AN ALGORITHM TO MEDIAN SHORTEST PATH PROBLEM (MSPP) IN THE DESIGN OF URBAN TRANSPORTATION NETWORKS

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Abstract
This paper proposes an efficient solution algorithm for realistic multiobjective median shortest path problems in the design of urban transportation networks. The proposed problem formulation and solution algorithm to median shortest path problem is based on three realistic objectives viz. route cost or investment cost, overall travel time of the entire network and total toll revenue. The proposed solution approach to the problem is based on the heuristic labeling and exhaustive search technique in criteria space and solution space of the algorithm respectively. The first labels each node in terms of route cost and deletes cyclic and infeasible paths in criteria space imposing cyclic break and route cost constraint respectively. The latter deletes dominated paths in terms of objectives vector in solution space in order to identify a set of Pareto optimal paths. The approach, thus, proposes a noninferior solution set of Pareto optimal paths based on nondominated objective vector and leaves the ultimate decision to decision-makers for purpose-specific final decision during applications. A numerical experiment is conducted to test the proposed algorithm using artificial transportation network. Sensitivity analyses have shown that the proposed algorithm is advantageous and efficient over existing algorithms to find a set of Pareto optimal paths to median shortest paths problems.

Keywords: Median shortest path problem; Pareto optimal paths; Feasible paths; Dominated paths
Topic area: C6 Network Design, Optimal Routing and Scheduling

1. Introduction

The better design of transportation networks in recent years has initiated the need to consider simultaneously several objectives in the analysis and solutions of shortest path problems. Transportation networks are huge investment infrastructure facilities whose design requires careful planning of the networks considering multiple and realistic objectives. The single criterion model is not enough to take into account many relevant aspects of reality and thus, multiobjective models seem to be more realistic. Steenbrink (1974) has claimed that "it is impossible to define a reasonable objective for transportation network optimization problem in which all relevant factors are included completely and consistently". Transportation network design problems are often combinatorial optimization problem and solving them with respect to more than one objective adds another dimension of difficulty. The conflicting natures of attributes make the problem formulation and algorithm even more difficult. The problems are compounded by the inability to generate efficient and adequate algorithms. Inclusion of multiple and conflicting vector of objective functions introduces another concept of multicriteria decision making and the concept of single optimal solution is no longer applicable. According to Coutinho-Rodrigues et al. (1999), there is no single optimal path in multiobjective path problem but rather, a set of nondominated solutions from which the decision maker select the most satisfactory. Current et al. (1987) have said that the most fruitful use of multiobjective programming lies in its implementation as a method to generate and analyze options rather than as a method to determine the most desirable option directly. From this set of Pareto optimal paths (POP), decision-makers upon specific applications make the final decision of the optimal path. Common multiple objectives to be considered for MSP in designing transportation network include route cost or construction cost (RC), overall travel time of the entire network (OTTEN), toll revenue (TR), social costs, environment, aesthetics, hazardous, regional equity, political goal of user satisfaction and so on.

However, generating and presenting the whole set of options to a decision maker, in general, is not effective way because those options may be very large. Generating a set of more feasible options rather than all feasible options with efficient solution algorithm is more adequate to overcome those drawbacks. In this context, it is necessary to develop an efficient algorithm for finding a compromise solution of shortest path (i.e. Median shortest path abbreviated as MSP) that can be used for designing real transportation networks. Here, the name MSP is given to emphasize a compromise solution among different objectives rather than using only the shortest path.

Diversities of existing journals and articles dealing with multiobjective design of networks are from the field of operation research, economics, logistics, management science and industrial engineering, which are not sufficient to deal realistic transportation network problems. In the past few decades there has been a growth of research to incorporate multiple objectives to shortest path problems in transportation networks (Current et al., 1985, 1987, Martins, 1999). A brief summary of these types of problems can be found in Current and Min (1986). Hansen's (1980) 10 bicriterion path problems are based upon total path length and individual arc length. The conflicting objectives like OTTEN and TR are not considered in the formulation. Martins (1984) presented multiple labeling approach to generate non-inferior solutions set based on some linear combination of path cost parameters. The solution does not include conflicting objectives like OTTEN and TR obtained from the path.

Current et al. (1987) proposed a bicriterion median shortest path problem (MSPP) considering path cost and access cost as two conflicting objectives. This algorithm is good enough to understand the concept of conflicting objectives and obtain a compromise solution. The same problem is solved by Park and Nepal (2002) using heuristic vector labeling approach. However, in both works, the access cost is defined as the cost required for the total node demand to reach to the nearest node on the MSP. The objective is not realistic for designing real transportation network because all node demands of the entire network do not approach to the MSP. It depends on the distance from their node to the path and moreover, only those demands that can decrease their travel times from origin to destination by the use of MSP approach to the path and others do not. Thus, OD matrix is the essential input in order to determine the reduction in the travel times of all demands of all nodes to reach their destinations. Thus, OTTEN serves the reality in the transportation networks, which is the most important objective or aim of urban transportation planner to achieve. The purpose of improving path or providing expressway or providing improved network services is to reduce the OTTEN. Furthermore, Current et al. (1987) has solved by integer programming formulation using double sweep method of k-paths generation and exhaustive search for minimum access cost to these paths in order to obtain POPs. The algorithm is inefficient in the sense that generation of the entire noninferior solution set from k-path generated in the solution space of multiobjective shortest path problem is computationally intense. The solution approach by Park and Nepal (2002) seems to be more efficient than Current et al.'s (1987) with respect to computational time and memory space in computer. However, the considered objectives are not realistic and useful for transportation networks. Thus, defining a realistic objectives and proposing an efficient solution algorithm to MSPP for transportation networks is an area of current research, which is the subject treated in this paper. MSPP discussed in this paper is the special type of one-to-one node problem that deals with the efficient solution of three conflicting objectives viz. RC, OTTEN and TR. The RC attribute consists of the sum of all arc costs (construction cost or improvement cost of MSP), OTTEN attribute is the travel time of all demands of all nodes to reach their destinations and TR attribute is the revenue that can be obtained from the demand served by the MSP.

