MODELING OVERFLOW QUEUES ON URBAN SIGNALIZED INTERSECTIONS

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Abstract
Much effort has been spent in developing more efficient control systems for signalized intersections to adapt the capacity of the network to the variability of the demand. This variability is partly due to time-dependent factors but also to the stochastic nature of the demand itself. The available formulae can still be successfully adopted in undersaturated conditions, where the stochastic effects fade into one green phase, or in highly oversaturated cases, where the queues aren't comparable with their portions caused by these uncertainties. In cases where the value of the degree of saturation is close to one the stochastic effect plays a relevant role. This paper provides a novel heuristic formula able to model the transition phase when the study involves a variable demand. It particularly solves the queuing behavior of an oversaturated period on the successive undersaturated ones. The delay experienced by the users is modeled using a Markov Chain process. Based on this data the novel model is calibrated. The new model for queues is further applied in two case studies. The first involves a two arms intersection where users of the two OD pairs can only choose between different times of departure. The second involves one OD pair and two parallel routes, one faster but ending with a traffic light while the other is unsignalised. The paper discusses the change in users' decisions with respect to the travel times and the delays they perceive at signalized intersections both from the route choice and the departure time choice point of view. In this sense a Dynamic Traffic Assignment problem in an extended network can be applied and solved. From this research we gained a deeper knowledge of the relevant role played by the random nature of the queue evolution in time for the delay experienced by the users. By acquiring this knowledge a time-dependent queue function has been provided. It is now possible to model the queue as a continuous function with more accurate results than the ones provided by the simple deterministic method.

Keywords: Traffic control; Dynamic traffic assignment; Time-dependent queues
Topic Area: C3 Traffic Control

1. Introduction
Up to now traffic control has been extensively studied in the static context. Available optimization formulae like the Webster optimal cycle formula (Webster, 1958) are derived from steady state equations. During congestion periods the dynamics of the transportation system become relevant and often the system may not reach a steady state condition within these periods. To investigate the optimization problem during congested periods we need to evaluate the effects on the network characteristics in time. Microscopic models are best suited for analyzing the propagation effects of congestion and for evaluating delays in time. However, its low computing speed limits impede this technique to be used in an

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optimization context characterized by long iterative processes. Furthermore microsimulation models require several runs before giving an accurate estimate of the costs, since the stochastic components of the system produce variable costs.

This research aims at building up a complete framework of a dynamic optimal control policy using a macroscopic model. We will focus our research on queuing models instead of delay models for two reasons: first modeling queues permits easily to determine delays while from delays to queues the process isn't trivial. Second knowing the queue size we are able to solve problems like spillback effects, accumulation lane design etc. To do so, the research has focused on the consistency of the queuing models in time. When in oversaturated conditions the degree of saturation, considered invariant within a sub-period of the whole evaluation period, reduces itself from one sub-period to another the queue starts reducing as well. The available formulae model this phenomenon still using the linear deterministic formula (Catling, 1977).

To generate valid data we introduce a mesoscopic simulation technique based on the Markov Chain probability theory. This model is able to simulate the dynamics of the queue length by updating the probability distribution cycle by cycle. Such a model makes it possible to analyze in much more detail the dynamics of the queue and its variance, without the need of repeated simulations. Analyzing the data we will show the strong relationship that links the dynamic of the queue and the standard deviation of its probability distribution. If the standard deviation is small enough with respect to the mean the queue will have a deterministic behavior while if the standard deviation becomes comparable with the mean then the mean will follow an exponential evolution in time.

With the introduction of the Markov Chain technique we provide a faster generation of data than with microsimulation. In this way we are able to analyze the validity of the available formulae and to detect and quantify the errors committed when applying these formulae.

Among others, these limits preclude the utilization of the available analytic models to assignment and optimization problems:

- Time dependent queuing models are valid only if the mean flow rate is constant for the whole evaluation period;
- Only an initial queue equal to zero is admitted;
- The models don't cover the estimation of decreasing queues, occurring when the initial queue is larger than the equilibrium one;
- The models are usually suited for certain time steps, typically 15 minutes.

To assess the importance of the new model we compare first the results with the available models in a test scenario with time-varying flows. Later on we apply the novel model to a realistic scenario showing the differences that we obtain in the case of an assignment problem involving both route choice and departure time choice.

2. State of the art

Traffic control is firstly designed for reducing conflicts between flows but it can be a powerful tool, together with other DTM measures, for also adapting the network capacity according to the amount of flows. Moreover changes in the signal settings reduce or increase delays and consequently increase or reduce the quality of a route. If the settings are also modified in time then these changes in the user's preferences propagate in time as well. When evaluating the performance of an urban area delay at signalized intersections represents the most commonly used component for the users' costs.

Delay is usually defined as the difference between the travel time experienced by a vehicle passing an intersection and the hypothetical travel time experienced if the intersection wasn't controlled. According to the Highway Capacity Manual (HCM 2000), if the intersection is undersaturated (the degree of saturation is low), delay has a deterministic and a stochastic component.

The more the degree of saturation increases the more likely the fluctuations of the arrival rate may produce “cycle failures”, this means that some vehicles may not be served within one green phase producing an overflow queue for the starting of the next red phase. The presence of this initial queue produces an additional delay introduced in the Highway Capacity Manual only in his latest version (HCM 2000). The HCM 2000 specifies then three terms for calculating the delay:

$$d = d_1 \cdot PF + d_2 + d_3 \tag{1}$$

where
d = average delay per vehicle (s/veh)
d_1 = uniform delay (s/veh)
d_2 = random delay (s/veh)
d_3 = initial queue delay (s/veh)
PF = adjustment factor for including non-uniform arrivals

The term d_3 depends on the assumed initial value. A good estimation of the delay is then implied by a good estimation of the queue. The manual doesn’t provide an accurate model concerning the evolution of the queue but suggests the use of the linear deterministic model. When the value of the degree of saturation floats around 1 the initial queue delay is enormously larger than the uniform and the random components. It’s straightforward understanding that only a good estimation of the overflow queue can lead to a good estimation of the initial queue delay. This reasoning leads to the focalization of the research on modeling queues instead of delays, looking for a systematic, easy-to-apply method to evaluate the behavior of the queue in a general context.

