HETEROGENEOUS TRAFFIC FLOW MODELLING USING CELLULAR AUTOMATA

P. J. Gundaliya, Tom V. Mathew, S. L. Dhingra[1]
Transportation Systems Engineering, Department of Civil Engineering.
Indian Institute of Technology Bombay, Mumbai-400076, India.
Ph: 91-22-25767329, Fax: 91-22-25767302/25723480,
E-mail: gundaliya@iitb.ac.in; vmtom@civil.iitb.ac.in; dhingra@civil.iitb.ac.in

Abstract

Developing countries have heterogeneous traffic pattern which gives considerable difficulty in modelling traffic flow phenomenon. Due to the complexity in develop mathematical models for heterogeneous traffic, simulation based approach is adopted by many researchers. Cellular automata (CA) based modelling has emerged as an efficient tool for homogeneous traffic flow modelling. In this study, an attempt has been made to develop a heterogeneous traffic flow model using CA. In this approach the road stretch is divided into cells of uniform size and vehicles are represented as cells, including clearance required for each vehicle type. Vehicles are updated at each time step based on simple rules, namely acceleration, deceleration, and randomization. The arrival pattern, arrival speed distribution, classified volume, and driver behaviour probability are the required inputs for the model. With these inputs, the model generates vehicles and follows the CA rules for updating the vehicle position at each time step. A small road stretch is taken for validating the model. The observed and simulated speeds for each type of vehicle are compared and found satisfactory. A simulation experiment is also done to get the speed-flow-density relationship for a given vehicle composition.

Keywords: Heterogeneous traffic; Traffic flow modelling; Cellular automata; Microscopic traffic flow modelling
Topic Area: C3 Traffic Control

1. Introduction

In recent years various investigations have contributed to the field of traffic flow modelling including operation research, applied mathematics, statistical physics, computer science, and psychology. However, mathematical models for heterogeneous traffic condition, particularly for an urban arterial, is still lacking due to the difficulty to incorporate interaction of different vehicle and stop-and-go kind of situation. Hence, most of the researchers preferred simulation based approach to model urban arterial for heterogeneous traffic flow condition. The simulation approach using CA is successfully used to model the homogeneous traffic flow in recent past. This concept is used here to model the heterogeneous traffic flow model for an urban arterial. The model developed will be useful for the microscopic and macroscopic parameters derivation in different situations. In CA model, the position and speed of the vehicles are assumed to be discrete. The speed of each vehicle changes according to its interactions with other vehicles, following some pre-assigned (stochastic) rules depending on the circumstances.

The evolution rules are set up for heterogeneous traffic conditions so that only the essential features of the real interactions will be taken into account. The model gives the

[1] Corresponding author

average vehicle travel time and average speed. The model also produces the speed-flow-density relationship for heterogeneous traffic flow. The actual traffic behaviour, e.g. arrival pattern, composition of vehicles type, speed-distribution at entry, and aggregate driver behaviour probability are the required input for the model. In addition to these, details of the vehicle size and maximum speed of the vehicle are also required.

2. Cellular automata

Cellular Automata is a dynamical system in which space and time are discrete. CA consists of a regular grid of cells, each of which can be in one of a finite number of *k* possible states, updated synchronously in discrete time steps according to a local, identical interaction rule. The state of a cell is determined by the previous states of a surrounding neighbourhood of cells. CA is an array of *cells* that interact with their neighbours. These arrays can take on any number of dimensions, starting from a one dimensional string of cells. Each cell has its own state that can be a variable, property, or other information. By receiving input from connected cells or general messages, a cell uses its own set of rules to determine what its reaction should be. This reaction is a change of state and can also be a trigger to send out its own messages. These messages are passed onto other selected cells which cause them to act likewise.

CA can act as good models for physical, biological, and sociological phenomena. The reason for this is that each person, or cell, or small region of space *updates* itself independently (parallelism); basing its new state on the appearance of its immediate surroundings (locality) and on some generally shared laws of change (homogeneity). CA was originally conceived by Ulam and von Neumann in the 1940s to provide a formal framework for investigating the behaviour of complex, extended systems. von Neumann (1959) was the first to propose a system for producing life like behaviour from simple rules. Creamer and Ludwig (1986) have used a CA model for simulation of traffic flow. They have developed a boolean model representing individual vehicles by 1-bit variables. Since the introduction of the Nagel and Schreckenberg (NaSch) model in 1992, CA has become a well-established method of traffic flow modelling.

3. Past studies on heterogeneous traffic flow models and CA traffic flow models

Researchers have been contributing to traffic science by developing models of traffic flow and drawing general conclusions about the basic principles governing traffic phenomenon by studying these models. The microscopic theories are mainly concentrating on the behaviour of driver and the individual vehicles. The vehicle speed depends on the time and space headway. Car following model forms the core of microscopic traffic flow theory. These microscopic theories are used in various simulation models for homogeneous traffic. However, few attempts have been made to develop simulation models for heterogeneous traffic flow. Marwah et al. (1978), Kumar and Rao (1996), and Pillai (1974, 1975) have developed simulation models for intersection. Marwah et al. (1983) developed simulation models for two lanes with signalized intersections. Palaniswamy (1983), Chalapati (1987), and Ramanayya (1988) developed simulation model for two-lane roads. Bhuvneshsingh (1999); Roy (2000); and Arasan and Khosy (2002) have developed simulation models which are capable of simulating different types of non-motorised and motorised vehicles. Khan and Maini (2000) have done comprehensive review of the studies carried out in South Asia on mixed, non-lane-based traffic. The most of these model developed are based on the car following theories. The limitation of car following theories becomes a limitation of the model developed.

