MEASUREMENT MODELS AND CONSISTENCY ANALYSIS OF STOP-START WAVES AT SIGNALIZED INTERSECTIONS

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ABSTRACT

Stop and start waves are two special cases of shock waves. According to the different speed-density relationships describing traffic flows, this paper presents various stop-start wave models at signalized intersections based on the shock wave model deduced from hydrodynamics. Then, this paper develops the measurement model of stop-start waves based on kinetic equations by analyzing the kinetic characteristics of neighbor vehicles in a platoon of vehicles in the process of propagation of stop-start waves. It is found that the two kinds of models can transform mutually by the relationships among traffic flow parameters. Subsequently, the theoretical consistency of stop and start waves is explained. Furthermore, the stop-start wave model based on kinetic equations is validated and the consistency of stop waves and start waves is analyzed by the survey data from some typical road segments in the city of Dalian, China. And the means of the observed stop-start wave velocities from the different survey locations are compared. Finally, the methods and steps for calibrating those parameters and processing those independent variables are remarked for the two kinds of stop-start wave models. The findings of this paper include: 1) stop-start waves at signalized intersections have the theoretical and observational consistency; 2) the stop wave is slower than the start wave; 3) the measurement method based on kinetic equations is effective; and

4) its measured independent variables can be gotten from detectors. Thus, the model can be applied in traffic engineering practice.

Key words: traffic engineering; traffic flow; stop-start waves; measurement model; kinetic equation

INTRODUCTION

The shock wave theory which originates from hydrodynamics is an important branch of traffic flow theory and describes the transition of traffic states. It was first applied to traffic flow and the solution of the conservation equation was proposed by Lighthill and Whitham (1955) and Richards (1956). The stop wave and the start wave are two special shock waves at signalized intersections. Some scholars applied the linear speed-density relationship suggested by Greenshields to embody the stop-start waves (Gerlough and Huber 1975), and some researchers presented the mathematical models to explain the experimental observations that showed the formation of stop-start waves (Pipes 1965; Gartner et al. 1996). Many academics used these models to investigate the formation and dissipation of queues on highways or freeways and at signalized intersections (Wang et al. 2002).

Recently, many scholars discussed the methods or techniques which can reproduce stop-start waves resulted from traffic jams. The typical achievements include the following. The microscopic traffic flow model based on the constant-time-headway policy and man-machine crossover model was designed and can reproduce the start and stop waves at intersections (Shi and Ziliaskopoulos 2002). The time-average sound level of waves which are formed in the process of vehicular start and stop was analyzed by applying program PROP9 and PROP10, respectively (Walerian et al. 2003; Walerian et al. 2005). The stochastic process generated from an ergodicity satisfying Markov chain was created to describe traffic flow states, including jam wave formation or dissipation and “stop and go” regimes (Sopasakis 2004). The optimal velocity car-following model was proposed for *n* cars on a circular single-lane road and can describe stop- and go-fronts between regions of uniformly flowing and stagnant traffic, namely stop and start waves (Orosz et al. 2005). A traffic microsimulator, namely, RoadSim which is capable of producing stop-start waves in traffic jams was developed (Artimy et al. 2005). The periodic traffic oscillations caused by network geometry was investigated by means of a microscopic simulation model Paramics and the stop- and go- waves can also be reproduced (Jin and Zhang 2005). The many-particle-inspired theory was proposed to analyze continuous and intermittent bottleneck flows and can describe stop-and-go wave phenomenon (Helbing et al. 2006). The nonlinear car-following model of highway traffic is considered and can reproduce spatial waves observed in real-world traffic flows (Helbing et al. 2006). These models reproduce the stop-start waves in traffic flow by adjusting the vehicular actions in order to more reliably describe vehicular motion or traffic flow. However, they cannot reveal the formation mechanism of stop-start waves.

