

### **COMBINED TRAFFIC ASSIGNMENT AND SIGNAL SETTING**

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## **Abstract**

The conventional traffic assignment models do not take into consideration intersection-related factors such as traffic signal timing, phasing, etc and the resulting delay. The interactions between links are also not considered. On the other hand the signals are designed using the flows on the approaching links, which gives a set of green times. But these flows change according to the delay, which is caused due of the signal. This variation is not considered during the design of the signal, thereby giving inaccurate results. To sum up, the interdependency of traffic assignment and signal setting hitherto is not considered.

In this study, a model combining the user equilibrium assignment with the signal setting is developed and presented. The model is formulated as bi-level programming problem. The lower level problem represents Wardrop's user equilibrium assignment, which incorporated driver's reaction to a given signal control pattern and is solved using the Frank Wolfe algorithm. The upper-level problem is to determine an optimal green split that minimizes the total network travel time. This takes into account the driver's route choice behavior in response to signal split changes.

 The model is tested with a numerical example and the results are presented in detail. The results obtained from the model are compared with the conventional user equilibrium assignment. The study on the example network demonstrated the working of the model for combined traffic assignment and signal control and the need for considering the link interactions for realistic traffic assignment.

Keywords: Combined traffic assignment; Bi-level programming; User Equilibrium Assignment; Signal Setting

Topic Area: G01 Transport and Urban Development Issues

### **1. Introduction**

The conventional traffic assignment models do not take into consideration intersection-related factors such as traffic signal timing and phasing and the presence and adequacy of turning lanes. The interactions between links are not considered. Also there is no temporal dimension to traffic assignment. Because the trip table is fixed, the entire table must be assigned from origin to destination, during the analysis period regardless of whether sufficient capacity exists.

The paths on a network cross at a number of nodes. Turning movements occur at these nodes. These turning movements are conflicting in nature, thereby causing accidents and congestion of traffic. Hence these nodes have some control measures in-place. One of these measures is a signalized intersection. The signal is designed using the flows on the approaching links. This gives a set of green times for each approach for a given cycle time.



The signal design is based on the flows approaching the junction. But these flows change according to the delay that is caused because of the signal. This variation is not considered during the design of the signal, thereby giving inaccurate results.

# **1.1 Objectives of the study**

 The interdependency of traffic assignment and signal setting or in other words the effects of signal settings on the traffic assignment and visa versa need to be considered. Hence the objective of the study is to develop a methodology for combined traffic assignment and signal setting. The traffic assignment is done using user equilibrium assignment. The delay at signal for a link is worked out using the Webster's uniform aggregate delay function with a constant cycle time and loss time. A combined methodology of the user equilibrium assignment with the delay at the signal is developed and tested on an example network.

### **1.2 Background**

The basic concepts of traffic assignment and signal settings have been discussed in the previous sections. The need for studying traffic assignment and signal setting has also been established. The basic assumption of the most commonly used traffic assignment models is taking the link capacities as fixed and known. However, this is not realistic whenever signal settings are flow-responsive. Similarly, in traffic engineering practice, flows are considered as known and fixed while doing the signal-setting computations, thus neglecting the effects of rerouting induced by implementation of signal plans (current). The mutual interaction between user route choices and signal control decisions to calculate network traffic equilibrium are studied by number of researchers (Meneguzzer, 1997).



Fig.1 Conceptual structure of combined traffic assignment and control problem (Meneguzzer, 1997).

For a road network (with flow responsive signal control and origin-destination travel demands fixed) consider **f** and **g**, which denote respectively, a vector of link flows and a vector of signal settings for the network; assuming that the signal plan structure is given (specified by number, type, and sequence of phases), signal settings may consist of cycle length's, green splits, and offsets. For a signal control policy P, in general any rule or procedure that can be used to determine the components of g once **f** is known. A traffic equilibrium, **f**<sup>e</sup> is a specification of **f** satisfying Wardrop's first principle (Meneguzzer, 1997).