Methods based on a CA approach have recently gained a lot of interest in traffic flow modelling. The CA Traffic flow modelling is extensively done for uninterrupted roads like freeway and arterial to model the traffic flow behaviour for homogeneous traffic. In this approach, cars are represented as points moving on a discretized road with only a small set of possible velocities and accelerations. CA was first implemented to produce the lifelike behaviour by von Neumann (1959). Creamer and Ludwig (1986) have first developed single-bit coding with the goal to make the simulation fast enough to be useful for real-time traffic applications. Since the introduction of the models by Nagel and Schreckenberg (1992, 1995), CA has become a useful tool for traffic flow modelling as it proves good for predicting traffic flow behaviour. Many researchers have developed single lane to multilane traffic flow model using CA; Emmerich and Rank (1997) and Freund and Poschel (1995), to name a few. The key features of the multi-lane traffic flow are the modelling of lane changing and overtaking behaviours. Rickert and Nagel (1996), Wagner et al. (1997), Nagel et al. (1998), Chowdhury et al. (1997), Chowdhury and Wolf (1997), and Fukui and Nishinari (2002) have proposed various rules for the lane changing and overtaking. Schreckenberg (2001), Rickert et al. (1996), and Wolf (1999) have extended the CA application to the network level. Chowdhury and Wolf (1997) and Knospe and Santen (1999) have developed a two-lane traffic flow model with two kinds of vehicles; each having different maximum-velocity, but with identical dimensions of the vehicle. Therefore, a car and a truck may differ in the maximum velocity, where as the vehicle is still represented by the same cell. This is not realistic as the density at the time of the congestion may lead to wrong representation.

As the vehicle composition and size of the vehicle differ (heterogeneous condition) in a drastic way the above model will not give the true picture of the traffic condition. In these study CA approach is modified for the heterogeneous vehicles having large variation in size. In basic CA model all the cells are either occupied by a single vehicle or with no vehicle. But in the modified CA model, a number of cells are clubbed to represent a particular vehicle based on its size. The updation of this vehicle depends on the cell in the front-left corner.

All measurements of speed, volume, and density indicate that traffic flow is a stochastic process which cannot be described completely by temporal and spatial development of macroscopic fluid variables. In online simulation approach, computational efficiency is the prime criteria to develop an efficient model. Most of CA based traffic flow models have developed for homogeneous traffic flow condition; hence there is a need to exploit heterogeneous traffic flow modelling. CA has proved it's capability to predict the behaviour of traffic under homogeneous condition and become popular for computer implementation and simulation due to the discreet nature. In addition to this, it is ideally suited for real-time computer simulations. This gives inspiration to develop CA based traffic flow model for heterogeneous condition. In the present study, a model is developed for heterogeneous traffic flow in an urban arterial using CA approach.

4. Methodology

In most of the Asian countries traffic on urban arterial is mostly heterogeneous. In the proposed simulation model, road space is divided in to small imaginary cells. The size of the cells is decided according to the type of vehicles so that each vehicle may occupy a cluster of cells. The size of this cluster should represent the actual size of vehicle as close as possible. The physical representation of the vehicle should be kept slightly more than the actual size of vehicle to give some clearance. The longitudinal dimension of the cell affects the acceleration and deceleration rate. This factor plays an important role in updating the cells at every time instant. The forward movement is defined as the cell per time instant. Hence, the vehicle speed will be represented as cells per time interval. In the present study, we have implemented rules for updating the vehicle similar to NaSch model.

Time scan procedure is adopted in the simulation. The scan interval is decided based on the length of the cell, average acceleration, and average deceleration. A cell size of 1.9 meter in length and a simulation time step of 0.5 second are adopted in this study. However, this time step looks to be very small for decisions like overtaking, lane changing, etc, which can be a function of reaction time. Note that the previous NaSch model used a cell length of 7.5 meter and a maximum speed of 5 cells per time step. The generated vehicles will be put at the entry point of the road stretch. If sufficient gap is available, then the vehicle enters the system with a speed generated by the given distribution. If sufficient gap is not available, then the vehicle will not be allowed to enter the system and will be kept in a queue. These vehicles will be released when gap is available and the entry speed of such vehicles will be zero, but accelerates to the desired speed governed by the CA rules. The lateral position of the vehicle on the stretch will be selected according to availability of the gap. The vehicular type is generated on the basis of percentage of the vehicle composition.

When vehicle moves over the road space it has to take two decisions: what would be the rate of change of the speed and what would be the direction of movement in the next time step. Once this decision is taken, it will continue to the next decision point. However, lane changing factor is not considered in the present study. Total six types of vehicles are used in this simulation study as observed from the survey. The length and width of the effective vehicular size is obtained by adding the clearance to the observed vehicular length and width. This clearance is the minimum gap available in the front and side of the vehicles while stationary. The length and width are then converted into corresponding number of cells in lateral and longitudinal direction and the vehicles will be physically represented as number of cells occupied. The cell in the front-left corner of each vehicle represents the position of the vehicle at each time step. In Table1, each type of vehicle dimensions is given and corresponding number of cell taken in simulation for representing the vehicle. The physical representation of vehicles on single lane is shown in Figure 1. As the cell size decreases, the physical representation of the vehicle will be better but it requires huge memory for computation. Hence, minimum possible cell representation is used considering clearance and vehicular dynamic aspect in mind. The width of the cell was taken as 0.9 meter and length of the cell is taken as 1.9meter. The location of the vehicle on the road stretch is considered with respect to x and y coordinates of the front-left corner cell of the vehicle. Consider a road stretch having a width of M cells and a length of N cells. Starting from the front-left corner, let any vehicle k located on this grid have row and column number i_k and j_k. The boundary of the vehicle at this instant of time will be a block reserved in coordinate (i_k, j_k) and (i_k+k_w, j_k+k_l).

