A STOCHASTIC MULTI-MODAL FREIGHT TRANSPORT ASSIGNMENT MODEL WITH RANDOM COEFFICIENTS

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Abstract

Freight transport assignment models are often purely deterministic models who search user or system optimal equilibrium. The models may follow a multi-class structure, i.e. allowing different weights on the attributes in the cost function for different modes and freight classes. However, few models follow a general utility maximum framework, that also allow for description of overlapping routes as well as heterogeneities within freight groups, i.e. random coefficients in the utility function. The paper present a stochastic schedule-based freight transport assignment modelling framework, that allow for both an error term dealing with the overlapping route problem, random coefficients dealing with heterogeneities, and cost functions dealing with capacity problems. This is done within different choice levels consisting of a set of possible mode and route alternatives.

Keywords: Freight transport, assignment models, schedule-based models, random coefficients, mixed logit, overlapping routes, probit-based assignment, multi-class assignment, user equilibrium
Topic Area: D2 Freight Transport Demand Modelling

1. Introduction

Many freight transport assignment models are purely deterministic which follows e.g. a user or system optimal approach. Many models have a specific focus, e.g. the widely used STAN model (INRO, 2002), which uses a system optimal approach, CUBE (Citilabs, 2003) which does not consider schedule based transport, or they are originally developed for passenger transport (e.g. the software packages TransCAD, VISSUM and VIPS),

The paper presents the results and recommended modelling framework resulting from a Danish pre-study for a national model (Nielsen *et al*. 2003a). Different parts of this were presented at Conference on National and International freight transport models (www.clgdk.xxx). The recommended approach from this study was to consider freight transport assignment, sub-mode and mode choice as a joint problem. This distinguishes between decisions made by transport buyers, choices of multi-modal chains, and finally path choice through a network with a given set of a priori modes. Section 2 exemplifies this approach on the Danish freight choices.

Section 3 describes how choice sets are generated; at an upper level in a pre-defined discrete choice set and a lower level by a choice set generation procedure for each of the upper level choices.

Section 4 discusses briefly how capacity can be addressed for freight transport. It is argued that the cost and time functions most often will reduce with volume, opposite the assumptions in most passenger models and car traffic assignment.

Section 5 deals with the utility functions for freight transport. It is argued that error components (random coefficients) are crucial for freight transport modelling. While it is also argued that stochastic elements may be as necessary as for passenger models, if not more.

Finally, section 6 describes how the assignment model is solved for scheduled based transport. The forthcoming Danish freight model will also include modelling of non-scheduled based transport within other of the lower level choices mentioned above. These follow for trucks a similar stochastic user equilibrium framework as described in Nielsen *et al*. 2002, however extended with a semi-dynamic model which allows for different travel times in broader time intervals.

The modelling framework focus on the Zealand and Skåne regions in East Denmark respectively Southern Sweden (see figure 1), for which it eventually will model both internal transport, to/from transport, and in an separate module also through transport. The area includes the capital region of Greater Copenhagen and the Malmö region of southern Skåne with about 3 millions inhabitants. The two regions Zealand and Skåne are separated by a sea sound with two competing ferry crossings and one toll bridge/tunnel. The later includes a rail link.

Traffic between Zealand and Germany has to cross the Femern belt sea, where there are several competing ferries – one with rail onboard, while access to west Denmark has to use the tolled Great Belt Bridge (also both road and rail) or ferries. There are alternative direct ferry lines between west Denmark and Sweden (but not directly to Skåne), and between Skåne and Germany (as well as Poland). The road network has several alternative routes within each region, while the rail network has only a few alternative routes – in many cases just one.