On the other hand, some scholars found that the observed speeds of start-stop waves at signalized intersections were much less than their theoretical speeds obtained by the linear model (Yang et al. 2006; Yang et al. 2008). This is because the linear model did not suit to

describe the start-stop waves at signalized intersections where traffic flow was often in congested state. Thus, both the start wave model and the stop wave model were modified by means of the Greenberg model which was used to describe congested traffic flow by them. Moreover, the Greenberg start wave model was investigated and improved using the survey data collected in Changchun, China (Yang et al. 2006) and the Greenberg stop wave model was verified by the survey data from the urban expressway in Beijing, China (Yang et al. 2008). But the factor in the revised Greenberg start wave model has no explicit meaning.
To obtain a stop-start wave model that has the explicit physical meaning and can explain the stop-start waves, this paper analyzes the mechanism of shock waves based on kinetic equations and develops a uniform kinetic model of stop-start waves. In this paper, the author also compares the various stop-start wave models from hydrodynamics and kinetic equations and explains the relationship and difference between both of them. Furthermore, the consistency of stop and start waves is analyzed and validated by the survey data.

STOP-START WAVE MODEL FROM HYDRODYNAMICS

Shock wave model

A shock wave is a discontinuity of flow or density, and has the physical implication that cars change speeds abruptly without time to accelerate or decelerate. Fig.1 sketches a shock wave, where S represents a wave surface, u_{w} is the shock wave velocity, u_2 and k_2 are respectively the speed and density of traffic flow before the shock wave, u_1 and k_1 are respectively the speed and density of traffic flow behind the shock wave. The shock wave model is based on hydrodynamics (Lighthill and Whitham 1955; Richards 1956)

$$u_{\mathrm{w}} = \frac{u_2 k_2 - u_1 k_1}{k_2 - k_1} \qquad (1)$$

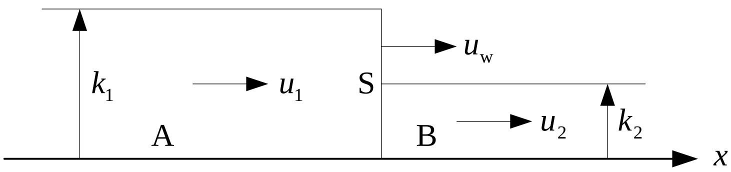

Fig.1 Sketch of shock wave

Stop-start wave model

Stop and start waves are two special cases of shock waves. For stop waves, traffic flow before a shock wave is in jammed state, namely $u_2 = 0$ and $k_2 = k_{\mathrm{j}}$. For start waves, traffic flow behind a shock wave is in jammed state, namely $u_1 = 0$ and $k_1 = k_{\mathrm{j}}$. Then, the stop and start wave models are respectively given as follows.

$$u_{\mathrm{wsp}} = -\frac{u_1 k_1}{k_{\mathrm{j}} - k_1} \qquad (2)$$

$$u_{\mathrm{wst}} = -\frac{u_2 k_2}{k_{\mathrm{j}} - k_2} \tag{3}$$

Where u_{wsp} and u_{wst} are respectively stop and start wave velocities and their units are *km/h*.

Eq.(2) and Eq.(3) are named the basic stop-start wave models (Yao 2007). In the model, the parameter k_{j} needs calibrating and the independent variables u and k need obtaining their values; and their units are *veh/km*, *km/h* and *veh/km*. Here u and k are speed and density when traffic flow is in running state, and k_{j} is jam density when traffic flow is in jammed state.

When traffic stream is in non-congested state, the Greensheilds model suits to describe the speed-density relationship. Then, Eq.(2) and Eq.(3) are transformed into

$$u_{\mathrm{wsp}} = -\frac{u_{\mathrm{f}} k_1}{k_{\mathrm{j}}} \tag{4}$$

$$u_{\mathrm{wst}} = -\frac{u_{\mathrm{f}} k_2}{k_{\mathrm{j}}} \tag{5}$$

Eq.(4) and Eq.(5) are named the Greenshields stop-start wave model (Gerlough and Huber 1975). In the model, the parameters u_{f} and k_{j} need calibrating and the independent variable k needs obtain its value; and their units are *km/h*, *veh/km* and *veh/km*. Here, u_{f} is the free-flow speed of traffic flow.