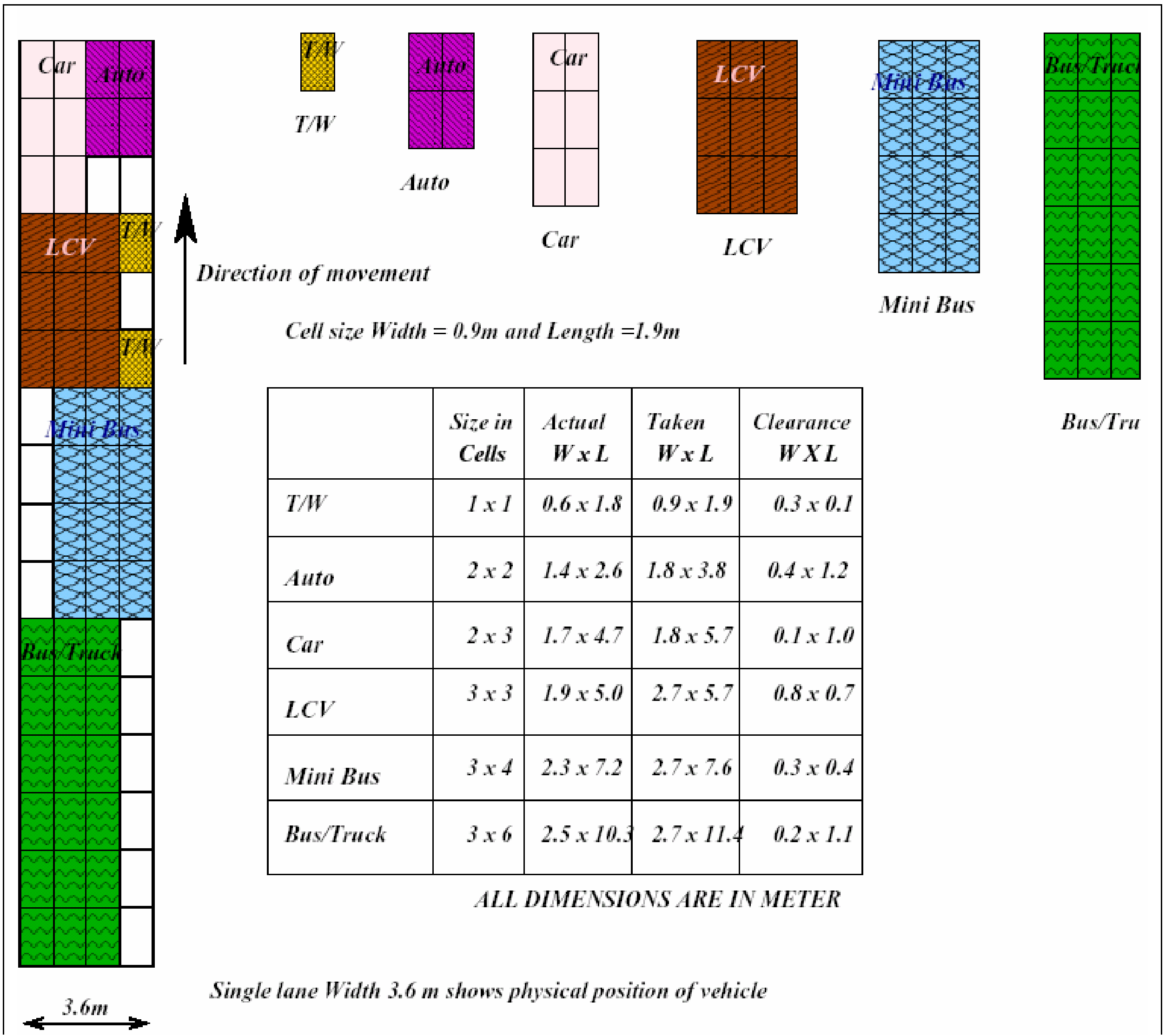

Figure 1. Physical Representation of Various Type of Vehicles on the Single Lane Road

	Size in Cells	Actual W x L	Taken W x L	Clearance W X L
T/W	1 x 1	0.6 x 1.8	0.9 x 1.9	0.3 x 0.1
Auto	2 x 2	1.4 x 2.6	1.8 x 3.8	0.4 x 1.2
Car	2 x 3	1.7 x 4.7	1.8 x 5.7	0.1 x 1.0
LCV	3 x 3	1.9 x 5.0	2.7 x 5.7	0.8 x 0.7
Mini Bus	3 x 4	2.3 x 7.2	2.7 x 7.6	0.3 x 0.4
Bus/Truck	3 x 6	2.5 x 10.3	2.7 x 11.4	0.2 x 1.1

4.1 Cellular automata rules for vehicle movement

In the CA models of traffic flow, the position, speed, acceleration as well as time are treated as discrete variables. In this approach, a lane is represented by one-dimensional lattice. Each of the lattice sites is represented by a *cell* which can be either empty or occupied by at the most one *vehicle* at a given instant of time. At each discrete time step, the state of the system is updated following a well defined rule. The computational efficiency of the discrete CA model is the main advantage of this approach over the car-following and coupled-map lattice approaches. The basic computation is defined on a one-dimensional array of total sites with open or periodic boundary conditions. However, to incorporate heterogeneity we have implemented two-dimensional array for physical representation of the vehicle. The forward movement of the vehicle is followed in a similar manner as considered in CA traffic flow model by NaSch model. This rule is made for forwarding movement, which is defined as per the type of the vehicle so as to incorporate different deceleration and acceleration. Each vehicle has an integer velocity with values between zero and the maximum speed of the vehicle. The speed of each vehicle can take one of the integer values out of like {0,1,2,3, ..., $v_{\max}$, in term of cells per time interval} Suppose, x_n and v_n denote the longitudinal position and speed, of the n^{th} vehicle respectively. Then, $gap_p^f = x_{n-1} - x_n$, is the gap between the n^{th} vehicle and the vehicle (*n-1*) in front of it at time *t* in the longitudinal direction. At each time step *t*, the arrangements of the all vehicles on Matrix of M x N are updated simultaneously, according to the following rules:

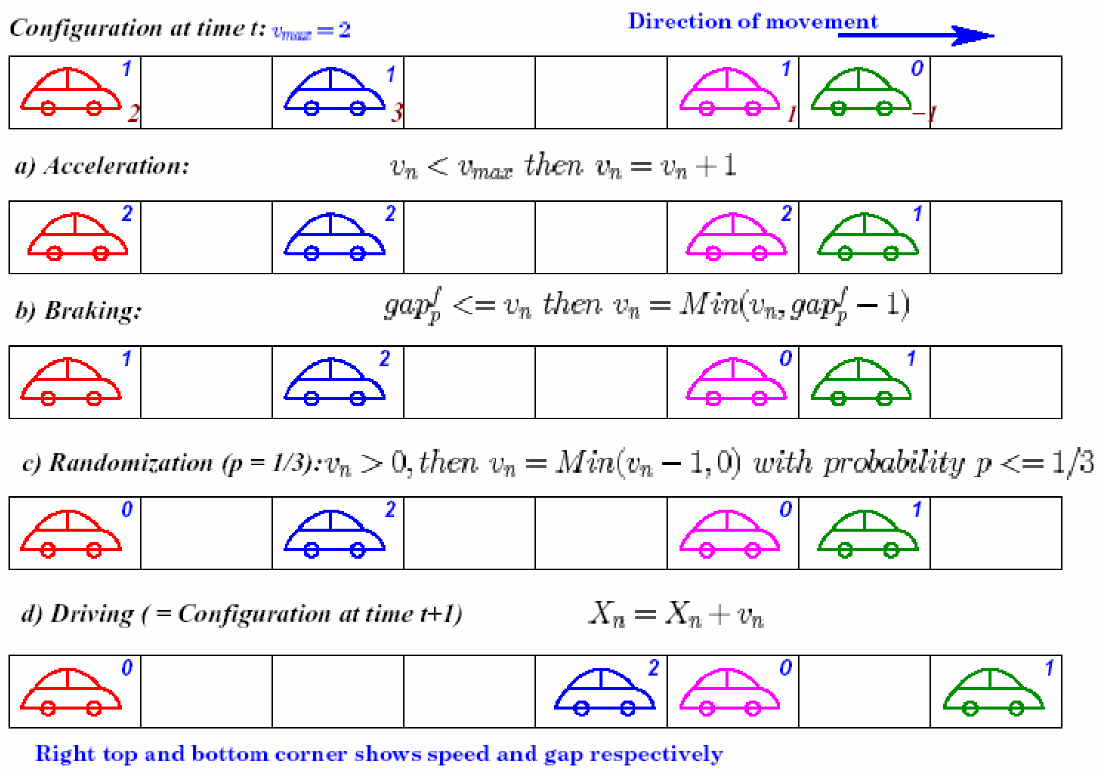

Figure 2. Step by step vehicle position and speed at different rule for each time step.

Step 1: Acceleration.

If $v < v_{\max}$ the speed of the nth vehicle is increased by one, $v_n = \min(v_n + 1, v_{\max})$ but v_n remains unaltered if $v_n = v_{\max}$,

Step 2: Deceleration due to other vehicles (vehicle ahead).

If $gap_p^f <= v_n$, the speed of the nth vehicle is reduced to gap_p^f -1, here gap is taken as 1 even a front vehicle is in the next cell. $v_n = \min(v_n, gap_p^f - 1)$, here gap_p^f is front vehicle gap in the present lane p.

Step 3: Randomization.

If $v_n > 0$, the speed of the nth vehicle is decreased randomly by one unit with probability p, but v_n does not change if $v_n = 0$, $v_n = \text{Min}(v_n - 1, 0)$ with probability p.

Step 4: Vehicle movement.

Each vehicle is moved forward according to its new velocity determined in Steps 1-3 , i.e.

$x_n = x_n + v_n$.

In model the acceleration and deceleration is considered as 1 cells/time step. In Figure 2 small example is explained taking numerical values. It is assumed $v_{\max} = 2$ and p=1/3 here. Therefore, on an average one third of the cars qualifying will slow down in the randomization step. The NaSch model is a minimal model in the sense that all the four steps are necessary to reproduce the basic features of real traffic. However, additional rules need to be formulated to capture more complex situations. Step 1 reflects the general tendency of the drivers to drive as fast as possible, if allowed to do so, without crossing the maximum speed limit. Step 2 is intended to avoid collision between the vehicles. The randomization in Step 3 takes into account the different behavioural patterns of the individual drivers, especially, non deterministic acceleration and overreaction while slowing down; this is crucially important for the spontaneous formation of traffic jams. Even changing the precise order of the steps of the update rules stated above would change the properties of the model. e.g. after changing the order of steps 2 and 3 there will be no overreactions at braking and thus no spontaneous formation of jams. This model may be regarded as a stochastic CA. In CA based traffic flow models like the NaSch model, the updating is done parallels rather, as it can lead to a chain of overreactions. Consider the case of a vehicle which slows down due to the randomization step. If the density of vehicles is large enough, this might force the following vehicle also to brake in the deceleration step. In addition, if p is larger than zero, it might brake even further in Step 3. Eventually this can lead to the stopping of a vehicle, thus creating a jam. This mechanism of spontaneous jam formation is realistic. Further order of each step also plays key role in movement of vehicle. The vehicle movement rules for the forward movement are same as the single lane rules of the NaSch model in the present model. Study also carried out for the homogeneous case to validate the model in the case of the homogeneous traffic and its behaviour. However the previous models have considered purely based on CA concept where the vehicle represents as cell only. Most of the cases the cell size is taken as 7.5 m and time step taken as 1 second. Due to this size of the cell the acceleration and deceleration are large than the observed. In the present study, cell size taken as 0.9meter and 1.9 meter. The simulation time step is taken as 0.5 second. This size of cell and time step is having to main two advantages. One is that that it is possible to take wide range of the speed which was not possible in previous CA traffic flow model. This range is almost double. The second is that acceleration and deceleration are quite close to real life. The different kind of the vehicle represent in number of cells in lateral (width) and longitudinal (length) direction.

4.2 Model description

The model is developed using C programming language. In order to run a simulation program, it is required to provide a description of the road topology (length and width in cells), arrival pattern of vehicle in the system, classified volume, vehicle size (in cells), Initial density (percentage), Maximum speed or free speed (cells per time step) of the vehicle, Speed distribution at entry, Road width, Road length, Driver behaviour probability and simulation time. In a simulation run, the initialization of the vehicle to the cells is generated by random variables considering initial density at time t=0. The position of the each vehicle will be represented using co-ordinate of the left corner of the vehicle with respect to road left most corner. A velocity is also assigned randomly to each of the generated vehicles satisfying the maximum velocity criteria. At time t=1 vehicles update their speeds according to vehicle gap ahead and follow the order of rules.. This updation is carried out up to the specified time-intervals. The detail procedure is explained in Figure 3.

The model developed addresses the stochastic and dynamic nature of the heterogeneous traffic flow. It is a discrete simulation model using interval scanning technique with the fixed interval time in advance. The road length and width can be varied as per the multiplication of the cells. The different kind of the vehicle can be incorporate in the model giving it size in cells and the free speed in cells per time step. In the case of the initial density vehicle are generated according to the given density from the left most corner of the road and speed is randomly assign to the vehicle satisfying the maximum speed criteria. In the case of no vehicle at time t=0 the warm up period is required. Model developed also capable to generate vehicle according to the Negative exponential distribution, Shifted exponential distribution and Erlang distribution. The model measure the speed of the vehicle at each time step, number of the vehicle at each time step in the system and average speed of the vehicle at that time instant. The speed –density –flow relationships during the simulation run will be plotted. Model gives headway distribution plot of generated and given.

