INFLUENCE OF PROBE VEHICLE SAMPLE SIZE ON EXPRESSWAY TRAVEL SPEED ESTIMATION

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Abstract

In this paper, both real-world and simulation data of urban expressway networks are used to evaluate estimation performance of probe only and detector only travel speed estimation. A sampling statistics analysis shows that the accuracy of probe only estimates depends on two parameters: probe sample size *n* and traffic density *D* of corresponding expressway sections. A new concept form for MAPE (Mean Absolute Percentage Error), MAPE $(n, D) = a(D) + b/n$ is suggested, where *a* is a function of *D* and *b* a constant. Also, a study on detector only estimates shows that estimation MAPE is a function of *D* only. Based on the relative performance of probe and detector only travel speed estimates, a weighted average model is proposed to combine probe and detector only estimates. A case study shows the advantages of the proposed data fusion model and the future perspective of applying probe data in expressway travel speed estimation.

Keywords: Urban expressway network; Probe vehicle
Topic Area: E2 Performance Measurement

1. Introduction

One of the mostly used expressway travel speed data sources is vehicle detectors. A common characteristic of estimation methodologies based on detector data is that they almost always depend on spot-speed data. Good accuracy may be expected only when the assumption that the traffic condition in the section is either homogenous or a linear combination of two nearby points is met. Therefore it requires a high allocation density of vehicle detectors, which are expensive in both installation and maintenance.

Today new sources of traffic information have come into practical use, like probe vehicle technology. In 2002, the authors proposed a model to use a weighted average model to combine data from probe vehicle to improve the overall accuracy of vehicle detector based travel speed estimation (Nakamura and Wang, 2002). However, the combination methodology is only a naive average or substitution despite the fact that probe based travel speed estimation depends a lot on the sample size *n* of probe vehicles; and both probe and detector travel speed estimation accuracy varies a lot when traffic situation changes.

Probe vehicles can be regarded as a sample of traffic flow. The variance of sample means is an accuracy index of probe based estimation. Some earlier works (Sen, et al., 1997; Hellinga, et al., 2002) pointed out that this variance never goes into zero, even with 100% probe vehicle MPR (Market Penetration Rate), since the lack of dependence among probe data. Sen, et al (1997) also suggested out a form of the variance as $a + b/n$ where *a* and *b* were location specific constant parameters, and both positive.

In this paper, a detailed study on probe data in expressway travel speed estimation is carried on. Using sampling statistics of both simulation and real-world data, the important issue of the influence of probe sample size on travel estimation accuracy is investigated. The study partially supports the form suggested by Sen, et al. (1997) and further develop it into the form as *a(D)* + *b/n* where *D* represent the average traffic density of the study sections. On the basis of the findings, a model to fuse travel speed data independently estimated by vehicle detector data and probe data is developed. The advantages of the model are shown by a comparison study based on two month's data at 15 minute time interval.

2. Data and the network

Two sets of data are used in this paper to evaluate the performance of the proposed travel speed estimation methodologies and to investigate the influence of probe vehicle sample sizes on travel speed estimation accuracy.

From January 1 to March 31, 2002, a large scale Internet ITS Pilot Program was carried on in Nagoya city. During this filed test, probe data from 1,570 taxies had been collected around the entire city. The contents of the real-world probe data cover 29 types, including vehicle ID, realtime position, spot-speed, and so on.

Since the real world data are constrained to a low MPR only, it is the simulation data of the same network that is mainly used in this study. The simulation environment is AIMSUN2. The utilization of real world probe data is restricted for the validation of low MPR cases only.

A route of Nagoya Urban Expressway, as shown in Figure 1, is used in this study. Travel speeds of the study route are estimated under 4 different scenarios: 1) detector data only, and based on the original high detector allocation density, i.e. 453.3m (Det. High); 2) detector data only, and only the detectors on a middle section is used, i.e. a 2,720m allocation density (Det. Low); 3) probe data only (Probe); and 4) combination of detector and probe data (Comb.) by the proposed method. And Det. High is used as a benchmark through the study. An earlier study shows that Det. High on Nagoya Expressway may give a satisfied estimation accuracy (Wang and Nakamura, 2003a).