The equilibrium traffic signal setting is a pair  $(f^*, g^*)$  such that  $f^*$  is a traffic



equilibrium when signals are set at g\*.

$$
\mathbf{f}^* = \mathbf{f}^e \left( \mathbf{g}^* \right) \tag{1}
$$

where 
$$
g^* =
$$
 signal settings corresponding to  $f^*$  under specified control policy P;

$$
\mathbf{g}^* = \mathbf{g}^p(\mathbf{f}^*)
$$
 (2)

If there exists such a pair  $(f^*, g^*)$ , then link flows and signal settings are

$$
\mathbf{f}^* = \mathbf{f}^e \left[ \mathbf{g}^p(\mathbf{f}^*) \right] \text{ or } \mathbf{g}^* = \mathbf{g}^p \left[ \mathbf{f}^e \left( \mathbf{g}^* \right) \right] \tag{3}
$$

Sheffi and Powell (1983) have formulated the problem of optimally allocating the green splits over a transportation network as a (non linear) mathematical program. Let A be a set of links. Each node is assumed to represent a distinct intersection. Also, let Q denote the traffic sources and sinks, i.e., those network nodes where traffic originates or terminates, and let D denote the origin-destination (O-D) trip matrix with entries *Dw* to denote the trip rate between origin, and destination. Let  $x_a$  denote the (non negative) flow on link a, and let  $\lambda_a$ denote the green split. The travel time on a given link, is a function of the flow on this link and the green split at the downstream node (which determines the link's effective capacity i.e.  $t_a(x_a, \lambda_a)$  is a function which is non negative, increasing in x<sub>a</sub>, and decreasing in  $\lambda_a$ . The traffic signal timing problem is to choose the green splits (given cycle times) for each intersection, so as to optimize the total travel time over the network i.e.

minimize 
$$
\sum x_a t_a(x_a, \lambda_a)
$$
 (4)

Here x and  $\lambda$  are both variables and problem is to find the optimal values of  $\lambda$ , subject to the fact that the flows are at equilibrium. Now if we assume that the network is always in standard (Wardropian) user equilibrium, the equilibrium flows are the solution of the following equation:

$$
\min_{0} \int_{0}^{x_a} t_a(x_a, \lambda_a) dx, a \in A
$$
 (5)

subjected to flow conservation and non-negativity constraints. Now the optimal signal-setting problem can be formulated as

$$
\min \sum_{a \in A} x_a t_a(x_a, \lambda_a) \tag{6}
$$

where (\*) implies the optimal flows obtained by equilibrium constraints. This procedure after certain modifications in constraints can incorporate any one of the commonly used equilibrium frameworks.

Smith (1993) developed the traffic assignment / signal control, the theory of traffic equilibrium, which involves responsive signal control policies. In this theory drivers' route choice and the control policy's choice of green times are treated in a symmetrical manner. The iterative optimization assignment algorithm is regarded as a highly idealized model of the day-to-day dynamics of driver's route choices.

Yang and Yagar (1994) have formulated a general traffic corridor consisting of two subsystems of a freeway network and a surface network. The two systems are coupled by access ramps to provide multiple alternative routes for drivers from their origins to destinations. The lower-level problem represents a traffic equilibrium model involving explicitly ramp queuing, which predicts how drivers will react to any given on-ramp control pattern. The upper-level problem is to determine ramp-metering rates that optimize system performance criterion, taking into account drivers' route choice behavior.



Michael (1995) proposed delay on a link arise in three ways. First, as the density of traffic increases, its speed tends to fall in accordance with a relationship referred as the fundamental diagram. Second, signal controlled links are subject to periodic interruptions due to the signals. Third, residual queues from when capacity is exceeded.