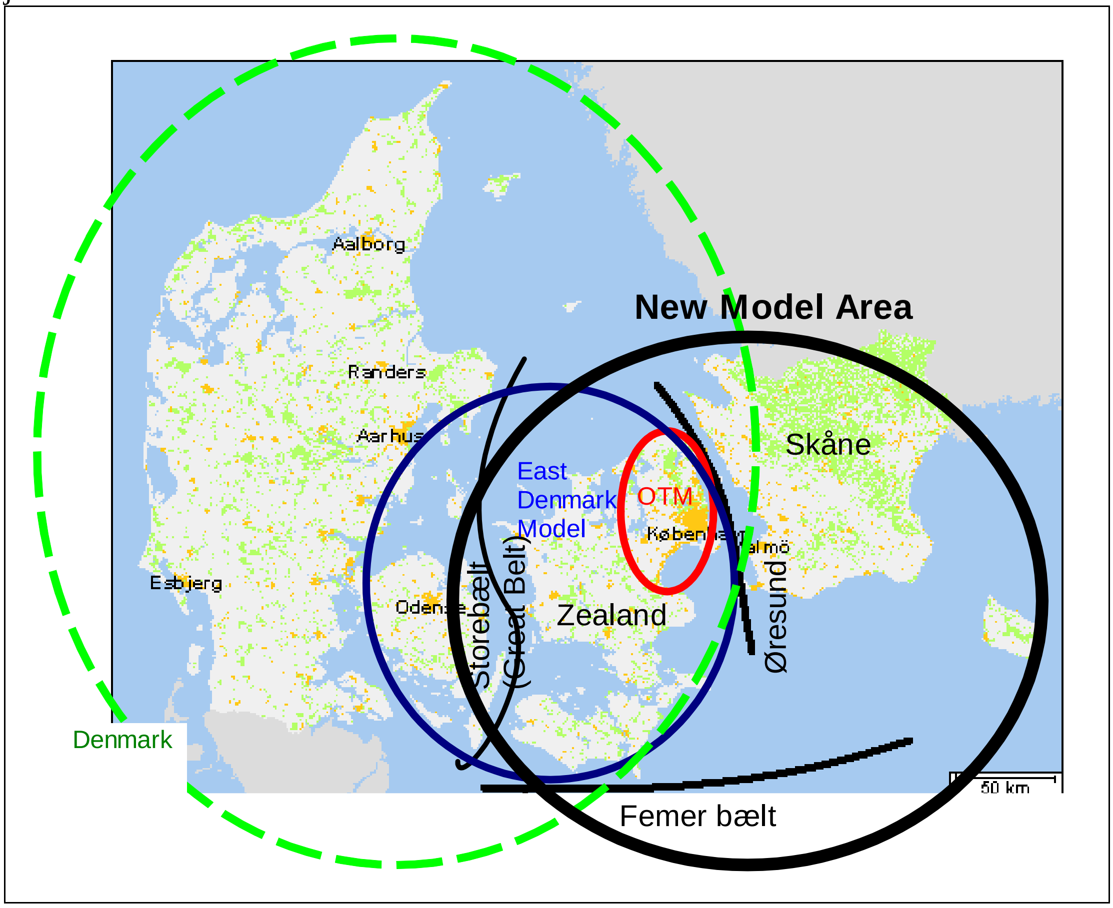

Figure 1 *The modelling area. OTM is an existing model (Paag et.al. 2001) that cower the metropolitan Copenhagen region, including van and truck assignment, jointly in a multi-class assignment together with passenger cars. The East Denmark model (Nielsen et.al. 2001a) provides some network data and matrices for the new model. The new models' core area is the Danish island Zealand (including Copenhagen) and southern Sweden (Skåne). The rest of Denmark and Sweden are described at a more aggregated level, while the remaining Europe is described as port-zones.*

The present state of the model is that the methodological foundation has been established, the software has been developed to a proto-type status, and that network data has been established. However, data for the model estimation has not yet been established, why only functionality of the methods - e.g. convergence, solving overlapping route problems, test of reasonable calculation times - can be evaluated on the full-scale. While specific values of coefficients etc. is assumed based on prior studies.

2. Choice levels

The Danish Prestudy recommended a 3-level hierarchy of mode and route choice modelling for freight transport (Nielsen *et al*. 2003a):

1. <u>Decisions at the level of transport buyers</u>, e.g. door to door truck transport versus multimodal transport, based on level of service, speed, delivery security, etc. These decisions restrict the choice set for the following transport decisions. Transport buyers' decisions can in most cases be dealt with by discrete choice models, e.g. by a nested logit model. Some choice sets are almost given already at this level (e.g. coal import to power plans by ships), while others are more open, e.g. multimodal truck-rail transport versus direct trucks.

2. <u>Choices in multi-modal networks between terminals.</u> At this level only connections (route segments) between terminals are described (Nielsen & Frederiksen, 2001b & 2001c). VISUM uses a similar level of aggregation (Friedrich *et al*, 2003) referred to as route legs, and Cube Cargo construct paths between so-called Transport Logistics Nodes (Citilabs, 2003). Route segments can be generated for schedule-based transport by a frequency based approach (to avoid overlapping route problem) as in Nielsen (2000) or time-table based as in Nielsen (2004). For non-schedule-based transport (typically trucks) one may use a static or dynamic assignment model. Decisions at this level are typically taken by the transport firm, and can be discrete choices, pure route choices, or a combination of the two.

3. <u>Choices of paths between terminals.</u> This may be described by a detailed route choice model. For trucks, decisions are typically taken by the driver (except cases with restricted/forced routes), while trains are typically directed by the rail administration (which may be simulated by separate models, as in Nielsen *et al*, 2001a).

It is noted, that especially 2) may consist of a mixture of route and mode-choice decisions, 1) restrict the choice-set for 2), while 3) is typically a uni-modal route choice problem.

There are a number of unresolved issues in the modelling of particularly route choice for freight transport. For the project at hand it is proposed to model mode and route choice in two stages. The first stage is a discrete choice model dealing with mode choice and also with route choices where there is a clear choice between a few routes. This is particularly relevant for the major ferry/bridge crossings of Øresund, Storebælt and Femer Belt in Denmark, who naturally restricts the choice set to a small finite numbers. The second stage simultaneously generates choice sets and routes using a probit-like model.

3. Choice set generation

Predefined choice sets has e.g. been implemented in connection with the Danish Storebælt (Fosgerau, 1996), Øresund (Øresundskonsortiet, 1999) and Femer Belt (FTC, 1999) forecast models which combine discrete choice models for specific corridors in conjunction with all-or-nothing in between (although the latter may easily be replaced with another type of model). In all these cases, the choice set was pretty obvious and of a reasonable size. Section 3.1 describes how predefined choice set is being defined in the present study.

When the number of alternatives becomes larger, it is more difficult to generate a choice set for discrete choice models. The choice set and route choice can be generated simultaneously by simulation, which makes the two consistent, but this may lead to larger run times and convergence issues. Alternatively a set of criteria can be used for the choice set generation, and another for the route choice. A risk of this approach is that the choice set generation process may not find all alternatives that would have been relevant and chosen in the route choice model. Section 3.2 discusses the choice set generation problem.

3.1 Choice of mode and crossing

The description of networks and demand matrices across Øresund contain some inhomogenities due to the two countries different statistical definitions, which may make it hard to estimate a more ambitious choice model. Therefore it was suggested to treat choice of mode and crossing in a hierarchical choice model and to model route choice in a separate assignment procedure. The following flows are dealt with in separate discrete choice models.

- Between Scandinavia and the continent. This can be structured as a nested crossing location and ferry choice model.
- Between Skåne and the remaining Sweden/Norway/Finland. This can be structured as a nested logit model with four modes.
- Between Sweden/Norway/Finland and Sjælland/Lolland/Falster. This includes both mode choice and point of crossing Øresund. A nested logit model is proposed, although estimation might pinpoint other tree-structures or even a cross-nested model (Sørensen *et al*, 2001a).
- The traffic between Sweden/Norway/Finland and Jutland/Fyn (figure 2) is further complicated, since it includes combinations of various crossings over Øresund as well as Storebælt. Being the most complicated choice set, this has been shown in the figure as an example. The main mode choice is modelled at the upper level. The lower level – which is especially complicated for trucks – includes direct ferries versus connections via Zealand. This again distinguish at the next level between the Øresund bridge or ferries (the latter has two competing companies with different sailing times, frequencies and pricing policies) and at the lowest level between the bridge and two ferry companies between Zealand and West Denmark.
- Finally, choices between Sjælland/Lolland/Falster and Jutland/Fyn are described in a separate nested logit model.

