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MULTICLASS TRAFFIC FLOW THEORY FOR MODELING OF MULTILANE TRAFFIC AT ON AND OFF RAMPS

D. Ngoduy, S.P. Hoogendoorn, Henk J. Van Zuylen
Transportation and Planning Section, Delft University of Technology
Stevinweg 1, 2628 CN Delft, The Netherlands
d.ngoduy@citg.tudelft.nl; s.hoogendoorn@citg.tudelft.nl; h.j.vanZuylen@citg.tudelft.nl

Abstract

The traffic control task on motorway and urban networks is complicated because it has to deal with the highly complex interplays between control measures applicable to multiple user groups and different user classes in different types of networks. To handle this complexity, it is essential to have a model-based approach that is able to give an insight into those processes and provide the relevant control measures. In the scope of this research, the macroscopic modeling is focused. Several macroscopic traffic models have been implemented and applied to freeways and urban networks. The common deficiency of these models is that they fail to represent the dynamics of mixed traffic on multilane motorways, therefore, fail to deal with the multiplicity of control objectives and user-classes. To overcome this, a new type of model needs to be implemented for operational anticipatory traffic information, guidance, and control by applying mathematical optimal control explicitly. The aim of this paper is to present a macroscopic model that is able to take into account explicitly the dynamics of traffic at on and off ramps by modeling lane-changing processes within these zones. Simulation and calibration results with real data from the freeway A1 in The Netherlands show a good agreement.

Keywords: Multiclass traffic theory; Multilane gas-kinetic models; Macroscopic traffic models
Topic Area: C3 Traffic Control

1. Introduction

Researches on multilane traffic and heterogeneous traffic on freeway have recently become more attractive for the purpose of using multi-agent control measures and optimizing traffic control of multiclass traffic users. These strategies may allow traffic planners to use the existing infrastructure more efficiently. Examples of those control measures are dynamic truck overtaking prohibitions, uninterrupted passage for buses by ramp metering at on and off ramps, dynamic lane allocation control.

The significant contributions to these research topics are the works by Helbing (1997), Shvetsov and Helbing (1998), Helbing et al. (2001), Hoogendoorn and Bovy (1999), Hoogendoorn (1999), etc. Most of these models are developed from the principle of the multilane gas-kinetic model for mixed traffic on multilane roadways. These types of model describe the evolution of the phase-space density of vehicles on a freeway in which the left hand side of the differential partial equations describes the continuous dynamics of the phase-space density function due to the motions of traffic flow and the right hand side describes the discontinuous changes of this function due to the events such as lane-changing, acceleration/deceleration, etc. However, the lane-changing maneuver at on and off ramps has not been taken into account explicitly so far. In this paper, we manage to capture the interactions of traffic at on and off ramps by determining the so-called *mandatory lane-*

changing rate from on ramps to the adjacent main lane (or from main lane to off ramps). Finally, the macroscopic traffic model is derived by the so-called *method of moments*.

Our paper is organized as follows. Section 2 reviews multilane and multiclass traffic flow model. In section 3, we derive the multilane multiclass macroscopic traffic flow model at on and off ramps based on the *method of moments*. Section 4 describes some simulation and calibration results with real data obtained from freeway A1 in The Netherlands. Finally, we conclude the paper with some further research directions in section 5.

2. Multilane and multiclass macroscopic traffic flow models

The multilane and multiclass (ML MC) macroscopic traffic model is developed based on the gas-kinetic theory. It consist of the following set of partial differential equations (Helbing et al., 2001) describing the time-space evolution of macroscopic traffic variables of class u ($u \in U$) on lane i ($i \in I$), such as flow rate $q_{u,i} = q_{u,i}(x,t)$, mean speed $V_{u,i} = V_{u,i}(x,t)$ and density $r_{u,i} = r_{u,i}(x,t)$

- *Conservation of vehicle equation*

$$\frac{\partial r_{u,i}}{\partial t} + \underbrace{\frac{\partial q_{u,i}}{\partial x}}_{\text{convection}} = \underbrace{\sum_{s=1}^{U} \sum_{i'=i\pm 1} \left(P_{u,i'} \Psi_{u,s,i'} - P_{u,i} \Psi_{u,s,i} \right)}_{\text{immediate lane\_changing}} + \underbrace{\sum_{i'=i\pm 1} \left(\Delta_{u,i'} r_{u,i'} - \Delta_{u,i} r_{u,i} \right)}_{\text{spontaneous lane\_changing}} \tag{1}$$

- *Momentum dynamics*

$$\frac{\partial q_{u,i}}{\partial t} + \underbrace{\frac{\partial E_{u,i}}{\partial x}}_{\text{convection}} = \underbrace{\frac{q^{e}_{u,i} - q_{u,i}}{\tau_u}}_{\text{acceleration}} + \underbrace{\sum_{s=1}^{U} \sum_{i'=i\pm 1} \left(P_{u,i'} \Phi_{u,s,i'} - P_{u,i} \Phi_{u,s,i} \right)}_{\text{immediate lane-changing}} + \underbrace{\sum_{i'=i\pm 1} \left(\Delta_{u,i'} q_{u,i'} - \Delta_{u,i} q_{u,i} \right)}_{\text{spontaneous lane-changing}} \tag{2}$$

While the left hand side (LHS) of equation (1) and equation (2) describes the continuous changes of traffic variables at location x and time instant t due to the convection term that reflects the movement of the vehicles flowing in or out of very small cell [x, x+dx) during time interval [t, t+dt), the right hand side (RHS) describes the discontinuous changes of traffic variables that consist of the following terms:

Acceleration term, reflecting the tendency of drivers to accelerate to desired speed

Immediate lane changing, reflecting the fact that the faster vehicles catch up with the slower ones and have to change their lane to avoid collisions.

Spontaneous lane changing, describing the tendency of drivers to use a particular lane.