Queuing theory can be used to evaluate the delay the travelers incur but can also be helpful for evaluating other characteristics not valuable with the delay itself. Spillback effects and the length of an accumulation lane cannot be modeled without the estimation of the queue length.

Several authors have formulated expressions that describe the dynamics of a queue from an initial value that is smaller or larger than the equilibrium value. As for the delay models, queuing models can be deterministic or stochastic. Deterministic queuing models are typically used when the number of vehicles that can be served during a green interval is greater than the number of arrivals per cycle. In this case the fluctuations generated by the randomness of the arrivals are mostly covered by the available green time. Deterministic models are also in good agreement with reality when the intersection is highly oversaturated. In this case the component of delay generated by the random fluctuations is negligible with respect to the uniform component.

If the intersection is slightly under or oversaturated only stochastic models can help at estimating delays and queues. By assuming a statistical distribution the stochastic models calculate the delay incurred by the users for the variability of the degree of saturation. If the number of the arrivals is, on the average, less than the number that can depart during the green the intersection is undersaturated, but due to stochastic variations in the number of arrivals, there is a finite probability that still one or more cars will have to stop twice because the green phase could not handle the whole queue. The queue that evolves in this case is defined as the *overflow queue*.

The formulae proposed under steady state conditions require an infinite time period of stable traffic conditions to be achieved. In this ideal condition if the degree of saturation is exactly one the expected equilibrium queue is infinite. In reality flows are stationary for a limited period of time and it is more relevant estimate the finite queue at the end of the evaluation period more than the hypothetical one reached after an infinitely long evaluation period. If the degree of saturation is low the equilibrium queue is reached after few cycles and steady state formulae can be still valid. If the degree of saturation is nearly one it may take too long to reach the equilibrium and the knowledge of the behavior of the queue during this transition phase becomes relevant.

For the situation that initially the queue is zero, Kimber and Hollis (Kimber and Hollis, 1979) developed a suitable heuristic analytic model for queue length and delay using the coordinate transformation technique. Their model has been applied in e.g. the TRANSYT program. Akcelik formulated the expression that is most frequently used by practitioners. He provided a continuous formulation of the queue evolution using the coordinate transformation technique:

$$Q(T) = \frac{cT}{4}\left(x - 1 + \sqrt{(x-1)^2 + \frac{12 \cdot (x - x_0)}{cT}} \right) \quad (2)$$

where

T is the time step (expressed in number of cycles);

Q is the number of vehicles in queue at the starting of a green phase;

$c = S.g / C$ is the capacity of the arm, equal to the saturation flow S multiplied by the green ratio g / C;

x is the degree of saturation, ratio between the flow and the capacity;

$x_0 = 0,67 + \frac{S \cdot g}{600}$ is the limit value of the degree of saturation above which the stochastic effects are relevant.

This formula is probably the most used model in case of flows approaching capacity and initial queue Q_0=0. On the other side it fails in all other cases were the queue starts from a non-zero initial value limiting its applicability. A recent extension of the former model for including the effect of an initial queue can be found in the aaSIDRA manual (Akcelik, 2002). The authors add to the flow rate q a term that takes the residual queue generated in the previous intervals into account. The model uniformly distributes the extra flow over the calculation period adding Q_0/T where Q_0 is the expected residual queue from the previous time step and T is the time step length. A more detailed description of existing models can be found in Rouphail et al. (Rouphail et al., 2000)

The case of decreasing queues hasn't more accurate analytical formulation than the one provided by Catling (Catling, 1977) who models the evolution as a linear function that follows at the starting the deterministic decrease represented by the linear function:

$$Q = Q_0 + (x-1)cT \quad (3)$$

until it reaches the equilibrium value that depends on the degree of saturation and not on the initial value (Miller, 1968). The aaSIDRA program uses this function to integrate the Akcelik's function.

The authors have already attempted to associate a heuristic formula to the decreasing part of the queue in two previous papers (van Zuylen and Viti, 2003, Viti and van Zuylen,

2004) but still the formula was valid only in cases where the flow rate was constant and with standard deviation equal either to the mean or zero.

3. The Markov Chain process model

Several traffic models have been provided in the past to estimate the costs the users incur when experiencing a trip. Both optimization problems and assignment problems require the evaluation and comparison of different scenarios in order to choose respectively the best performing and the most realistic ones.

There are three techniques to simulate the possible scenarios and to calculate the performance of management strategies: macroscopic, mesoscopic and microscopic simulation. Macroscopic models provide aggregate results: deal with flows as a continuum entity. Microscopic models try to model the vehicles as separated. Characteristics that distinguish one vehicle from another are usually easier modeled with the latter. Mesoscopic models use probability distributions for the microscopic states. The choice of a simulation model defines the level of accuracy by which the costs experienced by the users are computed. A microsimulation model can associate a realistic cost to each vehicle. A macroscopic model cannot often detect the fluctuations due to the random nature of the demand and the supply characteristics.

The evaluation and optimization of DTM measures requires information of the reaction of road users. Since the reaction depends, among others, on the perceived utility, a realistic estimate of perceived utility is necessary. An optimization procedure requires the computation of a relevant number of scenarios. From this point of view only macroscopic models are suitable, since the microscopic ones require long computation times. Mesoscopic models can represent a valid trade-off but the research is not evolving at the moment on this path.