The objective of this research work is to develop an efficient algorithm to MSPP by defining realistic objectives for transportation networks and conduct sensitivity analysis. The proposed algorithm is based on the combined heuristic labeling and exhaustive search technique. The basic notion of the proposed algorithm is taken from Aneja et al. (1979), Current et al. (1987), Park et al. (2001) and Nepal (2002). The cyclic and infeasible paths are deleted for each path of each node based on the cyclic break (CB) and route cost constant (RCC) and a number of acyclic feasible paths are generated from O to D. The exhaustive search technique based on the objective vector (RC, OTTEN, TR) is employed on these acyclic feasible paths in order to delete all dominated paths and obtain a set of POPs. The concept of RCC is introduced in order to delete the far infeasible paths (far infeasible paths are those paths that have the RC attribute significantly greater than the shortest route from origin to destination).

This paper first introduces the notations used in this paper and formulates the problem considering above-mentioned objectives. Secondly, the proposed algorithm is discussed in detail along with an example network. The application of the proposed algorithm and analysis of the results are done using Current et al.'s (1987) transportation network and hypothetical OD demand. Lastly, summary and concluding remarks are followed.

2. Notations

O	Origin;
D	Destination;
k	Index for sub-path number from O to each node;
M	Maximum allowed route cost of Pareto optimal paths (percent with respect to the shortest route cost);
α	Maximum allowable route cost of Pareto optimal paths (route cost constraint);
L	Set of feasible sub-paths, which are temporarily labeled;
E	Set of feasible sub-paths, which are permanently labeled;
i^k	k^{th} sub-path from O to node i;
$e(i)$	Number of feasible sub-paths from O to node i;
RC_O^i	Route cost of sub-path from O to node i;
RC_j^D	Route cost of shortest path from node j to D;
RC_O^D	Route cost of the shortest path from O to D;
r_{ij}	Route cost of the link (i, j);
t_{ij}	Travel time on the existing link (i, j);

t'_{ij}	Travel time on the link (i, j) on MSP;
Z	The objective vector;
Z_1	Total route cost (RC) of the MSP;
Z_2	Overall travel time of the entire network (OTTEN);
Z_3	Total toll revenue (TR) obtained from demand satisfied by MSP;
N	Number of nodes in the network;
A	Number of links in the network;
Q	Node sets on MSP;
$\lvert Q \rvert$	Cardinality of the node set Q;
$D_{ij}^{o,d}$	Demand through the link (i, j) from (o, d) pair of (O, D);
δ_{ij}	Toll of link (i, j);
P	Set of OD pair.

3. Modeling assumptions

Prior to the formulation of MSPP based on objectives considered, the following assumptions are made. These assumptions are not uncommon to model the transportation network in the static and deterministic network environment even though some of them are too simplified. These assumptions can be released using appropriate algorithms in some parts of the whole algorithm based on the application site.

. Demand exists only at nodes and must be satisfied;
. The MSP must pass through the nodes;
. OTTEN is measured in dynamic measure of the demand-time unit (vehicle-time, passenger-hr, passenger-min, passenger-sec etc.) considering sum of link demand multiplied by link travel times in all links in the entire network;
. Total TR is calculated by summing all link demand multiplied by link toll imposed on each links on MSP;
. Free flow speed is assumed. There is no congestion effect. The driver always chooses the shortest travel time route from origin to her destination;
. Link costs are non-negative;
. The link attributes and OD demands are deterministic and static.

4. Mathematical formulations

Let $G(N,A)$ be a transportation network with nodes N and links A. Let us denote α as a maximum allowable RC of POPs with respect to the shortest route form O to D. Then, the problem of identifying MSP is the minimization of objective vector as follows:

$$\textbf{V-minimize} \quad Z = [Z_1, Z_2, Z_3] \qquad \textbf{(1)}$$

Subject to,

$$\sum_{(i,j)\in A} r_{ij} X_{ij} \le \alpha \qquad \textbf{(2)}$$

$$\sum X_{ij} - \sum X_{ji} = \begin{cases} 1 & for \quad i = O \\ 0 & for \quad i = N - \{O, D\} \\ -1 & for \quad i = D \end{cases} \qquad (3)$$

$$\sum_{(i,j)\in A} X_{ij} = Q - 1 \quad (\forall \text{ arcs } (i, j) \in A) \qquad (4)$$

where,

$$Z_1 = \sum_{(i,j)\in A} r_{ij} \quad X_{ij} \qquad (5)$$

$$Z_2 = \sum_{(o,d)\in P} \sum_{(i,j)\in A} t'_{ij} D^{o,d}_{ij} X_{ij} + \sum_{(o,d)\in P} \sum_{(i,j)\in A} t_{ij} D^{o,d}_{ij} (1 - X_{ij}) \qquad (6)$$

$$Z_3 = -\sum_{(o,d)\in P} \sum_{(i,j)\in A} \delta_{ij} D^{o,d}_{ij} X_{ij} \qquad (7)$$

$$X_{ij} = \begin{Bmatrix} 1, \text{ if arc } (i, j) \text{ is an arc on median shortest path} \\ 0, \text{ if otherwise} \end{Bmatrix} \forall \text{ arcs } (i, j) \in A$$

Constraint (2) ensures that a path is not a feasible path if its RC is greater than RCC. Constraint set (3) ensures that origin and destination nodes should lie on MSP and if an arc on MSP enters to an intermediate node, one arc on the path should leave that node as well. The constraint (4) removes all cyclic paths.