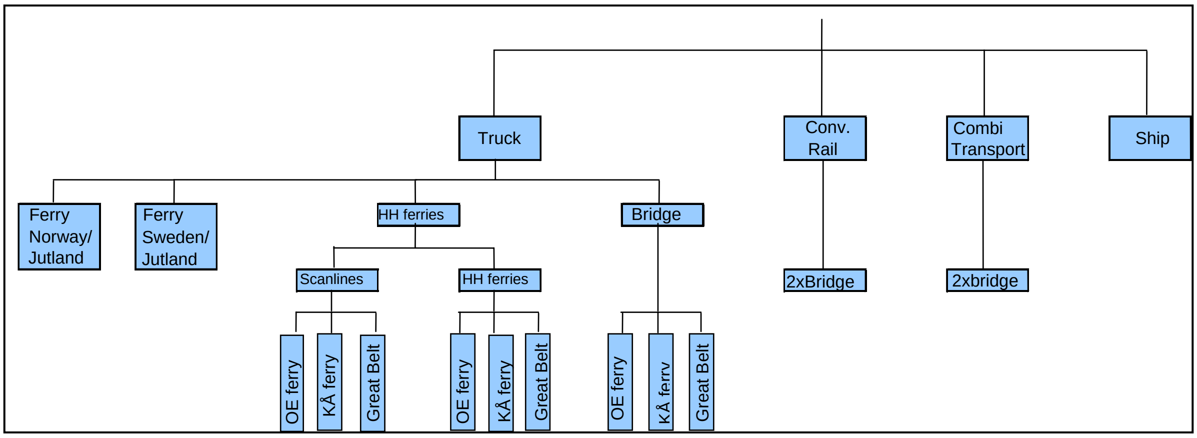

Figure 2 *Model for choice of mode and crossing for flows between Sweden/Finland and Jutland (West Denmark)*

3.2 A priori choice set generation

When a traffic network has a certain (not necessary very large) size, the number of choice combinations becomes very large. Predefined choice-sets can accordingly only be used in some cases or at an upper aggregate level, as for the major choices of modes and crossings described in section 3.1. An essential part of route choice models is however to obtain efficient methods to generate and handle alternatives before to – or simultaneous with – the final route choice and assignment of trips at the more detailed level of network modelling.

The simplest approach is all-or-nothing, which is too restricted for most choice combinations. It is none-the-less used in many route choice models, e.g. the Danish/German Senex model or the Swedish SAMGODS (2001) which within each class assume all-or-nothing path choice (unless there are capacity problems, which are very rare in Sweden). Extending this to a k-shortest path algorithm does not solve the problem on larger network, since this will often lead to a number of alternatives to one main route and may overlook other main alternatives (unless 'k' is very large).

A commonly used approach is to use a set of deterministic rules to generate a choice set, e.g. fastest, shortest or cheapest route. However, it is not certain that this will include all relevant alternatives for the following choice model, e.g. if the optimal route is neither the fastest nor cheapest. Da la Barra, *et al.* (1993) and Bekhor *et al.* (2000) have tested different search algorithms to generate choice sets; however the latter case did not fit observed choices particularly well. Ramming (2002) improved this method, while Fiorenze-Catalano m.fl. (2003) showed that a rule-based choice set generation method works for intercity passenger traffic in most cases.

Another approach to generate choice set is to search the entire solution space of combinations of coefficients in a deterministic utility function, which for a limited number of variables turned out to be efficient (Nielsen & Jovicic, 2003b). This is however an infeasible approach if the utility function has more than 4 variables. Ibid. limited it to length, cost, free flow time and congestion time. Naturally, this approach ignores potentially stochastic elements, i.e. an error term. However, this turned out to be of marginal importance in Ibid (i.e. the routes could be reproduced without this)

The Dutch SMILE freight-model (Tavasszy, 1998a) generates choice sets within apparently 12 different predefined mode/terminal combinations (5 networks and several transport forms per net). The choice between the alternatives is then done by assuming a distribution of the good-categories' value of time. A somewhat unclear aspect is that each of the 12 shortest paths would be different assuming different VoT. There might accordingly be an inconsistency with the following choice function.

Several types of discrete choice models can consider the overlapping route problem, if a discrete choice-set has been generated and enumerated. Examples are C-logit (Cascetta et. al., 1996), path-size logit (Ben-Akiva & Bierlaire, 1999), and Network GEV (Bierlaire, 2002).

The Biogeme software (Bierlaire, 2003) has an attached module BioRoute, which for medium-sized network can generate choice sets that then can be modelled by using the network GEV in Biogeme. This can be a way to model main route or mode choice in an aggregated transport network. As this is a new possibility, it has not yet been adapted in Danish studies though.

3.3 Simultaneous generation of choice sets and routes

Discrete choice-sets may not be complete for larger network sizes, even though they still may be very large. The modelling frameworks in the Copenhagen region therefore use a method that simultaneously generates paths and assign traffic to them. Different methods that do not need a priori choice set generations have been proposed in the literature;

1. Dial (1971) developed an algorithm to solve a logit-based assignment in a network. It is here assumed that all routes are uncorrelated (the general assumption behind the logit model). This is however doubtful in most real network (Sheffi, 1985). The generation of the full choice set and path probabilities are very efficient (the calculation complexity is near Dijkstra, which is typical used to solve all-or-nothing path search). Dial's algorithm can also be used to generate a choice set (although the calculation complexity increases if all paths are stored), followed by a recalculation of the choice probabilities by a path size logit.

2. Dial (1997) formulated a so-called bicriterion algorithm, which makes it possible to solve a route choice model with both time and cost in the choice function, where the value of time (VoT) follows a distribution. The model does however not include an error term (does not consider the general overlapping route problem) and is restricted to two variables only (cost and time can be the sum of a number sub-variables, but only one distribution can be included). The method is however useful in network with few overlapping routes – typical in multi-modal networks on a larger geographical scale.

