HIGHWAY MAINTENANCE MARGINAL COSTS: WHAT IF THE FOURTH POWER ASSUMPTION IS NOT VALID? *

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** This paper is based on a study previously reported in Transport Policy (Anani and Madanat, 2010).*

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ABSTRACT

Highway maintenance marginal costs have been estimated in the literature using the perpetual overlay indirect approach. This approach uses the equivalent single axle load (ESAL) as the unit for traffic loading, which implies that pavement deterioration caused by an axle is proportional to the fourth power of the axle weight. This paper answers the following question: how inaccurate are maintenance marginal cost estimates when a highway agency uses ESAL? We find that the inappropriate use of ESAL does not affect the sum of maintenance marginal cost prices paid by all vehicles, but it affects its distribution among vehicles, which reduces efficiency and equity.

Keywords: marginal cost, highway maintenance, traffic loading units

INTRODUCTION

In this paper, we analyze the sensitivity of pavement maintenance, rehabilitation and reconstruction (MR&R) marginal cost to the units of traffic loading. The units used to measure traffic loading are referred to as *deterioration equivalence factors* (DEF).

In evaluating pavement deterioration, total vehicle weight does not accurately describe the pavement deterioration caused by a vehicle. The weights on the individual axles (axle loads or *axle weights*) are better predictors of deterioration. Therefore, DEFs are calculated for each axle, both in the literature and in practice. The number of DEFs resulting from a given axle expresses how damaging that axle is. The number of DEFs resulting from a vehicle is the sum of DEFs resulting from each of its axles. A DEF increases pavement deterioration by the same amount, regardless of which vehicle it comes from. For example, a vehicle with a traffic loading of *n* DEFs causes *n/m* times the amount of deterioration caused by another vehicle with *m* DEFs.

The appropriate expression to use for the DEF depends on the indicator of deterioration being considered. One commonly used DEF is the *equivalent single axle load (ESAL)*, which assumes that the pavement deterioration caused by a given axle is proportional to the fourth power of the axle weight. This fourth power rule comes from the American Association of State Highway Officials (AASHO) Road Test (Highway Research Board, 1962), where deterioration was defined as loss of pavement serviceability (measured using the Present Serviceability Index), and the number of DEFs was found to be proportional to the fourth power of the axle loadings. This test was conducted near Ottawa, Illinois between 1958 and 1960 and provided an experimental data set that was both comprehensive and reliable (Highway Research Board, 1961; Prozzi and Madanat, 2004). The estimation procedures used to develop the fourth power rule are questionable (Bruzelius, 2004, p. 45). Prozzi and Madanat obtained a power of 4.15, with the same data set but using random effects estimation instead of a simple OLS estimation (Prozzi and Madanat, 2004).

Different highway agencies use different indicators of deterioration. For example, deterioration might be defined as the increase in roughness, cracking, or rutting. Depending on the indicator of deterioration, the fourth power may not be suitable for expressing the DEF, in which case the traffic loading should not be expressed as the number of ESALs. Prozzi and Madanat obtain a power of 3.85 when deterioration is defined as roughness (Prozzi and Madanat, 2004). Archilla and Madanat (2000) specify and estimate a model that predicts the increase in rutting. Although the power for tandem axle weight is 3.89, the power for the single axle weight is 2.98, which is closer to 3. Therefore, the use of ESAL as a DEF is appropriate only when deterioration is defined as the loss in serviceability, and even in that case, the fourth power is only a rough approximation.

Highway agencies generally use highway pavement MR&R strategies that are *condition-responsive*. In other words, the highway agency performs a given MR&R activity each time a given measure of pavement condition reaches a predetermined trigger level. Under such a condition-responsive strategy, an increase in traffic loading leads to an indirect increase in the MR&R total cost incurred by the highway agency, as Figure 1 shows. An increase in traffic loading accelerates pavement deterioration, which brings forward all future MR&R activities which, in turn, increases their present value.

Following Small et al. (1989), Vitaliano and Held (1990) and Lindberg (2002), an additional DEF is defined as an event that recurs annually, and the *MR&R marginal cost* is defined as the change in the annualized cost of all future MR&R. Such an additional DEF will be referred to as a *recurring additional DEF*.

The MR&R marginal cost represents only one component of the marginal social cost. Other components of the marginal social cost include the private marginal cost (the increase in own vehicle operating cost), and the highway user marginal cost (the increase in the cost of subsequent vehicles as a result of worse pavement condition). This paper only focuses on the MR&R marginal cost component.

The different approaches used in the literature to estimate MR&R marginal cost are surveyed by Bruzelius (2004). Among these approaches, the *perpetual overlay indirect approach* is the most detailed, in that it explicitly accounts for all links A, B and C in Figure 1. Bruzelius (2004) refers to it as the “indirect approach”. This approach assumes that pavement deterioration is deterministic. It also assumes that pavement overlay (resurfacing) costs dominate MR&R costs, and it ignores all other MR&R costs. It uses an infinite analysis horizon and assumes that a pavement is overlaid as soon it deteriorates to a predetermined trigger level (Newbery, 1990; Small et al., 1989). It first relates changes in traffic loading (additional DEF) to changes in overlay frequency (an additional DEF brings forward the future overlays), and possibly changes in the overlay intensity (thicker overlays in anticipation of higher traffic loadings in the future). Then, it relates these changes in overlay frequency (and intensity) to MR&R marginal cost.

To relate traffic loading to pavement deterioration (link A in Figure 1), the perpetual overlay indirect approach often assumes that the pavement deterioration caused by an axle is proportional to the fourth power of the axle weight. In other words, the literature defines the DEF as ESAL (Lindberg, 2002; Small et al., 1989; Vitaliano and Held, 1990).

This paper answers the following question: how inaccurate are the MR&R marginal cost estimates when a highway agency uses ESAL instead of the appropriate DEF? It is organized as follows. The next section describes the methodology used to quantify the errors resulting from incorrectly using the fourth power to estimate highway maintenance marginal costs. We then apply this methodology to hypothetical scenarios, where we look at three assumed distributions of axle weights, and to field data obtained at a weigh-in-motion station. Finally, the conclusions section summarizes the findings, discusses policy implications and mentions future research needs.

METHODOLOGY

Let DEF_p denote the DEF computed using power *p,* such that p>0. For a given single axle that weighs *w* (kN), let N_p denote the number of DEF_p units. The value of N_p is the ratio of *w* to a standard weight (80 kN) raised to the power *p*:

$$N_p = \left(\frac{w}{80\ kN} \right)^p \qquad (1)$$

DEF_4 is commonly known as ESAL. Therefore, N_4 is simply the number of ESALs for the single axle.

Consider one lane of a flexible pavement section of a highway. Let constant L_p, such that L_p>0, be the annual traffic loading for this section measured using power *p*. It has units of (DEF_p/year). In particular, L_4 is the annual traffic loading in (ESAL/year).

In order to simplify the analyses, a given axle group of any type (steering, single, tandem or tridem) is converted into (equivalent) single axles by dividing its weight equally among the single axles that make it up. Then, only a stream of single axles needs to be considered. Of course, this requires making the assumption that an axle group with k single axles causes as much damage as k single axles each carrying a fraction (1/k) of its weight. This assumption can be relaxed in future studies.

Also, consider a highway agency that uses a simple MR&R policy with only one type of MR&R activity, namely an overlay of constant intensity that is triggered by a specific pavement performance measure, *M*. The pavement section receives an overlay each time *M* reaches trigger level M_f. Assume that pavement deterioration and improvement are deterministic.