5. Results and discussions

The fundamental relationships of basic parameters of traffic flow are investigated through simulation run with the different arrival rate and initial density. The results for homogeneous traffic show resemblance to past studies and the basic relations given in HCM 2000. However, unstable condition shown in the fundamental diagram is created hypothetically by assuming very high initial density. In this case also, the results are found to be identical to that of many established mathematical models. This assumption may not show real traffic behaviour as they do not consider the driver behaviour and incident occurrence phenomenon. Many simulation runs were made with same characteristics as mentioned previously with varying driver behaviour probability. The results indicates realistic traffic phenomenon like shock waves where vehicles changes their speed regularly depending on the traffic condition. However, the probability should be decided in such a way that it represent the real traffic movement characterised by overall travel time as close to the actual travel time for a given flow.

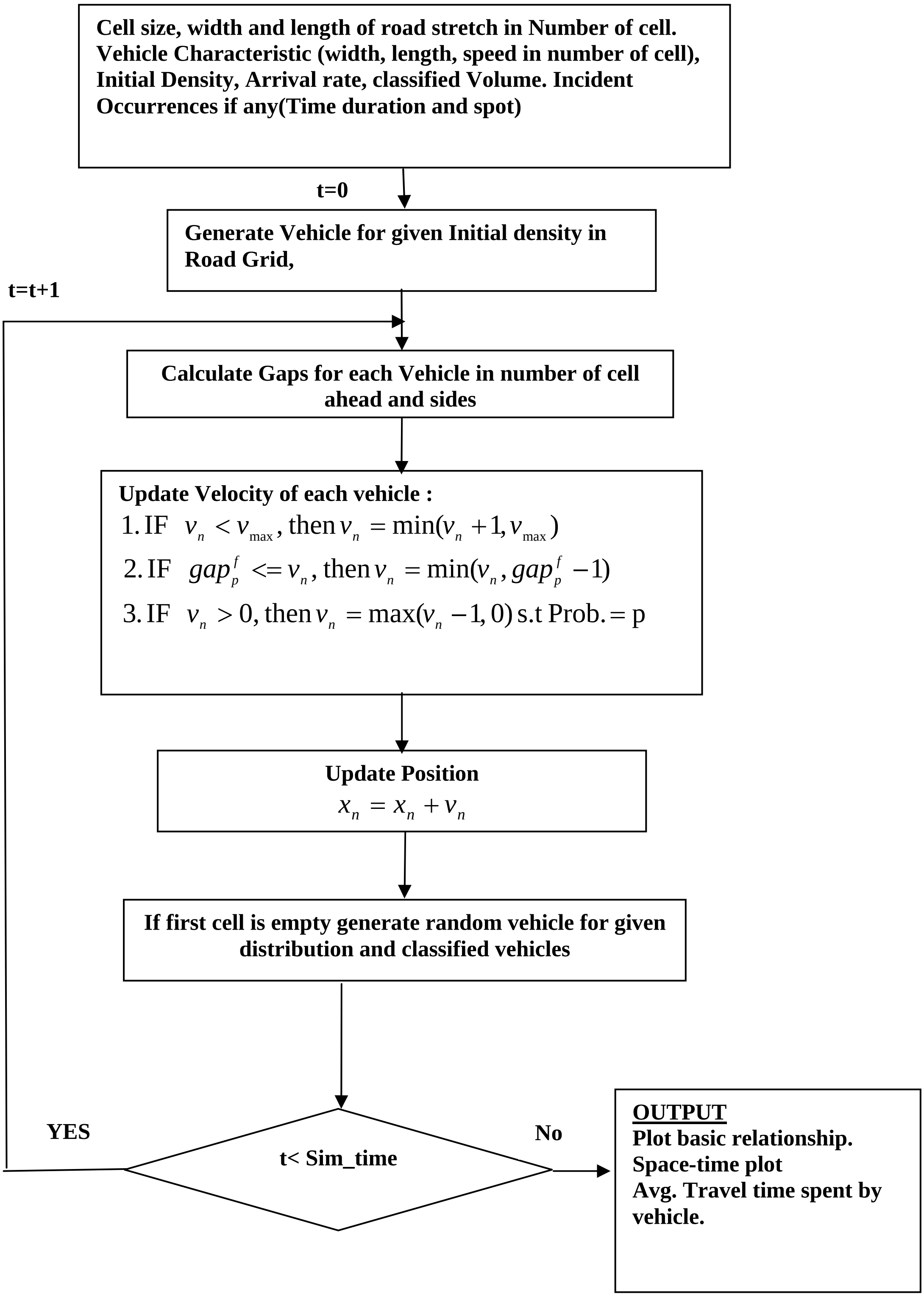

Figure 3. Flow Chart for Heterogeneous Traffic Flow Model

In order to compare with homogeneous model, a single vehicle type (car) is given as the input. The validation is done without randomization rule for a cell size of 7.5 meter, maximum speed of 5 cells/time interval, number of cells 300, a time duration of 300 sec, and an initial density of 0.6 (Schadschneider, 2000). Fundamental diagrams (Speed-Density, Speed-Flow and Flow-Density) for this are shown in Figure 4, Figure 5, and Figure 6 respectively.

Similar vehicle composition is taken for the present model for heterogeneous traffic. The following input data are taken for the experiment of homogeneous traffic flow. Vehicle considered for experiment is car. The simulation results presented here are for 1200 cells (2.28 km) and time duration of 600 time step (300 sec). Maximum velocity of car taken as 10 cells/time step (136.8 km/h) and initial density is taken as 0.6. Fundamental diagrams produced for (Speed-Density, Speed-Flow and Flow-Density) are shown in Figure 7, Figure 8, and Figure 9 respectively. These results show that the developed model represents traffic behaviour similar to the model developed by Schadschneider (2000).

The model is also compared to the heterogeneous traffic for a 50 meter road stretch (Arasan and Koshy, 2002). The survey was conducted for very sort time duration and hence full range of the density was not achieved. The classified volume observed on the road stretch is given in Table 2. The free flow speed is assumed on this road is given as the maximum speed for each type of vehicle. The experiment is carried out with the classified vehicles as given in Table 2. In this case, the initial density is taken as 0.6 and free speed of the vehicle is taken as shown in Table 2. The arrival rate is given as observed in the field. The fundamental diagrams (Speed-Density, Speed-Flow and Flow-Density) are shown in Figure 10, Figure 11, and Figure 12 respectively at probability p=0.