Optimizing (1) for all objectives is impossible due to their conflicting natures i.e. objectives Z_1, Z_2 and Z_3 cannot be minimized simultaneously. To solve (1), heuristic labeling algorithm (Dijkstra, 1959) is used to generate all acyclic feasible paths constrained by CB and RCC. In order to check the feasibility of the RC, the shortest RC routes between all pairs of nodes are calculated using traditional label correcting shortest path algorithm (Moore, 1957). The exhaustive search technique is used to delete all dominated

paths based on the vector of optimizing objectives to all these acyclic feasible paths. Assume that the travel time on the links of the MSP after upgrading it to the MSP is not equal to the travel time of the links of the existing network before upgrading it to MSP due to improved transport service conditions. This assumption is not uncommon because the purpose of proving expressway or toll way or making improvements on the existing transportation services is to reduce links travel times. This difference in travel times on MSP will change the link demand on all links in the entire network. Considering updated value of travel times on the links of MSP, the shortest travel time routes between all pairs of nodes are calculated using level correcting shortest path algorithm (Moore, 1957). Then, the link demands for all links of the entire networks are calculated using appropriate traffic assignment algorithm. OTTEN and total TR are calculated using the equations (6) and (7) respectively.

5. Proposed algorithm

The proposed algorithm is based on the combined heuristic labeling and exhaustive search technique. It consists of mainly five steps. In first step, the shortest RC routes between all pairs of nodes are calculated using traditional label correcting shortest path algorithm (Moore, 1957). In the second step, RCC (α) is determined based on the shortest route from origin to destination. The definition of terminology; cyclic break and route cost constraints are as follows:

Cyclic break (CB): CB removes cyclic paths in the network. It has significant importance in order to delete looping paths at criteria space producing less number of paths to expand during node expansion. The repeating nodes or arcs of the same path from origin are not common in general practice and driver, generally, does not want to return to the same path /node twice during her journey to destination.

Route cost constraint (α): RCC is defined as the maximum allowable RC of POPs. The route cost ratio (RCR) is defined as the ratio of RCC and the RC of the shortest route. The RCC is expressed as the RC of the feasible path, which is to some extent greater than the shortest route as follows;

$$\alpha = RC_O^D \times (1 + M\,(in\ decimals)) \tag{8}$$

For example, let us consider the path that has RC less or equal to 20% (M = 20%)more than the RC of the shortest route as feasible paths. Then, the RCC as per the expression above is 1.2 R_O^D and RCR is 1.20. The constraint is meaningful and has logic in the sense that MSPs having RCR very far from 1.00 are infeasible because our interest is to find the set of optimal paths, which have the RC almost similar to the shortest route from O to D as far as possible. The RCC directly removes all lingering paths from probable solution and drastically removes infeasible paths at criteria space, making small searching domain.

In step 3, labeling algorithm is applied similar to label setting shortest path algorithm (Dijkstra, 1959) in traditional shortest path problem. There may be multiple sub-paths for each node. The label of each path to each node is set based on the RC of each sub-path. Each sub-path at each node is then checked to ensure that the path is acyclic and RCC is satisfied. The sub-path, which does not satisfy these two conditions, is deleted. The mathematical expression to infeasible path is shown below;

$$RC_O^i + RC_i^D > \alpha \tag{9}$$

Step 3 ends when all acyclic and feasible paths from origin and destination are obtained. In step 4, first generated path in step 3 is taken and travel times of the links on MSP are updated. The shortest travel time routes between all pairs of nodes considering updated values of the travel time on the links in MSP are then calculated using level correcting shortest path algorithm (Moore, 1957). The link demand to each link of the entire network is calculated using appropriate traffic assignment algorithm. OTTEN is calculated using links demands and links travel times. Similarly, the total TR is calculated by using link demand and link toll. This process is repeated for all paths generated in the step 3. In the fifth step, the dominance checking is employed to all paths obtained in step 4 based on the objective vector (RC, OTTEN, TR) in order to delete dominated paths. The algorithm stops when all POPs from *O* to *D* are obtained.

Consider a simple network as shown in Figure 1 where each link consists of route cost and route travel time and each node consist of node number. The OD demand matrix is tabulated in Table 1. Traditional label correcting algorithm (Moore, 1957) is used in order to obtain the shortest RC routes between all pairs of nodes. The shortest route from *O* to *D* has the RC attribute {5.5}, that passes via node 2 and shortest RC routes between all pairs of nodes are tabulated in Table 2.