In equations (1) and (2), $P_{u,i}$ denotes the expected probability of vehicle class u to be able to change from current lane i to either adjacent lanes i±1. $E_{u,i} = r_{u,i}\left[\left(V_{u,i}\right)^2 + \Theta_{u,i}\right]$ is the traffic energy, where $\Theta_{u,i}$ is speed variance, defined as $\Theta_{u,i} = \alpha\left(r_{u,i}\right)\left(V_{u,i}\right)^2$, and $q^{e}_{u,i} = r_{u,i} V^{e}_{u,i}$, where $V^{e}_{u,i}$ denotes the equilibrium speed , determined by the expression below:

$$V^{e}_{u,i} = V^{0}_{u,i} - \left(1 - P_{u,i}\right) \sum_{s=1}^{U} \Pi_{u,s,i} \tag{3}$$

In equation (3), $V^{0}_{u,i}$ denotes the free speed of vehicle class u on lane i.

The so-called pre-factor density dependent function α is calculated as:

$$\alpha(r^{u}_{i}) = \alpha^{u,0}_{i} + \delta\alpha^{u}_{i} \left[1 + \exp\left(\frac{r^{cr,u}_{i} - r^{u}_{i}}{\delta r^{u}_{i}} \right) \right]^{-1} \tag{4}$$

In expression (4), $\alpha_i^{u,0}$, $\delta\alpha_i^u$, δr_i^u are the lane and class specific model parameters, and $r_i^{cr,u}$ denotes the lane and class specific critical density, being estimated.

The functions $\Psi_{u,s,i}$, $\Phi_{u,s,i}$, $\Pi_{u,s,i}$ in equations (1), (2) and (3) are the so-called Boltzmann factors functions, which describe the influence of interactions on traffic dynamics. Here, $\Psi_{u,s,i}$ determines the lane changing term due to interactions in conservation equations, $\Phi_{u,s,i}$ determines the lane changing term due to interaction in momentum dynamics equation, and $\Pi_{u,s,i}$ determines the braking term reflecting the fact that the faster vehicles cannot overtake or change their lane and have to slow down. These functions are determined as follows:

$$\Psi_{u,s,i} = \chi(r'_{u,i}) r_{u,i} r'_{s,i} \sqrt{S_{u,s,i}} \left[\phi\left(\delta V_{u,s,i}\right) + \delta V_{u,s,i} \varphi\left(\delta V_{u,s,i}\right) \right] \tag{5}$$

$$\Phi_{u,s,i} = \chi(r'_{u,i}) r_{u,i} r'_{s,i} S_{u,s,i} \left[\frac{V_{u,i}}{\sqrt{S_{u,s,i}}} \phi\left(\delta V_{u,s,i}\right) + \left(\frac{\Theta_{u,i}}{S_{u,s,i}} + \frac{V_{u,i}}{\sqrt{S_{u,s,i}}} \delta V_{u,s,i} \right) \varphi\left(\delta V_{u,s,i}\right) \right] \tag{6}$$

$$\Pi_{u,s,i} = \chi(r'_{u,i}) r_{u,i} r'_{s,i} S_{u,s,i} \left[\delta V_{u,s,i} \phi\left(\delta V_{u,s,i}\right) + \left(1 + \left(\delta V_{u,s,i}\right)^2\right) \varphi\left(\delta V_{u,s,i}\right) \right] \tag{7}$$

Where $\delta V_{u,s,j} = \dfrac{V_{u,j} - V'_{s,j}}{\sqrt{S_{u,s,j}}}$ denotes the so-called normalized speed difference factor between interacting vehicle class u and class s on lane i with $S_{u,s,j} = \Theta_{u,j} + \Theta'_{s,j}$

The standard Gaussian distribution function $\phi\left(\delta V_{u,s,j}\right)$ and error function $\varphi\left(\delta V_{u,s,j}\right)$ in expressions (5), (6) and (7) are calculated as below:

$$\phi\left(\delta V_{u,s,j}\right) = \frac{e^{-\left(\delta V_{u,s,j}\right)^2/2}}{\sqrt{2\pi}} \text{ and } \varphi\left(\delta V_{u,s,j}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\delta V_{u,s,j}} e^{-z^2/2} dz \tag{8}$$

The effective cross section factor χ reflects the increase of the effective number of interactions in dense traffic and is calculated by equation (9)

$$\chi(r_i) = 1 + \frac{V_i^0 \left(T_i\right)^2}{\tau_i \alpha(r_i^{\max})} \frac{r_i}{\left(1 - r_i / r_i^{\max}\right)^2} \tag{9}$$

Note that the variables without class index u represent the weighted averages of the variables belonging to the different user class, which are calculated, for example, as:

$$T_i = \sum_u T_i^u \frac{r_i^u}{r_i} \tag{10}$$

In the MLMC model mentioned above, the so-called *mandatory lane changing* term that reflects *the lane changing maneuvers at on/off ramps, lane drop, etc*. needs to be taken into account explicitly. To this end, the modified model should be able to capture the following situations:

All vehicles must move to the target lane due to the blockage at the end (on-ramp acceleration lane)

The drivers are willing to accept shorter gaps than immediate lane changing and spontaneous lane changing, hence, increase the braking process and disturb the traffic flow on main lane.

The vehicles must slow down and stop if there are not sufficient gaps on the main lane

This implementation will be described in the next section.