Let X_p, such that X_p>0, be the number of DEF_p units to failure for this pavement section measured using power *p*. Let *T* (year) be the overlay life, i.e. the time between two consecutive overlays.

$$T = \frac{X_p}{L_p} \tag{2}$$

Although Equation (2) is derived for the case of one lane, it can also be applied to the case of a highway section with multiple lanes, provided that *all* lanes are only overlaid at the same time, in which case these lanes can be treated as a system of lanes. When Equation (2) is used for a system, X_p and L_p should include the combined number of DEF_p units for all lanes. The exact definition of failure for this system depends on the highway agency (for example, the agency might overlay the system each time any lane fails), and it will affect the value of X_p.

We ignore the effect of weathering, so X_p is independent of L_p. To account for weathering, we would have to use a specific deterioration model[1]. By keeping the methodology general and simple, it is easier to gain intuition about the effect of using appropriate power *p*.

Under constant annual traffic loading L_p, the values of X_p and *T* depend on the underlying pavement structure, the climate and the value of the trigger level (M_f). These three factors are held constant. Assume that the highway agency already knows accurately the current value of *T* from records of previous overlays.

Our hypothetical highway agency assumes that *p*=4 and knows the value of L_4 from previous measurements. This knowledge could have been acquired as follows: assume that the numbers and weight distributions of axles remain the same each year. For a period of one year, the highway agency measured the weight of each axle, and converted each into a number of ESALs (N_4), without keeping record of the measured weights. By summing up the values of N_4 resulting from each axle over one year, the agency obtained the annual traffic loading in ESALs (L_4).

Had the highway agency kept record of the axle weight data for the section, it would have been able to generate its “axle load spectrum” for single axles, which is the “normalized frequency” (or probability mass function) of single axle weights (Lu et al., 2002).

Let *U* be the unit cost ($/kilometer) for an overlay. The value of U should be consistent with the values of X_p and L_p; for example, if X_p and L_p are defined for a system of lanes, then U should be the unit cost for overlaying all lanes. Figure 2 shows the cash flow diagram for all future overlays. Present time is time 0.

Let *r*, such that *r*>0, be the discount rate per annum. Let *V* be the present value of all future overlays ($/kilometer).

$$V = \frac{U}{\exp(r.T) - 1} \tag{3}$$

The additional DEF is defined as a *recurring additional DEF* (Lindberg, 2002; Small et al., 1989; Vitaliano and Held, 1990). Using continuous discounting[2], the annualized cost of all future MR&R actions equals $(e^r-1)\cdot V$. Then, it can be shown that Equation (4) gives the MR&R marginal cost ($/$DEF_p$/kilometer):

$$MC_p := \frac{d\left(\left(e^r - 1\right).V\right)}{dL_p} = \frac{\left(e^r - 1\right).r.U.T.\exp(r.T)}{L_p.\left(\exp(r.T) - 1\right)^2} \tag{4}$$

When *p* is four, Equation (4) is essentially a simplified version of Small et al.'s equation (1989, equation 2-9b with m=0) and Vitaliano and Held (1990, equation 8, with theta=1 or m=0), when the effect of weathering is ignored and continuous discounting is used consistently.

Let α be the appropriate power that should be used for traffic loading when the overlays are triggered by performance indicator *M*. Assume that the highway agency currently uses *marginal cost pricing*, and it currently uses ESAL (i.e. DEF_4) as the unit for traffic loading. Define DEF_α as the appropriate unit for traffic loading, when performance indicator *M* is used. We are interested in finding out how much difference it makes to use the appropriate power α instead of 4.

Let $PRICE_p$ ($/kilometer/axle) be the MR&R marginal cost price paid by a given single axle that weighs w, when the power *p* is used. Note that MC_p is measured per unit of DEF_p, whereas $PRICE_p$ is measured *per axle* of weight *w*. Since this axle contributes N_p units of DEF_p:

$$PRICE_p := N_p.MC_p \tag{5}$$

Using Equations (1), (4) and (5):

$$PRICE_p = \left\{\frac{1}{L_p}.\left(\frac{w}{80}\right)^p\right\}.\left\{\frac{\left(e^r - 1\right).r.U.T.\exp(r.T)}{\left(\exp(r.T) - 1\right)^2}\right\} \tag{6}$$

The first term in Equation (6) depends on *p*, and it represents the fraction of L_p that the axle contributes. The second term does not depend on *p*, and it represents the sum of prices paid by all axles in the year (which have an annual traffic loading L_p). Basically, the second term gives the total revenue, and the first term gives the fraction of this total revenue paid by an individual axle. The important thing to note is that, the total price paid by all vehicles would be the same regardless of the assumed power (only the allocation among axles would change). Of course, if pricing is implemented, responses will take place that change the total revenue in the long run. These responses will probably depend on the individual prices, which are a function of the assumed power.

In the extreme case where all the axles have the same weight, the assumed power *p* makes no difference at all on the price paid by an axle. In this case, the first term in Equation (6) reduces to simply 1/R. This is intuitive because, by symmetry, the total amount (second term) is equally divided among all *R* axles, regardless of the assumed power.

The highway agency can evaluate how closely the current price, $PRICE_4$, matches the appropriate price, $PRICE_\alpha$, by looking at the ratio of the two. Using Equation (6) twice:

$$\frac{PRICE_\alpha}{PRICE_4} = \frac{L_4}{L_\alpha} \cdot \left(\frac{w}{80}\right)^{\alpha-4} \tag{7}$$

It is this ratio, rather than the individual prices, that really matters for the purpose of this study. As Equation (7) shows, calculating this ratio does *not* require data on pavement structure, climate or maintenance trigger values. Only the appropriate power (α) and the traffic loadings are needed. As for the axle weight *w*, it will be varied parametrically. This ratio can be seen as the appropriate price, normalized by the current price.

In order to compute L_α, it is not sufficient to know L_4. It is also necessary to know the distribution of axle weights.

Let G denote the set of axle group types (e.g. steering, single, tandem, tridem). Let n_g be the number of single axles in an axle group of type g (n_g= 1 for steering and single, 2 for tandem, and 3 for tridem). Upper case (W) and lower case (w) will be used to denote the weights of axle groups and its equivalent single axles, respectively. If the weight of an axle group of type g is W, then this axle group is replaced in the analyses with n_g single axles each weighing ($w=W/n_g$). Let R_g be the annual repetitions of axle groups of type g. Then, the annual repetitions of single axles resulting from axle group type g equals ($n_g.R_g$). Let *R* be the number of equivalent single axle repetitions from all types of axle groups.

$$R = \sum_{g \in G} n_g . R_g \tag{8}$$

Let $f_{Wg}(W)$ denote the probability density function for the weights of axle groups of type g. Let f(w) be the probability density function for the weight of an equivalent single axle (taken across all types of axle groups). Then it can be shown that:

$$f(w) = \sum_{g \in G} \frac{n_g^2 . R_g}{\sum_{i \in G} n_i . R_i} . f_{Wg}(n_g . w) \tag{9}$$

Then, the annual traffic loading can be expressed as the product of the total number of equivalent single axle repetitions (from all types of axle groups) and the expected number of DEF_p's per equivalent single axle repetition (taken across all types of axle groups):

$$L_p = R.\int_0^{\infty} \left(\frac{w}{80}\right)^p . f(w).dw \tag{10}$$

Where, R and f(w) are given by Equations (8) and (9), respectively.