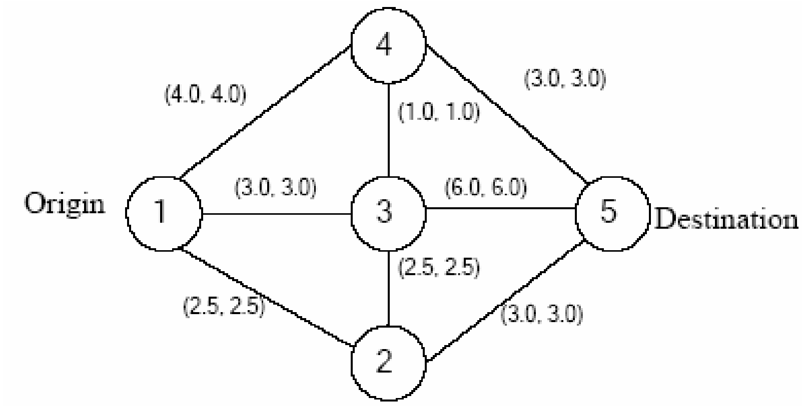

Figure 1 Example Network

Table 1 OD Demand of the Example Network

		To Node					
		1	2	3	4	5	**Total**
From Node	1	0	3	5	4	3	**15**
	2	4	0	7	5	4	**20**
	3	10	7	0	2	6	**25**
	4	11	10	3	0	6	**30**
	5	5	7	14	4	0	**30**
	Total	**30**	**27**	**29**	**15**	**19**	**120**

Table 2 Shortest RC Routes between All Pairs of Nodes

		To Node				
		1	2	3	4	5
From Node	1	0.0	2.5	3.0	4.0	5.5
	2	2.5	0.0	2.5	3.5	3.0
	3	3.0	2.5	0.0	1.0	4.0
	4	4.0	3.5	1.0	0.0	3.0
	5	5.5	3.0	4.0	3.0	0.0

Considering M as 30 % (assumed), the RCC is 7.15. The label setting algorithm is then applied from O (node 1) to D (node 5). When applying labeling algorithm, the label of origin node $1^1 = \{0\}$ is permanently labeled. The labels of nodes 2, 3 and 4 which are directly connected by the arcs from O would be $2^1 = \{2.5\}$, $3^1 = \{3.0\}$ and $4^1 = \{4.0\}$ respectively and number of feasible paths to origin node e(1) =1. These three sub-paths are acyclic and feasible because they do not repeat node twice and also they satisfy $RC < \alpha$. The sets of temporary and permanent paths are $L = \{2^1, 3^1, 4^1\}$ and $E = \{1^1\}$ respectively. The first feasible sub-path in L (2^1) which has the least RC from O is permanently labeled and updated. There are three links emanating from node 2. The sub-path to node 1 is cyclic and hence, it is deleted. The vector labels of nodes 3 and 5 are $3^2 = \{5.0\}$ and $5^1 = \{5.5\}$ respectively. The sub-path 3^2 is infeasible because 5.0 + 4.0 > 7.15. Hence, it is discarded. The sub-path 5^1 is feasible because 5.5 + 0.0 < 7.15. Hence, it is kept. The sets of temporary and permanent paths are, then, $L = \{3^1, 4^1, 5^1\}$ and $E = \{1^1, 2^1\}$ respectively. The number of feasible paths are e(1) = 1, e(2) = 1, e(3) = 1, e(4) = 1 and e(5) = 1.

Select the first sub-path of node 3 (3^1), with least RC in L is labeled permanently. Expanding this sub-path, the sub-path to node 1 is cyclic and labels of nodes 2, 4 and 5 are 2^2 = {5.5}, 4^2 = {4.0} and 5^2 = {9.0} respectively. The sub-paths 2^2 and 5^2 are infeasible because they do not satisfy RCC and sub-path 4^2 is feasible because 4.0 + 3.0 < 7.15. Hence delete 2^2 and 5^2 and keep 4^2. The temporary and permanent path sets are then L = {4^1, 5^1, 4^1} and E = {1^1, 2^1, 3^1} respectively. The number of feasible paths are e(1) = 1, e(2) = 1, e(3) = 1, e(4) = 2 and e(5) = 1. Updating the first sub-path of node 4 (4^1), the sub-path to node 1 is cyclic and sub-paths to nodes 3 and 5 have the labels {5.0} and {7.0} respectively. These both sub-paths are feasible because they satisfy α . Now, the temporary and permanent sub-path sets are L = {5^1, 4^2, 5^2} and E = {1^1, 2^1, 3^1, 4^1} respectively. The number of feasible paths are e(1) = 1, e(2) = 1, e(3) = 1, e(4) = 2 and e(5) = 2.

The only one sub-path to update, 4^2 in L , is taken and labeled permanently. The sub-paths to node 1 and 3 are cyclic and node 5 has the label 5^3 = {7.0}. This sub-path to node 5 is feasible path because 7.0 + 0.0 < 7.15. Then, the temporary and permanent node sets are, L = {5^1, 5^2, 5^3} and E = {1^1, 2^1, 3^1, 4^1, 4^2} respectively. The number of feasible paths are e(1) = 1, e(2) = 1, e(3) = 1, e(4) = 2 and e(5) = 3. Since the temporary set contains, only the paths from O to D with following RC attribute, the approach goes to step 4.

- 5^1 = {5.5} = 1-2-5
- 5^2 = {7.0} = 1-4-5
- 5^3 = {7.0} = 1-3-4-5

In STEP 4, the OTTEN and total TR is calculated on these acyclic feasible paths obtained from STEP 3 as discussed here. Assume, for this example, that the travel time on the links on MSP are 50 % of the existing links and unit toll on the link on MSP. Using all or nothing traffic assignment, OTTEN and total TR on paths 5^1, 5^2 and 5^3 are respectively, {384.00,19.00}, {370.00, 22.00} and {356.00, 47.00}. Then the objective vector to be minimized of these 3 paths are summarized as under:

- 5^1 = {5.5, 384.00, -19.00} = 1-2-5
- 5^2 = {7.0, 370.00, -22.00} = 1-4-5
- 5^3 = {7.0, 356.00, -47.00} = 1-3-4-5

In STEP 5, the dominated paths obtained from the STEP 4 are removed. Here, the path 5^3 dominates the path 5^2. Hence, under the assumed conditions, the obtained POPs with attribute vector, which are of the output of the algorithm, are:

- 5^1 = {5.5, 384.00, -19.00} = 1-2-5
- 5^2 = {7.0, 356.00, -47.00} = 1-3-4-5

The proposed algorithm is summarized as follows:

STEP 1. Identify the shortest routes between all pairs of nodes based on route costs in the transportation network using traditional label correcting shortest path algorithm.