The estimation time interval used in this study is 15 minutes. For the two month (59 days) study period, travel speeds of the all together 5,664 time intervals are estimated by following the **above three scenarios.**

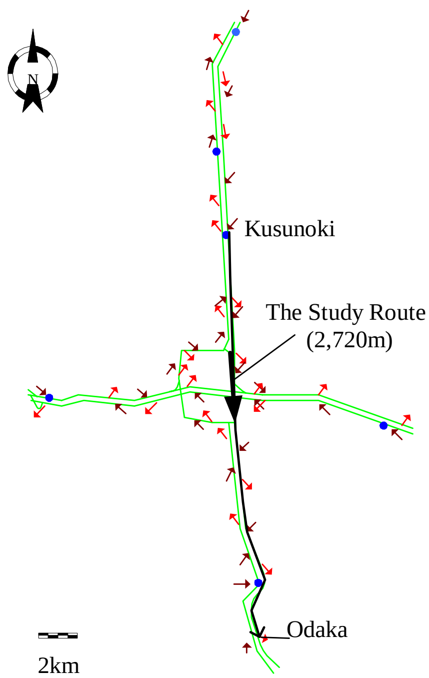


Figure 1. The Study Route (Nagoya Expressway)

3. Travel speed estimation methodologies

3.1 Methodology for detector data (V_{det})

The segment spot-speeds are obtained by the weighted average of 1-minute spot speed data by lane, while the weights are decided according to the corresponding traffic volumes. Segment travel speed V_{det} can be calculated by using the 15 minute mean value of the speeds to divide the length of the segment.

3.2 Methodologies for probe data (V_{probe})

According to the different types of data applied, two methods are suggested in this paper to use probe data in the estimation of section travel speed. Method (a) uses average probe travel speed which is estimated by applying the probe position and time data, and (b) the harmonic mean probe spot-speed. And after the mean speeds are estimated through one of the above two methods, probe travel speed V_{probe} can be calculated by dividing the length of the section by one of the two mean speeds, (a) or (b).

4. Accuracy of travel speed estimates and probe sample size

4.1. Theoretical analysis

4.1.1. An idealized situation

As for travel speed estimation involving probe vehicle data, it is in fact to use the mean travel speeds of n probe vehicles (or a sample of traffic flow) as an estimate of the mean travel speeds of the total N vehicles of the traffic flow (or the population) at a certain time interval. Let y represent the travel speed of a vehicle here to agree with the statistics tradition and for convenience. If probe vehicles can be regarded as a random and independent sample of the population, then the sample mean $\bar{y}$ is an unbiased estimator of the population mean.

$$\bar{y} = \frac{1}{n}(y_1 + \cdots + y_n) = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

Then the variance of the estimator $\bar{y}$ is

$$\operatorname{var}(\bar{y}) = (\frac{N-n}{N})\frac{\sigma^2}{n} = (1-\frac{n}{N})\frac{\sigma^2}{n} \tag{2}$$

where σ^2 is the variance of the population.

Since it is very often that the population N is very large compared with the sample size n, so the Equation (2) can be further simplified as,

$$\operatorname{var}(\bar{y}) = \sigma^2 / n \tag{3}$$

Equation (2) and (3) imply that the variance of sample means, or average probe travel speeds, will always decrease with the increase of probe sample size. And it will go to zero, in an ideal situation when n is equal to N, or 100% probe percentage has been achieved.

4.1.2. "Variance never go to zero"[2]

However, probe reports can hardly represent a random sample of the traffic stream in practical world. Sen, et al. (1997) concluded that the variance of probe means never went to zero after a thoroughly statistical examination of the field probe vehicle data.

