is supposed that the lot size is not a basic factor in the mechanism of housing demand. This supposition can be understood from the fact that many people don't hesitate to live in high building residences.

NUMBER OF HOUSEHOLD MEMBERS

Using data from households which have only one worker, the average number of household members by the travel time to work is shown in Fig.2. In this

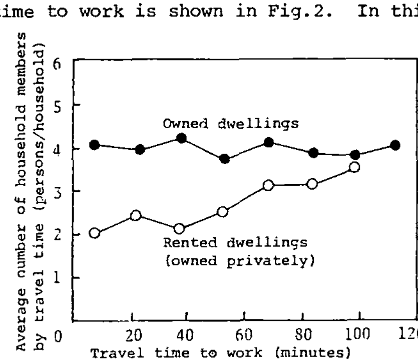

Fig.2
Average number of household members by travel time to work. (The interval of travel time is 15 minutes).

figure, the average number of household members in rented dwellings(owned privately) and owned dwellings are respectively shown. In this figure, the average number of household members of owned dwellings are nearly constant, independently of the travel time to work, but that of rented dwellings(owned privately) increase as the travel time becomes longer. These are interesting facts.

FLOOR AREA

In respect to the housing demand, it is considered that the per capita floor area is a more important factor than the floor area per household. So that, using the same data as the above clause, the per capita floor areas by the travel time to work are shown in Fig.3. In this figure, the rented dwellings (owned privately) and the owned dwellings are shown separately. Both of them

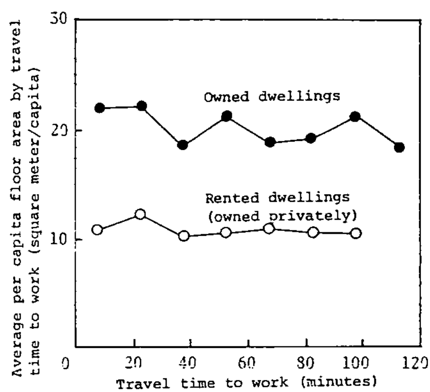

Fig.3
Average per capita floor area by travel time to work.

respectively are nearly constant, independently of the travel time to work. The average per capita floor area of owned dwellings is about 20 square meter per capita and that of rented dwellings(owned privately) is about 11 square meter per capita. The dwellings of the average Japanese are limited like this, so that some foreigners call Japanese dwellings "rabbit cabins".

HOUSEHOLD INCOME AND EXPENDITURE ON HOUSING

Inquiring into the relationships between the per capita monthly household income and the travel time to work, and between the per capita monthly expenditure on housing and the travel time, by the same method as the above clauses, , Fig.4 and Fig.5 are depicted. Both of the per capita incomes of the owned dwellings and the rented dwellings(owned privately) are considerably decreased as the travel time to work becomes longer. It is found that the decrease gradient of the owned dwellings is smaller than that of the rented dwellings (owned privately). One of the principal causes generating such a phenomenon may be due to the fact that households living in the rented dwellings(owned privately) have a tendency to be small families and to live near their work places.

Fig.5 illustrates that the per capita rents of the dwellings(owned privately) go considerably down as the travel time to work becomes longer. The expenditure on housing of the owned dwellings are unknown because it is very difficult to survey this expenditure accurately. So this expenditure is not shown in Fig.5.

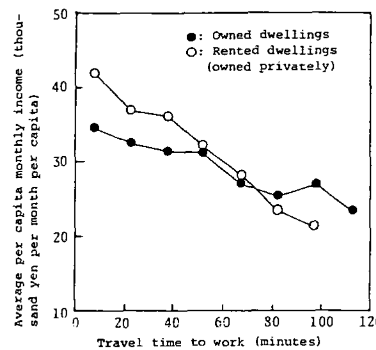

Fig.4
The relationships between average per capita monthly household income and travel time to work.

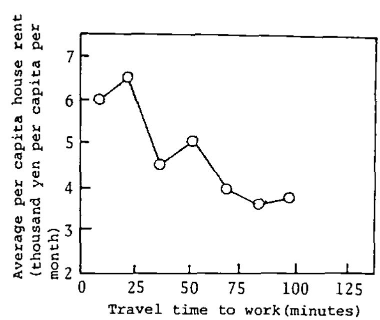

Fig.5
The relationships between average per capita monthly rent of rented dwellings(owned privately) and travel time to work.