The headway distribution given and generated is plotted in Figure 13. The average speed obtained from the model is found to be close to the observed at a probability of *0.18.* The results obtained from simulation studies and field observations for the road section are given in Table 2. Due to limited availability of the data, no statistical test is carried out.

Table 2. Classified Vehicle Details and Results

Type of Vehicle	Classified vehicle in (%)Observed	Classified vehicle in (%)Generated	Free Speed (Km./hour)	Speed Measured (Km./hour)	Speed Observed (km./hour) at p=0.18
Two-wheeler	54.50	54.940711	95.76	40	45.986
Auto	10.00	9.486166	82.08	39	47.76
Car	25.00	24.110672	123.12	48	48.46
LCVT	3.00	2.371542	82.08	41	46.18
Minibus	5.00	5.138340	82.08	44	45.302
Truck/Bus	2.50	3.952569	68.40	42	31.286

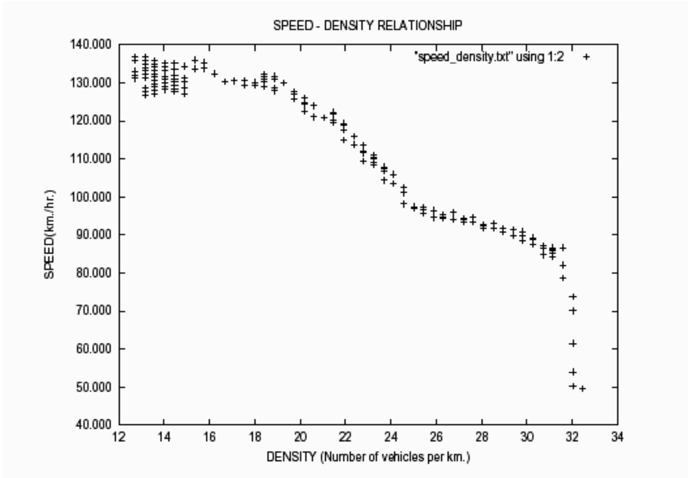

Figure 4. Speed- Density Relationship for Car Only Considering Initial Density as 0.6.

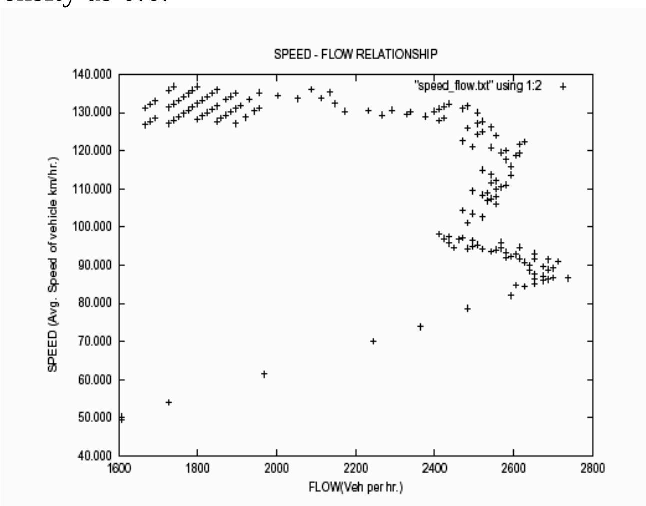

Figure 5. Speed- Flow Relationship for Car Only Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate**.**

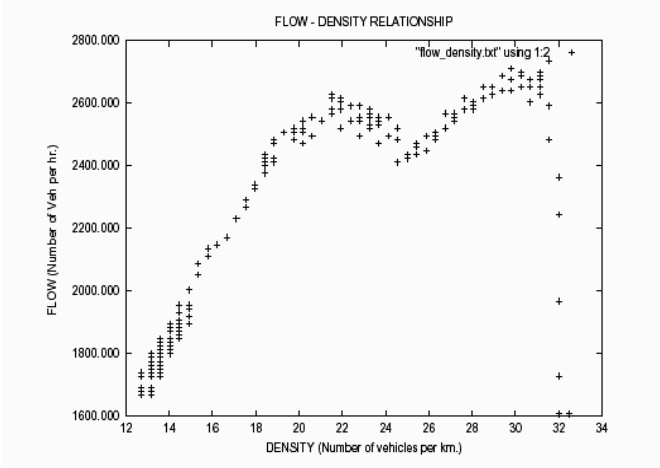

Figure 6. Flow – Density Relationship for Car Only Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate.

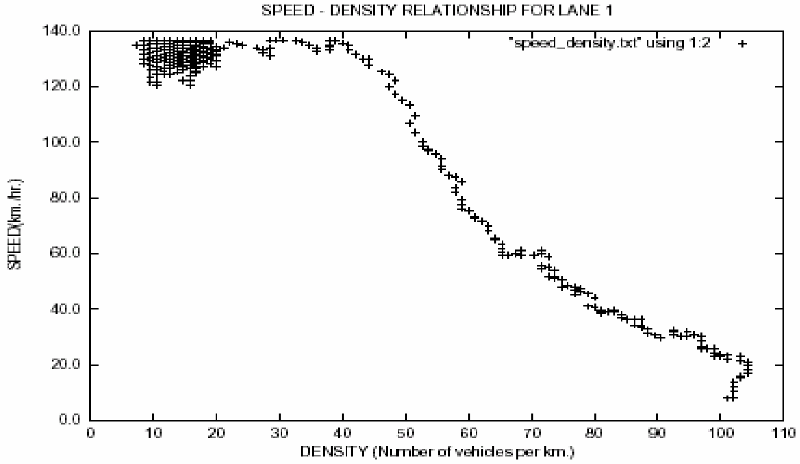

Figure 7. Speed- Density Relationship for Car Only Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate.