3. The third approach is to use simulation to simultaneously generate choice sets and route probabilities by an approximation to the probit model. The difference to the logit model family is that the error term is simultaneous normal distributed (opposite independent Gumbel) over alternatives, and that the variation of the error term is proportional to the mean utility of the alternative (opposite identical Gumbel). The principle in the solution algorithm (Sheffi, 1985) is that the error term is simulated in an inner loop while all-or-nothing routes are generated. An outer loop (the Method Successive Averages, MSA) are weighting the results with prior iterations. The distribution between routes approximates a probit model as the number of iterations increases. The calculation complexity is identical with the complexity of generating a choice set to a discrete choice model by simulation, and by using MSA one does not need to store paths (which can be infeasible for larger networks due to RAM restrictions).

Although a probit-based assignment model is fairly complex and calculation demanding, the model has been used in several large-scale models in Denmark with success, e.g. in the Harbour Tunnel model for cars, vans and trucks route choice (Nielsen et. al., 2002), frequency-based assignment in Nielsen (2000) and timetable-based in Nielsen (2004). Since this model type can combine both an error term, random coefficients in the utility function and flow-cost relationships, it has been chosen for the freight assignment in the present paper as well. As freight network assignment is usually done at a more aggregated level of detail (e.g. that fewer lines exist than in regional public transport) the calculation time and convergence even becomes a less critical issue than when used for passenger transport. The model is however more difficult to estimate and calibrate due to the complex utility functions. For this reasons the Danish freight model prestudy (Nielsen *et al.* 2003a) proposed a hierarchical model structure with MNL at the upper level, and only use of the probit model at the lowest level only.

3.4 Multimodal route choice models

Several research oriented studies have used multimodal route choice models, i.e. models who integrate choice of mode in the route choice model. Common for them is that they operate on a so-called multi-modal transport network, where different modes can use different part of the network, which are linked together at terminals. The generation of choice set and choices of routes are done by generalised network algorithms. The technique itself is not new; Sheffi (1985) refer e.g. to 'super networks' for the same problem.

There has been a considerable progress with multimodal assignment methods from the late 1990'ies. Catalano *et al*. (2001) and Lindveld (2003) provides overview of more advanced multimodal methods in general, while Southworth & Peterson (2000), Carlier *et al*. (2003), and Fernández *et al*. (2003) are specific examples on multimodal freight models.
The problem with multi-modal assignment has traditionally been calculation complexity, which have been solved today, and data management, which e.g. Nielsen & Frederiksen (2001b) and Southworth & Petersen (2000) have dealt with efficiently. Today it is therefore mainly the limited experiences on estimation and calibration of such models that have limited their use in practice. The problem is that discrete choices (e.g. mode choice) and near simultaneous choices (routes) are mixed together in the same model, which makes it difficult to describe correlated alternatives and different decision levels. This is naturally avoided in all-or-nothing or equilibrium approaches on multi-modal networks, e.g. in SAMGODS, which however does not appear reasonable for e.g. mode choice.

Due to the estimation problems, a multi-modal approach for simultaneous route and mode choice was rejected for the first phases of the Danish freight model.

4. Equilibrium principles

Wardrop's (1952) first principle defines the *user equilibrium* in a traffic network as the state in which no passenger can reduce his/her travelling time solely by changing route. Daganzo & Sheffi (1977) extended this principle to the Stochastic User Equilibrium (SUE):

An equilibrium is obtained where no traveller's perceived cost can be reduced solely by the traveller changing route.

The *perceived* corresponds to the fact that travellers may not have complete knowledge of the network, and to other random variation. The *cost* is a generalisation of only one variable, namely time consumption. This can be reformulated to a utility maximisation with any *utility function*, where the sign is reversed. Daganzo & Sheffi (1977) formulated SUE as a principle and mathematical programme, and Sheffi & Powell (1982) found an operational solution algorithm hereto (Method of Successive Averages, MSA). The East Denmark Model (EDM, Nielsen *et al*. 2001a) used an extended version of the user equilibrium:

An equilibrium is obtained where no traveller's perceived utility, determined by the traveller class' utility function, the traveller's preferences, and the type of vehicle (service) and its reliability can be increased solely by the traveller changing route at the desired time of travel.

This implies an equilibrium with a discrete number of traffic classes, each with its own utility function, variation of its coefficients (variation of preferences within each class modelled as random coefficients) and error term (unexplained variation, handling of overlapping routes).

The vehicle type (service) influences reliability as well as the Level of Service (LoS) within the utility function. The utility on a link is allowed to influence other links, e.g. if a full ship in a given departure prevents adding more containers on it (Nielsen *et. al*., 2001a, tested a similar approach for passenger transport). Finally, the LoS depend on the desired time of travel through the timetable, punctuality (delay distributions) and capacity.

Freight transport models often adopt principles from passenger models, e.g., network assignment following a user equilibrium approach, where travel times and costs increases with volume. STAN (Samgods, 2001) finds an system optimum (equilibrium). This may not correspond to freight haulers actual choice pattern, which is more likely to be stochastic user equilibrium. In the Danish model following freight model formulation is therefore suggested;

An equilibrium is obtained where no freight categories' generalised cost function determined by the freight class' generalised cost function, the transports' preferences (parameterised generalised cost function and random coefficients), the type of vehicle (service) and its reliability can be increased solely by the freight category changing route at the desired time of travel

Most assignment models assume increasing travel costs with volume. However, this seems improper at the level of route segments choices in freight transport;

- Travel times may reduce with volume, due to higher frequencies and maybe use of more efficient means of transport. This is often the case when there are no capacity restrictions. Infrastructure capacity is rarely a core issue for sea and rail transport, and few roads face rarely heavy congestion due to truck transport alone.
- Travel costs will in most cases reduce with volume (demand), since larger more efficient means of transport can be used, and they can be used more efficiently as volume increase. Examples are container ships, which have low unit prizes on high demand routes, and truck transport that is cheaper between regions with high demand and competition than between other destinations.
- Economic theory often assumes that cost reduces as demand rises and visa-versa.

The above issues can in advanced transport models be described as part of a transport logistics module (which is outside the scope of the present paper). However, a simplification is to assume simpler cost functions at a route segment level. Transformation from goods to trucks may be used to estimate the possible frequency and hereby hidden waiting time. And volumes may indicate possible train length and ship sizes and hereby cost per transported good unit. These functions must be calibrated for different modes and goods types.