Wong (1996) developed the group-based optimisation of signal timing for traffic control using TRANSYT program. The group-based control variables such as the common cycle time, the start time and duration of the period of right way for each signal group. Signal co-ordination among intersection over an area is called traffic control.

Wong (1997) developed the concept of reserve capacity has been used extensively for performance measure and timing design of individual signal-controlled intersections and can be used to determine signal setting for maximization of the network reserve capacity.

Van (2001) has presented two mathematically similar problems in transport network analysis: trip matrix estimation (ME) and traffic signal optimisation on congested road networks. These two problems are formulated as bi-level programming problem with stochastic user equilibrium assignment as the second-level programming problem.

Ziyou (2002) have presented the concept of reserve capacity with the continuous equilibrium network design problem. A bi-level programming model and heuristic solution algorithm based on sensitivity analysis are proposed to model the reserve capacity problem of optimal signal control with user-equilibrium route choice. The upper level problem address is to try to the find the maximum possible increase in traffic demand by both setting traffic signals and increasing the road capacity, the lower-level problem, which is a standard userequilibrium problem, accounts for the user's route choice behavior.

In general, network design problems are concerned with two groups network planners and network users. On the one hand the behavior of network users follows the user equilibrium principle of Wardrop, on the other hand network planners try to maximize capacity of the system. The next section discusses the methodology for achieving the objective of simultaneous traffic assignment and optimizing signal timing in saturated road networks. The model is formulated as a bi-Level programming problem, and described below.

## **2. Bi-Level Network Model**

The transportation network consists of links, junctions, origins and destinations. Vehicles enter the network at origins and leave the network at destinations. Origins are connected to destinations through links, which themselves are connected through intermediate junctions. Links represent real carriageways in the original network and can have differing properties, such as number of lanes, capacities etc. Junctions in the network can be controlled or uncontrolled, and the intersecting movements at junctions are supposed here to be able to filter through one another with no added delay. The model is formulated as a bi-level programming problem. The lower-level problem represents a network equilibrium model involving queuing explicitly on saturated links, which predicts how drivers will react to any given signal control pattern. The upper-level problem is to determine signal splits to optimize a system objective function, taking account of drivers' route choice behavior in response to signal split changes.

#### **2.1 Link travel time function**

The time required to travel on a link has to be computed. For this purpose some function, which reflects the reality is used. This function is termed as the "link travel time



function". Researchers have found many such link travel time functions. One of the most commonly used link travel time function is the BPR function, which is monotonically increasing, and continuous function. The function is given as follows

Link travel time function  $t_{\text{link}}$ 

$$
t_{\text{link}} = t^0 \left[ 1 + \alpha \left( \frac{x_a}{k} \right)^{\beta} \right] \tag{7}
$$

where  $t^0$  is the travel time on a link with 0 flow, x is the flow on a link, k is the capacity of the link,  $α$  and  $β$  are parameters.

As discussed earlier the effect of signal has to be incorporated in this function. The delay on a link due to signal

$$
f_{\text{signal}} = \delta_{a}.\tag{8}
$$

Combining the link travel time and the delay on a link due to signal is the most difficult part and is the focus of this study. There is no established rule for combining the two aspects. For this study we have assumed that –

$$
t_a = t_{link} + t_{signal} \tag{9}
$$

From the equations 7, 8 and 9 we get -

$$
\therefore t_a = t^0 [1 + \alpha \left(\frac{x_a}{k}\right)^\beta] + \delta_a \tag{10}
$$

### **2.2 Lower level problem**

According to Wardrop's (Thomas, 1991) principle each driver traveling from an origin to a destination will have perfect knowledge of the travel costs and queues via all routes and will choose the in a user-optimized manner. This problem is equivalent to the following nonlinear mathematical optimization program.

$$
\min \sum_{a \in A} \int_{0}^{x_a} t_a(x_a, \lambda_a) dx \tag{11}
$$

subject to some of the following flow conservation and non-negativity constraints. The link flows are computed from the path flow and the link flows on each link is estimates as below.

$$
\sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_w^w = x_a, a \in A \tag{12}
$$

Next, the total path flows should be equal to the total demand and as given below.

$$
\sum_{r \in R_w} f_r^{\ w} = D_w, w \in W,
$$
\n(13)

And finally the relevant non-negativity constraints given as below.

$$
f_r^{\ w} \ge 0, r \in R_{\omega}, w \in W. \tag{14}
$$

where  $D_w$  is the demand for O-D pair  $w \in W$ , R is the set of routes in the network.