Microscopic and mesoscopic models can also be used to find approximate expressions for macroscopic models. Aggregating the results of the former models in a macroscopic level we can compare the results obtained by using the random characteristics with the ones computed by the macroscopic model and associate to the difference a heuristic formulation. In this research we implemented a model based on the Markov Chain process to generate data valid for calibrating and validating the heuristic model. Some authors used already this technique to generate queue lengths (Brilon and Wu, 1990, Olsewski, 1990, Fu, 2000).

Olszewski (Olszewski, 1990) studied the queue length dynamics with a model in which the probability distribution of the queue length is calculated from cycle to cycle:

$$P(n,j) = \int_{-\infty}^{+\infty} ds' \frac{1}{\sigma\sqrt{2\pi}} e^{-(s'-s)^2/2\sigma^2} \sum_{l=0}^{j+s't_g} p_l P(n-1, j-l+s't_g) \qquad (4)$$

with

$P(n.j)$ = probability of a queue of j vehicles at the end of the nth green phase

σ = the variance of the saturation flow and

p_l = probability of l arrivals in the cycle.

Microscopic simulation can be an alternative to a Markov model for the probability distribution. It is rather unlikely that anything will come out of micro-simulation models that significantly differs from the Markov model since most microscopic simulation models use the same assumptions about the arrivals and departures of cars as the Markov model does. Olzewski has implemented the Markov model in a specific situation. He was interested in his research to show the behavior of the queue when it started from a non-zero initial value. In his research he considered the starting standard deviation equal to zero and flow rate constant for the whole evaluation period.

In this research we went further by extending the Markov model to a more realistic case. By considering a variable flow rate we implicitly removed also the hypothesis of zero standard deviation, clearly unrealistic especially for non-zero initial queues.

Our scope is to generate simulated data for assignment and optimization processes. For sake of simplicity we used only fixed time periods where the flow is considered constant in its average according to the common procedure in an assignment process. Figure 1 shows an example of how a variable flow rate modifies the queue behavior in time and how the standard deviation evolves accordingly.

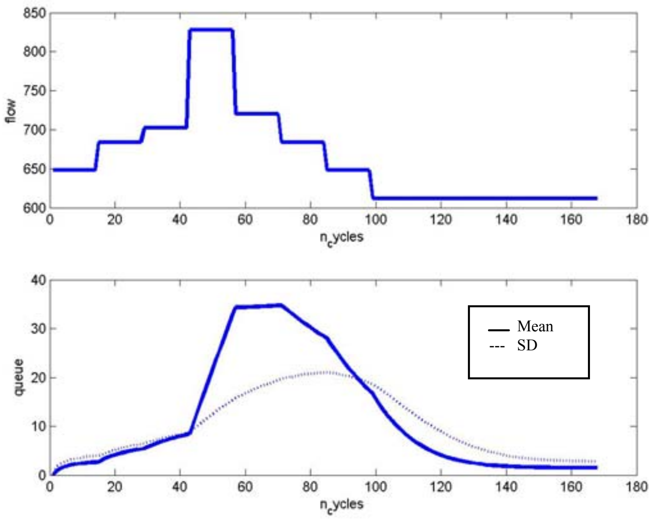

Figure 1: An example of overflow queue evolution and its standard deviation with variable demand

4. Proposed model

4.1. Influence of the variance-to-mean ratio

When queues at signalized intersection evolve in time, a relevant role in their behavior is due to the entity of the standard deviation with respect to the mean value of the queue. When the queue has a non-zero initial value coming from a previous oversaturated period, in the next periods the green interval is likely to be completely used for clearing the demand represented by these vehicles already in queue and the ones arriving during the same green interval. For high values of this initial value, since we assume that the capacity, and consequently the departures, is a deterministic function, there is a low chance that part of the green phase isn't used. The queue will then follow a deterministic behavior.

As long as the standard deviation becomes comparable with the mean queue there is a relevant chance that within one cycle the oversaturated queue and the arrivals are completely served and that there is even part of the green phase unused. In Figure 2 the mean value of the queue is displayed (bold curve) together with its value with added and subtracted the standard deviation (dashed curves). We notice that in the first cycles, since the standard deviation assumes a small value with respect to the mean one, there is a spare chance that the queue will completely disappear. This means that in the next cycle there will be still an overflow queue to be cleared. The arrivals, even though stochastic, will influence these cycles only with its mean and the result will be a linear evolution for a certain number of cycles.

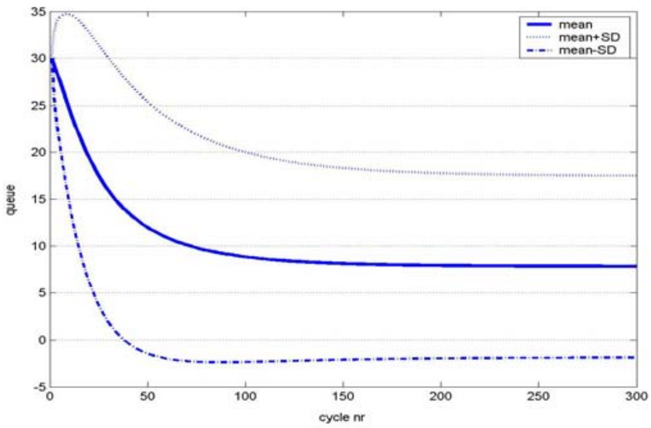

Figure 2: Evolution of the mean queue and confidence interval

After few cycles (around 40 in the picture) the probability that a zero overflow queue remains becomes non-negligible. In Figure 3a the probability distribution of the overflow queue is given for a queue starting from an initial value of 14 vehicles and degree of saturation of 0.95. The distribution is referred to the evolution after 5 cycles. The distribution, as also Newell (Newell, 1971) noticed is normal for all the values larger than zero but the probability of the queue being zero starts increasing.