2.3 THE BASIC FACTORS WHICH COMPOSE THE HOUSING DEMAND MECHANISM

As the result of the above analysis, it has been proved that the residential lot size may not be directly affected by the travel time to work. So, it is supposed that the residential lot size is not an attribute which makes up for a long travel time to work.

And, although the average per capita floor area of the owned dwellings and that of the rented dwellings(owned privately) respectively are nearly constant, , independently of the travel time to work, the per capita rent and both of the per capita household incomes decrease as the travel time to work becomes longer. From these facts, it is considered that the households whose per capita incomes are low would ask for inexpensive dwellings at a distance, in order to keep a certain floor area per capita. Therefore, other things being equal, it is supposed that the housing demand mechanism may be composed of the following five basic factors: Number of household members, Household income, Housing cost, Floor area and Travel distance to work. This conclusion on the housing demand mechanism may be available only in a country where it is made up of a single race such as Japan. In a multiracial country, the basic factors of the housing demand mechanism may not be as simple as in Japan.

2.4 THE MUTUAL RELATIONSHIPS AMONG THE BASIC FACTORS

When the travel time to work is fixed at any length, the higher the per capita household income rises, the wider the per capita floor area must become , because the rise of per capita income raises its solvency on housing cost. Using the same data as in the above clause, the relationship between the per capita floor area and the per capita monthly household income is shown accordingly in Fig.6, where the travel time to work is from 45 to 60 minutes. The owned dwellings and the rented dwellings(ownd privately) are taken up in this

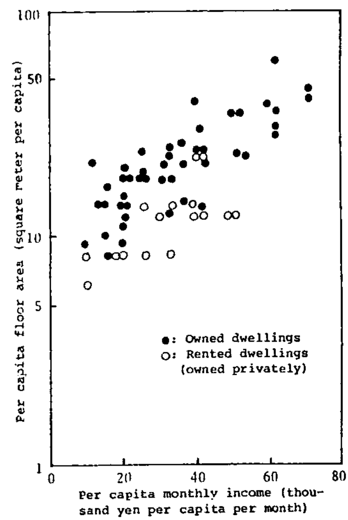

Fig.6
The relationships between the per capita floor area and the per capita monthly income, where the travel time to work is from 45 to 60 minutes.

figure. And the ordinate of this figure is graduated in a logarithmic scale.

The per capita floor areas of the owned dwellings and the rented dwellings(owned privately) respectively are monotonously widened as the per capita monthly household income rises on the figure. So, the relationship between the per capita floor area A and the per capita monthly income I is able to be represented as the following:

$$A = A_{*} \exp(\alpha I) \qquad (1)$$

where, A_{*} is the per capita floor area, which the household without any income asks for at a residential place where the travel time to work is t . The owned dwelling and the rented dwelling(owned privately) have their own A_{*}. And α is a coefficient.

Using the data of the rented dwellings(owned privately), the relationship between the per capita monthly rent and the per capita monthly income is shown in Fig.7. In this figure, the travel time to work is not fixed. This figure illustrates that the per capita monthly rent monotonously rises as the per capita monthly household income rises. And, in Fig.4 and Fig.5, it has been certified that both the average per capita monthly income and the average per capita monthly rent monotonously go down as the travel time to work becomes longer. From these facts, it is considered that the relationship between the per capita monthly rent P' and the per capita monthly household income I must be nearly linear, irrespective of the travel time to work. This phenomenon is represented as follows:

$$P' = \beta'(I - I_0) \qquad (2)$$

where, β' is a coefficient that stands for the propensity to expend on rented

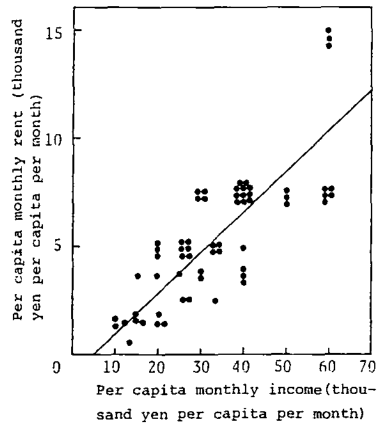

Fig.7
The relationship between the per capita monthly rent and the per capita monthly income of the rented dwellings (owned privately).

dwellings. And I_0 is the per capita monthly household income without any house rent. In the previous studies[2,3], this coefficient β' has not been fixed. They have thought that the expenditure on housing depends on the selection among the trade-offs with other living costs. But, as Fig.7 shows, the per capita rent rises nearly in proportion to the per capita household income. So , in this study, the coefficient β' is assumed to be constant. This assumption makes our theoretical consideration clear.