When traffic stream is in congested state, the Greenberg model suits to describe the speed-density relationship. Then, Eq.(2) and Eq.(3) are transformed into

$$u_{\mathrm{wsp}} = -\frac{u_{\mathrm{m}} k_1 \left(\ln k_{\mathrm{j}} - \ln k_1\right)}{k_{\mathrm{j}} - k_1} \tag{6}$$

$$u_{\mathrm{wst}} = -\frac{u_{\mathrm{m}} k_2 \left(\ln k_{\mathrm{j}} - \ln k_2\right)}{k_{\mathrm{j}} - k_2} \tag{7}$$

Eq.(6) and Eq.(7) are named the Greenberg stop-start waves model (Yang et al. 2006; Yang et al. 2008). In the model, the parameters u_{m} and k_{j} need calibrating and the independent variable k needs obtain its value; and their units are *km/h*, *veh/km* and *veh/km*. Here, u_{m} is the optimum speed corresponding to traffic at maximum flow rate.

STOP-START WAVE MODEL FROM KINETIC EQUATIONS

Stop wave model

To simplify the following explanation, the differences of individual vehicles are ignored and the spreading of a stop wave is investigated by analyzing the motion of neighbor vehicles. The propagation process of a stop wave between two neighbor vehicles can be described as in Fig.2. The distances from the nth and $(n+1)$th vehicles to the stop line are respectively x_n and x_{n+1} before the leader vehicle decelerates. Suppose that the nth vehicle at the speed of

u_{sp} at $t=0$ begins to decelerate with an average deceleration rate a and stops with the distance x'_n away from the stop line. The interval is assumed to be τ. Meanwhile, the ($n+1$)th vehicle repeats the same process but delays a time gap Δt and the distance away from the stop line is x'_{n+1} when it stops (Yao et al. 2007).

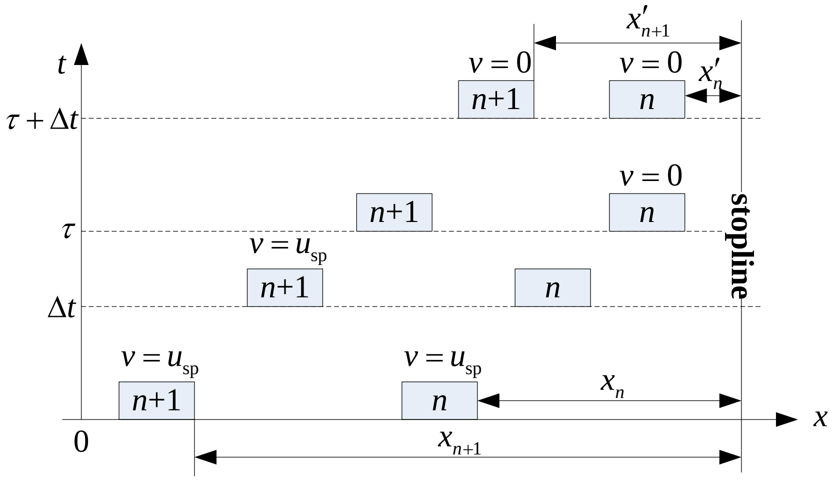

Fig.2 Propagation process of stop wave

According to the kinetic equation,

$$x_n - x'_n = 0.5a\tau^2 \tag{8}$$

$$x_{n+1} - x'_{n+1} = 0.5a\tau^2 + u_{\mathrm{sp}}\Delta t \tag{9}$$

Let Eq.(8) minus Eq.(9) and solving for Δt gives

$$\Delta t = \frac{x_{n+1} - x_n - x'_{n+1} + x'_n}{u_{\mathrm{sp}}} \tag{10}$$

According to the traffic flow theory, we have

$$x_{n+1} - x_n = \frac{1}{k} \tag{11}$$

$$x'_{n+1} - x'_n = \frac{1}{k_{\mathrm{j}}} \tag{12}$$

Substituting Eq.(11) and Eq.(12) into Eq.(10), yields

$$\Delta t = \frac{k_{\mathrm{j}} - k}{k_{\mathrm{j}} k u_{\mathrm{sp}}} \tag{13}$$

Let Δx denote the distance between the ($n+1$)th and n th vehicles after they stop and then the spreading distance of the stop wave in Δt is $-\Delta x$, where the minus sign means that the spreading direction of the stop wave is backward, then

$$-\Delta x = -\frac{1}{k_{\mathrm{j}}} \tag{14}$$

The spreading speed of the stop wave is

$$u_{\mathrm{wsp}} = -\frac{\Delta x}{\Delta t} \tag{15}$$

Substituting Eq.(13) and Eq.(14) into Eq.(15), gives

$$u_{\text{wsp}} = -\frac{u_{\text{sp}} k}{k_{\text{j}} - k} \quad (16)$$

Using $k = \frac{q}{u}$ and $q = \frac{1}{h}$, where k is density, q is volume, u is speed and h is headway, to rewrite Eq.(16), gives

$$u_{\text{wsp}} = -\frac{u_{\text{sp}}}{h_{\text{sp}} k_{\text{j}} u_{\text{sp}} - 1} \quad (17)$$

Eq.(17) is defined as the kinetic model of the stop wave. Here the units of u_{sp} and h_{sp} are *km/h* and *s*, respectively.