The variance of the estimator $\bar{y}$ can be also calculated by using Equation (4),

$$
\begin{aligned}
\mathrm{var}(\bar{y}) &= E[\bar{y} - E(\bar{y})]^2 = E\left\{n^{-1}\sum_{i=1}^{n}[y_i - E(y_i)]\right\}^2 \\
&= n^{-2}E\left\{\sum_{i=1}^{n}[y_i - E(y_i)]^2 + \sum_{\substack{i,j\\ i\neq j}}[y_i - E(y_i)][y_j - E(y_j)]\right\} \\
&= n^{-2}\left[\sum_{i=1}^{n}\mathrm{var}(y_i) + \sum_{\substack{ij\\ i\neq j}}Cov(y_i, y_j)\right]
\end{aligned}
\tag{4}
$$

where *Cov*( ) is covariance. Let $\eta = n^{-1}\sum_{i=1}^{n}\mathrm{var}(y_i)$ be the average variance of y_i's and let

$$v = [n(n-1)]^{-1}\sum_{\substack{i,j\\ i\neq j}}Cov(y_i, y_j) \tag{5}$$

be the average covariance of all pairs (y_i, y_j). Then Equation (4) may be written as

$$\mathrm{var}(\bar{y}) = n^{-2}[n\eta + n(n-1)v] = \eta n^{-1} + (1-n^{-1})v = v + n^{-1}(\eta - v) \tag{6}$$

When the sample is random, Equation (3) and (4) are the same since the covariance is equal to zero. However, the observation of probe vehicles is not independent. The estimates of travel speeds in a time interval had a standard error that would never smaller than a positive number. Thus Sen. et al. (1997) proposed a simple concept form of the variance of the mean travel speeds obtained from n probes for a same link over a fixed time period (Sen, et al., 1997),

$$\mathrm{var}(\bar{y}) = a + b/n \tag{7}$$

where $a = v$ and $b = \eta - v$.

4.1.3. Variance may go to zero – the insights of this paper

When traffic becomes more congested, drivers' freedom to maneuver may be more and more restricted and starts to be "significantly limited" (TRB, 2001). In more scientific words, the variance of travel speeds of all vehicles within the traffic flow, i.e. $\mathrm{var}(y_i)$, decreases as the traffic turns out to be more congested and so does the covariance $Cov(y_i, y_j)$. This can be also shown in the third step of Equation (4). At an extreme situation when all vehicles stop, $\mathrm{var}(y_i)$ and $Cov(y_i, y_j)$ will be equal to zero, and then the variance of probe means *May Go to Zero*.

Many researchers have noticed the phenomenon that the probe based estimation accuracy could be improved under more congested situations (Sen, et al., 1997; Srinivasan and Jovanis, 1996), given a same probe vehicle MPR. However, a typical explanation for it was often like that; "under high-congestion levels, because volumes would be higher, even low deployment rates (of probe vehicles) would usually achieve reasonable probe frequencies" (Sen, et al., 1997).

The explanation is correct in some perspectives, but it can not reflect the phenomenon under some really serious congestion levels, the traffic volumes would in fact be lower; still even better estimation accuracy can be expected from probe vehicles at a very same deployment rate.

Based on the earlier analysis, a new and more general form is suggested in this paper to reflect the relationship between probe estimation variance and probe vehicle sample size, under various traffic conditions. The form is shown in Equation (8),

$$\mathrm{var}(\bar{y}) = a(D) + b/n \tag{8}$$

where D represents the average traffic density of the study link.

Since $\mathrm{var}(\bar{y})$ is also an measure of estimation accuracy, the estimation accuracy index MAPE (mean absolute percentage error) can be assumed to follow a similar tendency of $\mathrm{var}(\bar{y})$. It can be calculated by Equation (9),

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - f_i}{f_i}\right| \tag{9}$$

where y_i is the i^{th} estimated travel value; n is the number of estimates; f_i is the *true value* which is the average travel speed of the population for simulation data, or the value estimated via original vehicle detectors for the real world data.