The gradient of rent reduction in Fig.5 must be what generates, when the per capita floor area is constant, because, in Fig.3, the per capita floor area is nearly constant, independently of the travel time to work. So that, when the per capita floor area is constant, the relationship between the per capita monthly rent P' and the travel time to work t is represented as follows:

$$P' = P_0 - v't \tag{3}$$

where, P_0 is a per capita monthly rent at the residential places where the travel time to work is nearly equal to zero, when the per capita floor area is equal to the average value. And, v' is the gradient of the per capita monthly rent for the travel time to work.

As a result of the above analysis, it is imagined that there are the relationships between the basic factors as shown in Fig.8 and Fig.9. Fig.8 illst-

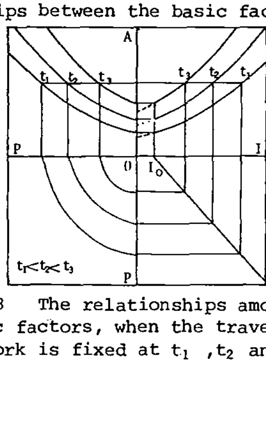

Fig.8 The relationships among the basic factors, when the travel time to work is fixed at t_1 ,t_2 and t_3.

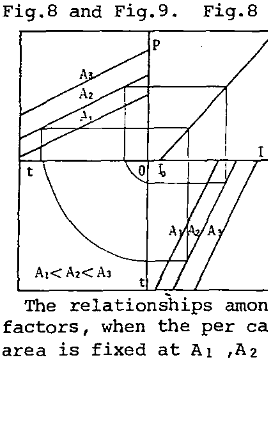

Fig.9 The relationships among the basic factors, when the per capita floor area is fixed at A_1 ,A_2 and A_3

rates the mutual relationships which may be observed, when the travel time to work is fixed at t_1,t_2 and t_3. And Fig.9 illustrates the mutual relationships which may be observed, when the per capita floor area is fixed at A_1,A_2 and A_3. And then, the curves in the first quadrant of Fig.8 are represented by Eq.1 and correspond to the phenomenon in Fig.6. The straight lines that are shown in the fourth quadrant of Fig.8 and in the first quadrant of Fig.9 are represented by Eq.2 and correspond to the phenomenon in Fig.7. And the lines that are shown in the second and fourth quadrants of Fig.9 correspond respectively to the phenomena in Fig.5 and in Fig.4. Equation 3 represents a line in the second quadrant of Fig.9.

3. THE PREMISE FOR THEORETICAL CONSIDERATIONS

Generally speaking, the equilibrium prices of goods or services are those at which the amounts willingly supplied and the amounts willingly demanded are equal. Housing is an economic commodity too, so that, this principle can be applied to housing. There is a view point that an owned dwelling is not an economic commodity, because the household living in it doesn't pay house or land rent. However, there is no difference in the building expenses between an owned dwelling and a rented dwelling. And also, there is no difference in the residential land prices between them. An owned dwelling is a kind of accumlated capital. So that, it must be considered that the households living in the owned dwellings essentially pay rent for their housing and land. For this reason, in the theoretical consideration of this study, it is assumed that all the households live in rented dwellings(owned privately) and pay rent for their dwellings.

In this paper, the monthly bid rent for a rented dwelling(owned privately) is called the housing demand price, and the product of the building expense of a dwelling and the monthly interest rate is called housing supply cost. Here, a payment for a residential lot is not included in the building expenses. Further, the friction cost of a work trip, so called in this paper, means the dissatisfaction incurred by any worker through time loss, money expense and human energy expenditure on his work trip. This friction cost of the work trip is given by the monthly total friction cost of a worker divided by the number of his household members.

The number of workers per household is not the same in every household. There are some households which have several workers. The ways in which residential location decisions are made in multiworker households must be very complicated. So, in this study, all households are assumed to have only one worker, in order to make easier the theoretical considerations.