STEP 2. Initialize constraints

2.1 Calculate minimum route cost from O to D

2.2 Set route cost constraint (α)

$$\alpha = RC_O^D \times (1 + M(in\ decimals))$$

STEP 3. Generate all acyclic and feasible paths using label setting algorithm with RC as attribute from O to D constrained by α.

3.1 For all paths of all nodes, let

$$i^k = \infty; \quad e(i) = 0; \quad L = \phi \text{ and } E = \phi;$$

3.2 Choose origin node and initialization

$O^1 = 0;\ e(O) = 1;\ L = \{O\}$ and $E = \phi;$

3.3 Determine the next path (for permanent labeling)

IF

• L contains all feasible paths from O to D, then GOTO STEP 4 (K- feasible paths are generated)

ELSE

• Choose any node, i, in L which has the smallest route cost

• Insert i into E (i.e. $E = E + \{i\}$)

• Delete node i from L (i.e. $L = L - \{i\}$)

3.4 Label updating

• For all nodes j connected with node i by link (i, j),

• Update route cost by

$RC_O^j = RC_O^i + r_{ij}$

• Cyclic path removing

IF the path travels through the same node twice, DELETE the temporal sub-path.

Then, $e(j) = e(j)$

ELSE

• Feasibility checking

IF $RC_O^j + RC_j^D > \alpha$, DELETE current temporal sub-path

Then, $e(j) = e(j)$

ELSE Insert the current temporal path as a feasible path. Then, $e(j) = e(j) + 1$

3.5 STOP Rule

• IF L contains all feasible paths from O to D, GOTO STEP 4

• ELSE, GOTO STEP 3.3

STEP 4. Calculate the OTTEN and total TR for feasible paths in STEP 3.

4.1 Set $n = 1$ (i.e. first feasible path), update travel time of each links on this feasible path.

4.2 Determine the shortest travel time routes between all pairs of nodes considering updated values of travel times on the path.

4.3 Calculate link demand of each link of the entire network using appropriate traffic assignment algorithm.

4.4 Calculate OTTEN using link travel time and link demand by equation (6).

4.5 Calculate total TR using link demand and link toll by equation (7).

• IF $n = K$, GOTO STEP 5

• ELSE, $n = n + 1$, GOTO STEP 4.1 STEP 5. Remove dominated paths and obtain a set of POPs.

6. Application of the proposed algorithm

6.1. Experimental study design

The proposed algorithm to MSPP is implemented using Current et al.'s (1987) transportation network and artificial set of OD demand. For sensitivity analysis purposes, RCR is varied from 1.00 to 2.00. The value of RCR equal to 1.00 means generated paths are only accepted when the RC of each generated path is equal to the shortest route from O to D. At this value, the obtained path(s) is the shortest route from O to D. Similarly, the value of RCR equal to 2.00 means the generated paths are feasible only when their RC is less or equal to the double of the shortest route. The value of RCR more than 2.00 is not significant for MSPP because the paths having RC attribute significantly more than the shortest route cost are, generally, not of interests for transportation network designs. Value of RCR has significant importance only when the values are near to 1.00. To examine the efficiency of the proposed algorithm, total generated paths (feasible paths), Pareto optimal paths and execution time of the algorithm are tested for different cases.

6.2. Test bed network

To demonstrate the algorithm of finding MSP, the proposed algorithm is tested and numerical results are analyzed using Current et al.'s (1987) network as test bed network and a set of artificial OD demand. The network consists of 21 nodes and 39 undirected arcs. The undirected arcs are treated as bi-directed arcs with the same attribute value in both directions because the coding of the algorithm is for directed network. Thus, in this paper, 21 nodes and 78 directed arcs are used for the analysis. The test bed network with link attributes is shown in Figure 2. The OD demand matrix is tabulated in Table 3. The MSP used in this paper is based on free flow travel time on existing links as well as links on MSP, 50% less travel time in the links of the MSP than in links on existing network, all or nothing traffic assignment algorithm and unit link toll. The computer code has the flexibility to adjust these values. In order to implement the algorithms other than all or nothing traffic assignment, computer code is to be changed accordingly.