3. Multilane and multiclass macroscopic traffic flow model at on-off ramps

This section illustrates the multilane gas-kinetic equation of the phase-space density (PSD) for heterogeneous traffic operations at on/off ramps, and then the resulting MLMC macroscopic traffic model is described. The equation describing the evolution of the PSD for heterogeneous traffic operations at on/off ramps is a generalization of the single gas-kinetic equation, which was first introduced by Prigogine et al (1971) and then by Paveri-Fontan (1975). Let ρu,i(x,v,t) be the so-called reduced multilane PSD of vehicle class u on lane i. The following equation is proposed (Helbing et al, 2001):

$$\frac{\partial \rho_{u,i}}{\partial t} + \underbrace{v\frac{\partial \rho_{u,i}}{\partial x}}_{\text{convection}} + \underbrace{\frac{\partial}{\partial v}\left(\rho_{u,i}\frac{V_{u,i}^{0}-v}{\tau_{u,i}}\right)}_{\text{acceleration}} = \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{\text{int}}}_{\text{interaction}} + \underbrace{\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}}_{\text{lane-changing}} \tag{11}$$

In equation (11), the lane-changing maneuver from on ramps to main lane or from main lane to off ramps is defined as mandatory lane-changing, where as the lane-changing within main lanes is defined as immediate lane-changing and spontaneous lane-changing. In this paper, the former one is focused on.

Let gu,i(v|x,t) denote the probability speed density function of vehicles class u at (x,t) on lane i and ru,i(x,t) denotes the density of vehicle class u at (x,t) on lane i, the following relations are defined:

$$r_{u,i}(x,t) \overset{def}{=} \int_0^{\infty} \rho_{u,i}(x,v,t)dv \text{ and } \rho_{u,i}(x,v,t) \overset{def}{=} r_{u,i}(x,t)g_{u,i}(v\,|\,x,t) \tag{12}$$

Let us consider a merging process from lane 0 to lane 1 (Figure 1). The decision to make a lane change is based on the distance hr between the subject vehicle and the vehicle behind (lag-gap) and the distance hf between it and the vehicle in front of it on the main lane (lead-gap). When both gaps suffice, the lane-changing maneuver will be performed; hence, the probability that a gap on the adjacent main lane is accepted depends on the joint probability distribution of lag-gap and lead-gap on that lane taking into account also the distance from the subject vehicle to the end of the on ramp. These gaps suffice if the space between the merging vehicle and the one behind it as well as the one in front of it is larger than certain threshold values of distance gap. Again, these threshold values should reflect the fact that the drivers are willing to accept smaller gaps when approaching closer to the end of the on ramp.

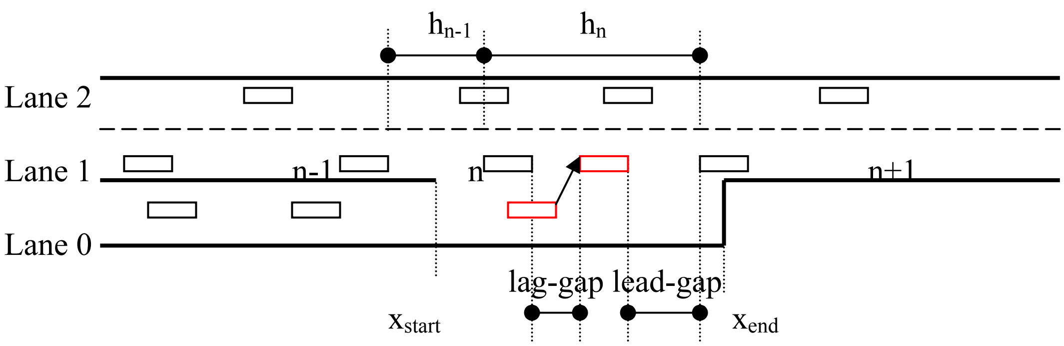


Figure 1 Lane- changing behavior at on ramp

Let h_u denote the threshold value of lag-gap and lead-gap for lane-changing manoeuvre, respectively. Assuming that the vehicle only reacts to the one in front $h_u = h_0 + L_u + T_u v$. Where v is the speed of the following vehicle, h_0 is the minimal distance reflecting the safety-margin acceptance of the drivers on the target lane, L_u and T_u is the length and reaction time of vehicle class u, respectively. Since approaching the end of the ramps, all drivers are willing to accept really smaller gap, therefore, they disturb traffic on the main lane significantly.

Assuming a linear relation of willingness-to-pay of drivers with respect to the remaining distance as shown by equation

$$h_u = h_0 + \frac{x_{end} - x}{x_{end} - x_{start}} T_u v = h_0 + \alpha T_u v \tag{13}$$

Now let us assume that the mandatory lane-changing rate is proportional to the number of vehicles stemming from on ramp or exiting at off ramp and that all those vehicles are forced to change their lane at the end of the ramp (at location x_{end}) as below:

$$\left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man} = \frac{P_{u,i}\rho_{u,i} v}{x_{end} - x} \tag{14}$$

In equation (14) $P_{u,i}$ denotes the expected lane-changing probability of vehicle class u from lane i to lane i+1 (i-1 in case of off ramp).

Let $A_u(v|x,t)$ be the event that a vehicle of class u driving with speed v merges into the main lane at (x,t), and B_u be the event that a vehicle is of class u.

$$P_{u,i} = \sum_{s=1}^{U} P(A_u \,\&\, B_s) = \sum_{s=1}^{U} P(A_u \mid B_s) P(B_s) \tag{15}$$

Let $P\left(h_r \geq h_1, h_f \geq h_2 \mid u, s\right)$ be the joint probability of lag-gap and lead-gap on the main lane between merging vehicle class u and any vehicle of class s.

$$P\left(B_s\right) = \frac{r_{s,i+1}}{\sum_{s=1}^{U} r_{s,i+1}} = \frac{r_{s,i+1}}{r_{i+1}} \text{ where } r_{i+1} = \sum_{s=1}^{U} r_{s,i+1} \tag{16}$$

The following relation is applied to the lane-changing probability at on ramps:

$$P\left(A_u \mid B_s\right) = P\left(h_r \geq h_s(w), h_f \geq h_u(v)\right) = \left[1 - \left\langle F_{rear}\left(h_s(w)\right)\right\rangle\right]\left[1 - \left\langle F_{lead}\left(h_u(v)\right)\right\rangle\right] \tag{17}$$