The solution to the lower level problem would satisfy the user equilibrium condition. This would in-turn give the flows on links based on the user equilibrium assignment. Now using the travel time function as depicted in equation 10 would mean that the delay on the link due to the signal has also been incorporated which is the basic aim of this study. The formulation is as shown below –

$$
\min \sum_{a} \int_{0}^{x_a} t_a dx \tag{15}
$$



$$
\therefore \min \sum_{a} \int_{0}^{x_a} [t_a^0[1 + \alpha \left(\frac{x_a}{k}\right)^\beta] + \delta_a] dx \tag{16}
$$

$$
\therefore \min \sum_{a} t_a^0 x_a + \frac{t_a^0 \alpha k}{\beta + 1} \left(\frac{x_a}{k}\right)^{\beta} + \delta_a x_a \tag{17}
$$

#### **2.3 Upper level problem**

Given the driver's route choice behavior described by the above equations, a model can be formulated to determine optimal signal splits such that a particular system performance criterion or objective function is optimized for a given origin-destination demand. A meaningful objective is to minimize the total network travel time expressed as the sum of running time, signal delay spent in the network by all vehicles over a given time period.

$$
F(\lambda, x(\lambda)) = \sum_{a \in A_i} x_a(t_a(x_a, \lambda_a)),
$$
\n(18)

where  $t_a$  is total travel time and  $\lambda_a$  is signal split.

Let I be the set of all signalized intersections in the network. The proportions of green times for links approaching a given signalized intersection  $i \in I$  should satisfy some linear constraints, the detailed form of which depends on specific phase structure. One of the possible constraint relations may be written as:

$$
\sum_{a \in A_i} \lambda_a = 1.0, i \in I,
$$
\n(19)

where Ai denotes the set of approaching links for intersection  $i \in I$ . Here it is assumed that lost times are constant for simplicity. Finally, the signal split parameter should be fitted into given bounds.

$$
\lambda_a^{\min} \le \lambda_a \le \lambda_a^{\max}, a \in A. \tag{20}
$$

where  $\lambda_a^{\min}$  and  $\lambda_a^{\max}$  are lower and upper bounds of proportion of green time for link  $a \in A$ .

### **2.4 Combined model**

To sum up, the global optimal signal setting problem under queuing network equilibrium conditions can be formulated as the following bi-level programming problems. In this two types of problems first one is lower level problem and second is upper level problem. In the lower level problem, signal split  $\lambda$  is taken as constant and is input to get the traffic flow (x). In the upper-level problem takes the flow vector from the earlier level as the input to give the set of signal split  $\lambda$ .

$$
MinF(\lambda, x(\lambda)) = \sum_{a \in A_i} x_a(t_a(x_a, \lambda_a)), \qquad (21)
$$

subject to

$$
\sum_{a \in A} \lambda_a = 1.0, i \in I,
$$
\n(22)

$$
\lambda_a^{\min} \le \lambda_a \le \lambda_a^{\max}, a \in A. \tag{23}
$$

where  $x(\lambda)$  is obtained by solving



$$
\min \sum_{a \in A} \int_{0}^{x_a} t_a(x_a, \lambda_a) dx \tag{24}
$$

subject to

$$
\sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_w^w = x_a, a \in A,
$$
\n(25)