It's straightforward to understand that the mean starts to be slightly larger than the mode and this difference increases as long as the time passes and the probability for zero increases. When the standard deviation is in its value comparable with the mean and the probability of being zero assumes a value of around 25% of the cumulative distribution the mean queue starts not changing its value in time and the steady state equilibrium holds. Figure 3b shows the distribution of the queue probability for the same example but after 50 cycles. This distribution doesn't change anymore and the mean remains around 7.8, stating the validity of the Miller (Miller, 1968) steady-state queue function.

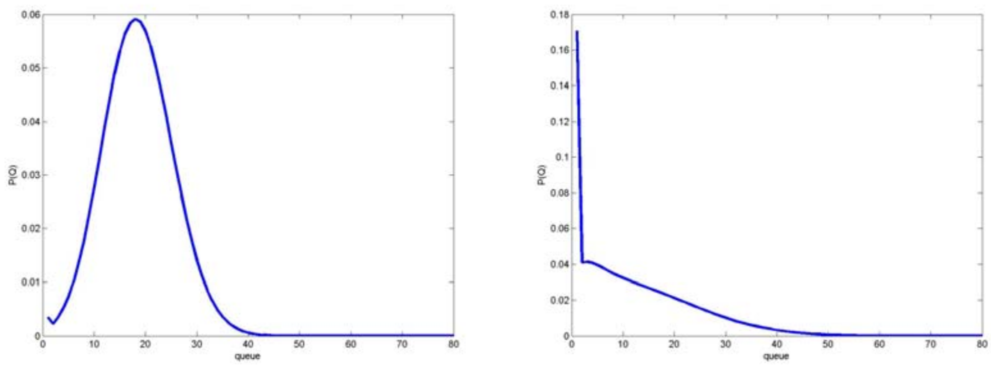

Figure 3a,b: Probability distribution of the queue after 5 and 50 cycles

The transition phase is then starting to evolve when the probability of being zero starts increasing and the trend gradually passes from the linear function to the exponential function introduced. In order to properly model this transition part we have to analyze the influence of the standard deviation. In van Zuylen and Viti (2003) and Viti and van Zuylen

(2004) the authors modeled the queue starting from an initial queue that can have a value of the standard deviation equal to zero or to the value of the mean.

If we suppose to divide a peak period in *n* sub-periods in which the flows arriving at the intersection are assumed constant in the mean and distributed as poissonian it may happen that within these periods the standard deviation is not even able to reach a value comparable with the mean. In Figures 4a, b, c we simulated a period of 30 minutes and we supposed this period to be divisible in two sub-periods. The mean degrees of saturation are respectively 0.95, 1 and 1.2 in the first and 0.90 in the second. The Markov Chain process model has been extended in order to capture the probability distribution at the end of one sub-period being the probability distribution for the initial queue at the starting of the following sub-period.

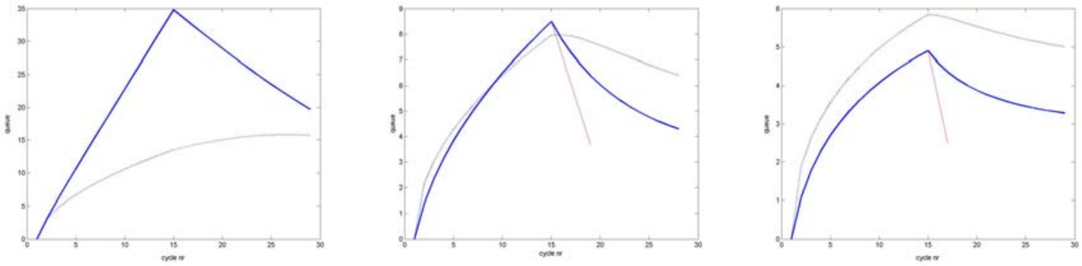

Figure 4a,b,c: Evolution of the queue with x=1.2,1,0.95 in the first 15min and x=0.9 in the next 15min

As shown, the behavior of the queue in the second period depends on the value of the standard deviation with respect to the mean. In the first case the variance is around 30% of the mean and the initial trend is clearly linear. The following two pictures show that the more the variance-to-mean ratio increases the less the evolution follows the linear trend. The various differences highlighted suggest to modify the approximate expression introduced in van Zuylen and Viti (2003) and Viti and van Zuylen (2004). We need to understand how the initial variance-to-mean ratio can be included in the analytical function such that if its value is low enough the evolution will initially follow the deterministic function while when this value is close to the unity or larger the function won't follow this trend but the transition phase will be different.

There should be also a relevant role played by the time interval chosen for the sub-periods. Obviously it will influence in the previous period the evolution of the variance-to-mean ratio. We will investigate on this path in future researches. In this way we are able to model in a very realistic way the queuing process within the hypotheses assumed. This novel model will have the following features that don't characterize the available formulae:

- It is valid for variable demand; it is then possible to be used for assignment processes and optimization problems;
- Models the queue also when the initial value isn't zero.

4.2.Extension to variable demand using the variance-to-mean ratio

The mean of the queue at the end of a sub-period and its standard deviation influence strongly the behavior of the queue in the next period, especially at the first cycles. The system tends, whenever possible, toward the equilibrium value both for the mean and for the standard deviation starting from an instable point represented by the initial queue and it's straightforward understanding that different standard deviation for the initial point lead to different evolutions.

We immediately notice that in Figure 4b and 4c the standard deviation assumes a value always comparable or higher than the mean value. This causes in the second sub-period a behavior that doesn't present any linear part. The reason is that in the first sub-period the queue has built up with a non-zero mean but the chance to have cases when the overflow queue isn't present is still high.

