# COMPARING MARGINAL COST PRICING WITH TOLL CORDON POLICIES FOR LARGE SCALE MULTIMODAL NETWORKS

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**Abstract**

In this paper, the possibility of applying simplified schemes, such as cordon pricing, as a second-best solution of the toll network design problem is investigated in the context of a multiclass equilibrium on multimodal networks with elastic demand. To this end a suitable equilibrium model is presented together with an efficient algorithm capable of solving it for large scale networks in quite reasonable computer time. This model represents an implementation of the theoretical framework proposed in a previous work on the toll optimization problem, where is stated the validity of marginal cost pricing for the context at hand. The application of the model to real cases shows, not only that through cordon pricing a relevant share of the maximum savings achievable with marginal cost pricing can be actually obtained, but also that in practice rationing is a valid alternative to road pricing which obviates to some of the relevant questions (technical, social, …) that the latter raises. As a result we developed a practical method for analyzing advanced pricing and rationing policies, which enables us to compare different operative solutions with an upper bound based on a solid theoretical background.

Keywords: Toll cordon; Marginal cost pricing; Equilibrium on multimodal networks
Topic Area: H9 Implementation of Pricing in Transport

## 1 Introduction

Road pricing is currently seen as one of the most powerful tool to manage transport demand in urban areas, in order to reduce traffic congestion and externalities. But, at present, there are very few examples of large scale applications world-wide and many uncertainties remain concerning features and conditions for successful implementation.

On the other hand, rationing policies (i.e. access restriction to particular areas during certain hours of the day) have been applied for many years in a large number of cities, proving to be very effective in reducing traffic flows in the involved areas and in increasing the speed and reliability of public transport vehicles (Filippi, Persia *et al.*, 1996). Some counteracting effects can arise. Traffic congestion and lack of parking in the surrounding area can increase, thus inducing unauthorized vehicles to violate the access restriction. Moreover, in some Italian cities this measure led to a dramatic explosion of the number of mopeds accessing the area, with very negative impacts on environment and, particularly, safety.

These questions and the need of financing the development and the maintenance of the transit system led the local authorities to investigate the introduction of advanced pricing policies, where different fare structures are imposed to different categories of users. The aim of our study is to present a methodology to simulate the impacts on the transport system of such policies, and evaluate their effectiveness in comparison with Marginal Cost Pricing (MCP) which acts as an upper bound. Other significant issues (e.g. fairness, acceptability, legal and institutional barriers, economic impacts) are not addressed in this paper.

The methodology is based on an implementation of the theoretical model developed in Bellei, Gentile and Papola (2002) and aims at making operative the great potentialities there expressed only in formal terms (multi-class traffic equilibrium model on multimodal networks with elastic demand and non-separable arc cost functions). The main result there obtained and here exploited is that, subject to the possibility of charging each arc of the network any real valued toll, a solution to the toll Network Design Problem (NDP) exists, coincides with a System Optimum (SO) where user travel choices can be prescriptively controlled, and can be determined by calculating a System Equilibrium (SE), which is defined by replacing in the User Equilibrium (UE) the cost function with the marginal social cost function.

In the specific implementation of the above equilibrium model to an urban network we shall represent transit line vehicles sharing road links with private traffic that are affected by the same overall congestion. This yields an asymmetric arc cost function Jacobian, which may potentially lead to the non-uniqueness of the equilibrium.

In Bellei, Gentile and Papola (2002) the concept of mode remains undeveloped as the main attention is devoted to the mathematical properties of the pricing optimization model. Here we develop that theoretical framework providing an operative specification of the supply model which can efficiently be applied to multi-class and multimodal large scale urban networks.

The traffic equilibrium model is formalized as a fixed point problem and solved through Bather's method (Bottom and Chabini, 2001), which is an accelerated averaging algorithm similar to the Method of Successive Averages (MSA).

The paper is organized as follows. In section 2, we present a discussion about cordon pricing and rationing policies in the context of multimodal networks with elastic demand. In section 3, we propose a suitable equilibrium model capable of catching the relevant aspects in the context at hand and provide a solution for the corresponding toll optimization problem based on MCP. In section 4, we sketch a solution algorithm for the above model. In section 5, we present some numerical results referred to the case study of Rome and discuss them.

## 2 Cordon pricing and rationing policies

Cordon pricing and rationing policies generally have very similar aims: to reduce private traffic in specific areas of particular value (mostly in the city centre) mitigating their environmental impacts and allowing public transport to improve its performance (speed and reliability) and its attractivity.

The principal difference between the two policies is that, while under rationing automobile access to a specific portion of the road network delimited by a cordon is allowed only to some categories of users with particular requirements (e.g. public utility, residence, work-related activities, walking problems) on the basis of a political evaluation by the Public Administration, under pricing the passage through the cordon is allowed only to users willing to pay. As a consequence, pricing also generates income for the Public Administration, revenue that in large part (a part is employed in the collection costs) can be reinvested to boost the supply of the transit system and, in general, of the most sustainable modes of transport.

In both cases, there exist sizable obstacles to the real implementation of such policies, first among them those related to "fairness" and "acceptability" on the part of public opinion.

Leaving aside deeper considerations of these aspects of the problem, it is relatively easy to note how the theme of equity and fairness is brought up mainly by the introduction of pricing policies. While, in fact, these determine a selection "by census" of those authorised to have automobile access, in the case of rationing the possibility of access is granted only for conditions of local necessity (residents, particular economic activities) or for services of public utility.

Equally complex is the theme of acceptability, which involves, in addition to the above considerations of fairness, a series of other factors, including economic (payment of the tariff, effects on local economic activities in the area) and behaviours (need to modify trip habits). All this implies that the reactions to the introduction of these two policies are different for different groups of users.

In Figure 1 are illustrated the results of a survey conducted in the centre of Rome to evaluate how acceptable each policy was. In general, rationing enjoys better acceptance than pricing. The residents are more in favour of the system of restriction and control of access than are the merchants, while the merchants appreciate more than residents a pure road pricing scheme.

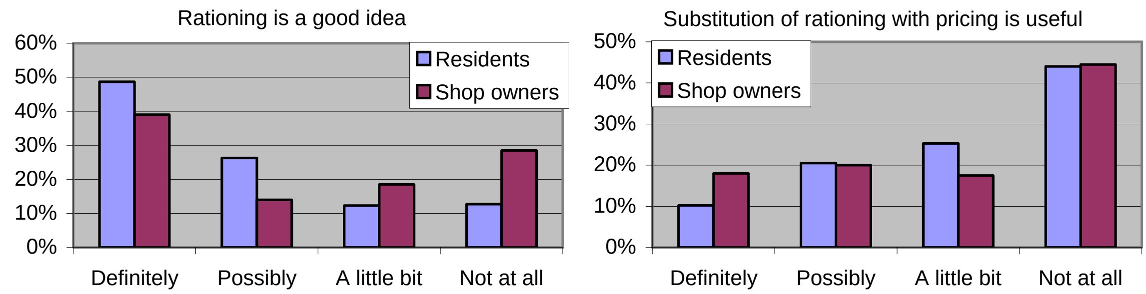



Figure 1. Acceptability of rationing and pricing in Rome's Limited Traffic Zone.

The difference in opinion on the part of the different user groups derives also from the difference in efficacy of the two approaches. Rationing policies bring about drastic reductions in vehicle congestion, with peaks very unlikely to be achieved with pricing policies (tariffs would need to be unacceptably high). That is, obviously, appreciated by the residents. Pricing policies, however, offer greater flexibility than rationing, having a lesser weight on the accessibility of the area, which makes the merchants happy.

The two policies also differ in complexity of planning on the part of the decision makers. Leaving aside here the technical issues for both cases, the problems related to the introduction of rationing schemes are principally in the choice of the authorised user groups and in the identification of "complementary measures" (e.g. improvement of the transit lines accessing the area) to avoid undesired effects (e.g. increase of the traffic in the surrounding area). Such issues are relatively simpler than those that accompany the introduction of pricing policies, which has been for years the subject of a technical and political debate over how to put them into practice.

For some decades, the cardinal point of the debate on the taxation of road use has been the theory of MCP, starting from the welfare theories of Marshall and Pigou of the beginning of the twentieth century. But it has gone in and out of favour (Rothengatter, 2003). According to this theory, the external costs (impact on environment and on other users) generated by the road user are to be "internalised", levying on the user a tax equal to the external marginal cost, that is, the variable cost induced on the use of the infrastructure by one additional user. As a consequence the tariff imposed on the user would have to vary in function of the place, traffic, moment, vehicle type, etc.

This idea was first applied to the traffic assignment problem by Beckmann, McGuire and Winsten (1959). Dafermos and Sparrow (1971) proved the optimality of MCP with respect to total costs in a mono-user monomodal deterministic context with fixed demand and separable arc cost functions. Dafermos (1973) extended this result to a multi-user context, while Smith (1979) generalized it to the case of elastic demand and asymmetric arc cost function Jacobian. In Bellei, Gentile and Papola (2002) the validity of MCP is extended to any stochastic traffic assignment on multimodal networks based on random utility theory.