In expression (17), F(.) denotes the cumulative gap distribution function and <.> denotes the mean value. By definition, for any function $\phi(x)$ one gets $\left\langle \phi(x) \right\rangle = \int_0^{\infty} \phi(x) g(x) dx$, where g(x) is probability density function

Let $f_{lead}(h)$ be the gap density distribution function of the leader and, for simplicity, assuming that it is exponentially distributed as equation

$$f_{lead}(h) = \frac{1}{E(h)} e^{-\frac{h}{E(h)}} \tag{18}$$

In expression (18), E(h) denotes the mean distance gap, determined by equation

$$E(h) = \bar{h} = \int_0^{\infty} h f(h) dh = \left(1 - \sum_s \left(h_0 + L_s + T_s V_s\right) r_s\right) \Big/ \sum_s r_s \tag{19}$$

By definition $F_{lead}(h) = \int_0^h f_{lead}(z) dz = 1 - e^{-h/\bar{h}}$ (20)

Let $f_{rear}(h)$ be the lag gap density distribution function and considering two situations. First, the nearest following vehicle moves into median lane in order to give way to merging vehicle with probability λ then the space for merging is $h_n + h_{n-1}$, and the lag gap distribution is denoted by $f_{rear}^1(h)$. This distribution is determined based on the convolution theorem (Morris, 1986). Secondly, if the nearest following vehicle is unable to change her lane to the median lane with probability (1- λ) then the space for merging remains h_n, and the lag gap distribution is denoted by $f_{rear}^2(h)$. Based on the renewal process (Cox, 1962) the distribution of lag gap is determined as follows:

$$f_{rear}^{1}(h)=\frac{1-F_{rear}^{1}(h)}{E(h_n+h_{n-1})}=\frac{1-\int_0^h f(z)\int_0^{h-z} f(x)dxdz}{2E(h)}=\frac{e^{-h/\bar{h}}\left(1+h/\bar{h}\right)}{2\bar{h}} \tag{21}$$

$$f_{rear}^{2}(h)=\frac{1-F_{rear}^{2}(h)}{E(h_n)}=\frac{1-\int_0^h f(x)dx}{E(h)}=e^{-h/\bar{h}}/\bar{h} \tag{22}$$

Hence $F_{rear}(h)=\lambda\int_0^h f_{rear}^{1}(x)dx+(1-\lambda)\int_0^h f_{rear}^{2}(x)dx=1-e^{-h/\bar{h}}-\frac{\lambda h}{2\bar{h}}e^{-h/\bar{h}}$ (23)

Substituting equations (18) and (23) into equation (17) we obtain the expected lane-changing probability of vehicles class u from on-ramps, irrespective of vehicle types on the main lane:

$$P_i\left(A_u\mid B_s\right)=\int_0^\infty e^{-h_u(v)/\bar{h}_{i+1}}g_{u,i}(v)dv\int_0^\infty e^{-h_s(w)/\bar{h}_{i+1}}\left(1+0.5\lambda\, h_s(w)/\bar{h}_{i+1}\right)g_{s,i+1}(w)dw$$

$$P\left(A_u\mid B_s\right)_i=e^{-(h_0+L_u)/\bar{h}_{i+1}}\int_0^\infty e^{-\alpha T_u v/\bar{h}_{i+1}}g_{u,i}(v)dv$$

$$e^{-(h_0+L_s)/\bar{h}_{i+1}}\int_0^\infty e^{-\alpha T_s w/\bar{h}_{i+1}}\left(1+0.5\lambda\left(h_0+L_s+T_s w\right)/\bar{h}_{i+1}\right)g_{s,i+1}(w)dw$$

$$P\left(A_u\mid B_s\right)_i=e^{-(h_0+L_u)/\bar{h}_{i+1}}\int_0^\infty e^{-\alpha T_u v/\bar{h}_{i+1}}g_{u,i}(v)dve^{-(h_0+L_s)/\bar{h}_{i+1}}$$

$$\left[1+0.5\lambda\left(h_0+L_s\right)/\bar{h}_{i+1}\int_0^\infty e^{-\alpha T_s w/\bar{h}_{i+1}}g_{s,i+1}(w)dw+0.5\lambda\alpha T_s/\bar{h}_{i+1}\int_0^\infty we^{-\alpha T_s w/\bar{h}_{i+1}}g_{s,i+1}(w)dw\right] \tag{24}$$

Using expansion $e^x=\sum_0^\infty\frac{x^n}{n!}$, equation (24) becomes

$$P_i\left(A_u\mid B_s\right)=e^{-(h_0+L_u)/\bar{h}_{i+1}}\sum_0^\infty\int_0^\infty\frac{\left(-\alpha T_u v/\bar{h}_{i+1}\right)^n}{n!}g_{u,i}(v)dve^{-(h_0+L_s)/\bar{h}_{i+1}}$$

$$\left[\left(1+0.5\lambda\left(h_0+L_s\right)/\bar{h}_{i+1}\right)\sum_0^\infty\int_0^\infty\frac{\left(-\alpha T_s w/\bar{h}_{i+1}\right)^n}{n!}g_{s,i+1}(w)dw\right.$$

$$\left.+0.5\alpha\lambda T_s/\bar{h}_{i+1}\sum_0^\infty\int_0^\infty w\frac{\left(-\alpha T_s w/\bar{h}_{i+1}\right)^n}{n!}g_{s,i+1}(w)dw\right]$$

$$P_i\left(A_u\mid B_s\right)=e^{-(h_0+L_u)/\bar{h}_{i+1}}\sum_0^\infty\frac{\left(-\alpha T_u/\bar{h}_{i+1}\right)^n}{n!}\int_0^\infty(v)^n g_{u,i}(v)dve^{-(h_0+L_s)/\bar{h}_{i+1}}$$