Start wave model

Similarly, the differences of individual vehicles are also ignored and the spreading of a start wave is also explained by analyzing the motion of neighbor vehicles. The propagation process of a start wave between two neighbor vehicles can be described as in Fig.3. The n th vehicle achieves the final running speed u_{st} with an average acceleration rate a in a time interval τ and the travel distance is x_n when it passes the stop line at time t_n. Meanwhile, the ($n+1$)th vehicle repeats the same process but delays a time gap Δt and the travel distance is x_{n+1} when it passes the stop line at time t_{n+1} (Qu et al. 2008).

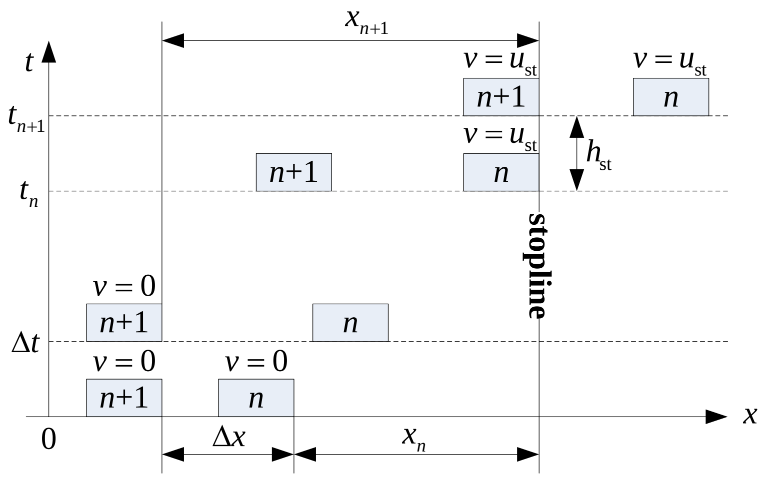

Fig.3 Propagation process of start wave

According to the kinetic equation,

$$x_n = 0.5a\tau^2 + u_{\text{st}}\left(t_n - \tau\right) \quad (18)$$

$$x_{n+1} = 0.5a\tau^2 + u_{\text{st}}\left(t_{n+1} - \tau - \Delta t\right) \quad (19)$$

Combining Eq.(18) and Eq.(19), generates

$$x_{n+1} - x_n = u_{\text{st}}\left(t_{n+1} - t_n - \Delta t\right) \quad (20)$$

The time gap of passing the stop line between the ($n+1$)th and n th vehicles is the saturated headway h_{st}, that is,

$$h_{\text{st}} = t_{n+1} - t_n \quad (21)$$

Let Δx denote the distance between the $n+1$th and nth vehicles before they start. The spreading distance of start wave in Δt is $-\Delta x$, where the minus sign means that the spreading direction of the start wave is backward, then

$$-\Delta x = -\left(x_{n+1} - x_n\right) \tag{22}$$

Substituting Eq.(21) and Eq.(22) into Eq.(20), yields

$$-\Delta x = -u_{\mathrm{st}}\left(h_{\mathrm{st}} - \Delta t\right) \tag{23}$$

From the traffic flow theory, Δx is the reciprocal of jam density k_{j}, namely

$$\Delta x = \frac{1}{k_{\mathrm{j}}} \tag{24}$$

Substituting Eq.(24) into Eq.(23), generates

$$\Delta t = h_{\mathrm{st}} - \frac{1}{k_{\mathrm{j}} u_{\mathrm{st}}} \tag{25}$$

Then, the spreading speed of the start wave is

$$u_{\mathrm{wst}} = -\frac{\Delta x}{\Delta t} \tag{26}$$

Substituting Eq.(24) and Eq.(25) into Eq.(26), gives

$$u_{\mathrm{wst}} = -\frac{u_{\mathrm{st}}}{h_{\mathrm{st}} k_{\mathrm{j}} u_{\mathrm{st}} - 1} \tag{27}$$

Eq.(27) is called the kinetic model of the start wave. Here the units of u_{st} and h_{st} are *km/h* and *s*, respectively.
Eq.(27) is similar to Eq.(17), so they are unified as the kinetic stop-start wave model.