Though recommended by the European Commission as late as 1998 (in the White Paper “Fair and Efficient Pricing of the Transport Infrastructure”) as a rule of thumb for setting tariffs for the transport infrastructures, application of the theory of MCP shows gross limitations for different reasons (Verhoef, 2002a), for example: the tariff structure would be excessively complicated and extremely difficult for users to understand; the current technology, while greatly evolved in recent years, is not yet able to support so complex a tariff structure; it is almost never possible to impose road pricing on the entire network, and, at least in an initial phase, the pricing scheme should be introduced on a part of the network; the tariff level may not be politically acceptable.

Such obstacles gave rise to the identification of different second-best solutions which take account of the limits imposed by the practical necessities of implementation (Verhoef, 2002b; May and Milne, 2000), even if this means the process will become less efficient (Liu and McDonald, 1999). One of the most common second-best solutions is *cordon pricing*, where the access to a given portion of the road network is charged a toll. This coincides with a particular configuration of road pricing where all arcs whose tail and head are respectively out and in the Limited Traffic Zone (LTZ) are charged a same toll. Rationing can be regarded as a particular case of cordon pricing where the toll equals the fine for entering the LTZ multiplied by the probability of getting it.

In the following we present some example of MCP, aimed at showing that referring to a multimodal context with elastic demand is necessary in order to grasp the actual potentiality of road pricing on real transport networks. Recall that MCP is proved to be the best solution of the *network pricing optimization* problem, where all arcs can be charged any toll, and thus is an upper bound for the toll NDP, where only a given set of arcs can be charged.

Specifically, Figure 2 shows the improvement due to MCP (that is the relative increment of social surplus with respect to the case of no policy) obtained with reference to the multimodal version of the classical test network of Sioux Falls for different levels of demand (that is scaling the travel demand uniformly through a demand multiplier) in three different contexts: fixed demand, elastic demand (users can stay at home), multimodal demand (users must travel but can choose the transport mode). The ratio between toll revenues and total costs is depicted to give an idea of the social impact of MCP. In the case of elastic demand we report also the percentage of the original demand still making the trip.

We can see that in the fixed demand context MCP is poorly effective and leads to very high tolls. On the contrary, it is very effective in the elastic demand context, although tolls are still high. In the multimodal demand context, which is the more realistic for cities during the peak hour, MCP is still very effective and the tolls are less high.

As expected the real target of road pricing is mode choice and not only route choice. In this respect, the question of which arcs of the network should be charged, which is fundamental for directing route choice, becomes less crucial, so that simplified schemes, such as cordon pricing, can be investigated, and the classical road traffic assignment models with fixed demand become an improper tool for this investigation.

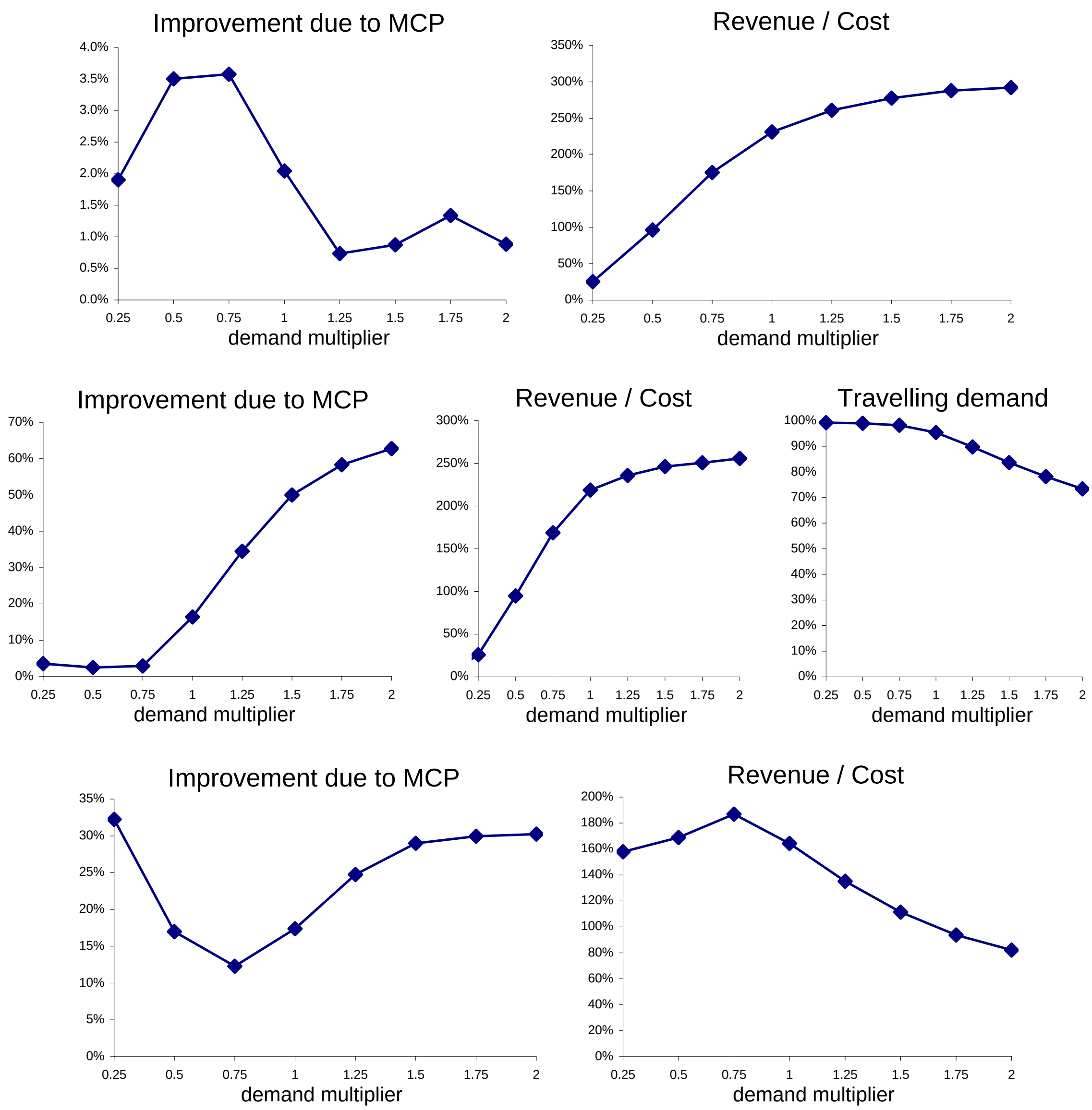



Figure 2. The effects of MCP in three different demand contexts: fixed, elastic, multimodal.

## 3 Modelling methodology

In this section we provide a specification of the multi-class equilibrium model on multimodal networks with elastic demand proposed in Bellei, Gentile and Papola (2002), preserving all the properties sufficient for the existence and the uniqueness of the solution.

### 3.1 The demand model

In modelling travel demand we follow the behavioural approach based on random utility theory (see, for instance, Ben Akiva and Lerman, 1985), where it is assumed that the user is a rational decision-maker who, when making his travel choice: considers a positive finite number of mutually exclusive *travel alternatives* constituting his choice set; associates with each travel alternative of his choice set a utility, not known with certainty and thus regarded as a random variable; selects a maximum utility travel alternative. On this basis, user’s behaviour is described in terms of the probability that each of his travel alternatives has maximum utility.

The users are grouped into *demand components*. The $N_i > 0$ users constituting the demand component $i \in D$, where $D$ is the set of the demand components, are assumed to be identical to each other with respect to any characteristic influencing travel behaviour and externalities, and share the same choice set $J(i)$ of travel alternatives. Specifically, these users travel between the origin $o(i) \in C$ and the destination $d(i) \in C$, where the set of the centroids $C$ is a subset of the node set $N_R$ of the base network, which will be introduced in the next subsection.

The demand components are grouped into *user classes*. All the demand components belonging to a same user class $u \in U$, where $U$ is the set of the user classes, share the same set of individual attributes characterizing the user as a trip consumer (e.g. age, value of time, and purpose of trip) and the same set of attributes specifying the production of trip externalities such as congestion and pollution (e.g. vehicle type and occupancy rate). Moreover, they have available the same subset $M(u) \subseteq M$ of modes, where $M$ is the set of the transport modes.

It turns useful in practical applications that only the following subset of the demand components is active in the sense that it is actually considered in the computation of the equilibrium: $\{i \in D$: $DM_{u(i)} \geq 0$, $ACT_{o(i)} \geq 1$, $ACT_{d(i)} = 1\}$, where $ACT_c \in \{0, 1, 2\}$ is a label referred to centroid $c$, $DM_u$ is the demand multiplier of the user calss $u$, $u(i)$ is the user class of the demand component $i$.