Since the highway agency has not recorded weight data, Equation (9) cannot be used; instead, the distribution of equivalent single axle weights, f(w), needs to be assumed. A previous study by Lu et al. (2002, App. B) is consulted in an effort to determine what a realistic distribution might be. Their study shows axle load spectra (distributions) for different highway sections, and many of these have multiple peaks. Therefore, it is difficult to come up with a typical distribution. Instead, we try three hypothetical distributions for the weight of a given axle, *w*, and check whether the assumed distribution has a big effect on the conclusions. The first is a uniform distribution between *a* (10 kN) and *b* (90 kN), as shown in Figure 3. The second is a symmetric triangular distribution, with a minimum value of *a* (10 kN) and a maximum value of *b* (90 kN), as shown in Figure 4. The third is a Burr type XII distribution, with the following probability density function for *w*:

$$f(w) = \frac{k.c.\left(\frac{w}{s}\right)^{c-1}}{s.\left(1+\left(\frac{w}{s}\right)^{c}\right)^{k+1}} \quad \text{, for } w \geq 0 \text{, and zero otherwise.} \tag{11}$$

Where, shape parameters *c* and *k* equal 1.5 and 4, respectively, and scale parameter *s* equals 80. This probability density function is unimodal, positively skewed and heavy-tailed, as shown in Figure 5.

Table 1 shows the expressions for the annual traffic loading for a general power *p* for each assumed distribution, which are based on Equation (10). In particular, we interested in p=4 and p=α. Plugging the expressions for L_4 and L_α into Equation (7) gives us the equations for the $PRICE_\alpha/PRICE_4$, which are shown in Table 2.

RESULTS USING ASSUMED DISTRIBUTIONS

This section compares the current price ($PRICE_4$) with the appropriate price ($PRICE_\alpha$) under the three aforementioned distributions. The appropriate power, α, depends on the type of performance indicator used by the highway agency to trigger MR&R.

Figure 6, Figure 7 and Figure 8 show the ratio of the appropriate price to the current price under different values of α and w, for the uniform, symmetric triangular and Burr Type XII distributions, respectively.

As each of these figures shows, when α=4, the price ratio ($PRICE_\alpha/PRICE_4$) equals one. For every other value of α, there is a unique axle weight w^*_α, for which the ratio equals one. For the uniform and triangular distributions, the value of w^*_α is not very responsive to changes in α, and all five curves seem to meet at one point in Figure 6 and Figure 7.

As Figure 6 and Figure 7 show clearly, if the performance indicator M used by the highway agency is associated with a power α that is greater than 4, for example 4.5, the ratio increases as the axle weight increases. For axle weights that are below w^*_α, the current price $PRICE_4$ is too high. These represent the majority of the axles. On the other hand, $PRICE_4$ is too low for axles that weigh over w^*_α. On the other hand, when the appropriate power α is smaller than 4, for example 3.5, the ratio decreases with the axle weight. The current price $PRICE_4$ is too low for axle weights that are below w^*_α, and too high for axle weights over w^*_α.

In order to better understand Figure 6, Figure 7 and Figure 8, the price ratio can be broken down into two factors. Equation (5) shows that the price equals the number of DEF units multiplied by the marginal cost per DEF. Therefore, the price ratio ($PRICE_\alpha/PRICE_4$) is the product of the number-of-DEF ratio (N_α/N_4) and the marginal cost ratio (MC_α/MC_4). As Equations (7) shows, the number-of-DEF ratio reduces to $(w/80)^{\alpha-4}$, and the marginal cost ratio reduces to L_4/L_α. These two factors are shown in Figure 9 and Table 3, respectively, and they both depend on the power α. The first factor, which is depicted in Figure 9, is intuitive: Equation (1) clearly shows that the number of DEF_p units, N_p, for a given single axle decreases with p for axle weights less than 80kN, does not depend on p for w=80kN, and increases with p for w≥80kN. All curves pass through the same point because N_α/N_4 equals 1 when w=80kN for every α.

Figure 9 can be used as a first step for obtaining the previous Figure 6, Figure 7 and Figure 8. The second step would be to multiply each value in Figure 9 by the corresponding marginal cost ratio shown in Table 3, which shifts each of the curves in Figure 9 vertically by a different scale. As a result, the curves in Figure 6, Figure 7 and Figure 8 no longer meet at exactly the same point. In the case of uniform distribution, the marginal cost ratios are very close to 1, so the resulting price ratios (Figure 6) are very similar to the number-of-DEF ratios (Figure 9). The marginal cost ratios do not vary much with α for the uniform and triangular distributions, so the curves are shifted vertically by similar factors and seem to still meet at a common point. This is not the case for the Burr distribution. Another interesting thing to note in Table 3 is that the marginal cost ratio increases with α under the uniform and triangular distributions, whereas it decreases with α under the Burr distribution.

At a first glance, Figure 8 might appear completely different from Figure 6 and Figure 7. The reason is that the values of w^*_α do not appear on the horizontal axis (they are greater than 130 kN). The Burr distribution assumes that *w* ranges from zero to infinity, and the horizontal scale (0 to 130) represents approximately 99% of the axles. For this vast majority of axles (lighter than w^*_α), the current price $PRICE_4$ is too high if α is greater than 4, and too low if α is smaller than 4, similar to the other two graphs. For all of the studied values of α, less than 1% of the axles weigh more than w^*_α. If one extends the horizontal axis, the graph would look more similar to the previous two graphs (except that the values of w^*_α would show more variance). It is still true that for the *very few* axles that weigh more than w^*_α, the current price $PRICE_4$ is too low if α is greater than 4, and too high if α is smaller than 4. These very rare occurrences are not captured by the limited horizontal scale of Figure 8.

The Burr distribution is unrealistic because it assumes that there is no upper limit on axle weights. That can cause problems. For example, it might appear from Figure 8 that $PRICE_3$ exceeds $PRICE_4$ for all axles. That might seem to contradict the earlier conclusion, drawn from Equation (6), that the sum of prices paid by all axles is the same regardless of the assumed power. There is no contradiction if one considers the very heavy axle weights, as well. For all axles that are heavier than 144 kN, $PRICE_4$ exceeds $PRICE_3$, and although these axles have a low probability, they can have a large impact on total price because the heavier the weights, the higher the prices.

The important thing to note is that the specific probability density function used has, qualitatively, no effect on how the price ratio ($PRICE_\alpha/PRICE_4$) varies with w and α. This means that the results shown in this section are robust to uncertainty regarding the true axle load spectrum, and are thus quite general.

RESULTS USING WEIGH-IN-MOTION DATA

The results so far are based on hypothetical weight distributions. This section, on the other hand, uses data measured at a weigh-in-motion (WIM) Station 97, along route 83 at postmile 5.7 (kilopost 9.2), in Chino, San Bernardino County, California. There are two lanes in each direction (Lu et al., 2002, p. 24). Assume that all four lanes always get overlaid together, and refer to these four lanes as a system.