Capacity restrictions (normal speed flow curve) are then mainly relevant for the transport network as one would normally expect that the market will provide sufficient transport capacity in terms of means of transport. In Denmark, capacity restrictions for road transport are concentrated in the Copenhagen region and the Danish model will not consider capacity elsewhere.

5. Utility functions for freight transport assignment

The utility function for a given transport is described by the class of transport it belongs to. High value goods for just in time production has e.g. higher values of time and demands to precision compared to low value heavy goods. The preferences (coefficients) within each class may also differ, which is modelled by random coefficients (the functional shape can be estimated or assumed a priori).

The line-based transport network refer to the physical network by using an object oriented GIS-model similar to Nielsen & Frederiksen (2001b). The lines may refer to the same links that are used for individual traffic assignment, e.g. that line-based trucks use the same road as door-to-door trucks. Or lines may run on its own network (rail typically)

Each (sub)mode i is described by specific coefficients on the travel time, as well as possible other attributes X_{ji}, where j indicates the vector of attributes. Random coefficients are included in the utility function to consider different preferences within groups. These are added as stochastic terms ξ_j to the traditional coefficients β_j. It is often assumed that the distribution of one coefficient is uncorrelated with the others. However, one may assume that a high value of one coefficient j (e.g. free flow time) is correlated with another coefficient j' (e.g. congestion time). The general framework therefore allow for a matrix of correlation, i.e. j x j':

$$U_i = \sum_j \left(\beta_j + \xi_{j\,x\,j'}\right) X_{ji} + \varepsilon_i \,, \tag{1}$$

Where the error term ε_i (Gumbel distributed) can be generalised to allow for a nested logit formulation to employ e.g. different Stated Preference (SP) datasets used for the estimation.

It is noted, that the choice context is many alternatives within the same main mode. The choices are accordingly not only between a discrete set, but rather due to different preferences regards to attributes within a set of combined choices and (sub)mode-chains. The random coefficients describe different weights and priorities between attributes, while the random coefficients describe choices between main alternatives (e.g. binary choices in a Stated Preference experiment used to estimate the model).

The choice situation in real schedule-based networks includes varying knowledge on the network. Arc-based variation, $\varepsilon_{a,}$, are accordingly added to describe the choices in the network;

$$U_R = \sum_j \left(\beta_j + \xi_{j\,x\,j'}\right) \sum_{a \in R} X_{ja} + \varepsilon_R + \sum_{a \in R} \varepsilon_a \,, \tag{2}$$

Where a indicates the vector of arcs, e.g. along a route R between a zone pair. Note that the choice-set consists of routes R instead of alternatives, i.

A problem with (2) is, that ε_R is unpractical in network algorithm for large-scale networks, which seldom explicitly collects all routes before assigning traffic to them (since this would make the algorithm time-inefficient). Furthermore, because many of the routes are overlapping, the ε_R*'s* do not fulfil the IID assumption for ε used in (1). To solve this problem, ε_R can – with some approximation - be considered additive in application. It can hereby be skipped from the assignment procedure by calibrating a larger ρ (variation of ε_a).

5.1 On the random coefficients

Importance of heterogeneous preferences is widely recognised in passenger transport models. Freight or firms do not have preferences or utilities like people do. Still it is very likely that heterogeneity is just as important for freight, whereby the generalised cost components may need random parameters in the generalised cost functions as well. In general, the Danish pre-study (Nielsen *et al.* 2003a) recommended to model as much heterogeneity as possible through a parameterisation of the general cost function. Nevertheless, there might be enough remaining heterogeneity that explicit modelling of this is needed, due to the following reasons:

- In general large heterogeneities can be found even within a sector (e.g. value of goods). Transport of fresh fish for sushi will e.g. most likely have a higher value of time than fish remains to be used in the animal food industry, although both are within the same (statistical) goods categories. This would apply for many other sectors, e.g. designer furniture (a major Danish industry) versus mass consumption furniture (like Ikea).
- There might be variation of preferences within a certain category of goods / freight transport, e.g. due to different business strategies and concepts.
- There might be variation of time restrictions due to different production principles, e.g. just-in-time production versus in-house warehouses, in-house production versus outsourcing, etc.

Although most random coefficient models in practice use normal distributions, this is not done in the present model for three reasons;

1. The normal distribution provides also negative values, which is in conflict with the assumption that e.g. time use represents a cost,

2. The negative values cannot be part of route choice models, since this will increase the algorithmic complexity considerably (Dijkstra can e.g. not be used, refer e.g. to Bertsekas, 1998)
3. The Value of Times (VoT) are undefined with normal distributed cost coefficients (Nielsen & Jovicic, 2003b) (if the VoT is calculated as a normal distributed time coefficients divided by a normal distributed cost coefficients. This follows a Gauchy distribution which is undefined in mean and variance).

An often used alternative to the normal is the logarithmic normal distribution, which has a fairly uncomplicated functional form and is multiplicative which means that the VoT is also log.normal if the cost and time coefficients are log.normal. The first example of the use of this for route choice applications (according to the author's knowledge) was Ben-Akiva *et al*. (1993), who described a number of applications and characteristics of a mixed logit model with log-normal distributes coefficients.

The work on Danish data indicates that log.normal distributions seem to be the best choice in most cases, and that especially the distributions of time-coefficients are highly correlated. The log.normal distribution has the advantage of being non-negative (short travel time is preferred to long) and multiplicative. The latter makes it possible to calculate the value of time directly by calculating the two parameters in the joint distribution of the time coefficient over the cost coefficient and then calculating the mean (NOT by taking the ratio between the two coefficients). This is a benefit compared to the normal distributed coefficients, where the VoT is only defined, if the cost coefficient is forced to be fixed.

In some cases it could be argued, that error components may compensate for too few segments in the model. It is recommended that test of segmentation in trip purpose or other attributes should be made prior to adding coefficients.