$$\left[\left(1+0.5\lambda\left(h_0+L_s\right)/\bar{h}_{i+1}\right)\sum_0^\infty\frac{\left(-\alpha T_s/\bar{h}_{i+1}\right)^n}{n!}\int_0^\infty(w)^n g_{s,i+1}(w)dw\right.$$

$$\left.+0.5\alpha\lambda T_s/\bar{h}_{i+1}\sum_0^\infty\frac{\left(-\alpha T_s/\bar{h}_{i+1}\right)^n}{n!}\int_0^\infty(w)^{n+1} g_{s,i+1}(w)dw\right]$$

$$P_i\left(A_u \mid B_s\right)=e^{-\left(h_0+L_u\right)/\bar{h}_{i+1}}\sum_0^{\infty}\frac{\left(-\alpha T_u/\bar{h}_{i+1}\right)^n}{n!}\left\langle v^n\right\rangle_{u,i}e^{-\left(h_0+L_s\right)/\bar{h}_{i+1}}$$
$$\left[\left(1+0.5\lambda\left(h_0+L_s\right)/\bar{h}_{i+1}\right)\sum_0^{\infty}\frac{\left(-\alpha T_s/\bar{h}_{i+1}\right)^n}{n!}\left\langle w^n\right\rangle_{s,i+1}+0.5\alpha\lambda T_s/\bar{h}_{i+1}\sum_0^{\infty}\frac{\left(-\alpha T_s/\bar{h}_{i+1}\right)^n}{n!}\left\langle w^{n+1}\right\rangle_{s,i+1}\right] \quad (25)$$

Let $M_k=\left\langle\left(v-V\right)^k\right\rangle$ denotes the kth moment and assume that the probability density function of speed is a Gaussian distribution, this leads to M2i+1 = 0, i = 1, 2, … Using the definition of average values for speed V, variance Θ as follows: $\left\langle v\right\rangle$ = V, $\left\langle\left(v-V\right)^2\right\rangle=\Theta$.

$$M_2=\left\langle\left(v-V\right)^2\right\rangle=\left\langle v^2\right\rangle-V^2=\Theta$$

$$M_3=\left\langle\left(v-V\right)^3\right\rangle=\left\langle v^3\right\rangle-V^3-3V\Theta=0$$

$$M_k=\left\langle\left(v-V\right)^k\right\rangle=\left\langle v^k\right\rangle-V^k-\frac{k(k-1)}{2}V^{k-2}\Theta=0$$

Then we end up with the general formula for $\left\langle v^k\right\rangle$:

$$\left\langle v^k\right\rangle=V^k+\frac{(k-1)k}{2}V^{k-2}\Theta,\qquad k\geq 2 \quad (26)$$

Substituting equation (26) into equation (25), after a lengthy but straightforward algebra calculation we end up with the following equation for mandatory lane-changing probability from on ramp to the adjacent main lane

$$P_{u,i}=e^{-\left(h_0+L_u+\alpha T_u V_{u,i}\right)/\bar{h}_{i+1}}\left[1+\frac{\Theta_{u,i}\left(\alpha T_u/\bar{h}_{i+1}\right)^2}{2}\right]\sum_{s=1}^{U}\frac{r_{s,i+1}}{r_{i+1}}e^{-\left(h_0+L_s+\alpha T_s V_{s,i+1}\right)/\bar{h}_{i+1}}$$
$$\left\{\left(1+0.5\lambda\left(h_0+L_s\right)/\bar{h}_{i+1}\right)\left[1+\frac{\Theta_{s,i+1}\left(\alpha T_s/\bar{h}_{i+1}\right)^2}{2}\right]+\right. \quad (27)$$
$$\left.+0.5\alpha\lambda T_s/\bar{h}_{i+1}\left[V_{s,i+1}+\frac{\Theta_{s,i+1}\left(\alpha T_s/\bar{h}_{i+1}\right)\left(2-\alpha T_s V_{s,i+1}/\bar{h}_{i+1}\right)}{2}\right]\right\}$$

From equation (27) it can be seen that the mandatory lane-changing probability depends on a lot of variables such as density on the main lane, the speed of merging traffic and main traffic as well as the speed variance, etc. Besides, it is also dependent on the safety margin that reflects the willingness of the subject drivers to accept smaller gaps when approaching the end of the ramps

In the case of off ramp, it is easy to show that the probability of diverging flow is dependent only on the vehicles in front, therefore we have

$$P_{u,i}=e^{-\left(h_0+L_u+\alpha T_u V_{u,i}\right)/\bar{h}_{i-1}}\left[1+\frac{\Theta_{u,i}\left(\alpha T_u/\bar{h}_{i-1}\right)^2}{2}\right] \quad (28)$$

To derive the macroscopic equations from equation (11), one needs to determine the right hand side (RHS) of equation (11), and multiple both sides with v^k (k=0,1) then integrate them over the speed v ∈ [0,∝), this is the so-called method of moments.

$$\frac{\partial}{\partial t}\int_0^\infty \rho_{u,i} v^k dv + \frac{\partial}{\partial x}\int_0^\infty \rho_{u,i} v^{k+1} dv + \frac{\partial}{\partial v}\int_0^\infty \left(\rho_{u,i}\frac{V_{u,i}^0 - v}{\tau_{u,i}}\right) v^k dv$$
$$= \underbrace{\int_0^\infty \left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{\text{int}} v^k dv}_{\text{interaction}} + \underbrace{\int_0^\infty \left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{spon} v^k dv}_{\text{spontaneous lane-changing}} + \underbrace{\int_0^\infty \left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man} v^k dv}_{\text{madatory lane-changing}} \tag{29}$$