In the following we refer to the generic demand component $i \in D$. The utility $U_j^i$ of the generic travel alternative $j \in J(i)$ is given by the sum of a finite systematic utility term $V_j^i$, and a zero mean random residual $\varepsilon_j^i$. The choice model is assumed to satisfy the following properties which permit to derive uniqueness conditions for the equilibrium (see Cantarella, 1997; Gentile, 2003): the random residuals have finite variance and their joint probability density function is independent of the systematic utilities, continuous, and strictly positive; the systematic utility of the generic travel alternative is given by subtracting the generalized cost $C_j^i$ of the associated route on the transport network to a constant utility term $X_j^i$ independent of congestion:

$$V_j^i = X_j^i - C_j^i. \quad (1)$$

The choice probability $P_j^i$ of the generic travel alternative is by definition the probability of $j$ being a maximum utility travel alternative among the choice set $J(i)$:

$$P_j^i = \Pr[V_j^i + \varepsilon_j^i \geq V_k^i + \varepsilon_k^i \text{: } k \in J(i)].$$

The above equation expresses the choice probability of the generic travel alternative as a function of the systematic utilities of all the travel alternatives of the choice set:

$$P_j^i = \mathrm{P}_j^i(V_k^i \text{: } k \in J(i)). \quad (2)$$

The flow $F_j^i$ of the travel alternative is given by multiplying its choice probability by the number of users of the demand component:

$$F_j^i = P_j^i \cdot N_i \cdot DM_{u(i)}\ ; \quad (3)$$

then, by definition, we have:

$$\textstyle\sum_{j \in J(i)} F_j^i = N_i \cdot DM_{u(i)}\ .$$

On the basis of (2) and (1), equation (3) defines the demand function, expressing the travel alternative flows in terms of the travel alternative generalized costs:

$$F_j^i = N_i \cdot DM_{u(i)} \cdot \mathrm{P}_j^i(X_j^i - C_j^i \text{: } k \in J(i)), \text{ in compact form: } \mathbf{F} = \mathrm{F}(\mathbf{C}), \quad (4)$$

where **F** and **C** are the $(n \times 1)$ vectors of the travel alternative flows and costs, with $n = \sum_{i \in D} |J(i)|$.

The *satisfaction* $W_i$ is by definition the mean value of the maximum utility:

$$W_i = E[\max\{V_j^i + \varepsilon_j^i : j \in J(i)\}].$$

On the basis of (1), the above equation defines the satisfaction function, expressing the satisfaction of the demand component in terms of the travel alternative generalized costs:

$$W_i = \mathrm{W}_i(C_j^i : j \in J(i)), \text{ in compact form: } \mathbf{W} = \mathrm{W}(\mathbf{C}).$$

where $\mathbf{W}$ is the $(|D| \times 1)$ vector of satisfactions.

As usual, we assume that the travel choice process can be decomposed into a sequence of mobility choices (in our case: to travel or not to travel, by which mode and following which route) represented by a *choice tree* (Oppenheim, 1995). A travel alternative is then a path on the user choice tree, specifying the choice made at each level (in our case: trip generation, modal split and route assignment), and its utility is the sum of the utilities associated with each arc of this path. In particular, each travel alternative (except for the "not-to-travel" alternative, when present) is associated with a single route connecting on a specific mode the origin of the trip to its destination.

The hierarchic structure of the choice model (i.e. the correlations between the utilities of the travel alternatives) is implicit in the form of the joint probability density function of the travel alternative random residuals. However, in practice the choice made at each level is described by means of a sub-model where the utility due to the choices made at lower levels is synthetically taken into account through a *satisfaction* variable. Specifically, the utility associated with each alternative of a sub-model is given by summing up a specific systematic utility term, a specific random error term, and the maximum utility obtainable from lower level, whose mean value is the satisfaction.

Each travel alternative $j \in J(i)$ is identified by a mode $m(j) \in M(u(i))$ and a route $r(j) \in R(o(i), d(i), m(j))$, that is $j = (m(j), r(j))$, where $R(o, d, m)$ is the set of routes available on mode $m \in M$ for travelling between the centroids $o \in C$ and $d \in C$. In order to model trip generation, the choice set $J(i)$ is given as: $J(i) = \cup_{g \in G(u(i)) \times \{0\} \cup \{1\}} J(i, g)$, where $G(u)$ is 1 if the user class $u \in U$ can choose whether to make the trip or not, and 0, otherwise. The set $J(i, 1)$ is the set of the actual travel alternatives: $J(i,1) = \{(m,r): m \in M(u(i)) , r \in R(o(i), d(i), m)\}$, while the set $J(i,0)$ is the "not-to-travel" alternative (0,0).

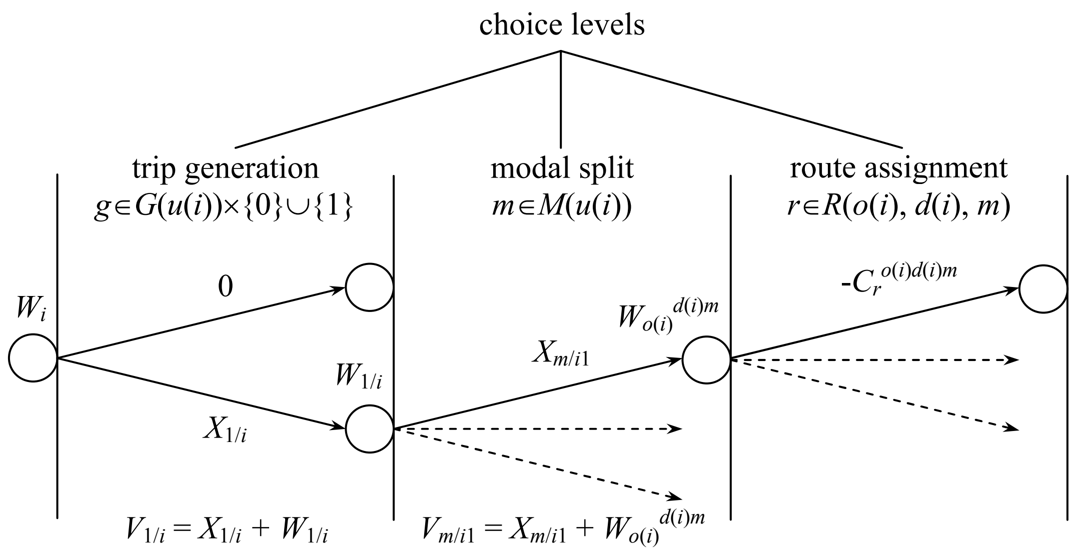



Figure 3. The choice tree.

The choice model has then the following structure:

$P_j^i = P_{1/i} \cdot P_{m(j)/i1} \cdot P_{r(j)}^{o(i)d(i)m(j)} \quad j \in J(i, 1)$ , $P_j^i = (1 - P_{1/i}) \quad j \in J(i, 0)$,

$P_{1/i} = \mathrm{P}_{1/i}(V_{1/i})$, $W_i = \mathrm{W}_i(V_{1/i})$, $V_{1/i} = X_{1/i} + W_{1/i}$ ,

$P_{m/i1} = \mathrm{P}_{m/i1}(V_{m/i1}: m \in M(u(i)))$, $W_{1/i} = \mathrm{W}_{1/i}(V_{m/i1}: m \in M(u(i)))$, $V_{m/i1} = X_{m/i1} + W_{o(i)}^{d(i)m}$,

$P_r^{odm} = \mathrm{P}_r^{odm}(C_r^{odm}: r \in R(o, d, m))$, $W_o^{dm} = \mathrm{W}_o^{dm}(C_r^{odm}: r \in R(o, d, m))$,

where the satisfactions are denoted $W$, the specific systematic utilities $X$, the systematic utilities $V$, and the probabilities $P$. For $j \in J(i, 1)$, it is: $X_j^i = X_{1/i} + X_{m(j)/i1}$ and $C_j^i = C_{r(j)}^{o(i)d(i)m(j)}$ .

Specifically, a Logit model is utilised for trip generation and modal split; thus we have:

$$P_{1/i} = \frac{1}{1 + \exp\left(\dfrac{-V_{1/i}}{\vartheta_{u(i)}}\right)}, \quad W_i = \vartheta_{u(i)} \cdot \ln\left[1 + \exp\left(\frac{V_{1/i}}{\vartheta_{u(i)}}\right)\right],$$

$$P_{m/i1} = \frac{\exp\left(\dfrac{V_{m/i1}}{\theta_{u(i)}}\right)}{\sum_{m \in M(u(i)))} \exp\left(\dfrac{V_{m/i1}}{\theta_{u(i)}}\right)}, \quad W_{1/i} = \theta_{u(i)} \cdot \ln\left[\sum_{m \in M(u(i))} \exp\left(\frac{V_{m/i1}}{\theta_{u(i)}}\right)\right],$$

where $\vartheta_u$ and $\theta_u$ are, respectively, the trip generation and the modal split Logit parameter of the user class $u \in U$. A Probit model, where the variance-covariance matrix results from the assumption that the correlation among routes derives from their overlapping in terms of arcs, has been adopted for the route assignment (see, for instance, Cascetta, 2001). This is not presented here as, actually, the variables of the route choice sub-model are never handled explicitly; in fact, as it will be seen, the equilibrium problem is solved through an implicit route enumeration algorithm.

The following definition of the specific systematic utilities completes the specification of the demand model:

$X_{1/i} = TU_{u(i)}$ ,

$X_{m/i1} = -DU_{u(i)}^m - (NET_m\text{-}2) \cdot RDC_{u(i)} \cdot (RD_{o(i)} + RD_{d(i)}) - (NET_m\text{-}1) \cdot TDC_{u(i)} \cdot (TD_{o(i)} + TD_{d(i)})$ ,

where $TU_u$ is the travel utility for the user class $u \in U$, and $DU_u^m$ is the ASA disutility term of mode $m \in M(u)$. When $m$ is a road mode ($NET_m = 1$, see the next subsection), a disutility term is added in order to take into account the accessibility by private vehicles of the trip terminals, for example in terms of parking costs, where $RDC_u$ is the road disutility coefficient of the user class $u \in U$ and $RD_c$ is the road disutility of the centroid $c \in C$. When $m$ is a transit mode ($NET_m = 2$), a disutility term is added in order to take into account the lack of accessibility of high performance transit systems from the trip terminals, where $TD_u$ is the transit disutility coefficient of the user class $u \in U$ and $TDC_c$ is the transit disutility of the centroid $c \in C$. More in general, this terms are added to simulate any effect on the mode specific systematic utility that depends on the location of the trip terminals.