$$
\sum_{r \in R_w} f_r^w = D_w, w \in W,
$$
\n<sup>(26)</sup>

$$
f_r^{\ w} \ge 0, r \in R_{\ w}, w \in W. \tag{27}
$$

where, *A* is the set of links in the network, *W* is the set of origin-destination pairs, *Rw* is the set of routes between origin-destination pair  $w \in W$ , *da* is the queuing delay at link  $a \in A$ ,  $f_r^w$ is the flow on router ∈*Rw*, t<sub>a</sub>( $x_a, \lambda_a$ ) is the travel time on link a ∈ *A* described as a functional of link flow  $x_a$  and green split ,  $\lambda$ , and  $\delta_{ar}^w$ , is 1 if route r between O-D pair w uses link a, and 0 otherwise.

#### **3. Methodology**

The use of combined traffic assignment has been discussed in the previous section. This section briefly gives the methodology developed for combining the user equilibrium assignment with the signal setting. The methodology has been explained clearly with the help of flow charts.

### **3.1 Solution algorithm**

The input in the form of network data and O-D matrix is given. The value of delay for the first iteration is assumed to be zero. The user equilibrium assignment is done using this data and a set of flows is obtained. This is given as input to the signal-setting algorithm along with the signal design parameters. This algorithm gives the value of delay on each link. This delay is taken as input by the user equilibrium assignment algorithm. The whole process is iterated until the result converges. The step-by-step procedure of the solution algorithm is given below.

The notations used here are :  $\delta_a^0$  is the delay due to signal on link 'a' at the first iteration,  $t_a$  is the travel time on link 'a', where  $t_{\text{link}} = f(x_a^n)$  and  $t_{\text{signal}} = \delta_a^n$  and  $\delta_a^n = f(x_a^{n-1})$ ,  $x_a^n$  is the flow on link 'a' in the n<sup>th</sup> iteration,  $x_a^{n-1}$  is the flow on link 'a' in the (n-1)<sup>th</sup> iteration, and  $\delta_a^n$  is the delay due to signal on link 'a' at the n<sup>th</sup> iteration, m is the total number of links on the network,  $\lambda$  is the green split for a link which is a function of  $x_a^{n-1}$ . The stepwise procedure is as follows –

- 1. Assume that there are no junction delays (no signals )  $\delta_a^0 = 0$ .
- 2. Do User Equilibrium Assignment.  $t_a = t_{link} + t_{signal}$  and get  $x^* \implies (x_1, x_2, x_3,..., x_a,...)$  $x_m$ )
- 3. Do Signal Setting Get  $\lambda^* \implies (\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m)$  and then find  $\delta_a^n = f(\lambda^*, x_a)$
- 4. Go back to step 2.
- 5. Stop at convergence.



Fig.2 Flow chart of solution algorithm

## **3.2 Computation for user equilibrium**

The user equilibrium is computed using Frank Wolfe's algorithm. The network data and the O-D matrix are taken as input. The delay is assumed to be zero for the first iteration. The travel time is computed when the flows are zero. Then the shortest path is worked out. Based on the shortest path the flows are assigned on all or nothing assignment. The network is then updated with these flows. The solution is checked for convergence. If the criterion is met the iteration stops else the new travel times are computed and the procedure is repeated. This is explained with the help of flow chart given in fig.3. The notations used here are  $t^0$  is the travel time on a link with 0 flow, x is the flow on a link, k is the capacity of the link,  $α$  and  $β$ are parameters. The stepwise procedure is as follows -

1. Assume a link travel time function 't<sub>link</sub>': 
$$
t_{link} = t^0 [1 + \alpha \left(\frac{x_a}{k}\right)^{\beta}]
$$