As mentioned above, the amount of goods which will really be consumed is determined at the equilibrium intersection point where the supply and the demand match. However, this amount has two concepts. One is the size of goods. The other is the number of goods. These two concepts can be fitted in an equilibrium amount in the supply-and-demand of housing. Namely, one concept must be: How many households want to get their dwellings in any given residential zone ? The other concept must be: Where do the households demand their dwellings and How large must they be ? The former is a problem that is related to the distribution of the residences or the work trips. The results of research work on this problem will be presented some other time. The latter is a problem of the trade-offs between the floor area and the friction cost of work trip. The consideration in this paper focuses on this trade-offs.

4. THE HOUSING DEMAND PRICE WITH FIXED FRICTION COST OF WORK TRIP

4.1 MARGINAL DEMAND PRICE AND TOTAL DEMAND PRICE OF HOUSING

Generally the marginal demand price of goods tends to rise as the household income increases and to fall as the amount consumed of goods increases. This principle may be applied to the housing demand mechanism. Now, when the friction cost of work trips is fixed, it is assumed that the marginal demand price per capita dP/dA falls in inverse proportion to the floor area per capita A, and rises in proportion to the remainder between the household income per capita I and those without any house rent I_0. This assumption is expressed as follows:

$$dP/dA = \beta(I - I_0)/A \qquad (4)$$

where, β is a coefficient.

Integrating Eq.4, the total demand price per capita P can be expressed as follows:

$$P = \beta(I - I_0)\ln(A/{}_I a_T) \qquad (5)$$

where, ${}_I a_T$ stands for the smallest floor area per capita which the households, with the per capita household income I, ask for at the residential places where the friction cost of work trip is T.

4.2 THE DECISION OF HOUSING SIZE

Most housing suppliers never fail to pursue the maximum profit. So that, the sizes of housing supplied must be determined at the highest total demand price per unit floor area. From Eq.5, the total demand price per unit floor area is obtained as follows;

$$p = \frac{P}{A} = \frac{\beta(I - I_0)}{A} \ln \frac{A}{{}_I a_T} \qquad (6)$$

Differentiating Eq.6 by A and putting $dp/dA = 0$, the per capita floor area A at the highest total demand price per unit floor area is obtained as follows;

$$A = {}_I a_T \exp(1) \qquad (7)$$

So that, the per capita floor area A depends on the size of ${}_I a_T$.

A housing is a typical superior good related directly with our daily lives. So, as the per capita income rises, the smallest per capita floor area ${}_I a_T$ must accelerately increase to come up to a satisfactory floor area. Then, ${}_I a_T$ is assumed as follows;

$$ {}_I a_T = {}_0 a_T \exp(\gamma I) \qquad (8)$$

where, ${}_0 a_T$ is the smallest per capita floor area which the households without any income ask for at the residential places where the friction cost of work trips are T. And γ is a coefficient.

Substituting Eq.7 into Eq.6, the per capita floor area A is obtained as follows;

$$A = {}_0 a_T \exp(\gamma I + 1) \qquad (9)$$

Equation 9 must correspond to Eq.1, so that $\gamma = \alpha$, and $A_t = 2.7183\ {}_0 a_T$.

4.3 THE RELATIONSHIP BETWEEN THE PER CAPITA INCOME AND THE TOTAL HOUSING DEMAND PRICE PER CAPITA

When the housing suppliers sell their housings to the consumers who bid the highest total price per unit floor area p_{max}, the p_{max} is as the following. Substituting Eq.7 into Eq.6, p_{max} is

$$\mathcal{P}_{max} = \beta \, (I - I_o) \, / \, A \tag{10}$$

It is but just that the total housing demand price per capita P should be equal to $\mathcal{P}_{max} \cdot A$. So that P is

$$P = \beta \, (I - I_o) \tag{11}$$

Equation 11 has to correspond to Eq.2. So that, β must be equal to β' .

4.4 THE TOTAL HOUSING DEMAND PRICE PER CAPITA

Substituting Eq.8 into Eq.5, the total housing demand price per capita P is

$$P = \beta \, (I - I_o) \left\{ ln \left(\frac{A}{{}_o a_T} \right) - \gamma I \right\} \tag{12}$$

Equation 12 expresses a curved surface of the total housing demand price per capita in the three-dimensional space which is composed of three basic factors; the total housing demand price per capita P , the per capita household income I , and the per capita floor area A . The conceptional picture is shown in Fig. 10.