### 3.2 The supply model

The multimodal network of infrastructures and services which constitutes the transport supply is modelled here through an *oriented hyper-graph* $G = (N, A)$, where $N$ is the set of the *nodes*, each node representing a spatial location and possibly a trip state (e.g. arriving at a road intersection, start or end waiting at a transit stop), and $A$ is the set of the *hyper-arcs*, each hyper-arc representing a specific trip phase (e.g. driving throughout a road link, waiting at a transit stop). The generic hyper-arc $a \in A$ is identified by one initial node $TL(a) \in N$, referred to as *tail*, and by a set of ending nodes $HD(a) \subseteq N$, referred to as *head*: $a = (TL(a), HD(a))$. An *arc* is an hyper-arc $a$ whose head is a singleton, that is: $|HD(a)| = 1$.

In this framework, the route, if any, associated with a given travel alternative (no route is associated with the "not-to-travel" alternative) is represented through a *hyper-path* connecting on $G$ the origin of the trip to its destination. An hyper-path on $G$ with initial node $o \in N$ and final node $d \in N$, is a hyper-graph $r = (N_r , A_r)$ having the following properties: 1) $r$ is an acyclic sub hyper-graph of $G$; 2) $o$ has no entering hyper-arcs in $A_r$ and $d$ has no exiting hyper-arcs in $A_r$ ; 3) each node $i \in N_r \backslash \{d\}$ has exactly one exiting hyper-arc in $A_r$ ; 4) for every node $i \in N_r \backslash \{o, d\}$ there is at least one path on $r$ from $o$ to $d$ traversing it.

In the following we assume that $G$ is constituted by a road graph $G_R = (N_R , A_R)$ and by a transit hyper-graph $G_T = (N_T , A_T)$. Conversely, each arc $a \in A$ is associated with a hyper-graph $net(a) \in \{G_R , G_T\}$. The road graph represents the base graph. In general, only the subset $\{a \in A_R: GR_a \geq 0\}$ of the base graph is active, in the sense that it is actually considered in the computation of the equilibrium; $GR_a$ is simply a free integer label referred to arc $a \in A_R$ which turns useful to identify arc groups in the database. The transit hyper-graph includes a copy of the road graph which acts as the pedestrian network. The generic line $l \in L$, where $L$ is the set of the transit lines, is constructed starting from its line route $LR(l) \subseteq A_R$ , expressed in terms of a connected sequence of *support arcs* of the base graph, consistently with the scheme depicted in Figure 4. An access arc representing a transit stop is introduced for each arc of the base graph which acts as support arc for at least one line. In general, only the subset $\{l \in L: GR_l \geq 0\}$ of the transit lines is active, in the sense that it is actually considered in the computation of the equilibrium; again, $GR_l$ is simply a free integer label referred to line $l \in L$ which turns useful to identify line groups in the database.

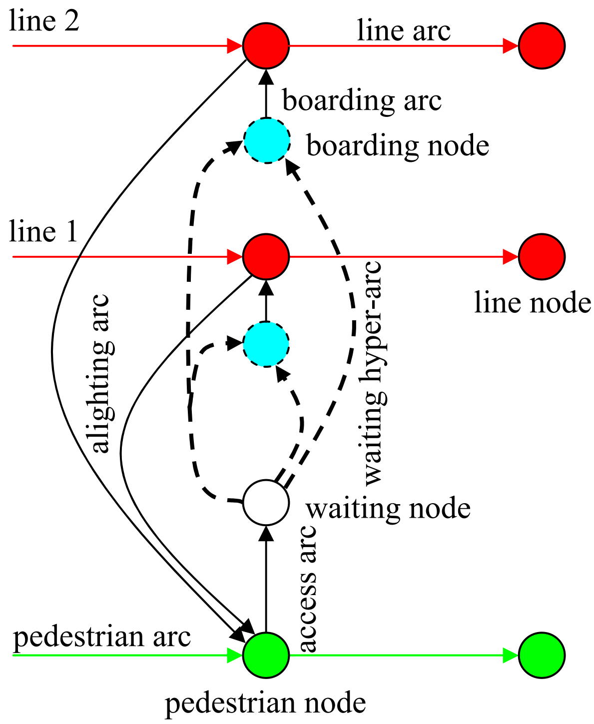



Figure 4. The transit hyper-graph.

Specifically, we define the following types of arcs: $AC$ road connectors, $AS$ street arcs, $AP$ pedestrian arcs, $AX$ access arcs, $AW$ waiting hyper-arcs, $AB$ boarding arcs, $AL$ line arcs, $AA$ alighting arcs. Thus we have: $A_R = AC \cup AS$ , $A_T = AP \cup AX \cup AL \cup AA \cup AW \cup AB$.

Note that the generic hyper-arc is defined as a one-to-many relationship between nodes. However, all the hyper-arcs here introduced are ordinary one-to-one relationships, that is simple arcs, except for the transit waiting hyper-arcs $AW$, where the tail is a waiting node and the head is a set of boarding nodes (Nguyen, Pallottino and Gendreau, 1998). The hyper-arcs are a useful tool to represent the adaptive choice occurring during a transit trip when the user waiting at a stop boards each line with a certain probability. Specifically, the generic hyper-arc $a \in AW$ identifies a set of lines on which the passenger is willing to board and a probability is associated with each boarding node belonging to the head of the hyper-arc. For example, if the passengers' behaviour is to board the first arriving carrier of a given set of lines, the probability associated with each line node is the probability that the line is served as first among the set of lines identified by the hyper-arc at hand. The presence of many hyper-arcs at the same stop (as in Figure 4) represents the possibility of choosing the most convenient set of lines on which the passenger is disposable to board, which is usually referred to as the *attractive set*. In theory, a waiting hyper-arc should be introduced for any disposition of the boarding arcs at a given stop. Fortunately, there is no need to enumerate the hyper-arcs, as they are regarded implicitly at the algorithm level, where the network is implemented through a standard graph by eliminating from the hyper-graph described in Figure 4 all waiting hyper-arcs and boarding nodes and then connecting the boarding arcs directly to the corresponding waiting node.

Starting from the road graph and the transit hyper-graph, several mode-specific networks can be constructed in order to simulate: different pricing policies for specified categories of users; different performances characterizing the various transport modes and vehicle classes; different behaviours in terms of cost perception. In general, the network associated to the generic mode $m \in M$ represents trips of users who perceive and produce costs and congestion in the same way and thus share the same route choice conditional probabilities.

This approach implies an extension of the concept of mode, which embodies elements of both the supply and the demand models and consists in an aggregation process aimed at optimizing the algorithm. Indeed, this permits, in the context of a multi-class and multimodal equilibrium, to accomplish in practice a thick segmentation of users on the demand side, as this is not expensive in terms of computational effort, while reducing to a minimum the segmentation on the supply side, which requires specific shortest paths and network loading procedures.

The generic mode $m \in M$ is referred to a specific hyper-graph $G(m) = (N(m), A(m))$, which coincides with $G_R$ or $G_T$ depending on the variable $NET_m$ that equals 1 if $m$ is a road mode, 2 if $m$ is a transit mode, and 0 if it is not active; in the following we will denote by $M(R) = \{m \in M\text{: } NET_m = 1\}$ and $M(T) = \{m \in M\text{: } NET_m = 2\}$ the subsets of the modes referred to the road graph and the transit hyper-graph respectively. Let $c_a^m$ and $f_a^m$ denote, respectively, the *arc cost* and the *arc flow* on the generic arc $a \in A(m)$. The following relations hold:

$$C_j^i = \sum_{a \in A(m(j))} c_a^{m(j)} \cdot \pi_{am(j)}^{ij} \quad \text{, in compact form: } \mathbf{C} = \Pi^{\mathrm{T}} \cdot \mathbf{c} \text{ ,} \tag{5}$$

$$f_a^m = \sum_{i \in D, j \in J(i)} F_j^i \cdot \pi_{am}^{ij} \quad \text{, in compact form: } \mathbf{f} = \Pi \cdot \mathbf{F} \text{ ,} \tag{6}$$

where: $\pi_{am}^{ij}$ is the *arc-alternative probability* that the arc $a \in A(m)$, with $m = m(j)$, is utilized by the users belonging to the demand component $i \in D$ as an element of the hyper-path associated with the travel alternative $j \in J(i, 1)$, otherwise $\pi_{am}^{ij}$ is equal to zero; $\Pi$ is the $(\nu \times n)$ matrix, with $\nu = \sum_{m \in M} |A(m)|$, whose elements are $\pi_{am}^{ij}$; $\mathbf{c}$ and $\mathbf{f}$ are the $(\nu \times 1)$ vectors whose generic component is respectively $c_a^m$ and $f_a^m$.