- 2. The delay on a link due to signal : ' $t_{signal}$ ' =  $\delta_a$ .
- 3. Combining the link travel time and the delay, we get  $t_a = t_{link} + t_{signal}$
- 4. Formulating the User Equilibrium function 'UE' =  $t^0 x_a + \frac{t^a \cdot dx \cdot k}{\beta + 1} \left( \frac{x_a}{k} \right) + \delta_a x_a$  $t^0 x_a + \frac{t^0 \cdot \alpha k}{\beta + 1} \left(\frac{x_a}{k}\right)^{\rho} + \delta_a.$  $\int_0^0 x + \frac{t^0 \alpha k}{\alpha k} \left( \frac{x_a}{x_a} \right)^{\beta} + \delta$ β  $\alpha k(x)$ <sup>β</sup>  $\left(\frac{x_a}{k}\right)^{\nu}$  + ⎝  $= t^0 x_a + \frac{t^0 \alpha k}{\beta + 1} \left(\frac{x_a}{k}\right)^{\beta} + \delta_a \cdot x_a$

Frank Wolfe algorithm has been used to the get the user equilibrium solution. (Ref flow chart in fig 3. Shortest paths over the network are found using the label correcting shortest path algorithm.



Fig.3 Flow chart for Frank Wolfe algorithm for user equilibrium assignment

## **3.3 Computation of signal delay** δ

The computation of signal delay is based on the link flows that have been obtained from the user equilibrium assignment. The cycle time and total loss time per cycle is assumed to be constant. The computation of signal delay on a link has been done using the aggregate uniform delay function.

#### **3.4 Aggregate uniform delay function**

The aggregate uniform delay ' $\delta_a$ ' is given by (Webster, 1958) -

$$
\delta_a = \frac{C_2 \left[ 1 - \left( \frac{g_a}{C} \right) \right]^2}{\left[ 1 - \left( \frac{V}{K} \right) \left( \frac{g_a}{C} \right) \right]}
$$
(28)

where C = Cycle time,  $g_a$  = green split time for link 'a', V = Flow, K = capacity.

$$
\lambda_a = \frac{g_a}{C} \tag{29}
$$

where  $\lambda i$  is the proportion of green time to cycle time for link 'i'.

Assuming  $V/K = 1$  and replacing equation 29 in equation 28 we get,



$$
\delta_a = \frac{C}{2} \left[ 1 - \lambda_a \right] \tag{30}
$$

Now  $\lambda_a$  is calculated as follows –

$$
\lambda_a = \frac{\left(\frac{V}{S}\right)_a}{\sum_{a's} \left(\frac{V}{S}\right)_a \left[\frac{C}{C - L}\right]}
$$
\n(31)

where Saturation Flow  $S = 625 \times 3.5 \times$  No. of Lanes and L is the total loss time per cycle.

### **4. Case study**

The solution algorithm for combining the user equilibrium assignment and the signal setting has been discussed in the previous section. This algorithm has been tested using an example network as a case study and the results have been discussed in this section.

### **4.1 Example network**

The proposed model is tested over a small network as shown in fig.4. The network consists of 6 nodes numbered as 1,2,…,6. The nodes are connected by 14 directed links and these links are numbered as 1,2,…,14. The Node no.2 and Node No.3 are signalized junctions. The link length, number of lanes, α, β, capacity and speed limit for each of the links are given in table 1. The O-D matrix is changed to get a set of results as discussed in the subsequent section.