In equation (29), the first two terms of LHS have been determined by Hoogendoorn and Bovy (1999), Helbing et al (2001). The last term is then determined from equation (27) and (28) for traffic at on and off ramps as below:

$$\int_0^\infty \left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man} v^k dv = \int_0^\infty \frac{P_{u,i}\rho_{u,i} v^{k+1}}{x_{end} - x} dv = \frac{P_{u,i}}{x_{end} - x}\int_0^\infty \rho_{u,i} v^{k+1} dv = \frac{P_{u,i} r_{u,i}\left\langle v^{k+1}\right\rangle_{u,i}}{x_{end} - x} \tag{30}$$

- *k = 0*

Equation (30) becomes $\int_0^\infty \left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man} dv = \frac{P_{u,i} r_{u,i}\left\langle v\right\rangle_{u,i}}{x_{end} - x} = \frac{P_{u,i} q_{u,i}}{x_{end} - x}$

- *k = 1 (momentum dynamics)*

Equation (30) becomes $\int_0^\infty \left(\frac{\partial \rho_{u,i}}{\partial t}\right)_{lc}^{man} v dv = \frac{P_{u,i} r_{u,i}\left\langle v^2\right\rangle_{u,i}}{x_{end} - x} = \frac{P_{u,i} E_{u,i}}{x_{end} - x}$

The macroscopic ML MC model for traffic flow at merging zones is obtained as below:

- *Conservation law*

$$\frac{\partial r_{u,i}}{\partial t} + \underbrace{\frac{\partial q_{u,i}}{\partial x}}_{\text{convection}} = \underbrace{\sum_{s=1}^{U}\sum_{i'=i\pm1}\left(P_{u,i'}\Psi_{u,s,i'} - P_{u,i}\Psi_{u,s}^{i}\right)}_{\text{immediate lane-changing}} + \underbrace{\sum_{i'=i\pm1}\left(\Delta_{u,i'} r_{u,i'} - \Delta_{u,i} r_{u,i}\right)}_{\text{spontaneous lane-changing}} + \underbrace{\sum_{i'=i\pm1}\left(\Gamma_{u,i'} q_{u,i'} - \Gamma_{u,i} q_{u,i}\right)}_{\text{mandatory lane-changing}} \tag{31}$$

- *Momentum dynamics*

$$\frac{\partial q_{u,i}}{\partial t} + \underbrace{\frac{\partial E_{u,i}}{\partial x}}_{\text{convection}} = \underbrace{\frac{q_{u,i}^e - q_{u,i}}{\tau_u^i}}_{\text{relaxation}} + \underbrace{\sum_{s=1}^{U}\sum_{i'=i\pm1}\left(P_{u,i'}\Phi_{u,s,i'} - P_{u,i}\Phi_{u,s,i}\right)}_{\text{immediate lane-changing}} + \underbrace{\sum_{i'=i\pm1}\left(\Delta_{u,i'} q_{u,i'} - \Delta_{u,i} q_{u,i}\right)}_{\text{spontaneous lane-changing}} + \underbrace{\sum_{i'=i\pm1}\left(\Gamma_{u,i'} E_{u,i'} - \Gamma_{u,i} E_{u,i}\right)}_{\text{mandatory lane-changing}} \tag{32}$$

where $\Gamma_{u,i} = \frac{P_{u,i}}{x_{end} - x}$

4. Simulation and calibration results

In this section, we simulate the developed model with data obtained from freeway A1 in the Netherlands. A dedicated numerical scheme is chosen for the simulation of the model (Ngoduy et al., 2004). The data used in this paper is obtained from the Dutch Ministry of Transport, Public Works and Water Management. This data contains the time-dependent (every 5 minutes) traffic flow rate and mean speed at each detector on freeway from KM 86.6 to KM108.6, and a part of this freeway from KM 86.6 to KM 90.0 during time period 14h00 to 19h00, 22nd October 2002, when the freeway was seriously congested, is used as shown in

Figure 2. The developed model is calibrated using a simple automated calibration procedure (e.g. Ngoduy and Hoogendoorn, 2003). In this procedure, a generalization of the simplex *Nelder-Mead* algorithm is applied to minimize the total mean square errors (TMSE) between observed speed (flow rate) and estimated model-based speed (flow rate).

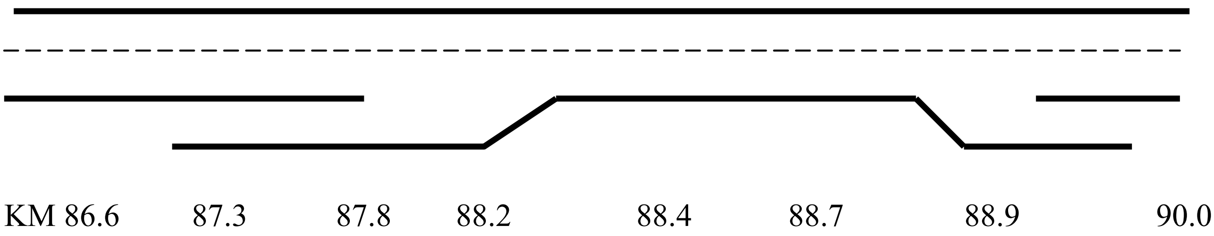


Figure 2: Layout of roadway for simulation

The objective function to be optimized is formulated as total mean square errors:

$$\text{Min} \quad z = \sum_{d}^{D}\sum_{t=1}^{N}\left\{\left[q_d(t)-q_{m,d}(t)\right]^2 \mu + \left[V_d(t)-V_{m,d}(t)\right]^2\right\} \tag{33}$$

In equation (33), q_d and V_d denote flow and mean speed predicted, $q_{m,d}$ and $V_{m,d}$ denote flow and mean speed measured at detector d, μ is weighting factor ($\mu = 0.1$ in this paper).