The main advantage of using this hyper-graph based network representation is the possibility of expressing in a purely additive form the travel alternative generalized costs in terms of the arc costs through (5) and the arc flows in terms of the travel alternative flows through (6), also when taking into consideration the adaptive choices taking place at the stops of the transit network, thus avoiding the introduction of non-additive alternative costs as in Cascetta (2001). It is to be noted that, where no on-line information is available at the stops, $\Pi$ is fixed as it depends exclusively on the line headway distributions (Gentile, Nguyen and Pallottino, 2003) which can be considered here an exogenous variable of the model (in Bellei, Gentile and Papola, 2000, this assumption is removed in order to analyze the case where the number of carriers operating each line is considered fixed, so that the frequencies become dependent variables), while if there is no need to represent adaptive choices, $\Pi$ becomes a classical arc-path incidence matrix.

### 3.3 Specification of the arc cost function

A desirable feature of the proposed equilibrium model is the possibility of simulating the simultaneous effect on traffic flows of applying different toll patterns on different modes. To this end it is convenient to separate the *tolls* $\mathbf{r}$ from the *performances* $\hat{\mathbf{c}}$, so that the arc costs are defined as follows:

$$\mathbf{c} = \hat{\mathbf{c}} + \mathbf{r} \,, \tag{7}$$

where the vectors $\mathbf{r}$ and $\hat{\mathbf{c}}$ have both the same structure as $\mathbf{c}$. The congestion phenomenon is then represented formally through the *arc performance function*:

$$\hat{\mathbf{c}} = \hat{c}(\mathbf{f}) \,. \tag{8}$$

In Gentile and Papola (2003) we provide a specification of (8) representing the main congestion phenomena affecting urban multimodal networks (including: transit vehicles sharing road arcs with private traffic and thus affected by the same overall congestion, the effect of line capacity on the waiting time, line carriers slowed down by each line stop where commuters' boarding and alighting time is taken into account) which is based on the concept of *equivalent flow* (Daganzo, 1983), where the function $\mathbf{s} = s(\mathbf{v})$, assumed continuous and monotone non-decreasing, expresses the arc standard time ($|A| \times 1$) vector $\mathbf{s}$ in terms of the equivalent flows $\mathbf{v}$. The proposed specification of the arc cost function holds the following property which is essential to establish the uniqueness of the equilibrium: the equivalent flows are linear functions with positive mode-specific and non arc-specific coefficients of the mode-specific arc flows; the mode-specific arc costs are linear functions with positive mode-specific and non arc-specific coefficients of the congested standard times.

$$v_a = \textstyle\sum_{m\in M} \xi_m \cdot f_a^{\,m} + cost \quad a\in A \;,$$

$$c_a^{\,m} = r_a^{\,m} + \eta_m \cdot s_a + cost \quad m\in M \,,\, a\in A \;,$$

where *cost* is any non-negative arc-specific and mode-specific term independent of flows. This condition defines the so called *undifferentiated congestion*:

### 3.4 User equilibrium, system equilibrium and toll optimization problem

The UE and the SE are formalized here as fixed point problems. On this basis, we address the problem of determining an optimal arc toll vector $\mathbf{m}$, assuming that it is possible to charge each arc of the network any real valued toll.

By combining (5), (4) and (6), we obtain the *network loading map*, which yields the arc flows as a function of the arc costs:

$$\mathrm{f}(\mathbf{c}) = \Pi \cdot \mathrm{F}(\Pi^{\mathrm{T}} \cdot \mathbf{c}) \,. \tag{9}$$

Then, for a given value of the arc tolls $\mathbf{m}$, a UE flow and cost pattern is determined by solving the fixed point problem obtained by combining (9), (7) and (8):

$$\mathbf{f} = \mathrm{f}[\hat{c}(\mathbf{f}) + \mathbf{m}] \,. \qquad (10)$$

The existence of a solution to the above problem is guaranteed by the continuity of the arc performance function. It is well known that, in the case of undifferentiated congestion, the uniqueness of multi-class UE is conveniently analyzed by reformulating the problem in terms of standard times and equivalent flows (Cascetta, 2001, pag. 330). Using this approach, in Gentile and Papola (2003) it is proved that a sufficient condition for the uniqueness of the solution to problem (10) is the monotonicity of the standard time function $s(\mathbf{v})$, moreover hypothesis on the coefficients of the supply model are drawn to ensure this condition.

The social cost and its gradient, referred to as the *marginal social cost*, can be expressed in terms of the class-specific arc flows respectively as follows:

$$sc(\mathbf{f}) = \hat{c}(\mathbf{f})^{T} \cdot \mathbf{f},$$

$$msc(\mathbf{f}) = \nabla_{\mathbf{f}}\, sc(\mathbf{f}) = \hat{c}(\mathbf{f}) + \nabla_{\mathbf{f}}\, \hat{c}(\mathbf{f}) \cdot \mathbf{f} \,.$$

By definition (see Bellei, Gentile and Papola, 2002), a SE flow pattern is determined by solving the following fixed point problem:

$$\mathbf{f} = \mathrm{f}[msc(\mathbf{f})] \,, \qquad (11)$$

obtained by replacing in the UE problem (10) the cost function $\hat{c}(\mathbf{f}) + \mathbf{m}$ with the marginal social cost function $msc(\mathbf{f})$. The existence of a solution to the above problem is guaranteed by the continuity of the marginal social cost function which required the arc performance function to be $C^1$. Unfortunately, the marginal social cost function does not satisfy the condition for undifferentiated congestion. Then, despite each component of the arc performance function is convex, the uniqueness of the above problem cannot be stated under the sufficient condition for the monotonicity of the marginal social cost function reported in Bellei, Gentile and Papola (2002).

The analysis of road pricing from the point of view of economic efficiency leads to the well-known result that the optimal flow pattern is a SE, which enables us to generalize the notion of SO current in the literature to the case of stochastic equilibrium. Specifically, with reference to the following toll optimization problem:

$$\max_{\mathbf{f},\,\mathbf{m}} S(\mathbf{f}, \mathbf{m}) = \sum_{i \in D} N_i \cdot W_i(\Pi^{T} \cdot [\hat{c}(\mathbf{f}) + \mathbf{m}]) + \mathbf{m}^{T} \cdot \mathbf{f}$$

$$\text{s. to: } \mathbf{f} = \mathrm{f}[\hat{c}(\mathbf{f}) + \mathbf{m}] \,, \qquad (12)$$

where the objective function is the social surplus and the UE constraint is expressed in terms of the fixed point formulation (10), in Bellei, Gentile and Papola (2002) it is proved that "for each arc toll vector that solves the problem there is another solution vector $mcp(\mathbf{f}) = \nabla_{\mathbf{f}}\, \hat{c}(\mathbf{f}) \cdot \mathbf{f}$ which leads to the same SE flow pattern $\mathbf{f}$" (proposition 5), and that "for an arc toll vector to solve the problem it is necessary for it to yield an SE flow pattern; if the SE is unique, then the condition is also sufficient and the solution is unique in terms of flow pattern" (proposition 6).

With specific reference to the arc cost function introduced in section 3.3, the MCP tolls relative to the generic arc $a \in A$ and mode $m \in M$ is:

$$mcp_a^{m}(\mathbf{f}) = \sum_{b \in A} \sum_{x \in M(net(b))} f_b^{x} \cdot \partial \hat{c}_b^{x}(\mathbf{f}) / \partial f_a^{m} \,.$$

Due to the particular structure of the arc performance function considered here, we have:

$\partial \hat{c}_b^{\,x} / \partial f_a^{\,m} = \eta_x \cdot \partial s_b / \partial v_a \cdot \xi_m \quad m \in M\,,\, x \in M\,,\, a \in A\,,\, b \in A$ .

On this basis we can rewrite the MCP tolls formula as follows:

$$mcp_a^{\,m} = \xi_m \cdot [\textstyle\sum_{b \in A} \tau_b \cdot \partial s_b / \partial v_a]\,, \tag{13}$$

where we have introduced the new symbol:

$$\tau_a = \textstyle\sum_{m \in M(net(a))} f_a^{\,m} \cdot \eta_m \quad a \in A\ .$$

The above expression shows that the mode-specific toll is obtained by multiplying a common arc toll by the weight of the mode on the equivalent flow, which reflects the "fairness" of MCP in the sense that the higher is the effect on the congestion the higher is the price that users must pay.

A solution satisfying the necessary conditions of the toll optimization problem (12) can then be determined by solving a UE problem, where the tolls $m_a^{\,m}$ are replaced with the MCP toll functions (13). Clearly, if the SE is unique, then the solution obtained this way is a global solution.

## 4 Solution algorithm

The implicit path fixed point formulations (10) of the UE problem and (11) of the SE problem can be solved through Bather's method as follows, where $K$ is the number of iterations to be performed before stopping. The issue of the stop criterion has been neglected on purpose as it requires a non-trivial discussion. The algorithm converges to the equilibrium, in case of uniqueness; otherwise it becomes a well performing heuristic.