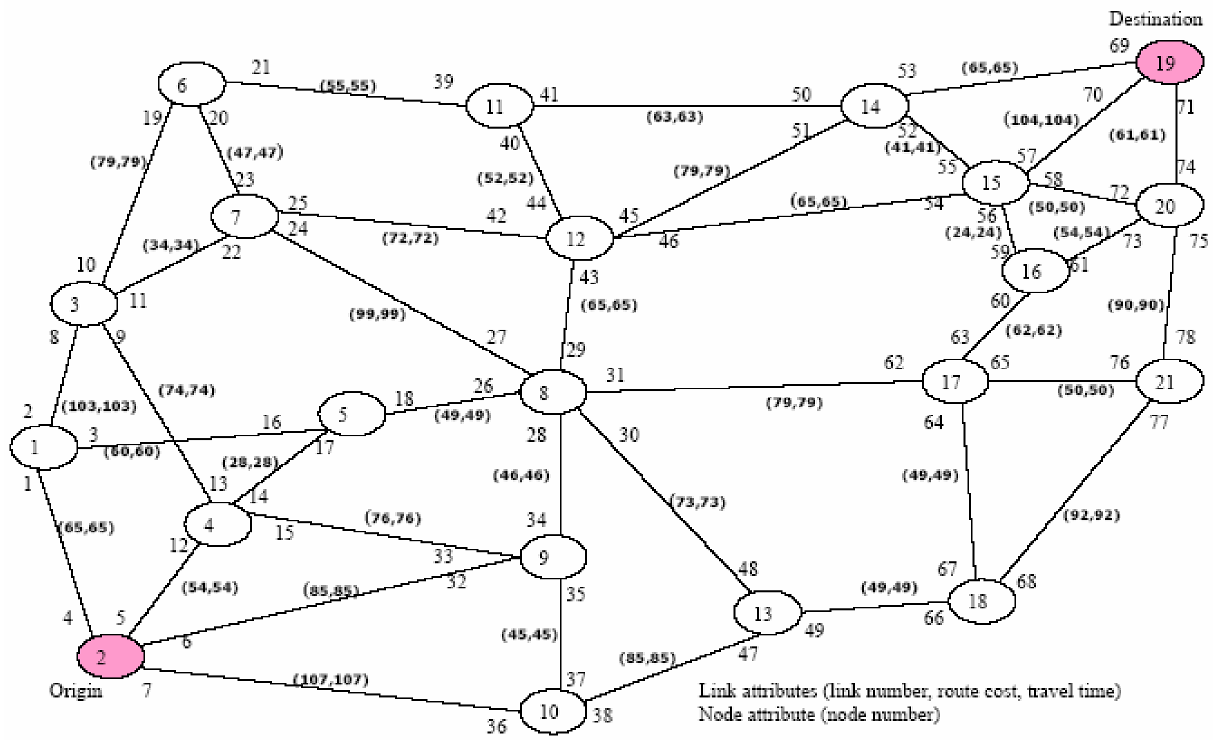

Figure 2 Test Bed Network (Current et al., 1987)

Table 3 Origin -Designation (OD) Matrix (10^3) for Test Bed Network

	To Node																					
From Node		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**Total**
	1	0	2	4	2	3	4	2	3	4	1	2	2	2	4	0	1	3	0	2	0	0	**41**
	2	12	0	15	20	30	21	16	34	12	9	14	27	25	14	18	19	21	23	28	13	14	**385**
	3	3	2	0	2	3	4	2	3	4	3	2	2	2	4	0	1	3	0	2	2	3	**47**
	4	6	15	8	0	15	14	20	7	6	5	11	14	12	14	3	8	7	6	12	2	8	**193**
	5	2	3	5	1	0	7	6	2	3	4	5	6	2	5	3	8	2	1	4	1	3	**73**
	6	9	36	15	22	23	0	11	12	40	13	28	8	9	11	23	21	22	10	14	18	10	**355**
	7	1	2	5	4	3	2	0	1	2	0	2	0	1	5	1	2	0	3	0	1	1	**36**
	8	16	18	32	50	22	34	34	0	24	28	30	31	24	13	15	21	28	19	44	40	42	**565**
	9	1	2	1	0	3	0	2	0	0	1	1	3	0	1	0	2	0	1	2	1	2	**23**
	10	0	2	2	1	1	0	3	1	2	0	0	1	2	0	2	1	0	3	0	2	3	**26**
	11	1	2	3	0	3	0	1	2	1	2	0	2	1	2	3	0	2	0	1	3	2	**31**
	12	2	3	2	5	3	2	1	4	1	3	2	0	1	5	4	3	2	1	5	2	3	**54**
	13	1	0	0	0	2	0	1	2	0	0	3	0	0	0	2	1	1	2	2	1	2	**20**
	14	50	30	28	30	21	21	14	51	19	18	15	25	30	0	40	45	41	21	18	32	25	**574**
	15	6	8	14	6	9	11	15	21	14	13	16	9	7	8	0	2	3	15	16	23	21	**237**
	16	2	10	7	4	5	3	6	5	2	8	3	5	9	2	4	0	3	2	5	6	2	**93**
	17	0	2	1	0	3	1	0	1	0	0	0	2	0	1	1	0	0	1	0	0	1	**14**
	18	0	2	1	0	3	1	0	1	0	1	0	2	0	1	1	0	0	0	2	0	1	**16**
	19	1	2	1	2	1	0	1	2	0	2	1	2	1	0	2	1	3	0	0	0	1	**23**
	20	4	2	3	5	10	8	5	2	6	14	5	12	6	8	1	6	5	4	1	0	8	**115**
	21	2	1	2	1	4	1	0	2	1	4	0	2	1	4	3	1	2	4	3	5	0	**43**
	Total	**119**	**144**	**149**	**155**	**167**	**134**	**140**	**156**	**141**	**129**	**140**	**155**	**135**	**102**	**126**	**143**	**148**	**116**	**161**	**152**	**152**	**2964**

7. Analysis of the results

This section deals with the computational behavior of the proposed algorithm on certain sets of constraints and parameters used in the formulations. Sensitivity analyses of proposed algorithm to MSPP with different values of constraints imposed in the formulations are carried out in order to evaluate the efficiency as well as the trend of change in parameter values. The major parameter values in the algorithms for sensitivity analysis are total number of paths generated, number of Pareto optimal paths and execution time of the algorithm. For this purpose, RCR is varied from 1.00 to 2.00. As explained already, the RCR value more than 2.00 is not of significant importance for transportation network planning and designing. Although the proposed algorithm can solve the problem for RCR values more than 2.00, the analyses are performed only for $1.00 \leq RCR \leq 2.00$ because this is the range of RCR, which is important for designing transportation networks.