A comparison of the three forms of MAPE: <u>under idealized condition</u>, <u>by Sen et al.</u>, and <u>by the proposed form in this paper</u> is demonstrated in Figure 2.

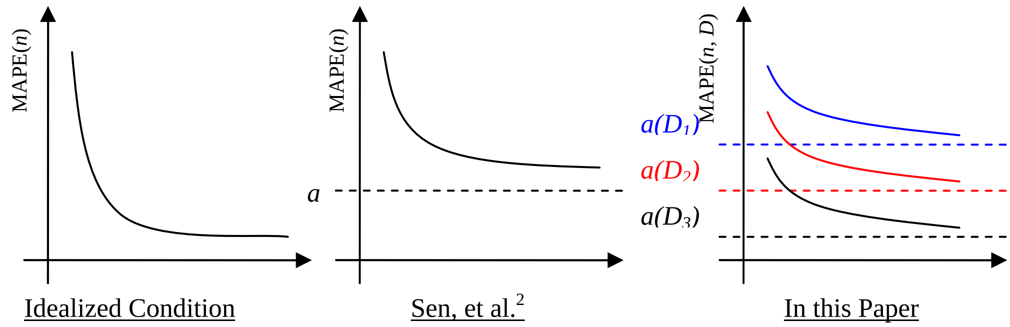


Figure 2: Comparison among Different Forms for MAPE

4.2. Empirical analysis

4.2.1 Results of the simulation analysis

Using the simulated data, travel speeds of two hour morning peak are estimated at a time interval of 5 minutes. The simulated traffic situation has covered all the 6 Levels of Service (LOS), A through F. The simulated probe vehicle MPRs include 0.5%, 1%, 3%, 5%, 10% and 20%.

Figure 3 shows the relationship between the estimation MAPE and the probe vehicle MPRs for different traffic flow LOS. Here the LOS is defined, according to the latest Highway Capacity Manual (TRB, 2001), on the basis of the average density *D* (pc/km/ln) of the simulated expressways. That is LOS A (<5), B (5-10), C (10-15), D (15-20), E (20-25) and F (>25).

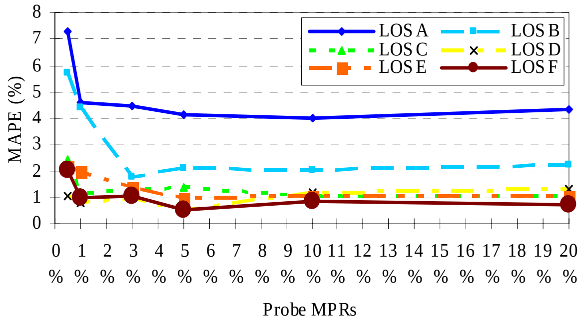


Figure 3: MAPE for Different LOSs and Probe MPRs

At each LOS, the relationship between estimation accuracy MAPE and probe vehicle MPRs agree with the Equation (7). However, this figure definitely shows that *a* is a function of D, here represented by different LOS. The proposed Equation (8) is thus justified.