MODEL COMPARISON AND CONSISTENCY ANALYSIS

Comparing Eq.(2) with Eq.(3), Eq.(4) with Eq.(5), Eq.(6) with Eq.(7) and Eq.(17) with Eq.(27) shows that the forms of the stop wave models are similar to ones of the start wave models. The directions of both stop and start waves are contrary to the running direction of a platoon. One side of wave surface of stop and start waves is running vehicular flow whereas the other side of wave surface of them is jammed vehicle flow. The wave velocity is relative to the jam density, the running speed and the headway of a platoon. The propagation process of stop waves is contrary to one of start waves. Therefore, the stop wave and the start wave have the consistent characteristics.
Comparing Eq.(2) with Eq.(16), they are identical. Similarly, Eq.(3) can be transformed into Eq.(27). It is shown that the stop-start wave models based on hydrodynamics and kinetic equations reach the same expressions for stop-start wave velocities although they are derived by means of two different theories.
But then, the stop-start wave models from hydrodynamics are not easy to be used because of the difficulty in obtaining the density when traffic flow is in running state. However, the stop-start wave model from kinetic equations can be applied in traffic engineering practice because the headway is easy to be obtained from detectors.

VALIDATION OF KINETIC STOP-START WAVE MODEL

Data collection

To validate the kinetic model and the consistency for stop-start waves, the following scheme is designed. A sketch of survey is given in Fig.4. Select a lane of an upstream road segment of a signalized intersection. Mark distance along the segment from the stop line to measure the position of the first and last stop vehicles for each signal cycle. Mark three profiles which are denoted as mark 1, mark 2 and mark 3 behind the stop line. The distances between mark 1 and the stop line, between mark 2 and mark 1 and between mark 3 and mark 2 are respectively D_1, D_2 and D_3. Set three observation stations. The first is a fixed one located between mark 1 and the stop line, which is used to calculate the final running speed and the saturated flow rate in the start wave model. The second is a mobile one located between mark 2 and the stop line, which is set for recording the stop times, stop locations and start times of both first and last stop vehicles and the number of observed stop vehicles in order to calculate the observed stop wave speed, start wave speed and jam density. The third one is a fixed one between mark 3 and mark 2, which is used to calculate the initial running speed and the headway in the stop wave model.

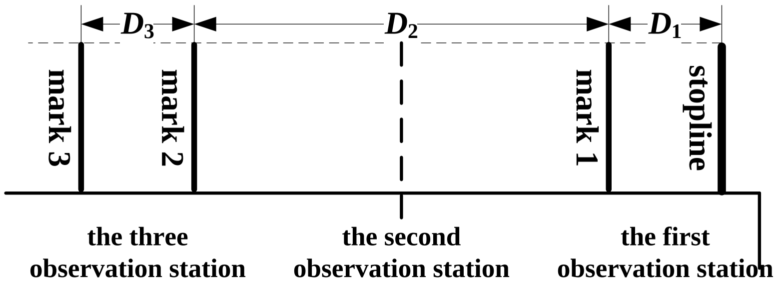

Fig.4 Survey sketch of stop-start waves

Results and analysis

According to the above survey scheme, traffic survey is carried out in the city of Dalian, China. We obtained 37 sets of effective data. The estimated and observed speeds for the stop-start waves are illustrated in Fig.5 and Fig.6. Table 1 lists the distribution of accumulative error of stop-start wave velocity.

From Fig.5, Fig.6 and Table 1, the relative errors of estimated and observed speeds of stop-start waves are acceptable. The percentage of these estimates whose relative errors are less than 30% is greater than 70% for the stop wave and 80% for the start wave. The precision for the start wave is slightly higher than one for the stop wave.

Furthermore, the collective means of the observed velocities of the stop-start waves from the different survey locations are listed in Table 2 in order to analyze the factors which have an effect on the stop-start waves. Many factors are considered here, such as survey location, survey time and sample amount.