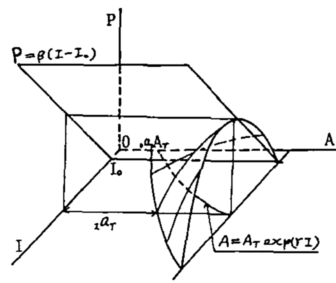

Fig.10
A curved surface of total housing demand price per capita and a plane of solvency on housing cost.

5. THE EFFECT OF THE FRICTION COST OF THE WORK TRIP ON THE HOUSING DEMAND PRICE

When the per capita household income I and the per capita floor area A are respectively fixed, the total housing demand price per capita P may go down as the friction cost of the work trip increases. Then, from Eq.12, the down slope dP/dT is

$$\frac{dP}{dT} = - \beta \, (I - I_o) \left(\frac{1}{{}_o a_T} \frac{d \, a_T}{d T} \right) \tag{13}$$

P must go down by an amount which is equal to the additional increase of T . So that, $dP/dT = 1$. Using Eq.12 and Eq.13, the P's function with respect to T is obtained as follows;

$$P = \beta \, (I - I_o) \left\{ ln \left(\frac{A}{a_o} \right) - \gamma I - \nu T \right\} \tag{14}$$

where, a_o is the smallest per capita floor area which the households without any income ask for from the residential place where the friction cost of work trips are zero. And ν is

$$\nu = 1 \, / \, \beta \, (I - I_o) \tag{15}$$

6. THE RELATIONSHIPS BETWEEN BASIC FACTORS

6.1 THE RELATIONSHIP BETWEEN FRICTION COST AND TRAVEL TIME TO WORK

In this study, it is assumed that a trip from home to work consists of an access trip(; walk to station), a line-haul trip, and an egress trip(; walk to work place), and that there is a relationship between the friction cost and the travel time to work as shown in Fig.11.

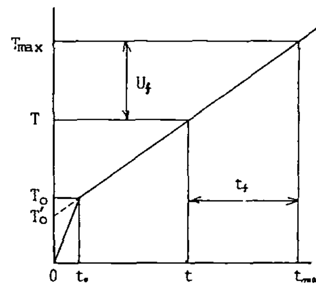

Fig.11
The relationship between the friction cost and the travel time to work.

Where, t_0 is the sum of the access time and the egress time.
T_0 is the friction cost which is taken with t_0.
t_{max} is the time length limit which is able to be spent on the work trip.
T_{max} is the friction cost limit which is taken with t_{max}.
t is the travel time to work.
T is the friction cost which is taken with the work trip.

Then, the free time t_f that is gotten by shortening the travel time to work from t_{max} to t is

$$t_f = t_{max} - t \tag{16}$$

Similarly, the total utility U_f which is produced by the shortening of the travel time is

$$U_f = T_{max} - T \tag{17}$$

Assuming that the total utility U_f, for $t \geqq t_0$, increases monotonously as the free time t_f becomes longer, the total utility U_f is represented as the following.

$$U_f = a\, t_f \tag{18}$$

where, a stands for the time value in the free time.

For $t \geqq t_0$, the relationship between the friction cost T and the travel time t is expressed as follows;

$$T = a t + (T_0 - a t_0) = a t + T_0' \tag{19}$$

On the other side, this a also means the friction cost of the work trip per unit travel time. And then, the contents of a is

$$a = \alpha + \dot{p} V + c\, e \tag{20}$$

where, α is the time cost per unit travel time.
$\dot{p}$ is the money cost per unit travel distance.
V is the travel speed.
e is the human energy expenditure per unit travel time.
c is a coefficient.

6.2 THE RELATIONSHIPS BETWEEN THE FRICTION COST AND THE OTHER FACTORS

From Eq. 11 and Eq. 14, the per capita floor area A, the per capita income I and the per capita housing demand price P are obtained respectively as the following;

$$A = a_0 exp(\gamma I + \nu T + 1) \qquad (21)$$

$$I = (1/\gamma)\{ln(A/a_0) - \nu T - 1\} \qquad (22)$$

$$P = (\beta/\gamma)\{ln(A/a_0) - \nu T - 1\} - \beta I_0 \qquad (23)$$

6.3 THE RELATIONSHIPS BETWEEN THE TRAVEL TIME TO WORK AND THE OTHER FACTORS

When Eq. 19 is substituted into Eq.21, we obtain,

$$A = A_0 exp(\gamma I + \eta t) \qquad (24)$$

where,

$$A_0 = a_0 exp(\nu T_0' + 1) \qquad (25)$$

$$\eta = \nu a \qquad (26)$$

When the travel time to work is fixed at a certain length, Eq. 24 must be consistent with Eq.1. We therefore have,