All Danish experiences with mixed logit models revealed so pronounced improvements compared to MNL or NL that they were statistically with no doubt preferable (by comparing the likelihood functions). The model type is also feasible today, since both the Biogeme and Alogit software can be used for the estimation (the latter in an unrealised test version received by the author from Andrew Daly; Rand Europe). Both can include simultaneous normal distributed as well as log.normal coefficients. Biogeme has in the latest version also possibilities to estimate more general mixed logit models. Both products have been used for model estimations with good results at CTT/DTU.

The Danish Harbour Tunnel model (Nielsen *et al*. 2002) included distributed coefficients for vans and trucks, which is also recognised in Williams *et al*. (2002), p. 74 'The use of a distribution of value of time as in the Copenhagen model is a potentially interesting approach to this'.

Limitations with prior model estimation software have now been overcome to a level, where the model is fully applicable for practical project evaluation. A problem though is that relatively few companies operate within each mode and goods sector. On the one hand this is an advantages since fewer interviews must be made. On the other this may provide too few observation for the statistical estimation. An example is cement for the concrete in the construction industry, where one firm now operates the entire national production of cement.

5.2 On the error term

The error term, ε, deal with two phenomena 1) unexplained differences in users choices, i.e. ε_R, and 2) the overlapping route problem (the traditional error term ε_a in route choice models). As mentioned after formula (2) the two are often calibrated and modelled as one to avoid route enumeration in application. In the following it is discussed why it is important to include these terms.

5.2.1 Why rational behaviour is not a valid assumption

In some circumstances it is a myth that freight transport act more rational in term of generalised transport cost than passenger transport, e.g. due to;

- The cost of knowledge acquisition (time use). If a logistic manager earns a high hourly wage, it is not rational to use a lot of time to reduce transport cost for infrequent transport tasks.
- Transport cost are often infinitesimal compared to production cost, capital cost, ware house costs, etc. Therefore transport costs may not be a core area of focus in many companies who buy transport (This has been revealed in several surveys carried out by participants in the Danish Research Centre on Logistics and transport, www. clgdk.com). Some stochasticity on the choices must accordingly be assumed.
- Transport buyers use often the same transport company due to habits or convenience, standard contracts (i.e. fixed partner of co-operation) or local preferences (this is e.g. fairly frequent in some provincial regions). A standard contract may provide general discounts and ease for the firm. While a few of the transport tasks could have been cheaper by using other suppliers, which is not realised by the firm.
- Some companies have their own transport departments that are used for all transport (only the marginal cost is considered for each transport).
- Each transport company may not have access to the full network (e.g. not to some terminals and transferring points), they may travel in some parts of the network very infrequent (e.g. foreign trucks with an infrequent delivery in Denmark), and they may only know the main network.

For these reasons it can be the case, that the error term has as large or larger variance compared to the mean utility than for passenger cars. This was e.g. the case in the Copenhagen assignment model (Nielsen, *et al*. 2002)

5.2.2 Solving the overlapping route problem

The logit-based stochastic route choice model (Dial, 1971) is based on the assumption that the individual routes are independent, which as mentioned in section 3.3 causes problems in traffic networks with overlapping (correlated) routes. Daganzo & Sheffi (1977) proposed to use probit-based models in order to avoid this problem. The probit model is based on an underlying assumption of Normal- instead of Gumbel-distributed error terms. Sheffi & Powell (1981) solved the capacity dependent stochastic user equilibrium (SUE) with the Method of Successive Averages (MSA).

Usually, it is preferred that the arc-based variation follows additive distributions in order to make the route choices independent of the specific digitisation and segmentation of the network. This is secured by;

1. Forcing the variance of ε_a to be proportional to the mean of the utility, i.e. E(·) = $\rho \cdot$ var(·). This can be done proportional to the entire utility including random coefficients and delays (the latter similarly to Sheffi, 1985). The argument for this is, that each category optimises its utility function, with its specific coefficients (a draw of a set of random coefficients) and considers the network at equilibrium. Cascetta (2001) however argues that the error term should only be proportional to the fixed part of the utility functions. The method proposed in the present paper allow for both configurations, however only the first formulation have yet been tested. The proportional factor is sometimes referred to as the error term in the assignment literature. However, to avoid confusion ρ is used in the following;
2. Using a distribution which is additive in mean and variance (van Vuren, 1995, and Nielsen, 1997).

Using a normal distribution fulfil this in the case without truncation (Sheffi, 1985). But the condition is violated if truncation is likely. The probability of truncation depends on ρ as well as the mean utility, since the probability increase with decreasing mean utilities. This is a problem, since some time-components (e.g. a one-minute dwell-time at a station) are small.

An alternative is the Uniform distribution that approximates the normal distribution on large networks with many iterations due to the central limit theorem. This works better than the truncated Normal (Nielsen, 1997), but it does not eliminated the problem with truncation fully, and posses a potential convergence problem.

The Gamma distribution fulfils the two conditions directly, since it is additive and positive, whereby truncation is avoided (Nielsen, 2004).

The overlapping route problem must be fulfilled for fixed coefficients β, i.e. on a given homogeneous user segment. When solving the equilibrium model in practice, this imply that the simulation of the stochastic coefficients must be carried out prior to the simulation of ε_a within each iteration in the solution algorithm.

When *applying* the model, ρ must be calibrated by simulation and compared with observed route choices. Accordingly, the estimation and calibration procedure can be formulated as:

1. Estimate utility functions (coefficients and random coefficients) as in formula (1);
2. Transfer these to the assignment model and calibrate ρ by simulation (this is an uncomplicated task as the problem is one-dimensional);
3. Carry out convergence tests for the selected ρ to determine the needed number of iterations in the assignment procedure, since this may depend on the level of variation in the model. Note that usual convergence measures such as comparison of the last two steps may underestimate the needed number of iterations compared to statistical tests where the model is run several times with different seed-values (Nielsen, 1997).

5.3 Discussion on the solution algorithm

Probit-based models and simulation solve the problem of overlapping routes, but converge slowly. Therefore, a number of alternative methods have been developed and presented in the literature. These can in practice primarily be applied in networks with relative few alternatives – e.g. intercity traffic, where the whole relevant set of choices can be found. Examples are the C-logit model (e.g. Cascetta *et al*, 1996), which has been used in practice e.g. in Nuzzolo *et al* (1997). Path-Size Logit (e.g. Ben-Akiva & Bierlaire, 1999) is a further development.