4.2.2 Results of the real world data analysis

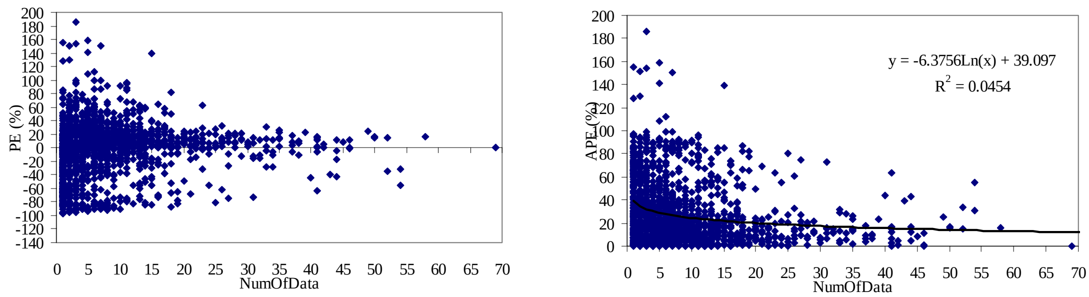
Within the two month study period (total 59 days), there are altogether 5,560 time intervals (each 15 minutes). Probe data are available for only 2,360 intervals among them. Although the scarcity of the real world probe data reports and traversals significantly restricts the utilization of them in this study, travel speeds of all the 2,360 time intervals with probe traversals are also calculated with the two estimation methods described in section 3.2.

In this paper, PE (Percentage Error) and APE (Absolute PE) are calculated by using Equation (10) to show the error characteristics of probe travel speed estimation.

$$PE = (y_i - f_i) / f_i, \qquad APE = |PE| \tag{10}$$

where f_i is the i^{th} *true* value by the high density detector data (Det. High); y_i is the i^{th} estimated value (Det. Low, Probe, or Comb.).

Figure 4 gives the probe only travel speed estimation PE/APE versus number of probe



(a) PE (b) APE

Figure 4. Probe Travel Speed Estimates APE versus Number of Probe Data

data (within one time interval).The result shows that Equation (7) in a common situation is proved to be valid. Probe-based estimation accuracy may be significantly improved by the increase of probe sample size, but it will come to a steady value, the *a* in Equation (7), when the sample size has become large enough.

Figure 5 shows that Probe estimation accuracy does not improve as the traffic density increases. This is different from what was found in the authors' earlier simulation study (Wang and Nakamura, 2003b). It may because that in this study the average probe sample size under free flow conditions (12) is much higher than it is under congested condition (only 6), as shown in Figure 6. Here sample size is defined as average number of probe reports per 1 time interval. The positive influence from more congested condition is complemented by the negative influence of a decreasing average sample size.

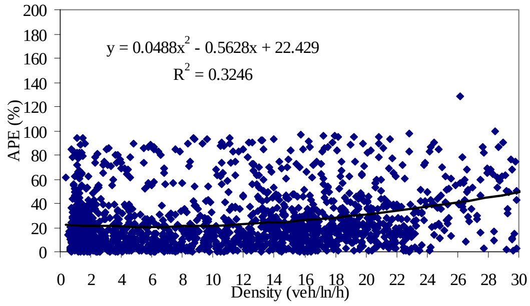


Figure 5. Probe Travel Speed Estimates APE versus Vehicle Density

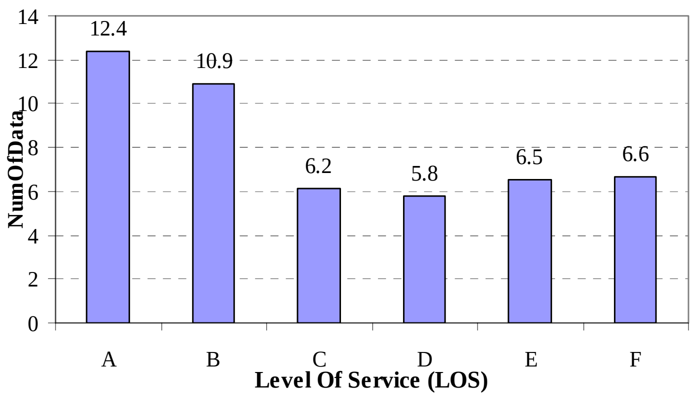
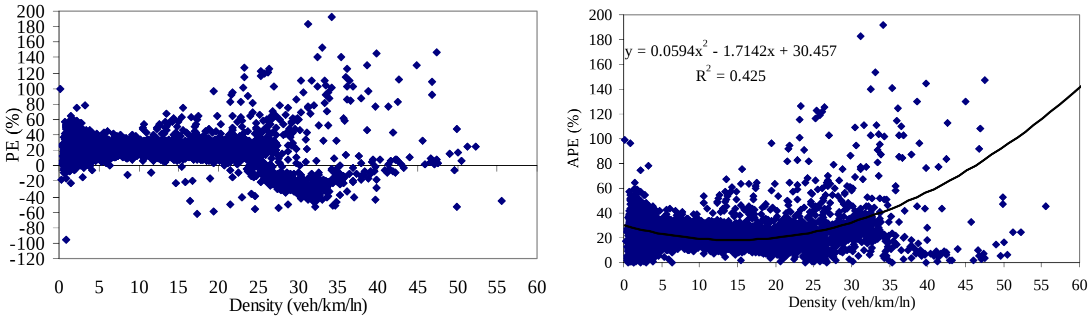