Let the set of axle group types be G=[steering, single_truck, tandem, tridem, single_nontruck]. The first four types are for trucks and buses. The last type includes motorcycles, passenger cars, pickup trucks, limousines, vans and recreational vehicles. Lu et al. (2002, p. H-141) provide (in the form of frequency plot known as load spectra) the weight distributions for the four truck axle group types at Station 97, which are shown in Figure 10. It is common practice for load spectra to ignore the nontruck vehicles, despite their high volumes, because they are believed to have a negligible effect on pavement deterioration, as indicated by the very small numbers of ESALs. However, in our study, we vary p that is used for calculating the DEF_p, and for small values of p, the contribution of nontruck vehicles to the total traffic loading might become larger. Therefore, besides Figure 10, we also include nontruck vehicles. In the absence of data, we make the simple assumption that half of the nontruck vehicles weigh 9kN and the other half weigh 18kN. These vehicles have 2 single axles each, and for simplicity, we assume even distribution of weight among both axles. As a result, half of the single_nontruck axles weigh 4.5kN, and the other half weigh 9kN. We will later test the assumption that the contribution of the single_nontruck axle group type to L_p is negligible for the studied values of p. If that is the case, then there is no need to refine the weight distribution for this axle group type.

The estimated annual repetitions of axle group types are shown in Table 4. These are calculated using multiple sources of data. The annual average daily traffic (AADT) for all the different classes of trucks at Station 97 in year 2001 are obtained from Lu et al. (2002, p. H-141). Nontruck AADT is calculated using the ratio of truck AADT to total AADT found on the Caltrans website (California Department of Transportation, 2008). Also, the (California statewide) average numbers of axle group types per vehicle of each class are obtained from Lu et al. (2002, p. G-8).

In order to find L_p, a different equation from Equation (10) will be used because the distribution is given as a frequency distribution (or probability mass function) as opposed to a probability density function. Equation (12) makes computer implementation simpler in this case. This equation is intuitive. The number of axle groups of type g that weigh W_i equals [$Prob_g(W_i).R_g$]. The number of DEF_p units from one axle group of type g that weighs W_i equals [$n_g.(W_i/n_g/80)^p$]. The product of these two quantities needs to be summed over all weight bins *i* and groups *g*.

$$L_p = \sum_{g \in G} n_g . R_g \sum_i \left(\frac{W_i / n_g}{80} \right)^p . \Pr ob_g (W_i) \tag{12}$$

Table 5 shows the resulting values of L_p for different values of *p*. It also shows the percentage that each axle group type contributes to L_p.

As Table 5 shows, over the studied values for p, the contributions of tridem and single_nontruck axle group types to L_p are negligible. Since nontruck vehicles only produce single_nontruck axle groups, the previous conjecture that these vehicles can be ignored (or not studied in detail) is validated, and there is no need to go into further details regarding the axle weight distributions.

Using the values of L_p in Table 5 and Equation (7), the ratio of the appropriate price to the current price is calculated under different values of α and single axle weight, w, and presented in Figure 11. This figure looks very similar to the previous figure that was obtained assuming a triangular distribution, i.e. Figure 7 (within the common range of w between 10 and 90kN). This is corroborated by Table 6, which shows that the marginal cost ratios at the Station 97 are very close to the values for the triangular distribution in Table 3. This does not mean, however, that it would be appropriate to assume the triangular distribution. Although this assumption does not have a significant effect on the price ratio (PRICEα/PRICE4), it can have a large impact on prices paid under a given power α. A highway agency should, therefore, measure axle weight distributions in order to obtain accurate estimates of marginal cost prices.

Besides understanding the ratios of prices, it is also important to get a feel for the values of the prices. In order to obtain prices, the values in Table 7 are assumed.

Figure 12 shows the resulting prices for two single axles that weigh 50kN and 85kN, under various assumed values of *p*. The heavier axle pays a much higher price than the lighter one, for all the assumed values of power.

It is clear from Figure 12 (or Figure 11) that the lighter axles (that weigh less than w^*_α) are better off (i.e. face lower prices) when the assumed power is high, whereas the heavier axles (with weights greater than w^*_α) are better off when the highway agency assumes smaller powers.

CONCLUSIONS

This study assesses the importance of using the appropriate units of traffic loading (i.e. appropriate DEF) in estimating the marginal costs of highway pavement MR&R. It considers a highway agency that uses a condition-responsive MR&R strategy and assumes that the agency currently uses DEF_4 (known as ESAL) as the unit for traffic loading, whereas the appropriate unit that is consistent with the MR&R strategy is actually DEF_α.

If the highway agency switches from DEF_4 to DEF_α, that affects not only the marginal cost (per DEF), but also the number of DEF units corresponding to a given axle of a weight *w*. Therefore, this study compares the values of MR&R marginal cost per axle, rather than per DEF, under different powers.

We find that under MR&R marginal cost pricing, the sum of MR&R prices paid by all axles remains the same regardless of the power *p* used for the DEF (at the current equilibrium). However, the value of *p* has equity and efficiency implications because it dictates how this total amount is distributed among the axles belonging to all the different vehicles.

The marginal cost price for an axle equals the product of the number of DEF units and the marginal cost per DEF for the axle. The value of *p* has two effects on this price. First, it obviously changes the number of DEF_p units resulting from the axle. Second, it affects the marginal cost per DEF_p by changing the annual traffic loading (L_p). For axle weights close to 80kN, the second effect dominates (because the first effect becomes negligible).

In order to estimate the annual traffic loading under different values of p, we look at three hypothetical distributions of axle weights, as well as one actual distribution at WIM Station 97 in Chino, California.

$PRICE_4$ refers to the marginal cost per axle of weight *w* if the highway agency continues to use DEF_4, whereas $PRICE_\alpha$ refers to the marginal cost per axle of weight *w* if the highway agency switches to the appropriate DEF_α. If the power α associated with the appropriate DEF is different from 4, the current estimate $PRICE_4$ will be very inaccurate (i.e. significantly different from the appropriate estimate $PRICE_\alpha$) for the relatively *small* values of *w*. Furthermore, if α significantly differs from 4, the current estimate $PRICE_4$ will also be inaccurate for the relatively *large* values of *w*. An example is when the scheduling of MR&R is based on rutting (α=2.98).

When $\alpha>4$, a pricing policy that is based on the fourth power leads to values of $PRICE_4$ that are too high for most axles (the relatively light ones). Using the fact that the total paid by all vehicles is constant (with respect to p), this means that the fewer remaining (relatively heavy) axles are currently highly “subsidized” by the lighter ones. Using similar reasoning, when $\alpha<4$, the values of $PRICE_4$ are too low for most axles (the relatively light ones), whereas the fewer heavier axles are currently overpaying. Therefore, it is important that a highway agency use the appropriate power α in order for marginal cost pricing to achieve equity and efficiency. Of course, there will be winners and losers from switching to α.

We find that the choice of the specific distribution has, qualitatively, no effect on the results regarding the price ratio ($PRICE_\alpha/PRICE_4$). In other words, these results are robust to uncertainty regarding the true distribution, and are thus quite general. The hypothetical triangular distribution gave quantitative price ratio results very similar to those obtained at WIM Station 97. However, the specific distribution of axle weights has a large impact, for a given α, on traffic loading and thus on $PRICE_\alpha$. Therefore, a highway agency should have information on the axle weight distribution in order to obtain accurate estimates of $PRICE_\alpha$.

The methodology described in this paper is practical because axle load spectra are becoming increasingly available as pavement design methods become more data-intensive, and WIM technology becomes more widespread. Furthermore, at locations where such spectra are not available, there is much to be gained from using rough estimates of weight distributions and using the appropriate power.