$$\alpha = \gamma \qquad (27)$$

$$A_t = A_0 exp(\eta t) \qquad (28)$$

Similarly, substituting Eq.19 into Eq.22 and Eq.23, we obtain,

$$I = (1/\gamma)\{ln(A/A_0) - \eta t\} \qquad (29)$$

$$P = (\beta/\gamma)\{ln(A/A_0) - \eta t\} - \beta I_0 \qquad (30)$$

The phenomena in Fig.8 and Fig.9 are illustrated by Eq.24, Eq.29 and Eq.30.

7. THE SUBSTITUTIVE RELATIONSHIP BETWEEN HOUSING SIZE AND FRICTION COST OF WORK TRIP

7.1 THE INDIFFERENCE CURVE AND THE BUDGET LINE

When the per capita household income is fixed, the indifference curve in the combinations of the per capita floor area and the friction cost of the work trip is represented by Eq.21. This indifference curve is dissimilar to the usual one in the combination of two goods. The usual indifference curve is convex(from below), but the per capita floor area by Eq.21 becomes wider as the friction cost of the work trip increases. It is because the work trip causes the commuters to suffer some disutility.

The budget line is introduced from the viewpoint that a worker who needs much free time t_f intends to choose his home close to his work place. When both of the two terminal travel times are fixed, the line is represented as the following;

$$E = pA + T_f = pA + a t_f \qquad (31)$$

where, E means the per capita budget to get the housing and the free time. p is the housing price per unit floor area.

When Eq.16 is substituted into Eq.31, the budget line is obtained as follows;

$$E'' = pA - T = pA - a t - T_0' \qquad (32)$$

where,

$$E'' = E - T_{max} \qquad (33)$$

This E'' means the budget to allow free time. And T_0' is shown in Fig.11.

7.2 THE DECISIONS OF RESIDENTIAL LOCATION AND HOUSING SIZE

If the consumption behavior of each household is rational, it is supposed that each household must decide his housing size and his location to achieve the greatest satisfaction. The greatest satisfaction is yielded at the equilibrium point($\hat{A}$, $\hat{T}$) at which the budget line just touches the indifference curve as shown in Fig.12. Then, the equilibrium per capita friction cost of

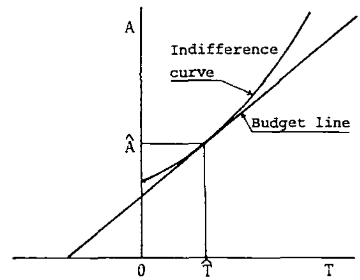

Fig.12
Consumers optimal equilibrium.

work trip $\hat{T}$ and the equilibrium per capita floor area $\hat{A}$ are obtained as follows;

$$\hat{T} = \beta (I - I_0) - E'' \qquad (34)$$

$$\hat{A} = a_0 \, exp(r I + v \hat{T} + 1) \qquad (35)$$

And, the equilibrium housing price per unit floor area $\hat{p}$ is

$$\hat{p} = \beta (I - I_0) / \hat{A} \qquad (36)$$

So that, when the budget of the free time E'' and the household income I are given, $\hat{T}$, $\hat{A}$ and $\hat{p}$ are decided.

Now, we inquire into the maximum amount and the minimum amount of the free time budget E'' . When a worker intends to get the whole of the free time that is produced by the shortening of his travel time to work, his budget for free time must attain the maximum amount E''_{max} and $\hat{T}$ must be equal to zero. Then, from Eq. 34, the maximum amount E''_{max} is

$$E''_{max} = \beta (I - I_0) \qquad (37)$$

If a worker has made no allowance for his free time, he is obliged to live at the furthest residential place where the friction cost T reaches to T_{max}. Then, from Eq.34, the maximum friction cost T_{max} is

$$T_{max} = \beta (I - I_0) \qquad (38)$$

Namely, the maximum friction cost T_{max} becomes equal to the expenditure on housing(: $\beta (I - I_0)$). Substituting Eq. 38 into Eq.34, we obtain

$$E'' = T_{max} - T = a \, t_f \qquad (39)$$

Equation 39 says that E'' is equal to the budget for the free time.