According to Figure 4a in the first sub-period the variance-to-mean ratio decreases in time. The more the mean increases, the less likely the queue can be zero since the standard deviation is considerably lower than the mean. This causes the behavior being deterministic. At the starting of the second sub-period even with the reduced flow ratio the chance the queue being zero is small enough to be neglected. The probability distribution of the queue at each cycle is then normally distributed and symmetric and the queue evolution is for the first cycles linear. As long as the standard deviation assumes a value comparable with the mean the evolution starts following the exponential behavior.

The results above shown are referred to sub-periods of 15 minutes. If we assume a different length, the variance-to-mean ratio at the end of each period will change. We will not stress this hypothesis at this point but we will focus our interests only on the influence of the variance-to-mean ratio with fixed sub-period intervals.

As introduced in van Zuylen and Viti (2003) and in Viti and van Zuylen (2004) the queue evolution in time for the decreasing part can be modeled with the following simple expression:

1. Initial linear decrease

$$Q = Q_0 - (x-1) \cdot c\, T \tag{5}$$

2. Transition between linear and exponential decrease, which can be approximated as:

$$Q = \alpha(T)\,(Q_0 - (x-1) \cdot c\, T) + (1-\alpha(T))\,[Q_0 + Q_e\,(1-\exp(-\beta T))] \tag{6}$$

3. Exponential approach to the equilibrium value

$$Q = Q_0 + Q_e\,(1-\exp(-\beta T)) \tag{7}$$

The parameter β, as shown in van Zuylen and Viti (2003) and in Viti and van Zuylen (2004), doesn't depend on the initial conditions and obviously neither on the variance-to-mean ratio so it's value is as stated:

$$\beta = \frac{(1-x)^2}{0.15} \tag{8}$$

The weight function $\alpha(T)$ is a continuous function that balances the linear deterministic behavior and the exponential function only in the transitional phase, assuming the value respectively 1 and 0 for the first phase and the third phase. The continuity of the function is assured by the continuity of the three functions and from its initial and final values. The function is then, even if divided into three regimes, continuous with continuous derivatives in all its points.

Two parameters, one representing the position and one the extension of the transition phase specify the logistic function. These parameters will depend on the variance-to-mean ratio. If the initial queue is zero the model can be extended by simply the following expression:

$$Q = Q_e \cdot (1\text{-}\exp(\text{-}\beta T)^2) \tag{9}$$

while the expression of the decreasing part can be applied also when the starting value is smaller than the equilibrium value.

Analyzing the data we notice that the transition phase tends to start when the variance-to-mean ratio assumes a value of approximately 0.30. Using this information we can always understand if and when the transition phase occurs and also depending on the degree of saturation how long it lasts. From the same analysis we notice that the queue is well represented by the exponential behavior when the variance-to-mean ratio is nearly the unity. If then at the end of a period we calculate a standard deviation that is in its value comparable with the mean we have to expect that in the following period the queue will have only the exponential evolution. If the value is in between 0.30 and 1 we will have in the first cycles a transition phase while if the initial value is smaller than 0.30 then we have to include in the behavior an initial linear part that follows the deterministic trend.

A physical explanation of this behavior can be attempted by considering Figures 3 and 4. The more the time passes the more the distribution of the queue assumes a shape well represented for its positive values by a normal distribution. This assertion is also confirmed in Newell (Newell, 1971).

If the standard deviation has a value smaller than 30% of the mean the probability that the queue at the end of the cycle is zero is nearly zero but surely negligible. This means that at the end of the cycle we have to expect in general a deterministic behavior. If the standard deviation is larger than 30% than the probability that the queue will be zero increases and the starts being larger than the mode, as it can be deduced by the above Figures. In this process the standard deviation attempts to reach the equilibrium as well as the mean. The exponential distribution is then the representation of the queue attempting to reach the equilibrium when both the functions have reached the equilibrium among each other. Viti and van Zuylen (2004) provide an expression of the standard deviation in the case of decreasing queues:

1. Initial square root behavior:

$$Q = \sqrt{c \cdot x \cdot T + Q_0 + 0.01 \cdot Q_0^{\ 2}} \tag{10}$$

2. Transition phase:

$$Q = (1\text{-}\alpha_{SD}(T))\sqrt{c \cdot x \cdot T + Q_0 + 0.01 \cdot Q_0^{\ 2}} + \alpha_{SD}(T)\ [Q_0^{SD} + Q_e^{SD}\exp(\text{-}\beta T)] \tag{11}$$

3. Exponential approach to the equilibrium value

$$Q = Q_0^{SD} + Q_e^{SD}\exp(\text{-}\beta T) \tag{12}$$

The formula is still applicable for increasing queues starting from zero (only the square root behavior will be present) and for variable demand.

5. Case studies

In this section we show how the application of the proposed formula leads to a user equilibrium different than the one that is found from the Highway Capacity Manual delay function. Firstly we assign on a test scenario a vector of flows representing a peak period. We assigned exactly the same vector of flows drawn in Figure 1 and we compare the results with the ones obtained from the Markov model. Secondly we apply the formula to the simple examples drawn in Figure 5, focusing only on the choice made by the users passing from west to east. We solved a simple Dynamic Traffic Assignment problem formulated as a Nash game where the users have both the chance to change their route and their time of departure. For reference on the assumed model see Viti et al. (2003).