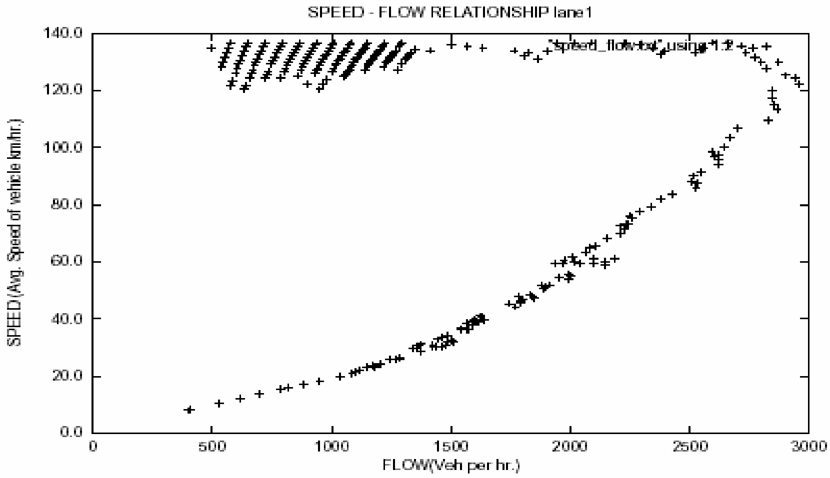

Figure 8. Speed- Flow Relationship for Car Only Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate.

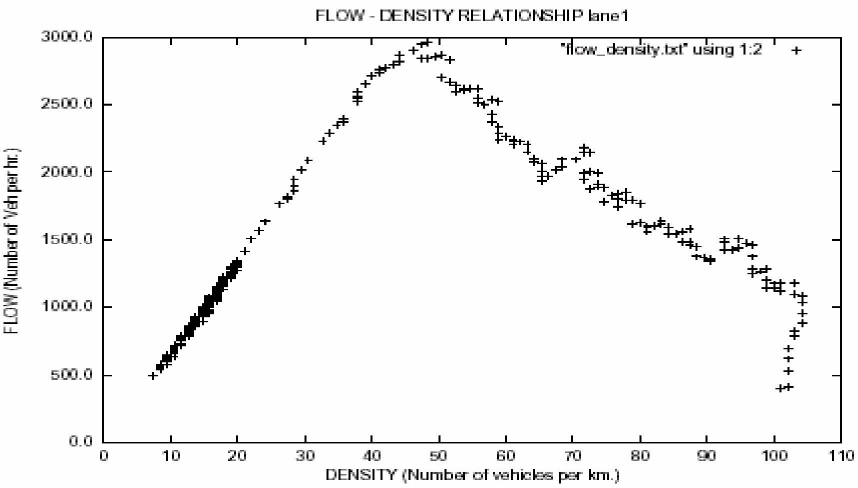

Figure 9. Flow – Density Relationship for Car Only Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate.

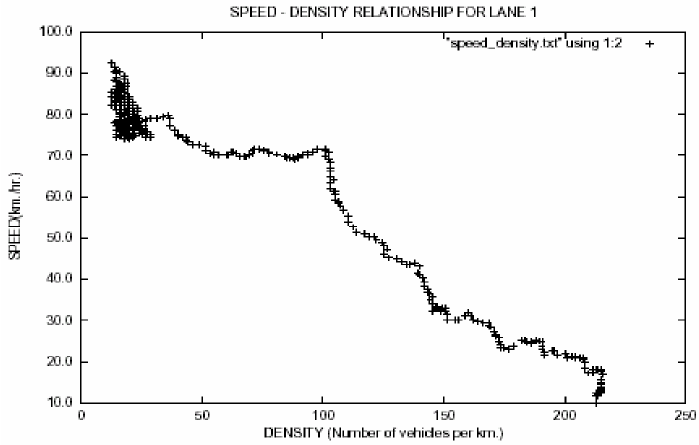

Figure 10. Speed- Density Relationship for Heterogeneous Traffic Considering Initial Density as 0.6 And 1600 Veh/Hr Inflow Rate.

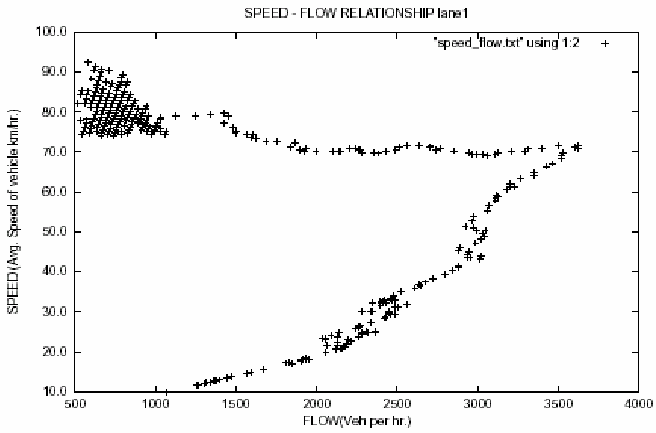

Figure 11. Speed- Flow Relationship for Heterogeneous Traffic Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate.

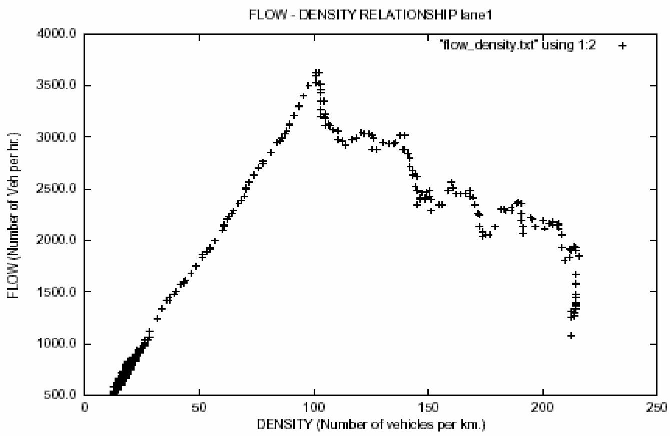

Figure 12. Speed- Flow Relationship for Heterogeneous Traffic Considering Initial Density as 0.6 and 1600 Veh/Hr Inflow Rate