When the budget for the free time(: E'') attains its maximum amount E''_{max}, the whole budget to get the housing and the free time(: E) attains the maximum amount E_{max}, too. So that, from Eq. 33, the maximum budget E_{max} is represented as follows;

$$E_{max} = E''_{max} + T_{max} = 2 \beta (I - I_0) \qquad (40)$$

Then, the maximum budget E_{max} is equal to twice as much as the expenditure on housing. Equation 40 means that a household living close to his work place pays first $\beta(I-I_o)$ for his residence and socond $\beta(I-I_o)$ for his free time. The minimum budget E_{min} is represented as follows;

$$E_{min} = \beta(I-I_o) \tag{41}$$

Equation 41 means that a household without any budget for his free time obliged to live at the furthest residential place and to pay $\beta(I-I_o)$ for his residence.

By the way, Eq.19 can be rewritten as the following;

$$T_{max} = \alpha t_{max} + T_o \tag{42}$$

When Eq.42 is substituted into Eq.38, the value of free time α is obtained as follows;

$$\alpha = \{\beta(I-I_o) - T_o\} / t_{max} \tag{43}$$

8. NUMERICAL EXAMPLES

8.1 INDEXES AND COEFFICIENTS

Using the data of the rented dwelling(owned privately) in which one worker's family lives, various indexes and coefficients are shown in Table 2.

Table 2 The indexes and the coefficients for rented dwellings(owned privately)

α or γ capita per yen	β	A_o m^2 per capita	I_o yen per capita	η per minute
0.390×10^{-4}	0.1875	2.200	5000	0.858×10^{-2}

8.2 THE LONGEST TRAVEL TIME TO WORK

From Eq. 15, Eq.26 and Eq.43, the furthest travel time to work t_{max} is expressed as follows;

$$t_{max} = \left\{1 - \frac{T_o'}{\beta(I-I_o)}\right\}\frac{1}{\eta} \tag{44}$$

When both the access and the egress travel time are equal to zero, the furthest travel time to work is calculated as follows

$$t_{max} = 1/\eta = 116 \text{ minutes}$$

this t_{max} is considered to be a reasonable value.

8.3 INDIFFERENTIAL CURVE AND BUDGET LINE

When the per capita household income is fixed, the indifference curve in the combination of the per capita floor area A and the travel time to work t is represented by Eq.24. And, substituting Eq.19 into Eq.32, the budget line is obtained as follows;

$$E' = E'' + T_o' = pA - \alpha t \tag{45}$$

where, $$E' = E - \alpha t_{max} \tag{46}$$

Let's consider the residential location of a household, the attributes of which are as follows; the per capita monthly income I is thirty thousand yen and the per capita monthly budget for free time is three thousand yen. From

Eq.34, the equilibrium friction cost $\hat{T}$ is obtained as follows;

$$\hat{T} = 1687.5 \text{ (Yen per month per capita)}$$

From Eq.15 and Eq.26, α is obtained as follows

$$\alpha = \eta / v = \beta (I - I_o) \eta$$

$$= 40.218 \text{ (Yen per minute per month per capita)}$$

When both of the terminal trip lengths are nearly equal to zero, the travel time to work t becomes 1687.5/ 40.218 = 42 minutes. And then, from Eq.24, the per capita floor area $\hat{A}$ is

$$\hat{A} = 10.16 \text{ (Square meter per capita)}$$

It is considered that these values may be adequate.

9. CONCLUSION

The purposes of this study posed in the introduction have been achieved. First, it has been found that, other things being equal, the basic factors which the mechanism of housing demand is composed of are the per capita income , the per capita floor area, the per capita housing expenditure and the per capita friction cost of the work trip. Second, it has been indicated that the mutual relationships between these basic factors may be represented by Eq.21, Eq.22 and Eq.23, or by Eq. 24, Eq. 29 and Eq.30. And thirdly, it has been confirmed that the per capita floor area may be substituted for the travel time to work when the per capita household income is fixed. Further more, it has been stated that the housing size and its location are decided at the equilibrium point at which the indifference curve in the combination of the per capita floor area and the friction cost of the work trip just touchs the budget line.

It has been considered that the facts which this study has found must be based on pure economic behavior, because Japan is basically a single racial country.

The methods of how to apply the results of this study to the estimations of some phenomena in an urban area, such as populaton distribution, trip distribution, modal split and residential land values have been accomplished. These results will be presented in another paper.

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