The length of the cells equals to 50 m, while the time-step equals to 1 s. The data used to feed the model is given at KM 86.6 and KM 90.0 of freeway and 87.8 of on-ramp (boundary conditions) and the objective function is calculated with the data at the remaining detectors. Figure 3 shows the input data for simulation. The model parameters to be calibrated are:

- Free speed V_0
- Jam density r_j
- Critical density r_{cr}
- The relaxation time τ
- Reaction time T
- Variance pre-factors α_0, $\delta\alpha_0$, δr

Calibration results

The automated calibration process is carried out with data set on 22nd October 2002 successfully for the developed model. The calibration results are shown in Table 1. These parameters are in good agreement with the one found by Helbing et al. (2001).

The outputs of simulation and calibration process are shown in comparison with real data in Figures 4, 5 and 6.

Table 1: Optimal model parameters

Free speed (km/h)	Jam density (veh/km)	Critical density (veh/km)	Relaxation time (sec)	Reaction time (sec)	α_0	$\delta\alpha_0$	δr (veh/km)	Total RMSE (%)
110	160	33	12	1.4	0.008	0.05	5.0	12%

It can be seen from figure 4, 5 and 6 that the developed model predicts the phase-space evolution of mean speed and density of the sample freeway A1 in good agreement with real data. We can also see from figure 6 that the results estimated by the proposed model at the bottleneck are very close to the real data. The oscillations in mean speed and density near congestion due to flow from on ramp were captured well, which means that the proposed model is able to describe accurately the traffic flow dynamics for freeway with on ramps.

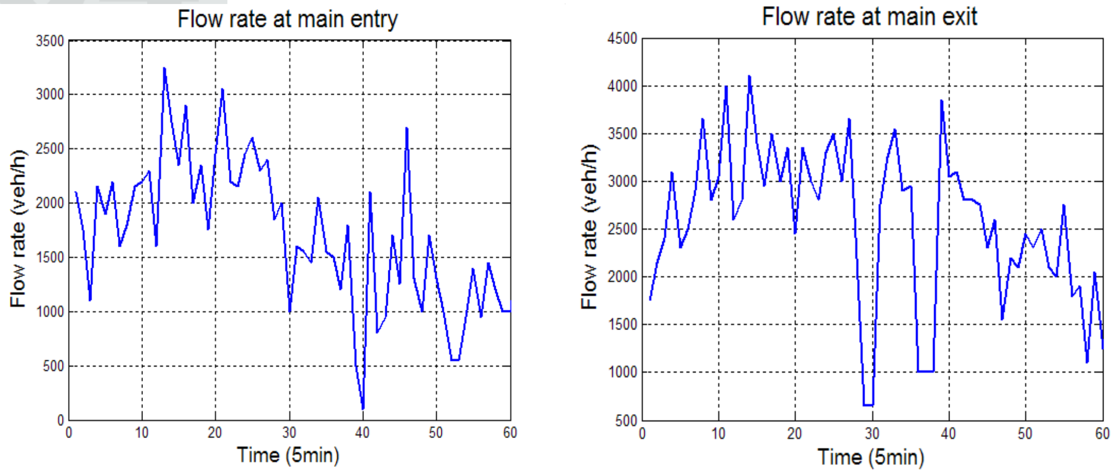
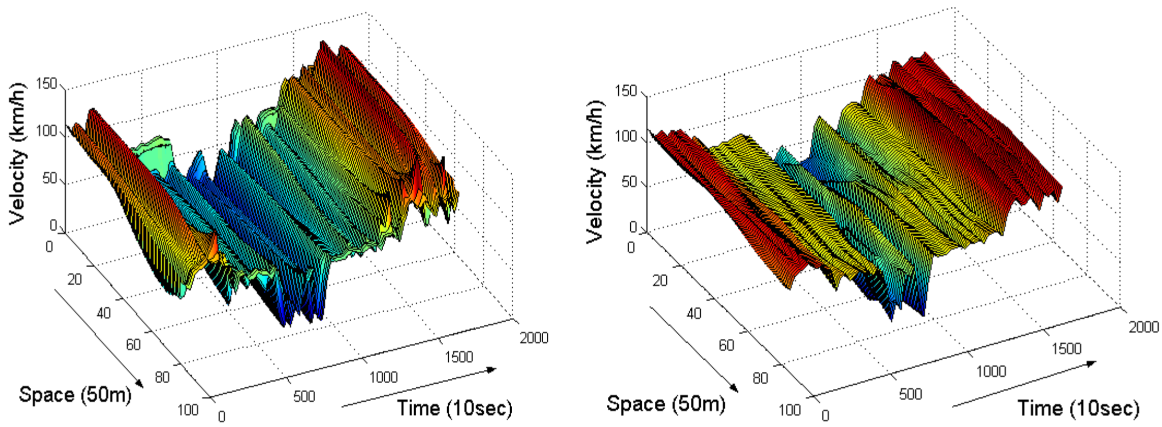
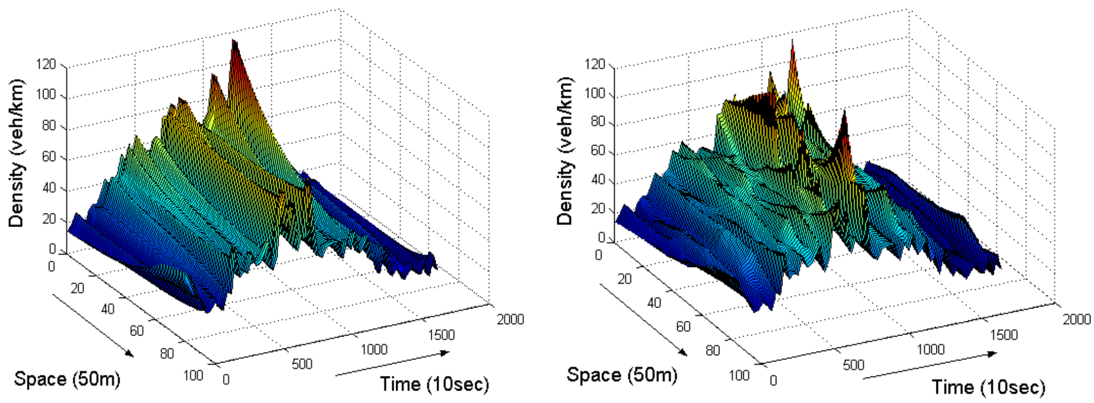


Figure 3: Data input for simulation and calibration process.