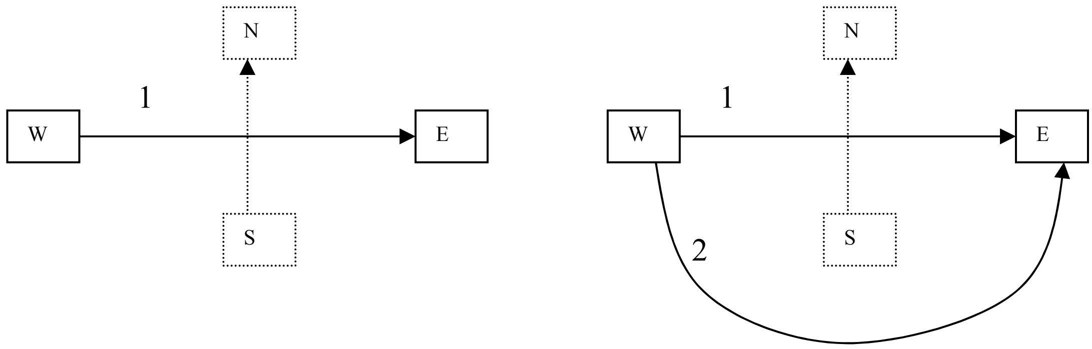

Figure 5: Case studies

In the first example we will obtain a difference only in terms of departure time choice since no alternative route is available. In the second we will obtain also route splits. Figure 6 compares the Markov model result redrawn from Figure 1 (crossed line) with the one obtained from the proposed method (bold) and the ones obtained applying the linear deterministic function (asterisks). In this example the Highway Capacity Manual does not calculate the queue behavior in the first cycles since it considers an overflow queue only when the degree of saturation is larger than 1. It also stops considering an overflow queue when the linear function decreases to zero.

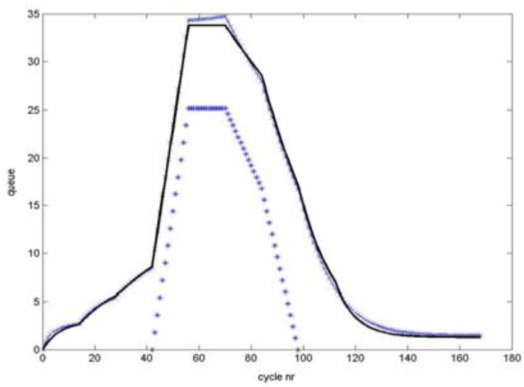

Figure 6: Example of queue evolution for variable demand

The model follows the Markov model with an estimation error in this example of maximum 1 vehicle while the error made applying the linear function is enormous. We now apply the queuing function in the delay function provided by the HCM 2000 by substituting the linear function with the proposed one. To evaluate the user equilibrium in the case studies drawn in Figure 5 we divided the whole evaluation period (4 hours) in sub-periods of 15 minutes each. The first scenario involves only departure time choice; this means that the users will switch their time of departure in order to maximize their individual utility. The second scenario includes for the W-E demand an alternative route representing a bypass. The utility function is represented by the following formula introduced by Arnott et al.(1990):

$$c_{rs}^{h}(t) = \alpha \cdot (t - PDT_{rs}) + \beta \cdot tt_{rs}^{h}(k) + \gamma \cdot SD_{rs} \tag{13}$$

where $c_{rs}^{h}(t)$ is the cost the user of direction rs perceives when choosing to depart at time t on route h, $SD_{rs} = t + tt_{rs}^{h} - PAT_{rs}$ is the scheduled delay, PDT_{rs} and PAT_{rs} are respectively his/her preferred departure time and his/her preferred arrival time, $tt_{rs}^{h}(t)$ is the travel time spent for traveling on path h and departing at time t and α , β , γ are weights.

For the path including the intersection the travel time function is represented by a free-flow travel time and a delay at the intersection. For the bypass we used the *BPR* function (TRB, 2000):

$$tt_{rs}^{bypass} = fftt_{rs}^{bypass} + \left(\frac{f_{rs}^{bypass}(t)}{s} \right)^{\delta} \tag{14}$$

where $fftt_{rs}^{bypass}$ is the free-flow travel time, $f_{rs}^{bypass}(t)$ is the flow departing at time t, s is the saturation flow and δ is a weight.

In the case studies we used the following values: the total demand is 8500 vehicles for the first example and 12000 for the second, *PDT* and *PAT* are respectively 8:00 and 8:30 in the morning, the free flow travel time for route 1 is 15 minutes while for route 2 is 30 minutes, s=1800 veh/h, g=24sec, r=60sec, α , β , γ are 0.5, 4 and 2 and δ=3. Figure 7 shows the different results in terms of degree of saturation obtained by applying the HCM 2000 or the proposed formula in the first scenario. To solve the problem we applied a Stochastic User Equilibrium assignment with MSA-FA convergence criterion.

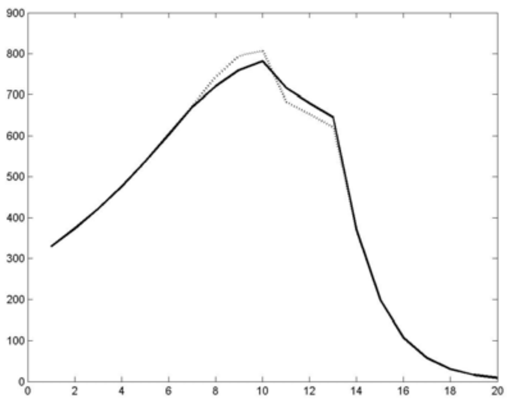

Figure 7: Equilibrium flow pattern applying the HCM 2000 and the proposed formula in case study 1

The differences in terms of degree of saturation are small. We notice that applying the proposed formula part of the flow accumulated in the central periods tends to be distributed in earlier periods. This difference gives enormous differences in terms of queue and delay. Figure 8a and b show the queue evolution according to the two user's equilibrium.