A promising contribution has also been applied by Cross-Nested Logit (Prashker & Bekhor, 1998) and link-nested models (Vovsha & Bekhor, 1998), which however remains to be solved for big networks.

Many of the mentioned models have been applied for car traffic assignment, but they can easily be modified to be used in schedule-based assignment with a solution algorithm as discussed in the present paper (opposite frequency-based models). C-logit has been used on intercity rail transport in Italy, and is used in the VISUM-package (Friedrich & Wekeck, 2002).

There are accordingly alternatives for simulation-based solution algorithms in cases where 1) a near complete choice set can be formulated, or 2) it is reasonable to reduce the choice set, and 3) the utility function can be brought to a form with a deterministic component and an error term. The first two criteria make it difficult to use the models on huge networks – e.g. the Greater Copenhagen area network. But for medium sized urban areas and intercity transport, path-size logit seem to be a good alternative, since this avoid simulation and hereby speed up convergence considerably.

6. The solution algorithm for schedule-based transport

The solution algorithm for schedule-based transport build on the work presented in Nielsen (2004), as the problem is quite similar for passenger and freight transport (except that different networks, utility functions and rules are used).

As in passenger transport, feeder mode links (walk, bike and car to park&ride) may not be schedule-based. In freight assignment trucks can be considered as a feeder mode to rail and ships, while trucks themselves can operate according to schedules as well (e.g. for parcel post and mixed cargo). Non-schedule-based freight trucks (and vans) can be modelled as in Nielsen *et al*. (2002), where they are assigned simultaneous with passenger cars. Schedule-based freight trucks are preloaded to the network, as buses also are in Ibid.

The upper level schedule-based problem is typically solved at the route segment level (Nielsen & Frederiksen, 2001b), i.e. where a route segment connects two terminals, and can be considered as an arc in the graph. At the lower level, i.e. the network modelling, the traffic is preloaded at the links and nodes along the segment in the graph before the non-schedule-based transport is assigned. A few other models and software also distinguish between route segments choices and network loading, e.g. VISSIM/PTV (Friedrich et. al. 2003) with the term route legs, and Cube Cargo (Citilabs, 2003) as path legs between Transport Logistics Nodes (Cube Cargo). With the emerge of GIS-based databases (Nielsen & Frederiksen, 2001b) or special software products (e.g. VISSIM), it is now fairly easy to establish and maintain the necessary data for such models.

The solution algorithm for the schedule-based assignment builds heavily on the work presented in (Nielsen, 2004). It was here found possible to optimise utility-based stochastic timetable-based models in three ways:

1. Optimisation of the implementation, especially building the graph by a pointer structure in memory, and pre-coding search routines as pointes, speeded up calculation time.
2. Optimisation of the shortest path algorithms by use of other methods than the classic Dijkstra, e.g. iterative algorithms, by reducing the size of the calculation graph (removal of irrelevant parts of the graph in a way effective for calculation), and by using the principle of event dominance (Florian, 2004).
3. Optimisation of the solution algorithms for the route choice model (MSA), by trimming the updating mechanism.

The method was tested on the large-scale East Denmark Model (EDM) network (Nielsen et al. 2001a), which was a passenger network of larger size than the Danish freight transport network. As a test is proved that the method can work for freight as well (whether a departure is by a freight train or passenger train does not matter from an algorithmic point of view).

6.1 Basic optimisation of the all-or-nothing algorithm

The optimisation of all-or-nothing algorithms in calculation graphs has attached considerable focus in research. And the different algorithms appear in textbooks within the area. For a basic introduction to network algorithms refer to Cormen *et al* (1998), whereas Bertsekas (1998), introduce different more complex methods.

The best-known solution algorithm is by no doubt Dijkstras algorithm, as it is efficient and has a predictable behaviour (it always finds the optimal solution). However, it is not necessarily the most efficient algorithm. The central operation in Dijkstra – resorting of a binary heap (or binary search tree) – is rather demanding, and the total calculation complexity is thus worse than linear (O(m*log n). This may lead to problems for big graphs, as in timetable-based networks. In certain types of networks there is an optimisation potential in other types of heaps**,** e.g. Fibonacci Heap, though the advantage in practice is often negligible due to the complicated operations on such heaps.

Nielsen & Frederiksen (2001c) investigated different alternative all-or-nothing algorithms. Especially the so-called iterative methods (label setting instead of search for optimum nodes) as L2Queue, LDeQueue, LXThreshold and LX2Threshold have some interesting characteristics (see e.g. Bertsekas, 1998). However, initial tests of label setting methods compared with Dijkstra only revealed marginal improvements on the EDM network.

6.2 Graph reduction in timetable-based networks

The primary way to reduce graphs for schedule-based assignment models is to sort the runs within each line. And to investigate only the first departure with a given line from a given node during the construction of a specific search tree.

The possibility that a given trip waits for the next departure of a specific line is hereby ignored. This is, however, only relevant if there is an transfer later during the trip, the waiting time in the next transfer is longer than the frequency for the current line, the decision maker feels confident that the next departure follows the timetable, and that the next terminal for one reason or the other is so undesirable (e.g. due to higher storage costs) that it is preferred to wait at the current terminal instead of at the next. An exception though is cases where the time of the first departure is flexible and that the feeder line (e.g. truck) has a higher frequency than the main line (e.g. freight train).

For certain cases and with certain value of time the graph reduction may accordingly lead to different results than optimal. However, this can easily be tested for by in a backward search to check if later departures with the prior line can reach the second line, and if the utility of this is better than the present option. This adds some calculation complexity to the problem – however still much less than a search on a complete graph.

By only using the first departure of a given line, the number of links in the graph can be reduced proportionately by the number of departures (runs). With a calculation complexity of $a{\cdot}\log(s)$, where a is the number of links (Arcs) and s is stops/nodes along the lines in the network, the reduction of the calculation time is proportional to the number of departures with each line. In EDM this corresponds to a graph reduction of 75% (note that the graph size is not necessarily proportionate to the calculation time, which also depends on search algorithms, memory structure, etc.). Adding a backward loop would add complexity to the remaining 25% of the graph. Though how much depends on the frequency of the different lines.