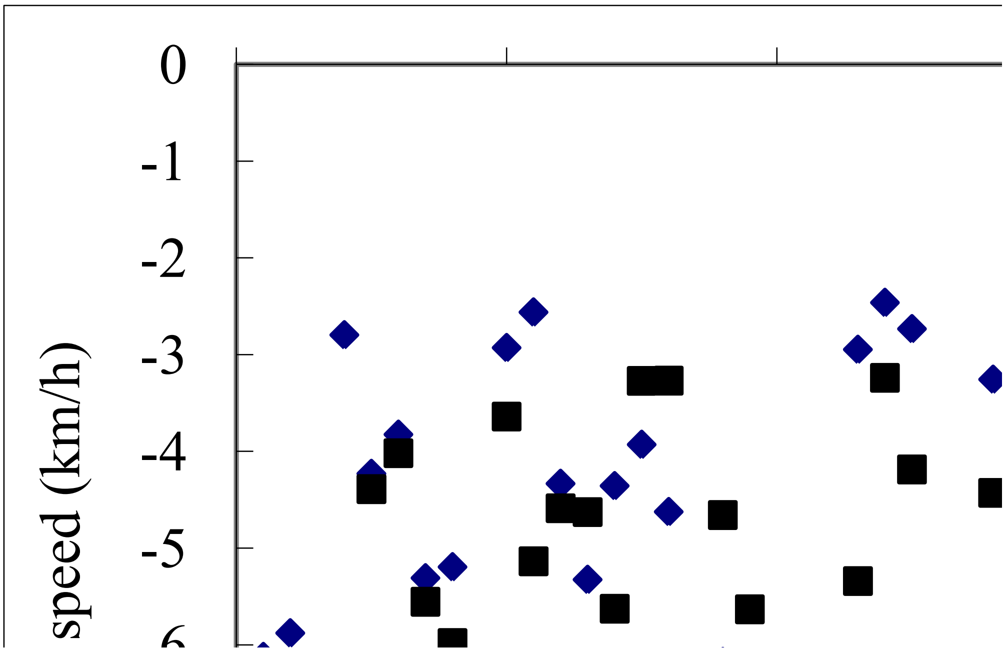

Fig.5 Estimated and observed stop waves speeds

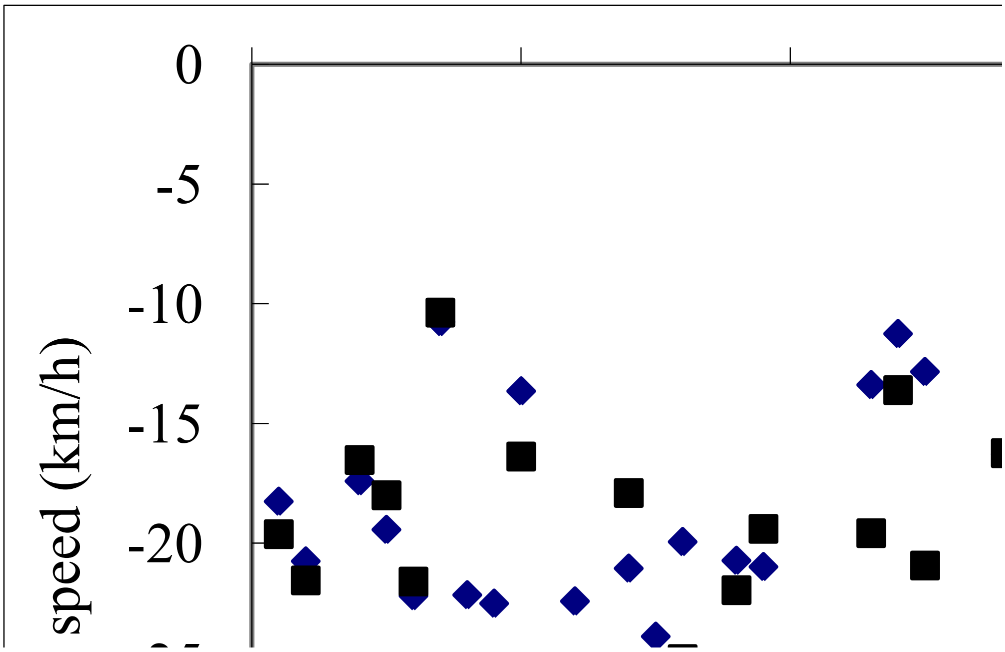

Fig.6 Estimated and start waves speeds

Table 1 Accumulative error distribution of velocity of stop-start waves

Relative error / Percentage	<5%	<10%	<15%	<20%	<25%	<30%
Stop wave	16.22%	24.32%	32.43%	48.65%	62.16%	70.27%
Start wave	27.03%	43.24%	56.76%	70.27%	78.38%	81.08%