In the first Figure the queues are comparable with each other and with the correspondent Markov Chain model. In the second picture we notice that if we calculated the queue with the deterministic model we would have obtained a queue 10 vehicles less than the Markov model. Moreover the maximum queue at equilibrium is around 15 vehicles less than the one calculated with the HCM 2000. The proposed formula mimics the Markov Chain model more accurately in both the examples.

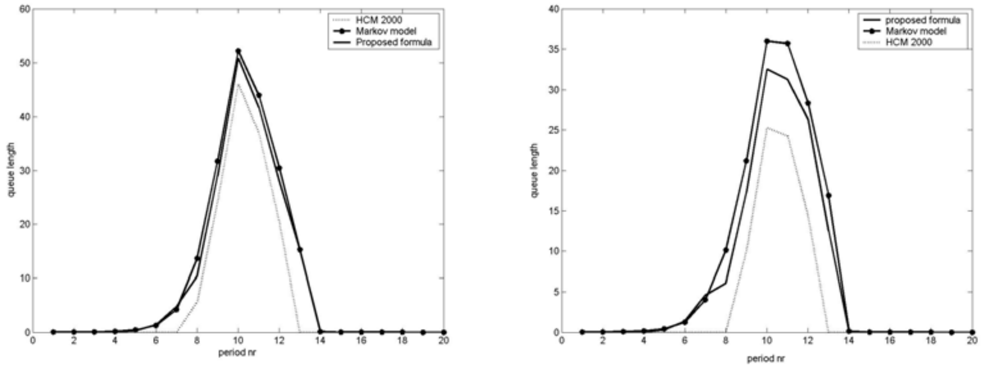

Figure 8a,b: Queue evolution at user's equilibrium applying the HCM2000 and the proposed formula

The same conclusion holds when comparing the delay, drawn in Figure 9a and b. The delay calculated with the HCM 2000 is underestimated with respect to the Markov Chain counterpart and in the first scenario the Markov maximum delay is nearly 400 s/veh while in the proposed formula solution the Markov maximum delay is less than 300 s/veh.

Figures 10 and 11a and b show the results of the second scenario. Comparing the result of the two methods, using the proposed formula part of the flows accumulated in the central sub-periods choose both to change time of departure and route. Figures 11a and b show that the equilibrium calculated applying the HCM 2000 method gives, using the proposed formula a delay around 100 s/veh less than the one calculated in the equilibrium with the applied proposed formula. Moreover the HCM 2000 estimates the delay 50% less than the Markov model in both the examples.

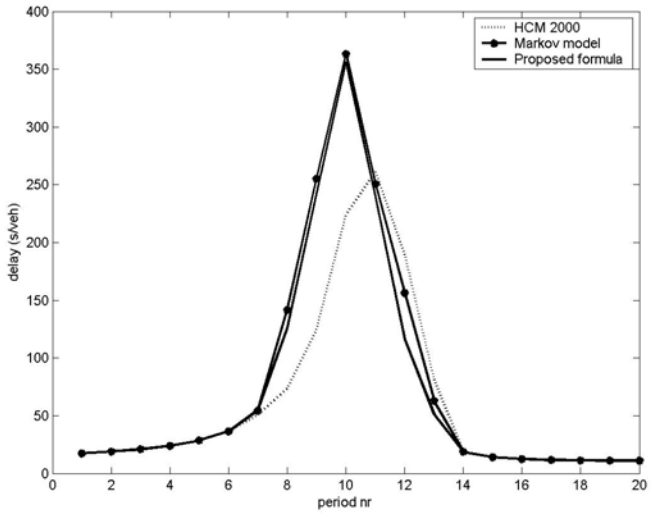

Figure 9a: Delay when user's equilibrium is calculated applying the HCM2000 compared with MC and the proposed formula

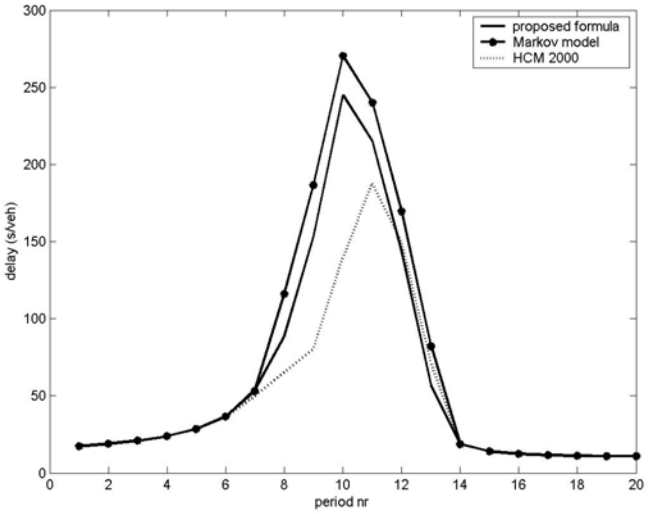

Figure 9b: Delay when user's equilibrium is calculated applying the proposed formula compared with the MC and the HCM 2000