Figure 6. Average Number of Data (per Time Interval) Corresponding to LOS



(a) PE (b) APE

Figure 7. Det. Low Travel Speed Estimates PE/APE versus Vehicle Density

5. Data combination methodologies

Figure 7 give the Det. Low travel speed estimates PE/APE versus average vehicle density D (veh/km/ln) of the study route. Here D is calculated by Equation (11):

$$D = Occ/(\bar{l} + C) \tag{11}$$

where $\bar{l}$ = average vehicle length, Occ is time occupancy, and C is the detecting length of vehicle detectors.

Figure 7(a) shows significant and systematic estimation bias of using Det. Low only data only. It reminds us the fact that vehicle detector is only a kind of point detection devices. When its allocation density is not high enough, no satisfied results can be expected. As shown in Figure 7(b), the estimation APE tends to become more significant when traffic flow becomes more congested, and this agrees with the earlier simulation studies by the authors (Wang and Nakamura, 2003b).

A data combination method is developed on the basis of the earlier analysis. Here we assume, at a certain time interval, V_i as a travel speed of expressway section i, $V_{\mathrm{det}i}$ as the section travel speed estimated by Det. Low, and $V_{\mathrm{probe}i}$ the section travel speed estimated by probe data only. A common way to estimate V_i by combining both detector and probe data is to use the weighted average of $V_{\mathrm{det}i}$ and $V_{\mathrm{probe}i}$, as the following formula:

$$V_i = [1.2 - w(n)]V_{\mathrm{det}i} + w(n)V_{\mathrm{probe}i} \qquad w(n) = \begin{cases} 0 & if\ \ n = 1; \\ 0.5 & if\ \ n = 2; \\ 1.0 & if\ \ n >= 3. \end{cases} \tag{12}$$

where n is the probe reports number, *w(n)* is weight. Its maxim value is bigger than 1 to help adjusting the positive bias of $V_{\mathrm{det}i}$.

6. Comparison study

In order to evaluate the efficiency of the proposed data combination method a comparison study is carried out. Travel speeds of the network are estimated by using ultrasonic vehicle detector data only and with two different allocation densities. The first density is the original high allocation density (Det. High) and its results are used as a benchmark in this comparison study. This density (Det. High) has been proved to be dense enough for correct travel speed estimation by an earlier study of the authors (Wang and Nakamura, 2003a). The travel speeds estimated via a low allocation density (Det. Low) detector data are to be combined with probe travel speed estimates applying Equation (12) and also two simple data combination methods, arithmetic "mean" and "substitution" methods proposed by the authors in an earlier paper (Nakamura and Wang, 2002).

Originally there are 16 vehicle detectors installed on 6 detector segments of the expressway section as shown in Figure 1, and it is also used as the Det. High scenario in this comparison study. Data from the 3 vehicle detectors installed on the middle segment only is used to estimate travel speeds this section too, which is the Det. Low scenario as shown in Table 1.