(i) Analysis of Total Number of Paths Generated, Pareto Optimal Paths and Execution Time for Different Values of Route Cost Ratio

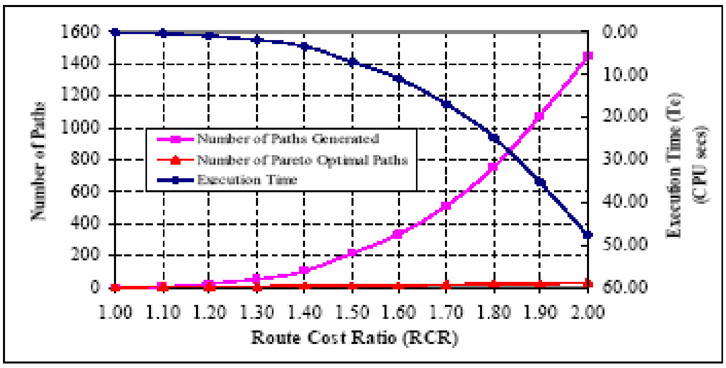

Figure 3 Number of Paths Generated, Pareto Optimal Paths and Execution Time

As shown in Figure 3, the total number of paths generated as well as the execution time of the algorithm increases exponentially with RCR whereas the increase in POPs is almost linear. At higher values of RCR (the paths having RC very large then the value of shortest route), more numbers of paths are generated and at the same time, there are more inferior solutions. The execution time has the same effect because time required to generate few feasible paths by controlling total generated paths is less, and also the search time required to calculate OTTEN and TR to each of these paths is less which results less execution time for lower value of RCR and vice versa. This result signifies the importance of imposition of RCR in the proposed algorithm because it shows directly the tradeoff between the number of alternatives and execution time required to generate them. Wide range of alternatives can be obtained for higher values of RCR but the execution time of the algorithm is high whereas few alternatives can be obtained from small values of RCR in less execution time. In simple words, in order to get few alternative POPs having RC very similar to the shortest route, the RCR is kept near to 1.00 and for wide range of alternatives, the high value of RCR is selected. Generally, for MSPP, only few alternatives are of interest and, therefore, the RCR is kept as low as possible and the execution time of the algorithm can be reduced significantly. Even if the value of RCR is very small, the obtained results are correct but that provides only few alternatives to the decision makers.

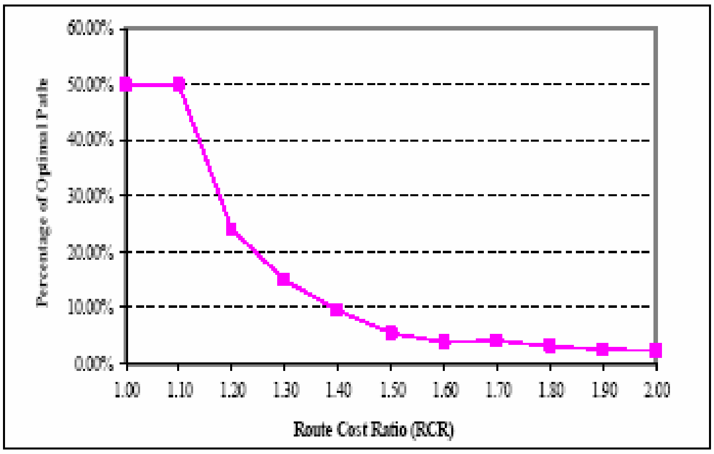

Figure 4 Percentage of Pareto Optimal Paths w. r. t. Total Paths Generated

The plot between percentage of POPs with respect to total number of paths generated and RCR as shown in Figure 4 shows that with the increase in the value of RCR, the percentage of POPs with respect to total number of paths generated decreases. For example, for RCR = 1.00, there are only two paths generated (two routes have the same RC from O to D) and one of them (50%) is POP. For RCR = 1.10, there are 8 paths generated and 4 of which (50%) are Pareto optimal paths. However, for RCR = 2.0, there are 1435 paths generated and only 34 of which (approx. 2.0 %) are optimal paths. It is obvious from the trend that, by controlling the value of RCR, total number of paths generated can be significantly reduced without making more effect on POPs. The plot between total number of paths generated (Ng) and the execution time (Te) as shown in Figure 5 shows the direct relationship between them. This means, execution time of the algorithm can be reduced directly by reducing total number of paths generated without making much effect on POPs because the effect on POPs is less for higher values of RCR than on total number of paths generated and execution time. Thus, by choosing RCR value judiciously, required output of the algorithm can be obtained efficiently.

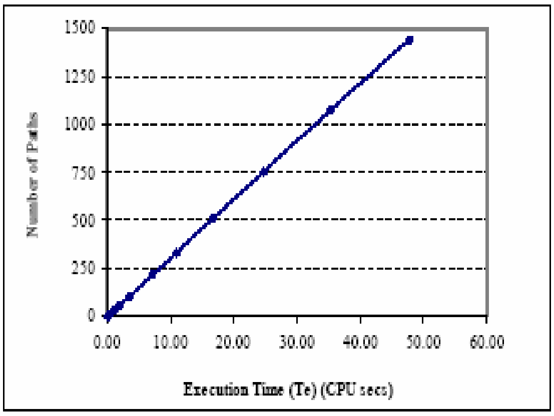

Figure 5 Total Numbers of Paths Generated versus Execution Time

(ii) Effect of number of objectives

In order to examine the effect of number of objectives, the third objective is removed and program is run. The output shows that as the number of objective increases, the number of POPs as well as the CPU time of the algorithm increases as shown in Figure 6 and Figure 7. For smaller values of RCR, the number of POPs and execution time are almost the same for both cases. For example, when RCR is 1.00, only one POP is obtained in 0.22 CPU seconds and when RCR is 1.10, 4 POPs are obtained in 0.38 CPU seconds. However, for higher values of RCR, the effect on POPs as well as on execution time is significant. For example, when RCR is 1.20, 7 POPs are obtained in 1.10 CPU seconds for three objectives case and 4 POPs in 1.05 seconds for two objectives case. Similarly, for the value of RCR = 2.00, 34 POPs are obtained in 47.78 CPU seconds for three 3 objectives and 18 POPs in 44.60 seconds for two objectives.