Table 2 Mean of observed velocities for stop-start waves from different survey locations

Shock wave	Survey location: City	Intersection	Approach	Lane	Survey time	Sample amount	Mean of observed wave velocities (km/h)
Stop wave	Chang chun	Renmin street & Huimin road	South	Through	Evening Peak hour	31	-11.76
	Chang chun	Xinmin street & Longli road	West	Shared	Evening Peak hour	41	-16.33
	Dalian	Wuyi road & Yanhe road	North	Through	Afternoon non-peak hour	37	-4.64
Start wave	Chang chun	Renmin street & Chongqing road	South	Through	Evening Peak hour	35	-21.77
	Dalian	Wuyi road & Yanhe road	North	Through	Afternoon non-peak hour	37	-22.53

According to the data from Table 2 and the investigators' experience, the following results can be presented. Firstly, an important factor for stop waves which results in the error is that driver in the follower vehicle follows not only his leader vehicle but also the signals at intersection; and an important factor for start wave which results in the error is that the follower vehicle starts up before its leader vehicle or the follower vehicle and its leader start up at the same time. Secondly, the speed of a start wave is higher than that of a stop wave, and their difference is smaller when traffic is congested whereas larger when traffic is non-congested. Thirdly, traffic volume has a larger effect on a stop wave than a start wave. That is, the range of start wave speeds is narrow and mainly lies on road characteristics. However, the range of stop wave speeds is wide and lies on traffic volume except for road characteristics.

CONCLUDING REMARKS

The stop-start wave model from hydrodynamics need the calibration of u_{f}, u_{m} and k_{j} and the values of u and k. The parameters u_{f} and u_{m} can be calibrated and the value of the independent variable u can be gotten by setting two profiles through which vehicles run. They can directly be calibrated or obtained by means of detector data. The parameter k_{j} can be calibrated by the average spacing of neighbor vehicles in stopped state. The independent variable k can be obtained by setting two profiles of which the distance need be adequately long when vehicles are in running state. However, this variable is hardly difficult to be estimated and cannot directly be gotten from detectors. The detailed methods and steps of calibrating these parameters and processing these independent variables are introduced in the related references (Yang et al. 2006; Yang et al. 2008; Yao 2007).

The stop-start wave model from kinetic equations needs the calibration of k_j and the values of u_{sp}, h_{sp}, u_{st} and h_{st}. The independent variables u_{sp} and u_{st} can be obtained by setting two profiles through which vehicles run. The independent variables h_{sp} and h_{st} can be gotten by setting one profile through which vehicles run. The parameter k_j can be calibrated by the average spacing of neighbor vehicles in stopped state. The independent variables u_{sp} and h_{sp} can be obtained through the two profiles. The independent variables u_{st} and h_{st} can also be gotten through the two profiles. These four variables can directly be given from detectors. The detailed methods and steps of calibrating this parameter and processing these independent variables are presented in the related references (Yao et al. 2007; Qu et al. 2008).

In this paper, the following contents are given. Firstly, the stop-start wave models based on hydrodynamics are investigated and the new stop-start wave model is developed according to kinetic equations. Secondly, the stop-start wave models from the two different model systems are compared and the consistency between stop and start waves is analyzed. Thirdly, the scheme of data collection is designed, and the proposed model is validated and the factors which have an effect on stop-start waves are analyzed by the survey data from the cities of Dalian and Changchun, China. The results indicate that the updated stop-start wave model is reliable and the obvious advantage of this model is that the required parameter and independent variables can be given from detectors. Therefore, the model can be applied in traffic control and other practices.

The contributions of this paper are that the stop-start wave models from hydrodynamics and kinetic equations are investigated and compared, and the consistency between stop and start waves is analyzed theoretically and validated practically.

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