Only traffic from each zone (tree) may be assigned by means of this technique, which has its drawbacks compared to assigning the entire matrix at once (dependent of computer memory, size of traffic network and choice of all-or-nothing algorithm). However, the assignment of a whole matrix reduces the number of stochastic simulations significantly, which due to the iterations in the outer loop (MSA) probably more than counteracts the benefit of assigning the entire matrix, cf. experience with the use of the deterministic schedule-based model in EMME/2 (INRO, 2002 and Florian, 2004).

Finding the first departure of a line is also connected with a calculation time. At fixed frequency this can be done by 1 search (direct calculation of first coming departure time) up to log[#departures] (binary search in a completely random timetable). However, even then the search might be optimised by a suitable construction of a time-dependent graph, which will however make implementation of transfer links more difficult.

In traffic networks with many departures of each line (typically in city areas or at the primary railway network), the above technique may reduce the calculation time significantly. But even in networks with low frequencies (e.g. main lines in sparsely populated areas) the technique is advantageous, as in the worst case it is as quick as a pure Dijkstra.

The technique requires similar stopping patterns for each line. A generalisation hereof might be to develop an aggregation method, where lines, which in a sequence of stops have same the stopping pattern and mode type, are aggregated. This aggregation may be performed once and for all for a given network. But the question is whether the benefit is worth mentioning, according to the discussion on frequency aggregation in frequency-based models in Nielsen (2000).

6.3 Event dominance

The principle in the prior section affect the forward search in the graph, i.e. decide to skip departures from a node. The idea of event dominance is, that arrivals to a node (in EMME/2 generalised to events and activities) is pre-processed and skipped (not put into the heap) if they are dominated in time and utility (in EMME/2 simplified to time and cost). This does not reduce the graph, but reduce the operations significantly on the heap, which is the most calculation demanding part of the search algorithm. The principle of event dominance is presented in Florian (2004), and applied in the EMME/2 deterministic timetable-based assignment.

Later arrivals with the same line will in most cases be rejected by the principle of event dominance, which then has somewhat the same impact as graph reduction. However, event dominance is checked one step further in the search process than in the graph reduction (after putting the run from the prior node into the heap, and then examining it when arriving at the next node). The graph reduction is accordingly more effective even in the case with backward search, since the entire operation on all runs with the line is done before it is decided which one to put into the heap.

However, many different lines may lead to the same node. And several runs with the next line may be used, if they are created from different paths leading to the node. The principle of event dominance will be able to reduce the graph further in these cases, and is accordingly clearly recommended as a supplement to the graph reduction.

6.4 Optimisation of the updating mechanism in MSA

Because of the time dimension in the timetable-based network, normally only a small part of the total time-space graph is visited in each iteration of MSA. The complexity of the stochastic simulation in SUE and the updating mechanism in MSA could therefore be reduced:

1. The stochastic simulation may be “moved” from the outer loop to be carried out simultaneous with the graph search in the inner loop, making it possible to reduce the stochastic simulation similar to the graph reduction in section 6.2. The stochastic simulation took up approx. 40-45% of the total calculation time in the network of the EDM, as some of the simulated distributions, e.g. Log.Normal or Gamma, require much calculation compared to the simpler operations in the all-or-nothing algorithm (typically arithmetic operations and if-else logic). With graph reduction of approx. 75%, this corresponds to approx. 30% reduction in calculation time.

2. Another benefit is to carry out the stochastic simulation for each tree separately (distribute each row of the matrix separately instead of the entire matrix in each step). This improves the convergence of the outer loop in MSA – especially for links used by many OD pairs – resulting in far more simulations (proportionate to number of zones) within each inner loop. It still remains to be tested how much the convergence is improved by this.

3. The updating in MSA may be limited to used paths in the time-space network in the specific iteration, reducing the number of updated arcs in each iteration considerably, as the paths used in one iteration can only follow one tree, and only one run along each line. Each updating is, however, a little more complicated: The iteration number for the time where the arc was last updated must be stored, as the updating for the iteration in question can only be computed with the previous iteration in a future iteration (contributions from the individual trees in the specific iteration must first be added, before being MSA updated with contributions from prior iterations). In addition, the entire network must be MSA updated after the last iteration (refer to Nielsen *et al*, 2001c). **Summary and conclusions**

The paper presents the methodological considerations behind the ongoing work on the Danish freight transport model. It was generally agreed (Nielsen *et al*. 2003) that the choice problem of mode and route should be formulated as a three level problem, i.e. decision made by

1. Transport buyers, i.e. choice of which main mode combinations to consider.
2. Transport suppliers, i.e. which combinations of route segments to consider.
3. Transport routes within each route segment.

In the Danish case it was possible to describe most of 2) as an a priori discrete choice set, since the sea sounds with ferry lines and bridges between East Denmark (Zealand) and West Denmark (Jutland), Southern Sweden and Germany constitutes natural barriers. Level 1) and 2) can be dealt with by traditional discrete choice models.

The main focus of the paper is accordingly level 3). It was argued that all-or-nothing as well as deterministic equilibrium methods are too simplified, since both heterogeneities in preferences within transport classes (random coefficients) and stochasticity (link-based variation) exists among transport buyers and suppliers. Each class of transport (mode and good specific) must have its own utility function of e.g. length, cost, time and congestion time, for which stochastic terms are included. Software and methods have now been developed that can estimate complicated utility functions. It is recommended to use correlated log.normal coefficients in these, since both empirical evidence justify this and it is preferable to use a non-negative distribution.

The paper argues that a user equilibrium is much more reasonable to assume for the freight sector than a system equilibrium, and that cost-volume relationships at the route segment level 2) may be lower cost, higher frequency and faster time with increased volume (due to larger and more efficient means of transport, less empty driving and more competition).

The solution algorithm is the Method of Successive Averages, where the stochastic parts of the utility function are simulated. Different suggestion to focus the stochastic simulation and updating mechanism in MSA to those paths being used in the present iteration can reduce the calculation time. The memory need and storage space can hereby be met by a standard PC even on a large network, why the method is fully feasible for real size models.

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