Fig.4 Example Network



From	<b>To</b>	Link	Length Km	No. of	<b>Alpha</b>	<b>Beta</b>	Capacity Veh/hr	<b>Speed</b> Limit
				Lanes				Km/hr
	$\overline{2}$	1	1.4	1	0.1	$\overline{2}$	1800	25
	3	$\overline{2}$	1.501	2	0.1	2	1800	50
$\overline{2}$		3	1.0	3	0.1	$\overline{2}$	1800	50
$\overline{2}$	5	$\overline{4}$	1.2	$\overline{2}$	0.1	$\overline{2}$	1800	25
$\overline{2}$	$\overline{4}$	5	1.5		0.1	$\overline{2}$	1800	25
$\overline{2}$	3	6	1.2	$\overline{2}$	0.1	2	1800	25
3		7	1.1		0.1	$\overline{2}$	1800	25
3	$\overline{2}$	8	1.4	$\overline{2}$	0.1	$\overline{2}$	1800	25
3	3	9	1.4		0.1	$\overline{2}$	1800	25
3	6	10	1.2	$\overline{2}$	0.1	$\overline{2}$	1800	25
4	$\overline{2}$	11	1.7	$\overline{2}$	0.1	2	1800	25
$\overline{4}$	3	12	1.7	1	0.1	$\overline{2}$	1800	25
5	$\overline{2}$	13	1.25	3	0.1	$\overline{2}$	1800	25
6	3	14	1.7	3	0.1	$\overline{2}$	1800	50

Table 1. Input Network Data

# **4.2 Model**

The model is developed by combining the user equilibrium assignment with the signal setting, which is obtained by using the aggregate uniform delay function. The model is tested for two cases firstly for a single OD pair and then for multiple OD pairs.

# **4.3 Case 1 (Single OD pair)**

A single OD pair is considered to demonstrate the effect of signal delay on the assignment and the subsequent changes in the link flows. Accordingly a single OD pair ( $q_{14}$  = 14266) is tested with and without considering the signal delay. The model is tested and the computational results are presented in tables 2 and 3.

The objective function value changes significantly with the addition of junction delay. The convergence of the objective function has been shown in fig. 5. The flows are assigned on paths 1-2-4, 1-3-4 and 1-3-2-4 after the UE assignment without considering the junction delay. When the junction delays are added the flows are assigned only on paths 1-2-4 and 1-3- 4. The paths with their respective travel times have been compared in table 3. (Ref. fig 8 and fig. 9). It is to be noted that link 8 has a flow of 471 when the signal delays are not considered. When the signal delay is added the flow on this link ceases to exist resulting in zero flow on link 8.

This clearly brings out the need for considering the signal delay while performing any assignment.



<b>System Travel</b> <b>Time</b>	<b>Flow on Link 8</b>	Delay at Junct 2	Delay at Junct 3
200714		856	889
204514	22	581	861
204526		570	860
204543		567	860
204542		567	860

Table 2. Comparison over 5 iterations

Table 3. Comparison of results for UE assignment with and without signal delay.





Fig. 5 System Travel Time vs Iterations





Fig.7 Plot Delay at Junktions 2 and 3 vs iterations





Fig. 8 Example Network after UE assignment



Fig. 9 Example Network after adding delay and iterating



# **4.4 Case2 (Multiple OD pair) -**

Multiple OD pairs are considered in this case (ref table 4 for the input data). The changes in the assignment and the subsequent changes in the links flows are observed. The model is tested and the computational results are presented in tables 5 and 6.

The user equilibrium converges to  $497114$  and the UE + Signal setting converges to 519474. The flows on the links vary so a to converge to a final value. The flows on some typical links are shown in tables 5 and 6 below and the flows on other links remain constant. A comparison of travel time on the path 1-5 has also been done as an example. It shows that the travel time on the path increases as soon as the signal delay is added (ref. table 6).

Node No.				
		1900	3700	3200
			1800	4000
	3950	1750		
	150	2050	3750	

Table 4. Input Demand Data (Case 2)





Table 6. Comparison of results for UE assignment with and without signal delay.







Fig.12 Flow on link 2 vs iterations









### **5. Conclusions**

A model for combined assignment and signal control is developed. The interdependency of equilibrium assignment and signal setting is demonstrated with the help of a case study. The results are compared for the cases with and without signal delay. The delay term is introduced in the link travel time function without changing its properties. The results suggest that there is a significant change in the flow pattern and the paths used when signal delay was considered. The model could be tested on a real network relaxing a few assumptions made, like constant cycle time.

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