One shortcoming of the present paper is that it only estimates MR&R marginal costs under current equilibrium for a pavement section without actually implementing any pricing. In other words, it estimates the marginal cost prices that would be charged had marginal cost pricing been implemented. It does not estimate behavioral changes that would result from the implementation of marginal cost pricing, such as shifting to vehicle types that cause less pavement damage (i.e. have fewer DEF units). Such behavioral changes affect the annual traffic loading, which in turn affects MR&R marginal costs. Anani (2008) extends the present paper by considering the entire roadway system in California and actually implementing MR&R marginal cost pricing, which leads to changes in the annual traffic loading on different types of roads, resulting from truckers changing truck types and shippers shifting between truck and rail. The MR&R marginal cost prices are assumed to replace all the current taxes and fees. The highway agency is assumed to constrain the time between MR&R activities to 10 years (and thus the pavement layer thicknesses depend on traffic loading). The results shown in the present paper still hold when one takes into account the shifts in trucker and shipper behavior: the assumed power has negligible effect on the MR&R total revenue, but there are important distributional effects. Furthermore, the MR&R total cost does not depend on the assumed power. Pricing leads to large decrease in annual traffic loading, despite the increase in distances traveled. As a result, pricing can lead to significant reductions in highway construction costs.

Another shortcoming of the present paper is that it uses a simplistic way to convert axle groups to equivalent single axles: a tandem (tridem) axle group that weighs W is replaced with two (three) single axles each weighing W/2 (W/3). Future research is needed in order to improve this conversion.

ACKNOWLEDGMENT

This research was funded by the University of California Transportation Center (UCTC), through a faculty research grant to the second author.

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TABLES

Table 1 – Annual traffic loading under the three assumed distributions

Assumed distribution	**L_p (DEF_p/year)** [a]
Uniform	$$L_p = \frac{R.\left(b^{p+1} - a^{p+1}\right)}{80^p.(b-a).(p+1)}$$ a=10 kN, b=90 kN
Symmetric triangular	$$L_p = \frac{4.R.\left(a^{p+2} + b^{p+2} - 2.\left(\frac{a+b}{2}\right)^{p+2}\right)}{80^p.(b-a)^2.(p+1).(p+2)}$$ a=10 kN, b=90 kN
Burr type XII [b]	$$L_p = \frac{R.k.s^p.\Gamma\left(k - \frac{p}{c}\right).\Gamma\left(\frac{p}{c} + 1\right)}{80^p.\Gamma(k+1)}$$ c=1.5, k=4, s=80

[a] Clearly, L_p= $E[w^p].R/80^p$. When p is an integer, the expectation $E[w^p]$ is the p^{th} moment of random variable w. Many textbooks provide expressions for the moments of distributions. These expressions should not be used without checking, especially that they are often derived assuming integer values. For this study, p is a positive real number.

[b] This can be written more compactly using the beta function (B), as follows: $L_p=R.k.s^p.B(k-p/c,\ p/c+1)/80^p$.

Table 2 – Price ratio under the three assumed distributions

Assumed distribution	**PRICE$_\alpha$/PRICE$_4$**
Uniform	$\frac{PRICE_\alpha}{PRICE_4}=\frac{(\alpha+1).(b^5-a^5)}{5.(b^{\alpha+1}-a^{\alpha+1})}.w^{\alpha-4}$ a=10 kN, b=90 kN
Symmetric triangular	$\frac{PRICE_\alpha}{PRICE_4}=\frac{(\alpha+1).(\alpha+2).\left(a^6+b^6-2.\left(\frac{a+b}{2}\right)^6\right)}{30.\left(a^{\alpha+2}+b^{\alpha+2}-2.\left(\frac{a+b}{2}\right)^{\alpha+2}\right)}.w^{\alpha-4}$ a=10 kN, b=90 kN
Burr type XII	$\frac{PRICE_\alpha}{PRICE_4}=\frac{s^{4-\alpha}.\Gamma\left(k-\frac{4}{c}\right).\Gamma\left(\frac{4}{c}+1\right)}{\Gamma\left(k-\frac{\alpha}{c}\right).\Gamma\left(\frac{\alpha}{c}+1\right)}.w^{\alpha-4}$ c=1.5, k=4, s=80. Γ is the gamma function.

Table 3 – Marginal cost ratio

	$MC_\alpha/MC_4 = L_4/L_\alpha$				
	α = 3	**α = 3.5**	**α = 4**	**α = 4.5**	**α = 5**
w ~ Uniform	0.90	0.95	1.00	1.04	1.07
w ~ Triangular	0.79	0.89	1.00	1.11	1.21
w ~ Burr	1.79	1.43	1.00	0.60	0.29

Table 4 – Annual repetitions of axle group types

Group (g)	Steering	Single_truck	Tandem	Tridem	Single_nontruck
Annual repetitions (R_g)	865,780	705,570	802,380	5,230	11,764,680

Table 5 – Annual traffic loading for Station 97

p	**L_p (DEF_p/year)**	**Contribution to L_p**				
		Steering	**Single_truck**	**Tandem**	**Tridem**	**Single_nontruck**
3	683,040	18.3%	26.4%	53.5%	0.4%	1.4%
3.5	596,969	16.3%	28.0%	54.8%	0.4%	0.5%
4	536,775	14.3%	29.3%	55.9%	0.3%	0.2%
4.5	492,833	12.6%	30.4%	56.6%	0.3%	0.1%
5	459,909	11.1%	31.5%	57.1%	0.3%	0.0%

Table 6 – Marginal cost ratios at Station 97

	$MC_\alpha/MC_4 = L_4/L_\alpha$				
	$\alpha = 3$	**$\alpha = 3.5$**	**$\alpha = 4$**	**$\alpha = 4.5$**	**$\alpha = 5$**
Station 97	0.79	0.90	1.00	1.09	1.17

Table 7 – Assumed values

Variable	**Description/units**	**Assumed**
r	Discount rate per annum	0.05
U	Overlay unit cost for four lanes ($/kilometer)	250,000
T	Time between two consecutive overlays (year)	5

FOOTNOTES

1. In order to take weathering into account, Small et al. (1989) use a specific deterioration model, and they derive the following: $T=X_o.\exp(-m.T)/L$, where m is an environmental coefficient (they use m=0.04), and X_o is the number of ESALs to failure under conditions of negligible weathering (i.e., when $L\rightarrow\infty$, so $T\rightarrow 0$). The actual number of ESALs to failure, $X=X_o.\exp(-m.T)$, depends on L (the smaller L, the smaller X).
2. The use of continuous discounting for annualizing V is consistent with its use for expressing V. Note that this approach diverges from some studies that have used a mixture of continuous discounting (for expressing V) and annual discounting (for annualizing V, i.e. r•V) (Lindberg, 2002; Small et al., 1989; Vitaliano and Held, 1990).