**function** UE
  $\mathbf{f} = \mathbf{f}^0$
  **for** $k = 0$ **to** $K$
    **if** SE **then** $\mathbf{r} = \text{mcp}(\mathbf{f})$ else $\mathbf{r} = \mathbf{r}'$ — pricing
    $\mathbf{c} = \hat{c}(\mathbf{f}) + \mathbf{r}$ — arc cost function
    $\mathbf{q} = f(\mathbf{c})$ — network loading map
    **if** $k = 0$ **then** $\alpha = 1$ **else** $\alpha = 1/k$ — step
    $\mathbf{f}^{\text{dev}} = (1-\alpha) \cdot \mathbf{f}^{\text{dev}} + \alpha \cdot (\mathbf{q}-\mathbf{f})$ — flow averaging
    $\mathbf{f}^{\text{avg}} = (1-\alpha) \cdot \mathbf{f}^{\text{avg}} + \alpha \cdot \mathbf{f}$ — flow averaging
    $\mathbf{f} = \mathbf{f}^{\text{avg}} + 1/\alpha^{1/3} \cdot \mathbf{f}^{\text{dev}}$ — flow averaging
  **loop**
**end function**

The Probit NLM is implemented through a Montecarlo simulation as follows:

**function** $\mathbf{h} = f(\mathbf{c})$
  $\mathbf{h} = \mathbf{0}_V$
  **for** $h = 1$ **to** $H$
    **for each** $m \in M$
      $\mathbf{g}^{\,m} = \mathbf{c}^{\,m} + \mathbf{\Phi} \cdot (\rho_m \cdot \chi)^{0.5}$ — perturbation of the arc costs
    **next** $m$
    $\mathbf{q} = f_D(\mathbf{g})$ — deterministic network loading map
    $\mathbf{h} = \mathbf{h} + 1/h \cdot (\mathbf{q} - \mathbf{h})$ — flow averaging
  **next** $h$
**end function**

where $\Phi$ is a diagonal matrix whose elements are extracted at each evaluation from a standard normal variable N(0,1), and $\rho_m$ is the ratio, assumed to be constant, between the variance of the error made by users on mode $m$ in evaluating the generic arc cost $c_a^m$ and a fixed arc cost $\chi_a^m = c_a^m(\mathbf{f}^0)$.

The computation of the Probit NLM requires calculating for $H$ times the deterministic NLM, which is implemented in the following procedure by combining an All Or Nothing assignment of the travel demand on shortest routes and the Nested Logit demand model introduced in section 3.1:

**function** $\mathbf{f} = \mathrm{f_D}(\mathbf{c})$
  $\mathbf{f} = \mathbf{0}_V$ , $\mathbf{n} = \mathbf{0}_{|C|\cdot|M|\cdot|N|}$
  **for each** $d \in C$
    **for each** $m \in M$
      $(\sigma^{dm}, \mathbf{W}^{dm}) = \mathrm{st}(\mathbf{c}^m, d, G(m))$
    **next** $m$
    **for each** $i \in D$: $d(i) = d$
      $u = u(i)$
      $o = o(i)$
      $w_{i1} = 0$
      **for each** $m \in M(u)$
        $w_{i1m} = \exp(W_o^{dm} + X_{i1m}) / \theta_u$
        $w_{i1} = w_{i1} + w_{i1m}$
      **next** $m$
      $V_{i1} = X_{i1} + \theta_u \cdot \log(w_{i1})$
      $P_{i1} = 1 / (1 + \exp(-V_{i1}) / \vartheta_u))$
      **for each** $m \in M(u)$
        $P_{i1m} = w_{i1m} / w_{i1}$
        $n_o^{dm} = n_o^{dm} + N_i \cdot P_{i1} \cdot P_{i1m}$
      **next** $m$
    **next** $i$
    **for each** $m \in M$
      $\mathbf{f}^m = \mathbf{f}^m + \mathrm{load}(\sigma^{dm}, \mathbf{n}^{dm}, d, G(m))$
    **next** $m$
  **next** $d$
**end function**

We used the following notation: $n_k^{dm}$ is the flow of users of mode $m$ travelling toward the destination node $d$ that pass through node $k$, $W_k^{dm}$ is the opposite of the minimum cost payable by the users of mode $m$ for travelling from node $k$ toward the destination node $d$, $\sigma_k^{dm}$ is the successive arc utilized by the users of mode $m$ at node $k$ when travelling toward the destination node $d$ (clearly, $TL(\sigma_k^{dm}) = k$), $w_{i1}$ and $w_{i1m}$ are just free variables utilized in the computation of the satisfaction and of the probabilities for the modal split.

Procedure st($\mathbf{z}$, $d$, $G$) provides the *shortest tree* on the hyper-graph $G \in \{G_R, G_T\}$ toward the destination node $d$ for given arc costs $\mathbf{z}$, yielding the successive arc (hyper-arc) and the minimum cost for each node. Procedure load($\mathbf{p}$, $\mathbf{x}$, $d$, $G$) performs the propagation on the hyper-graph $G \in \{G_R, G_T\}$ of the node flows $\mathbf{x}$ travelling towards the destination node $d$ for given successive arcs $\mathbf{p}$, yielding the flow for each arc. The two procedures are not specified here for brevity, as they solve problems that are very well known in the literature (see, for instance, Pallottino and Scutellà, 1998).

The proposed algorithm has been implemented in a computer application capable of solving large scale instances of the problems at hand in quite reasonable time, such as that concerning the case study of Rome which will be presented in the next section.

## 5 Numerical application to a large scale network

Access control in Rome was first applied in 1989, when restrictions were placed on vehicle entrances to the old city center (the Limited Traffic Zone, LTZ), an area of about 6 $km^2$ containing an enormous historical and artistic heritage. Permission to enter was given free of charge to residents of the area and to certain categories of users. In 1998 a pricing policy for authorized non-residents was introduced. They are now required to pay for their permits an annual fare of about 300 euros (the equivalent of 12 monthly public transport passes). Both measures are now enforced through an automated system based on 23 electronic gates able to identify the vehicle and to calculate the applicable fare for vehicle entrance into the restricted area.

Due to several problems occurring with the current regime (high level of congestion in the surrounding area and very high number of mopeds), the introduction of different pricing policies has been envisaged by the local authorities. The aim of our study is to simulate the effects of such policies, where different fare structures are imposed to different categories of users (residents, authorized, non-authorized, non-LTZ) by means of the proposed model. Specifically, we are interested in comparing cordon pricing and rationing with MCP, in order to establish how far these simplified schemes are from the SO upper bound and to which extent may thus represent a second-best solution of the toll NDP.

Despite the relatively limited area involved in the pricing intervention, the high attractivity of the LTZ has induced to consider in the traffic simulations a much larger study area (the whole Lazio Region). A graph with 16,878 directed arcs and 5,902 nodes, 854 of which are centroids, was used as the base network on which 680 transit lines with 19,626 line arcs (including the regional railway and bus systems) have been defined. The demand has been segmented in terms of trip purpose (systematic and non-systematic), personal attributes (5 combinations of age, sex, profession, car and motorcycle availability) and destination (ZTL and non-ZTL) giving rise to 1,534,788 demand components grouped in 20 user classes. Exploiting the concept of mode here introduced, this complex demand structure has been simplified at the route assignment level introducing only 3 modes (resident cars, authorized cars, non-authorized cars), while 2 additional modes have been introduced to simulate the multimodal urban network (mopeds, transit).

Consistently with the aim of the simulations, the choice of the destination is assumed to be relevant only when it involves the access to the LTZ. In fact, should road pricing be implemented, we expect that very few drivers not directed to LTZ will cross it; on the other hand, the availability of the car mode for travelling towards the LTZ, is likely to attract trips previously directed to other destinations. Then, one additional user class has been introduced representing non-systematic trips with destination in the LTZ, for which the demand is considered elastic up to the generation model, while for all other user classes the generation is assumed fixed. On this basis, the distribution model is implemented as a generation model toward the LTZ. Then, three choice levels have been actually considered: generation, modal split and route assignment. The demand model has been calibrated using RP-SP data as the introduction of a road pricing scheme involves changing the choice set of road users.

The computation of the above instance of the equilibrium problem by means of 50 iterations of the proposed algorithm with $H = 1$ takes about 10 minutes on a 1500 Mhz PC.

Some preliminary results showed that, even with high level of fares, no substantial changes can be expected, in terms of overall total cost and modal split, should the access limitation remain operational and only authorized cars be charged an increasing toll. This is due to the limited number of vehicles potentially subject to charges, compared to the whole number of users accessing the area, and to the usually high level of income of the user categories charged. More relevant modifications occur when the access restriction is extended to mopeds, that represent a relevant share (19%) of private traffic. This policy also reduces the through traffic generated by such a category of private vehicles (some 40% of the motorbike users currently accessing the LTZ have their destination outside this area).

In the following we present and discuss some results relative to the application of road pricing policies where the access restriction for non-authorized car users is removed: residents may travel free of charge through the LTZ, authorized cars pay a fixed toll of 1 €, non-authorized cars pay a variable toll up to the amount of the fine (here we assume that, thanks to the automated control system, the probability of getting the fine for the non-authorized vehicles violating the LTZ is 1).

In Figure 5 is depicted the percentage of the cost savings achievable through a SO on the whole network, that amount to 52% of the total costs in the non-intervention option (LTZ toll = 0 €), captured by means of different cordon pricing policies with an increasing toll to enter the ZTL. Although the LTZ covers only a very small part of the whole area of analysis and includes the destination for only 14% of the travel demand, up to 18% of the theoretical savings due to MCP can be achieved through a cordon pricing policy. On this regard we must recall that implementing MCP would involve very high construction and maintenance costs, so that the real savings due to MCP would be much less than the theoretical ones; moreover, in evaluating the impact of any pricing policy we ignored the negative effects on the activity system related to the reduction of accessibility in the charged area.