Table 1. Two Detector Allocation Densities

	Number of Sections	Number of Detectors	Mean Section Length	Total Length
Det.High	6	16	453m	2,720m
Det.Low	1	3	2,720m	

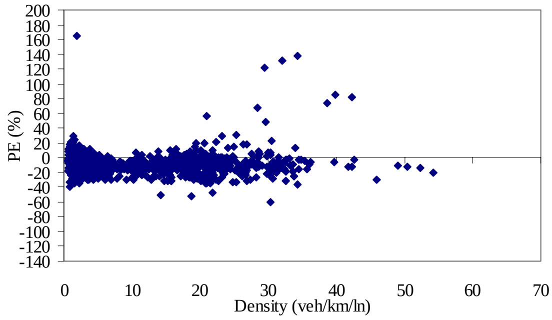


Figure 8. Probe Travel Speed Estimates PE versus Vehicle Density

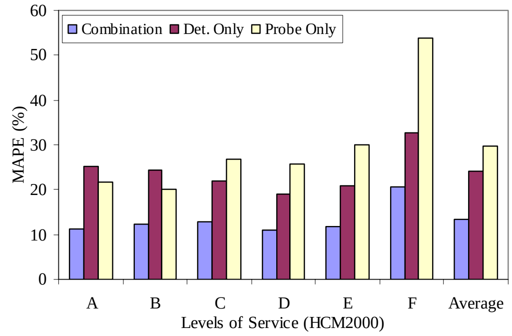


Figure 9. Comparison of Estimation MAPE Under Different Traffic Conditions

As shown in Figure 8, bias can hardly be found for travel speeds estimated by the combination of probe and detector data. This improvement in estimation bias helps to improve the estimation accuracy significantly, as shown in Figure 9. The MAPE (Mean Absolute Percentage Error) can be reduced 80.0 % if compared with detector only estimation and even 122.2% improved if compared with unsmoothed probe only estimation.

7. Conclusions

In this paper, a study on the issue of probe-based travel speed estimation is carried on. Using sampling statistics on the basis of both simulated and real-world data, the important topic of the influence of probe vehicle sample sizes on travel speed estimation accuracy is also carefully investigated.

Through an assessment of the relationship between accuracy of travel speed estimates and sample size of probe vehicles, it is found in this paper that the estimation accuracy is only convergent to a constant value, given a certain traffic density. As traffic density increases, the estimation accuracy will improve as the speed variances within the population decrease. The so called “constant” is in fact a function of traffic density. An empirical equation is given to describe the phenomenon.

After the accuracy comparison probe and detector travel speed estimates and under different traffic conditions, it is found that detector based travel speed estimates are very steady under un-congested flow conditions. However, this accuracy will be deteriorated as congestion develops. One inherent disadvantage of vehicle detectors is that they are capable of only point detection. It is shown in this paper that detector based travel speed estimation may also shows significant bias when the detector allocation density is too low. On the contrary, the estimation errors with probe based estimates are almost random even when the probe sample size is very low.

Based on these findings, a model fusing travel speed data separately estimated by vehicle detector and probe data is developed. In the model, probe and detector travel speed estimates are assigned different weights according to their relative performance under different traffic conditions and probe vehicle sample sizes.

A comparison study is also carried on to evaluate to proposed data combination method. It is shown that this method can significantly improve the travel speed estimates of low allocation density detectors by a combination of limit probe travel speed estimates. And obviously the proposed method outperforms the two simple combination methods, arithmetic mean and substitution. The proposed data combination model can reduce the estimation MAPE as much as 80.0-122.2%, if compared with the two naïve methods.

The results of the paper are especially useful to large cities of developing countries, where the coexistence of primitive and most advanced technologies is quite common.

Taking advantage of cutting-edged technologies to help skip some unnecessary stages experienced by developed countries is always welcomed.

Acknowledgments

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