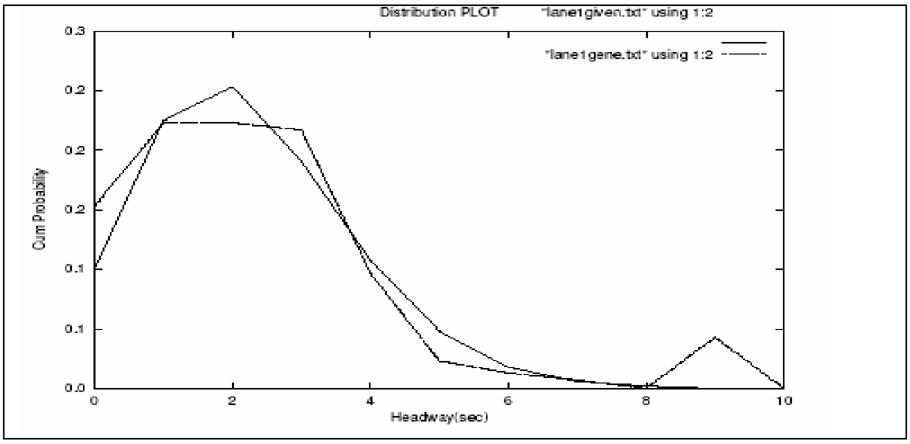

Figure 13. Observed and Generated Vehicle Headway Distribution

6. Conclusion

A cellular automata based microscopic traffic flow model for heterogeneous traffic is proposed in this paper. The key features of this model are the grid representation of the road stretch and the explicit treatment of individual vehicle characteristics. The model is tested for both homogeneous and heterogeneous traffic. The basic relations of the traffic flow parameters like flow, density, and speed obtained are close to the theoretical and empirical relations under homogeneous traffic conditions. The model has tested for a small road stretch having heterogeneous traffic and found that a probability of 0.18 give satisfactory result. In addition, the maximum flow, speed, and the fundamental diagrams for various vehicular compositions can be derived easily by assuming static conditions (no randomization rule). The study demonstrated the suitability of CA model for both homogeneous and heterogeneous traffic. However, the model needs further validation for real traffic of longer duration and long stretch of road. The multi-lane and heterogeneous traffic behaviours like lane changing, lane discipline, overtaking, incident management, etc need to be incorporated.

References

Arasan, V.T., and Koshy, R.Z., 2002. Modelling heterogeneous traffic flow on urban roads, in Transportation Systems and Directions:Proceedings of the national conference on transportation systems, Phonix publishing house Pvt.Ltd, New Delhi, pp.329-342.

Bhuvenesh singh., 1999. Simulation and Animation of Heterogeneous traffic on urban roads, Ph. D. Thesis, IIT, Kanpur.

Chalapati, M.V., 1987. Simulation of multi-lane unidirectional traffic, Ph.D. Thesis unpublished, I.I.T.Kanpur.

Chowdhury, D., Santen, L., Schadschneider, A., 2000. Statistical physics of vehicular traffic and some related systems, Physics Reports 329.

Chowdhury, D., Wolf, D.E., Schreckenberg, M., 1997. Particle hopping models for two-lane traffic with two kinds of vehicles: Effects of lane changing rules. Journal of Physica A, 235, 417-439.

Creamer, M., and J. Ludwig., 1986. A fast simulation model for traffic flow on the basis of Boolean operations. Mathematical and Computers in Simulation, 28, 297-303.

Emmerich, H. Rank, E. 1997. An improved cellular automaton model for traffic flow simulation, Journal of Physica A , 234, 676-686.

Freund, J., Poschel, T., 1995. A statical approach to vehicular traffic, Journal of Physica A , 219, 95-113.

Fukui, M., Nishinari, K., Takahashi, D., Ishibashu, Y., 2002., Metastable flows in a two lane traffic model equivalent to extended burger cellular automaton, Journal of Physica A , 303, 226-238.

Khan, S.I., and Maini, P., 2000. Modelling heterogeneous traffic flow, Journal of Transportaion Research Record., 1678, 234-241.

Knospe, W., Santen, L., Schadschneider, A., Schreckenberg, M., 1999. Disorder effects in cellular automata for two-lane traffic, Journal of Physica A, 265, 614-633

Kumar,V., and Rao., S.1996. Simulation modelling of traffic operations on two-lane highways, Highway research Bulletin, No.54, Indian Road Congress, highway Research board , pp.211-237.

Marwah, B., and Ramaseshan, S., 1978. Interaction between vehicles in mixed traffic flow using simulation, Indian Roads Congress, Highway research Bulletin, 8, 1-15.

Marwah, B., and Bandyopadhyay., S.,1983. Development of a traffic simulation model for an Indian city street, Indian Roads Congress Journal , 463, 655-695.

Nagel, K., and Schreckenberg, M., 1995. Traffic jam dynamics in stochastic cellular automata, Technical Report LA-UR 95-2132, Los Alamos National Laboratory, University of California.

Nagel, K., and Schreckenberg, M.,1992. Cellular automata model for freeway traffic, Journal of phsique, I, 2-22.

Nagel, K., Wolf, D.E., Wagner, P., Simon, P., 1998. Two-lane traffic rule for cellular automata: A systematic approach, Physical Review E, 58, 1425-1437.

Palaniswamy, S.P., 1983. A generalized simulation model for vehicular behaviour under heterogeneous traffic conditions, Simulation research project, IIT, Kanpur.

Pillai, K., 1975. Behavior studies on the simulation model for a zebra crossing, zebra, Indian Roads Congress, Highway Research Bulletin, 1, 1-12.

Pillai, K.S., 1974. Simulation techniques to study road traffic behaviour, Indian Highways, 2, 6-17.

Ramamayya, T.V., 1988 . Highway capacity under mixed traffic conditions, Traffic Engineering and Control , pp.284-287.

Rickert, K., Nagel, M., Schreckenberg, M., Latour, A., 1996. Two lane traffic simulations using cellular automata, Physica A 231, 534.

Rickert, M., and Wagner, P., 1996. Parallel real-time implementation of large-scale, route-plan-driven traffic simulation, Int. J. Mod. Phys. C, 7:133.

Roy Sudip Kumar., 2000. Effectiveness of Auto rickshaw as a fixed route public transport mode - case study Calcutta metropolitan area, Ph. D. Thesis, IIT, Kharagpur.

Schadschneider., A., 2000. Statistical physics of traffic flow, Journal of Physica A, 285,101-120.

Schreckenberg, M., Neubert, L. Wahle, J., 2001. Simulation of traffic in large road networks, Future Generation Computer Systems , 17, 649-657.

Von Neumann, J., 1959. Theory of automata: Construction, reproduction, homogeneity. In von Neuman, J., Theory of Self-Reproducing Automata ed. and completed by Arthur W. Burks, U. of Illinois Press, Urbana, Ill., Pt. Two, 91-3801

Wagner, P., and Nagel, K., Wolf, D.,1997 . Realistic multi-lane traffic rules for cellular automata, Physica A 234, 687.

Wolf, D.E., 1999. Cellular automata for traffic simulations, Journal of Physica A, 263, 438-451.