FIGURE CAPTIONS

Fig. No.	Caption
1	Indirect effect of traffic loading on MR&R cost
2	Future overlay costs
3	Assumed uniform probability density function for axle weight
4	Symmetric triangular probability density function for axle weight
5	Burr type XII probability density function for axle weight
6	Ratio of appropriate price to current price (uniform distribution)
7	Ratio of appropriate price to current price (triangular distribution)
8	Ratio of appropriate price to current price (Burr distribution)
9	Number-of-DEF ratio
10	Distributions of axle group weights (From Lu et al., 2002, p. H-141)
11	Ratio of appropriate price to current price at Station 97
12	Prices paid by two single axles

FIGURES

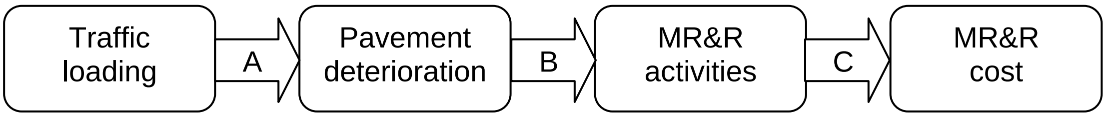

Figure 1 – Indirect effect of traffic loading on MR&R cost

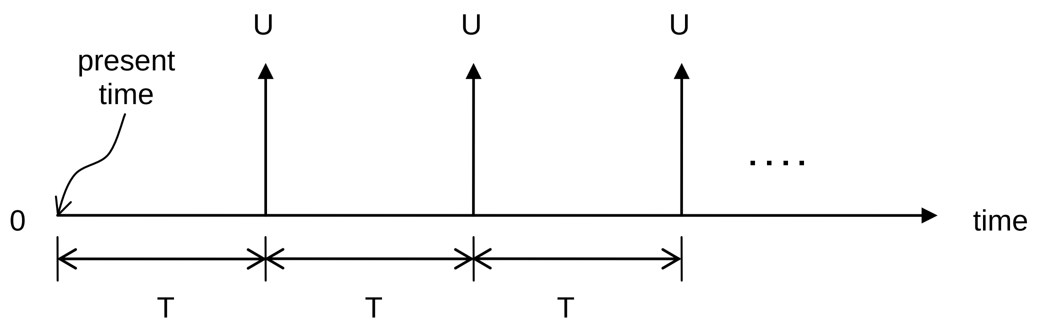

Figure 2 – Future overlay costs

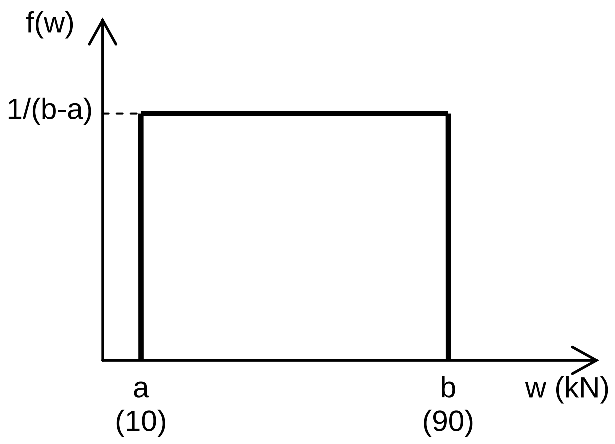

Figure 3 – Assumed uniform probability density function for axle weight

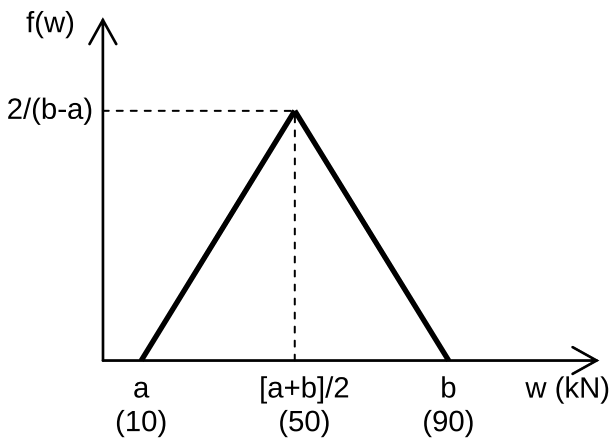

Figure 4 – Symmetric triangular probability density function for axle weight

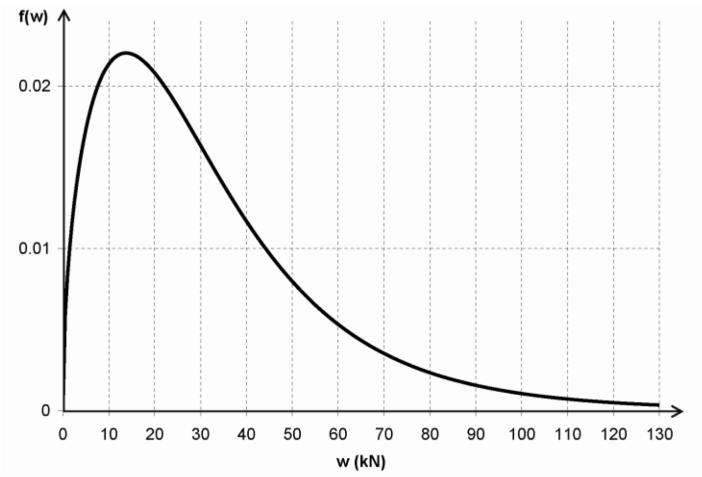

Figure 5 – Burr type XII probability density function for axle weight

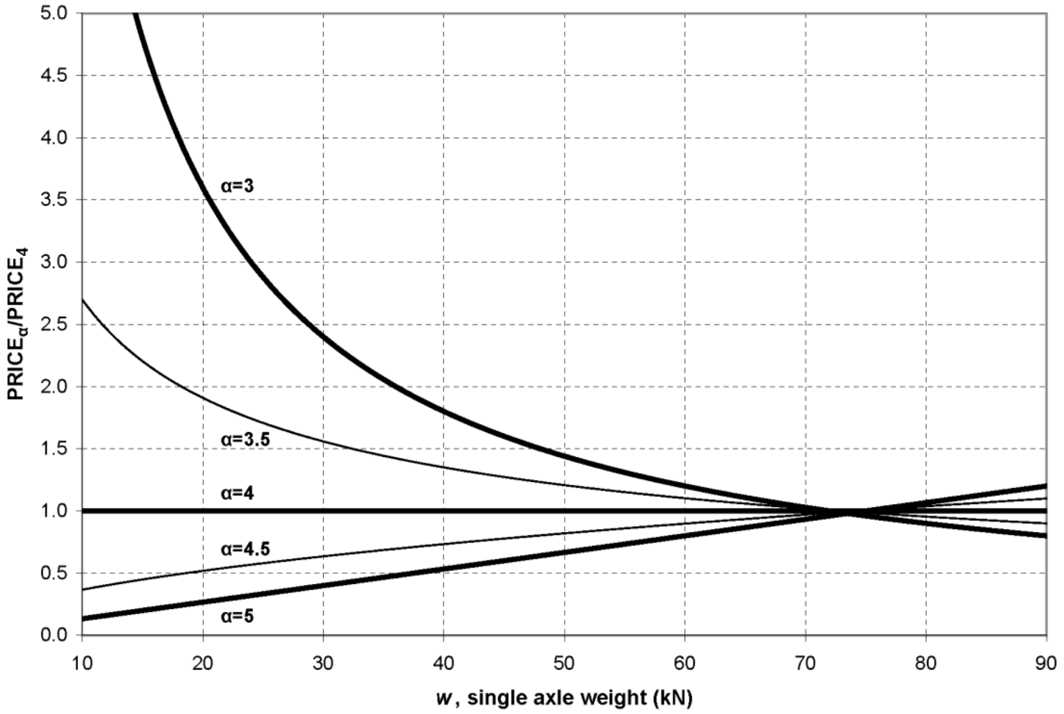

Figure 6 – Ratio of appropriate price to current price (uniform distribution)