Calibration **Data**

Figure 4: Phase-space evolution of speed, model calibration vs. real data



Calibration **Data**

Figure 5: Phase-space evolution of density, model calibration vs. real data

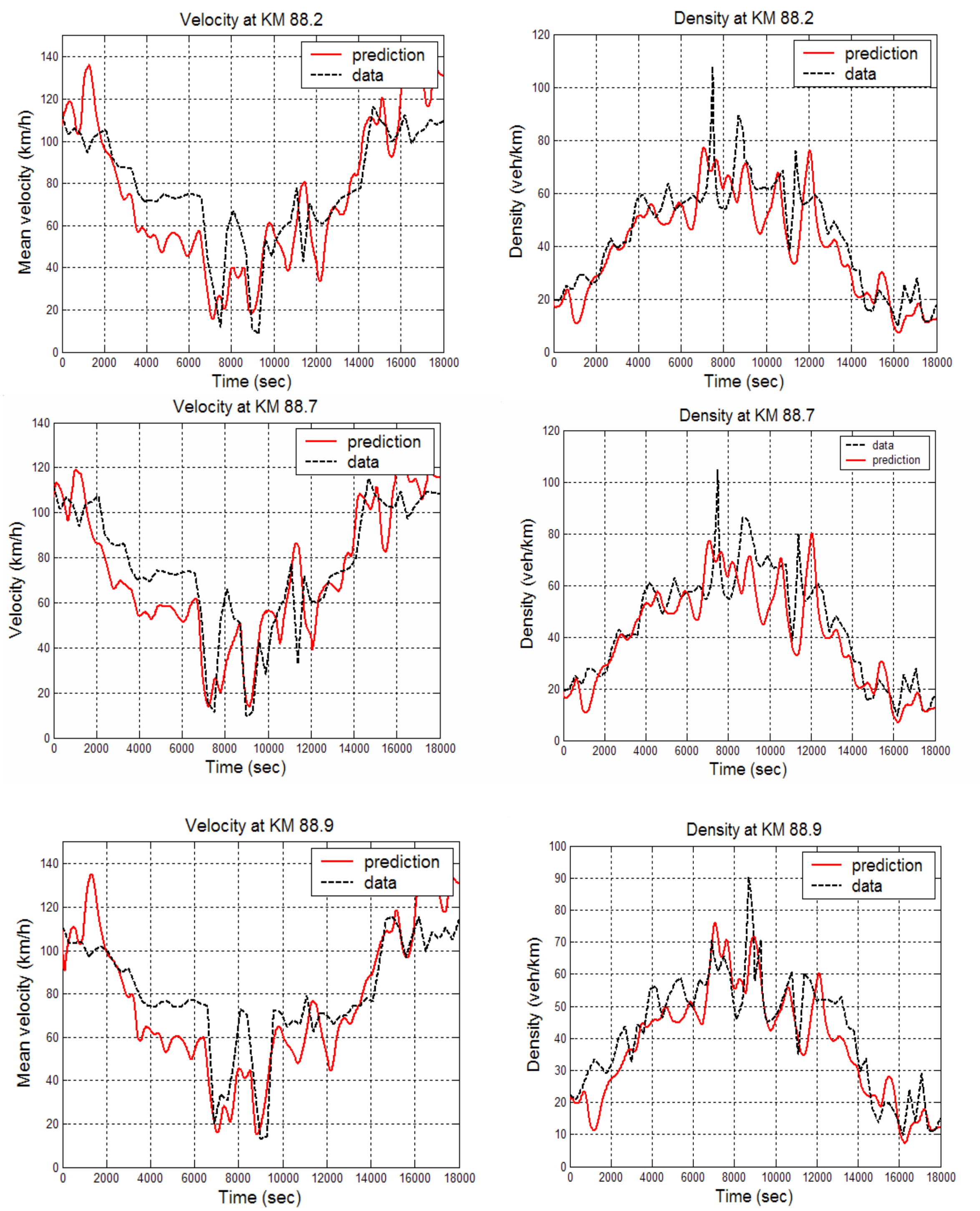


Figure 6: Time evolution of speed and density, model calibration vs. real data

5. Conclusions and further research

In this paper, we have developed a gas-kinetic flow model for mixed traffic at on and off ramps and derived the corresponding macroscopic model based on the so-called method of moments. In this model the lane changing maneuvers between on/off ramps and freeways have been explicitly taken into account by the renewal process. We have found that the lane-changing probability depends on a lot of factors as density, speed, speed variance and vehicle compositions on the target lane.

We have also calibrated successfully the developed model using empirical traffic data collected in Dutch freeway A1. The resulting model estimations are in good agreement with the observed flow and speed, especially the fluctuations of mean speed and density at on ramp. It reflects satisfactorily the impact of merging flow on main flow.

Our current work is to investigate and calibrate/validate the ability of the model to predict the evolution of traffic variables in heterogeneous freeways. This is a difficult task due to the lack of detail data. We expect that the model be able to describe the interplays between vehicle types and their effects on the traffic evolution in main lanes. The further work is to take into account the reduction of speed of vehicle on the shoulder lane to create a gap for merging vehicle. A cross-comparison of the improved model with a simple assumption of traffic interaction at on and off ramp will be made.

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