The optimal toll appears to be 12 €, which is consistent with the toll actually charged in other European cities (in London about 8 € are requested to access the LTZ). The rationing policy (LTZ toll = 100 €) is also effective, as it is capable of capturing one third of the savings due to the best cordon pricing policy, while it may require much smaller implementation costs depending on the control system adopted. The toll yielding the break-even in terms of total costs between pricing and rationing appears to be around 2 €.

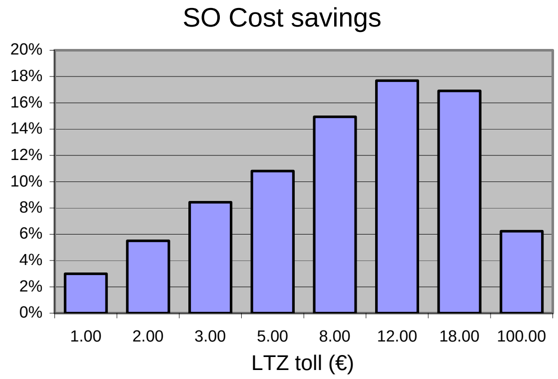



Figure 5. Percentage of the SO savings versus cordon pricing toll.

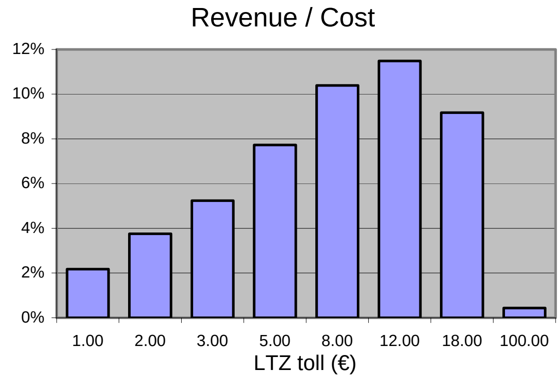



Figure 6. Ratio between total revenue and total cost versus cordon pricing toll.

In Figure 6 is depicted the ratio between the total revenue and the total cost for a number of different cordon pricing tolls. Interestingly, the policy achieving the maximization of the total revenue (around 500.000 € per day) coincides with the optimal one in terms of total costs; although, clearly, this is not a general result, and by thickening the values of the tolls this coincidence would vanish. Note that MCP implies a very high revenue-cost ratio (295%), and recall that in defining the total costs we did not take into account the impact of the pricing policies on the land use due to the reduction of accessibility. For this reasons, MCP is probably not acceptable from a social point of view, while the cordon pricing policies, although involving a restricted group of users, have a much softer impact on the network travel cost pattern (the revenue-cost ratio is at the most 12%).

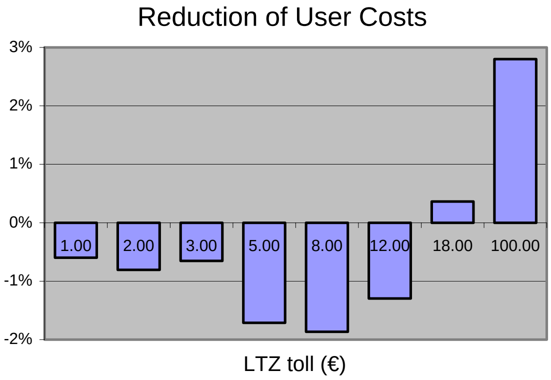



Figure 7. Relative reduction of user costs versus cordon pricing toll

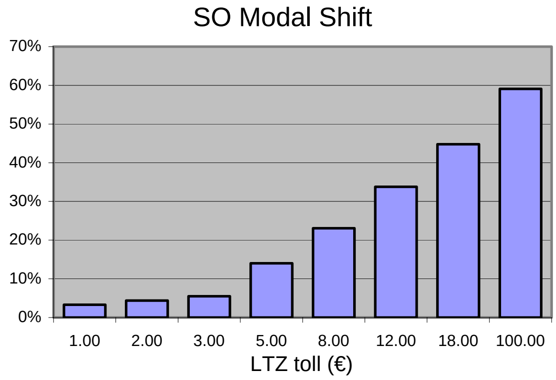



Figure 8. Percentage of the SO modal shift versus cordon pricing toll.

In Figure 7 is depicted the relative reduction of user costs assuming as point of reference the non-intervention option. Note that not all the pricing policies considered are *internally* effective, in the sense that they are convenient, not only in terms of total costs, but also in terms of user costs. In the latter respect, with reference to the specific case study at hand, rationing turns out to be the best policy, and this is not a foregone conclusion. On the other hand, with reference to the systematic mobility with destination in the LTZ, whose travel demand is assumed rigid, the increase of monetary costs due to the tolls is not at all compensated by the time savings. Indeed, the advantages of the pricing policies are to be found in the reduction of congestion and externalities for all users.

In Figure 8 is depicted the percentage of the increase of transit share achievable through a SO (equal to 4%) captured by means of increasing cordon pricing tolls, assuming as point of reference the non-intervention option. Clearly, the policy with the best modal shift is pure rationing, yielding here a 59% relative improvement.

In Figure 9 are depicted the percentages of speed increases achievable through a SO equal to (66% for cars, 10% for mopeds, and -6% for transit) captured by means of increasing cordon pricing tolls. The high speed increase enjoyed by cars, which corresponds to a travel time decrease of 40%, is due to a decrease of traffic flows of almost 20% (the car modal share diminishes from 48% to 40%; 4% switches to transit and 4% to the moped mode, whose relative weight on the main traffic stream is low) on a highly congested road network and reflects the non-linearity of the supply model. As expected, the policy with the best private speeds is rationing, yielding a 41% and a 23% relative improvement for cars and mopeds, respectively. The speed reduction characterizing the transit mode is due to the side effects of the cordon. In fact, in Rome's city centre the bus lines benefit from dedicated lanes, so that the LTZ does not induce on transit vehicles any relevant speed improvement. At the same time, just outside the LTZ, where the congestion increases due to the discontinuity in the network cost pattern produced by the toll cordon, the line carriers share the road links with private vehicles. The effectiveness of road pricing would be higher (in terms of total costs and modal split) and the social impacts softer (in terms toll values), should the policy be coupled with an improvement of the transit performances. Indeed, in the case of Rome, this is a priority and in particular the protection of the bus lines from road traffic outside the LTZ is a highly recommended intervention.

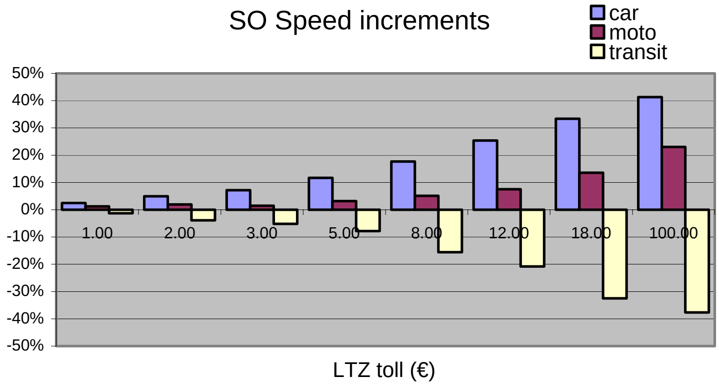
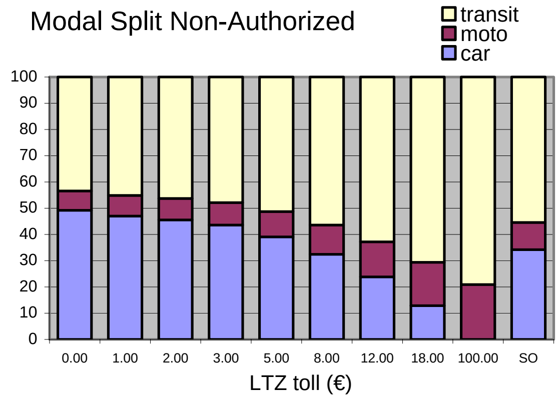



Figure 9. Percentage of the SO speed increment versus cordon pricing toll.

In Figure 10 are depicted the modal shares and in Figure 11 the travel times for different cordon pricing tolls, with reference to non-authorized users travelling towards the LTZ. This is the category of users to which the pricing policies are mainly directed, and non surprisingly their effects are remarkable. Specifically, the transit share, starting from 43% in the non-intervention option, goes up to 79% in the case of rationing, while the car share decreases from 49% to zero. On the other hand, the motorcycle share increases from 7% to 21% with a relative increment of 200% and thus a high negative impact on safety. The main effect on travel times involves the car mode, for which the commercial speed increases up to 50% in the 18 € solution (the value of car travel time for the rationing policy is not significant, as the modal share is too small); the improvement for mopeds, which are less affected by the congestion, is not as significant.