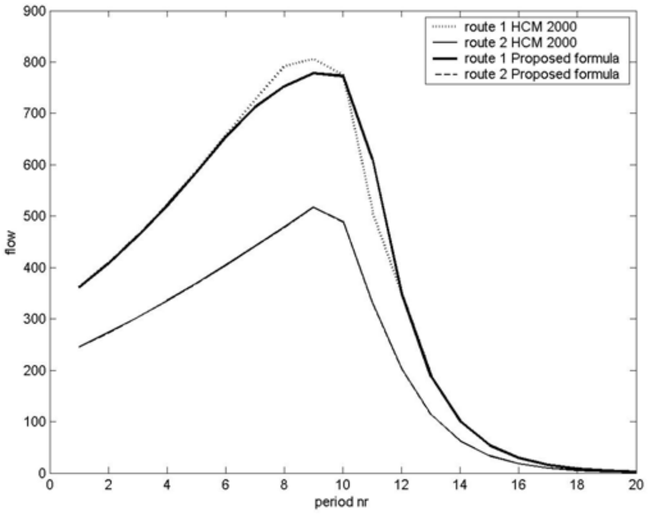

Figure 10: Flow pattern of the two alternative routes using the two methods

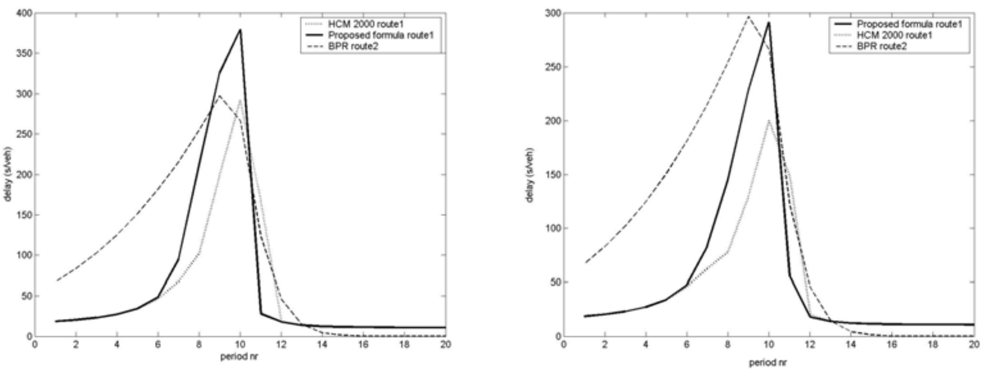

Figure 11a,b: Delay of the two alternative routes using the two methods

6. Conclusion and further research

In this paper we showed the possibility to use the Markov Chain process to generate data whose validity is comparable with the ones produced by a microsimulation program. Starting from previous works we implemented a Markov Chain process able to deal with variable demand.

Furthermore we provided heuristic, easy-to-use formulae that properly mimic the data both for the mean and for the standard deviation of the queue. We tested these formulae in a scenario where the peak hour flows have been represented by a step function and compared it with the Markov model and the deterministic model.

The formula improves highly the available analytical models, showing a better fit than the linear relationship when the degree of saturation floats around 1, giving the possibility to evaluate queues and delays in a dynamic scenario. Finally we showed a potential application of the model in an assignment process, showing that the function gives different results with respect to the ones computed using the Highway Capacity Manual delay formula.

Further research will attempt to improve the formula to include the cycle length and the sub-period lengths as parameters and to extend it to arterial networks. We also intend to show the results of an optimization problem and compare them with the available optimal green and cycle time formulae, study the effect of a responsive control and of signal coordination for evaluate both the efficiency and reliability of such measures with respect to the fixed, uncoordinated system.

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References

Akcelik, R., 1980. Time-Dependent Expressions for Delay, Stop Rate and Queue Length at Traffic Signals. Australian Road Research Board, Internal Report, AIR 367-1.

Akcelik, R., 2002. aaSIDRA Traffic Model Reference Guide. Akcelik & Associates Pty Ltd.

Arnott, R., de Palma, A. and Lindsey, R., 1990. Departure Time and Route Choice for the Morning Commute. Transp. Res. B, 24B (3) 209-228

Brilon, W., Wu, N., 1990. Delays at fixed time signals under time-dependent traffic conditions. Traffic Engineering and Control 31 (12) 623-631.

Catling, I., 1977. A Time Dependent Approach to Junction Delays. Traffic Engineering and Control, 18, 520-526.

Fu, L., Hellinga, B., 2000. Delay variability at signalized intersections. Submitted for presentation at the 79th Annual Meeting of the Transportation Research Board January 2004, Washington D.C. and for publication in Transportation Research Record.

Kimber and Hollis, 1979. Traffic queues and delay at road junctions. TRL LR 909.

Miller, A.S., 1968. The capacity of signalized intersections in Australia, Australian Research Board bulletin 3.

Newell, G.F., 1971. Applications of Queuing Theory. Monographs of Applied Probability and Statistics. Chapman and Hall, London.

Olzewski, P., 1990. Modeling of Queue Probability Distribution at Traffic Signals. Transportation and Traffic Theory, ed. M.Koshi, Elsevier, Amsterdam, pp. 569-588.

Robertson, D,I., 1969. TRANSYT: A Traffic Network Study Tool. Road Research Laboratory Report LR 253, Crowthorne.

Rouphail, Tarko, N.A., Li, J., 2000. Traffic flows at signalized intersections, Chap. 9 of the update of Transportation Research Board Special Report 165, “Traffic Flow Theory”, 1998.http://www-cta.ornl.gov/cta/research/trb/tft.html.

TRB, 2000. Highway Capacity Manual. Special Report No. 209, National Research Council, Washington D.C.

Van Zuylen, H.J., and Viti, F., 2003. Uncertainty and the Dynamics of Queues at Signalized Intersections. Proceedings CTS-IFAC conference 2003, 6-8 August, Tokyo. Elsevier, Amsterdam

Viti, F., and Van Zuylen, H.J., 2004. Modeling Queues At Signalized Intersections. Submitted for presentation at the 83rd Annual Meeting of the Transportation Research Board January 2004, Washington D.C. and for publication in Transportation Research Record.

Viti, F., Catalano, S.F., Li, M., Lindveld, C., Van Zuylen, H.J., 2003. An optimization problem with dynamic route-departure time choice and pricing. Submitted for presentation at the 82nd Annual Meeting of the Transportation Research Board January 2003, Washington D.C. and for publication in Transportation Research Record.

Webster, F.V, 1958. Traffic Signal Settings. Road Research Laboratory Technical Paper No. 39, HMSO, London.