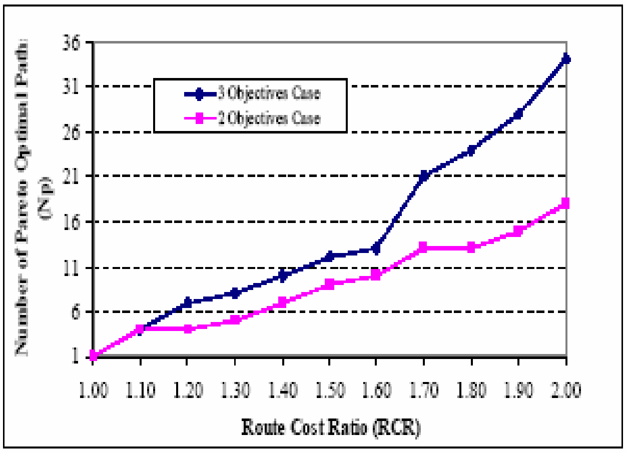

Figure 6 Number of Pareto Optimal Paths for Two and Three Objective Cases

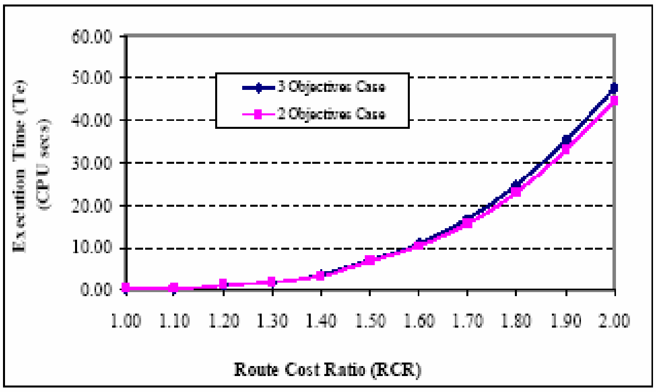

Figure 7 Execution Time of the Algorithm for Two and Three Objective Cases

Thus, the sensitivity analysis of the proposed algorithm shows that imposing CB and RCC during the labeling phase of the algorithm (criteria space) is very effective to make the solution more efficient in terms of execution time and the memory space in the computer not only for generating feasible paths but also for making small searching domain. The set of Pareto optimal paths with attributes vector (RC, OTTEN, TR) facilitates the final decision-making.

8. Advantages of the proposed algorithm

The proposed approach of the MSPP has many advantages than existing approaches. Firstly, it defines the realistic conflicting objectives for transportation planning and designs. Secondly, it does not need the tradeoff relationship between objective functions like utility maximization and can identify the alternative paths with vector of objectives. Thirdly, addition of any other new objectives to objective vector does not require intensive modification on the computer codes. Moreover, it is based on labeling and search algorithm, which is simple to understand and ease in application. Finally, it does not require full k-shortest path problems to generate all paths rather it uses CB and RCC to generate only feasible paths. It is expected that the proposed algorithm would be the milestone in the planning and designing of transportation network.

The proposed algorithm has been proposed considering transportation networks design problems as a multiobjective shortest path problem, therefore explicitly incorporating the distinct evaluation alternative aspects into the formulation. The developed algorithm computes nondominated paths by optimizing objective functions in terms of a compromise solution, which is the essential decision making tool for multicriteria decision making for many transportation network design applications.

9. Summary and conclusions

This paper presents a theoretical formulation and modeling algorithm for solving MSPP for designing static and deterministic transportation networks. Unlike the past practices to solve transportation network problems using the existing algorithms from other fields, which are not realistic to transportation networks, this study defines the realistic objectives for transportation network and solution algorithm for the same. The algorithm itself is also efficient over existing approaches because it uses heuristic labeling approach to identify feasible paths in labeling space and exhaustive search on these feasible paths in order to determine a set of POPs in solution space. Using labeling algorithm deletes the cyclic paths and infeasible paths at each node during labeling phase (criteria space) using CB and RCC respectively, which identifies all acyclic feasible paths form origin to destination. The exhaustive search on the feasible paths obtained with respect to the objective vector deletes dominated paths resulting a set of POPs. The approach is tested and validated using Current et al. (1987) network. It is found from the sensitivity analysis that the proposed approach can reduce the total number of generated paths as well as execution time by controlling RCR judiciously.

Beyond the limits of this study, there is much room for expansion. Since the static and deterministic transportation networks are the ideal cases in many urban transportation network systems, the improvement of the proposed approach to dynamic and stochastic transportation network would be more valuable for solving real transportation network problems. The proposed approach is expected to modify to include other objectives as safety, comfort, and environmental concerns as well as the political goal of user satisfactions. It remains open the practical efficiency of the proposed algorithm and, therefore, the model is to be tested for real world transportation networks.

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