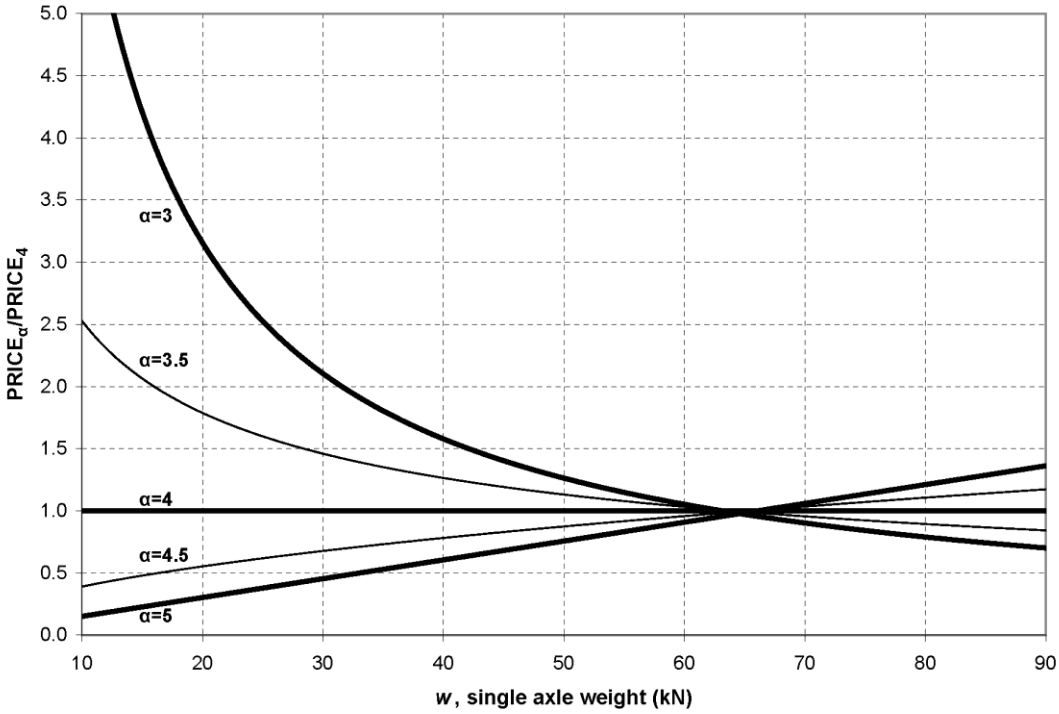

Figure 7 – Ratio of appropriate price to current price (triangular distribution)

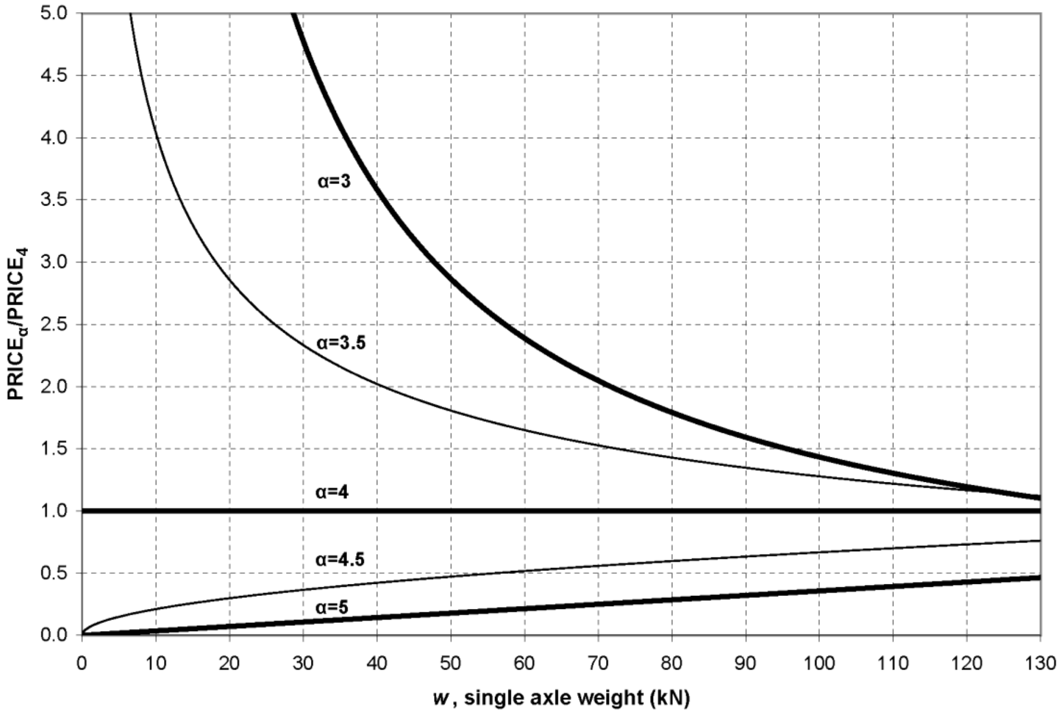

Figure 8 – Ratio of appropriate price to current price (Burr distribution)