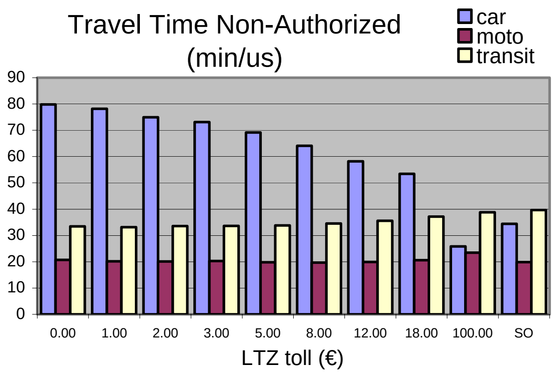




Figure 10. Mode shares for non-authorized users.

Figure 11. Travel times for non-authorized users.

In Figure 12 are depicted the mode travel times for different cordon pricing tolls, with reference to non-LTZ users. As the speed of private modes increases substantially with the toll, the initial conjecture regarding the side effects of the rationing policy results to be partially incorrect in the sense that the increment of congestion experimented right outside the LTZ is more than compensated by the benefits on the rest of the network.

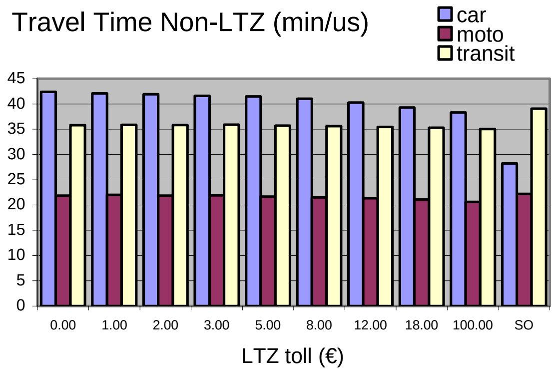



Figure 12. Mode travel times for non-LTZ users

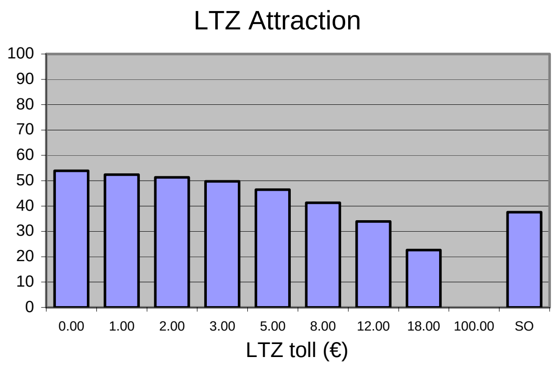



Figure 13. Trip generation for non-systematic users potentially directed to the LTZ.

In Figure 13 is depicted the share of non-systematic users potentially travelling to the LTZ who actually make the trip. This user type is introduced to simulate implicitly the effects on the demand distribution pattern as previously explained. Note that the rationing policy induces all users to choose other destinations than the LTZ.

## 6 Conclusions

Starting from a theoretical framework proposed in a previous paper, in this work we present an effective specification of a multi-class equilibrium model on multimodal networks for which the existence and uniqueness of the solution is proved under operative conditions on the parameters of the arc performance function. We also propose an efficient algorithm, capable of solving the model for large scale networks, which has been implemented in a computer program and applied to the case study of Rome.

The model is capable of simulating the simultaneous effect on traffic flows of applying different fare structures to different categories of users in urban networks with a variety of transport modes interacting among each and, in particular, where transit line vehicles sharing road links with private traffic are affected by the same overall congestion. In order to make practically possible a thick segmentation of users, on the demand side, and the introduction of many transport modes, on the supply side, while reducing to a minimum the calculation of specific shortest paths and network loading procedures, we provide an extension of the concept of mode which embodies elements of both the supply and the demand models and consists in an aggregation process aimed at optimizing the algorithm.

The proposed model has been then utilized for analyzing advanced pricing and rationing policies where different combinations of the two policies are imposed simultaneously to different categories of users. In particular, we have investigated the possibility of applying simplified schemes, such as cordon pricing, as a second-best solution of the toll network design problem. The analysis has been conducted with reference to the case study of Rome, where there is an automated control system based on 23 electronic gates that constitute the cordon of a Limited Traffic Zone.

In cases like these where the aim of the policy is to improve the modal split, it is not possible to evaluate any intervention by means of a fixed demand equilibrium model, while a classical monomodal elastic demand model does not reproduce correctly the context at hand. To meet such a requirement, a high-performance equilibrium model has been specified, capable of taking into account, in practical terms, variations in travel demand and modal split caused by broad-ranging measures, such as those considered here, and an efficient algorithm has been implemented, which allows simulating in quite reasonable computing times the different reactions of the different categories of users.

In evaluating the different operative policies, marginal cost pricing has been taken into account as an upper bound constituting an helpful point of reference, despite its being purely theoretical and not achievable in practice. When examining the effectiveness of a specific policy by comparison with the upper bound, two main aspects are to be considered: the implementation costs of marginal cost pricing are prohibitive with the available technologies; the extent of the charged area is very small compared with the whole network. In the light of the above, the results obtained can be considered definitely satisfactory: although the LTZ includes the destination for only 14% of the travel demand, up to 18% of the theoretical savings due to MCP can be achieved through a 12 € toll generating a total revenue of about 500.000 € per day and 1/3 of the modal shift produced by marginal cost pricing; rationing is capable of yielding 1/3 of the savings due to the above pricing policy and 60% of the modal shift produced by marginal cost pricing, while it may require much smaller implementation costs depending on the control system adopted, so being a valid alternative to road pricing, at least in the specific case of Rome.

The results are also promising for future investigation, where, removing the limits of the case study at hand (position, size and shape of the toll cordon), more general policies and schemes can be considered, so making optimal use of the power and versatility of the model.

## References


Beckmann, M., McGuire, C. B. , Winsten C.B., 1956. Studies in the economics of transportation, Yale University Press, New Haven, CT.

Bellei, G., Gentile, G. , Papola, N., 2000. Transit assignment with variable frequencies and congestion effects, in Proceedings of the 8th Meeting of the Euro Working Group Transportation EWGT, ed.s Bielli M. and Carotenuto P. , Rome, Italy, 525-532.

Bellei, G., Gentile, G., Papola, N., 2002. Network pricing optimization in multi-user and multimodal context with elastic demand, Transportation Research B 36, 779-798.

Ben Akiva, M., Lerman, S., 1985. Discrete choice analysis: theory and application to travel demand, MIT Press, Cambridge, MA.

Bottom, J., Chabini, I., 2001. Accelerated averaging methods for fixed point problems in transportation analysis and planning, in Preprints of Tristan IV, Azores, Portugal, 69-79.

Cantarella, G.E., 1997. A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand, Transportation Science 31, 107-128.

Cascetta, E., 2001. Transportation systems engineering: theory and methods, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Dafermos, S.C., 1973. Toll patterns for multiclass-user transportation networks, Transportation Science 7, 211-223.

Dafermos, S.C., Sparrow, F.T., 1971. Optimal resource allocation and toll patterns in user-optimized transport network, Journal of Transport Economics and Policy 5, 198-200.

Daganzo, C.F., 1983. Stochastic network equilibrium with multiple vehicle types and asymmetric, indefinite link cost Jacobians, Transportation Science 17, 282-300.

Filippi, F., Persia, L. et al., 1996. Public transport prioritization. Transport Research APAS Urban Transport Report, DG VII, Office for Official Publications of the European Commission, Luxembourg.

Gentile, G., 2003. Sufficient conditions for the uniqueness of the solution to the Stochastic User Equilibrium problem, submitted for publication to Transportation Science.

Gentile, G., Nguyen, S., Pallottino, S., 2003. Route choice on transit networks with on-line information at stops, submitted for publication to Transportation Science, Technical Report TR-03-14, Università di Pisa, Dipartimento di Informatica, Pisa, Italy.

Gentile, G., Papola, N., 2003. Some questions about the monotonicity of the arc cost function and the uniqueness of the stochastic user equilibrium in multimodal urban contexts, Technical Report, Università degli Studi di Roma "La Sapienza", Dipartimento di Idraulica, Trasporti e Strade, Rome, Italy.

Liu, L.N., McDonald, J.F., 1999. Economic efficiency of second-best congestion pricing schemes in urban highway systems, Transportation Research 33B, 157-188.

May, A.D., Milne, D.S., 2000. Effect of alternative road pricing systems on network performance, Transportation Research 34A, 407-436.

Nguyen, S., Pallottino, S., Gendreau, M., 1998. Implicit enumeration of hyperpaths in a logit model for transit networks, Transportation Science 32, 54-64.

Oppenheim, N., 1995. Urban Travel Demand Modelling, Wiley-Interscience, New York, NY.

Pallottino, S., Scutellà, M.G., 1998. Shortest path algorithms in transportation models: classical and innovative aspects, in Equilibrium and advanced transportation modelling, ed.s Marcotte P. and Nguyen S. , Kluwer Academic Publisher, Boston Hardbound, MA, 245-281.

Rothengatter, W., 2003. How good is first-best? Marginal cost and other pricing principles for user charging in transport, Transport Policy 10, 121-130.

Smith, M.J., 1979. The marginal cost taxation of a transportation network, Transportation Research B 13, 237-242.

Verhoef, E.T., 2002a. Marginal cost based pricing in transport: key implementation issues from the economic perspective, paper prepared for the IMPRINT Project.

Verhoef, E.T., 2002b. Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points, Transportation Research 36B, 707-729.