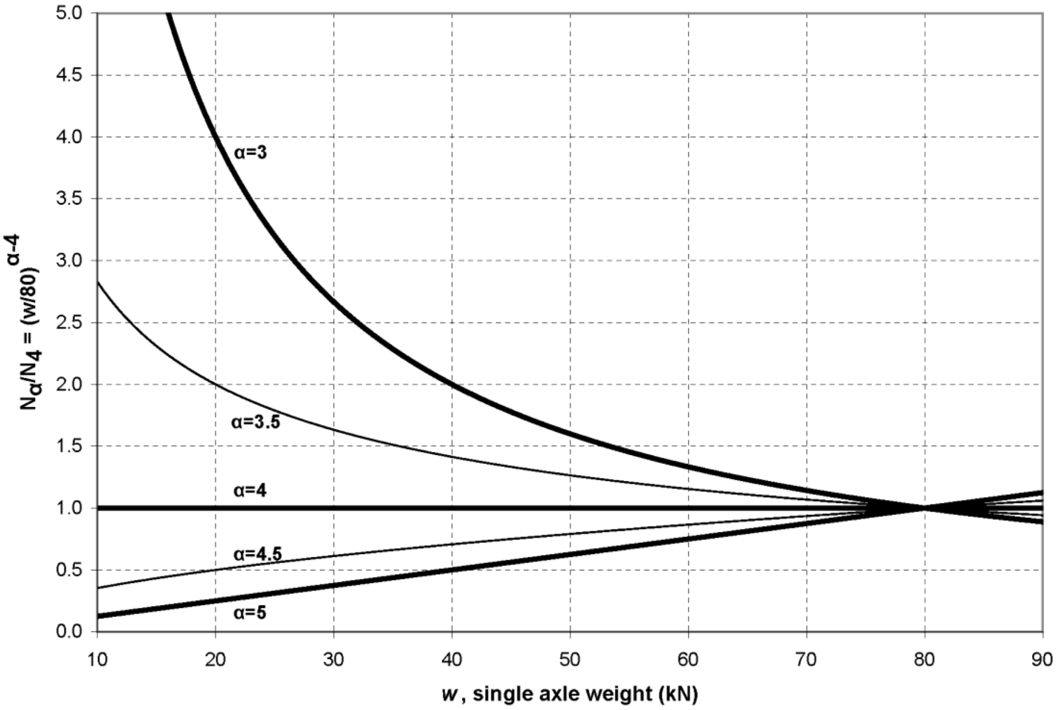

Figure 9 – Number-of-DEF ratio

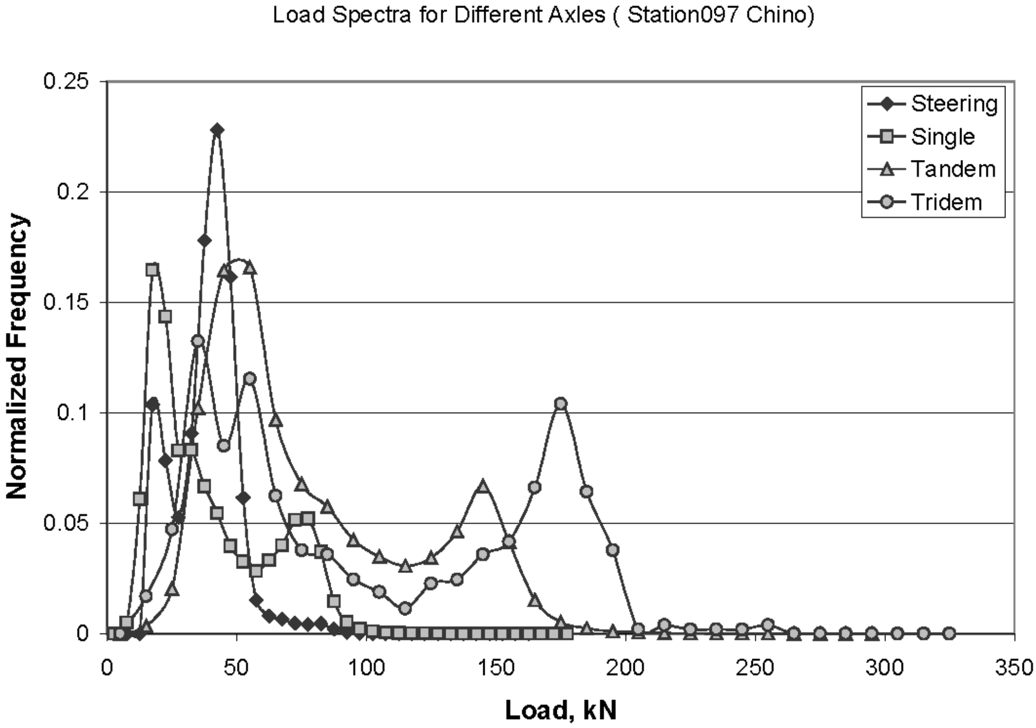

Figure 10 – Distributions of axle group weights (From Lu et al., 2002, p. H-141)

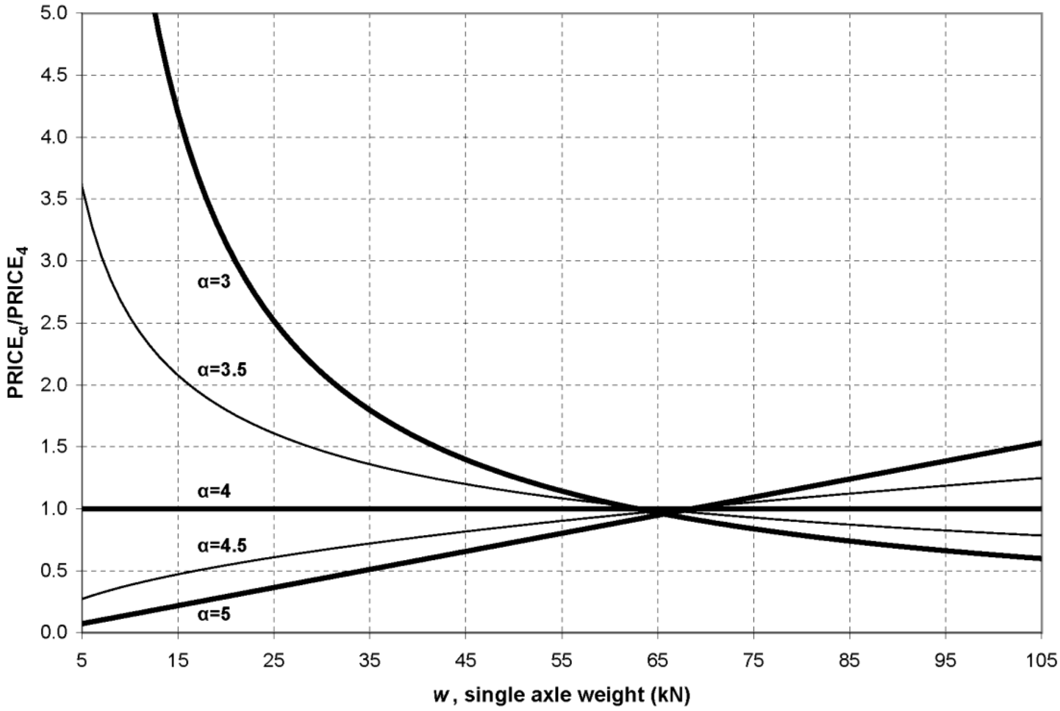

Figure 11 – Ratio of appropriate price to current price at Station 97

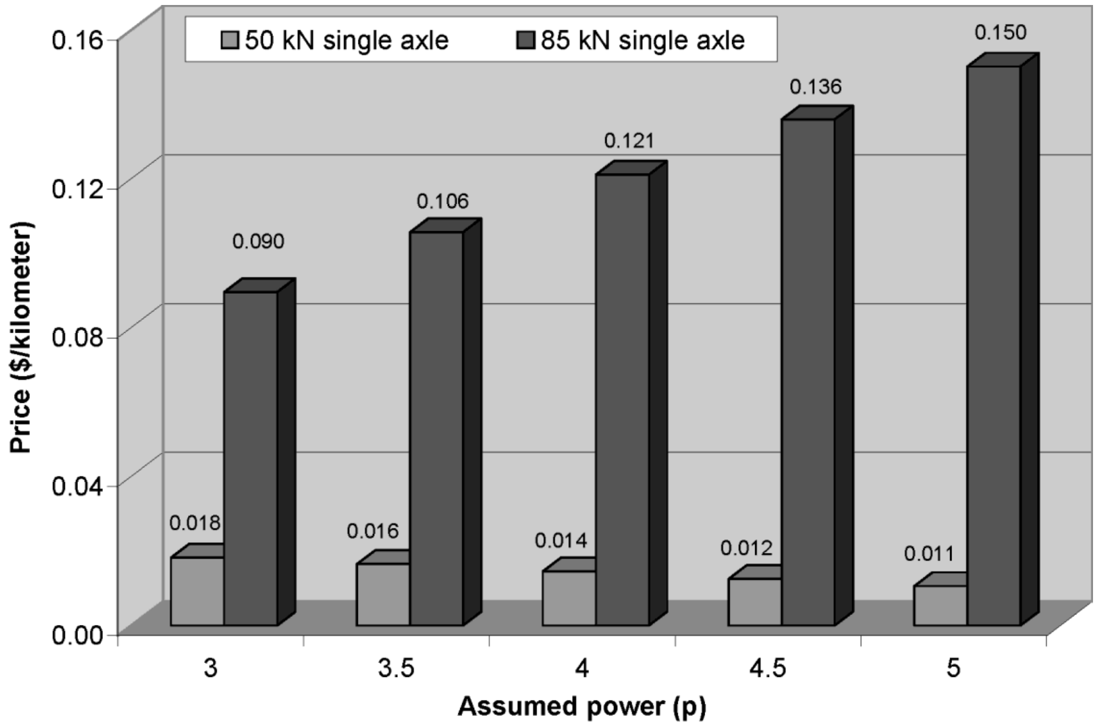

Figure 12 – Prices paid by two single axles