DIMENSIONING AND LOCATING URBAN FACILITIES - A SPATIAL AND TIMEWISE DECISION SUPPORT MODEL.

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ABSTRACT:

The objective of this paper is to present decision support model developed to help the urban planner to dimension and locate urban facilities, as well as to define their expansion phases. This model can be used in several areas such as school systems, hospitals, health centers, fire brigades, telephone centers, ambulance services, police patrol, etc. It is a practical system and it only requires information provided by municipal, state and federal departments of geography and statistics.

Keywords: Urban Management Systems, Urban Planning

INTRODUCTION

The most common questions formulated by urban planners are: How many facilities should be implemented? How big they should be? Where they should be located? Given the fact that the facilities are congested, when it would be appropriate to expand the system?

To answer these questions it is necessary to have a deep knowledge on how the system works, what is the population of each served area, how and what is the demand degree for these services in the region, how the demand levels will grow over time, etc. Moreover, it is necessary to understand the relationships among such variables, how they interact, and what variables present the greatest priority.

Since the demand for the services commonly increases over time, it is natural that after a period of time the available equipments cannot meet the demand adequately, in other words, are overburdened. The propose model aims at determining the time lapse after which the system should be reviewed, based upon the evolution of the service demand curve on the many districts of the area of study.

In order to define the time interval after which the size of the urban facilities should be reviewed as a result of the demand expansion, the model takes into account the mathematical forecasting function adjusted to each design area. Based on such forecasting functions, the computer model estimates the instant where the existing facility will not be able to handle the users with the pre-required service level or, in other words, when it will reach its saturation point.

The earliest saturation point obtained by analyzing the different districts will define the system review epoch. Taking this time instant, the demand function of each district will furnish the prospective load level of the individual facilities. With this information a new solution can be obtained from the model, getting, in advance, a new distribution of the facilities.

Considering the lack of statistical information and the difficulties in gathering data in developing countries such as Brazil, one important advantage of the proposed model is the easy way of introducing data and obtaining results. Thus, with the common information available from government agencies, the model described in this paper generates results reasonably close to the optimum.

SPATIAL ALLOCATION MODEL

For the execution of this project an adaptation of the p-median model was used, which, according to Drezner (1995) and Love (1988), has the advantage of possible reduction of costs associated with p facilities, which is very convenient in countries with low financial resources, like Brazil, where normally there is a limited number of equipments to serve all the potential users in the area of study. Moreover, only some units are available, which should be distributed in the best possible arrangement.

$$Z = min\left(\sum_{j=1}^{n}\sum_{i=1}^{m}d_{ij}x_{ij}a_{i}\right)$$
(1)

Subject to:

$$\sum_{j=1}^{n} y_j = p \tag{2}$$

Where:

 $y_i = 1$ if a vehicle (unit) is based on the location j, or 0 in other instances.

 a_i weight factor of cost to serve district i.

 x_{ii} = 1 if the facility in site j serves zone i, 0 in other instances.

 d_{ii} is the cost for covering zone i from the station j.

It means that the number p of units to be located is pre-arranged.

$$\sum_{j=1}^{n} x_{ij} = 1$$
i = 1, 2, ... *m* (3)

It guarantees that all districts will be provided by an emergency unit.

$$y_j \ge x_{ij}$$
 $i = 1, 2, ... m$ $j = 1, 2, ... n$ (4)

It assures that zone i can be served only by established units, which is, if xij = 1 so yj also must equal 1.

$$y_j, x_{ij} \in (0,1)$$
 $i = 1, 2, ... m$ $j = 1, 2, ... n$ (5)

Besides, if some district j, for some reason previously established, must count on a service installation, the correspondent y_j yj is set to 1; while, if it is not possible to locate the facility on that site, y_j is to equal zero.

On the other hand, according to Mirchandani and Reilly (1987), if a fixed cost f_j is established to place a facility in any site j and if the number of equipments to be installed is made to be dependent of the minimization of the units locating total cost, then it becomes necessary to add an additional term in the objective function replacing the restriction (2).

$$\sum_{j=1}^{m} f_j y_j \tag{6}$$

Another relation that can be drawn between the facilities implementation cost and the service use cost is adding a cost component in the objective function and removing the restriction of p facilities.

The devised problem is then:

$$.Z = min\left(\sum_{j=1}^{m}\sum_{i=1}^{n}d_{ij}x_{ij} + \sum_{j=1}^{m}f_{i}y_{j}\right)$$
(7)

Subject to the restrictions (3) to (5) where f_i is the settled cost to establish a facility in site j.

Another practical consideration may require that at least c of the existing facilities located in the set C of viable sites should remain in the new project. Then an additional restriction is needed.

$$\sum_{y_j \in C} y_j \ge c \tag{8}$$

In order not to obtain an optimal solution where the number of equipments to be located is overblown, the following restriction may be included, which will limit the maximum operation cost of the units that can be located in a zone.

$$x_{ij}d_{ij} \le d_i^*$$
 $i = 1, 2, ..., m; j = 1, 2, ..., n$ (9)

Where d_i^* is the maximum acceptable cost for service in zone i. The value of d_i^* can be specified by the system administrator.

According to Souza (2006), there are basically three methods to solve the problem, namely:

- Exhaustive enumeration;
- Mathematical programming and
- Heuristic approximations.

The exhaustive enumeration process requires the computation of the objective function to each possible combination of p in the n sites viable for the facilities implementation. However, the number of possible combinations of p in n sites may be very high, although not all possible sites for units' implementation are viable in view of the body of restrictions. Thus, the exhaustive enumeration method may require an excessive calculation volume. It is evident that the exhaustive search process grants a better equipment location arrangement, but in systems with a high number of districts and many units to be distributed, the number of possible combinations reaches such high figures that the processing time is so long it makes its utilization impracticable.

Approximations by the means of mathematical programming are not readily available for the type of devised in this article, because, on one hand, the objective function is non-linear, with complex development in view of the decision variables, and on the other, the number of restrictions and decision variables is too high, even related to problems of moderate size.

There are different heuristic approximations to solve the problem. An alternative that can be easily applicable to the model intended to be implemented in this project is the one proposed by Teitz and Bart (1968), known as "site-substitution approach". Essentially, this method starts with an initial set of elements, which are replaced one by one over time, exchanging the elements inside the current set for elements outside the current set. If the replacing of a given viable element for the facility installation reduces the objective function value, then the new site is included and the old site is cast off.

The Teitz and Bart procedure can be established as follows:

- 1. Select an initial set of *p* sites, known as set *P*;
- 2. Compute the objective function for the set *P*;
- 3. Choose any site *vb*, outside *P*;
- 4. For each site in *P*, replace *vb* and compute the value of the objective function for the new set of *p* sites.
- 5. Find the site *vk* in P so that, when it is replaced by *vb*, the reduction on the objective function value is larger than in any other of the p elements.
- 6. If a site that meets the criterion of step 5 is found, replace *vb* for *vk*, keeping this replacement from violating any of the restrictions. If there is no site, keep the set *P* and resume the procedure.
- 7. Select another element from outside the current set and that was not previously chosen as candidate for entering the set; repeat the steps 4 to 6.

- 8. When all sites outside *P* have been tried, define the resulting set of the existing sites as *P*' and repeat steps 2 to 7, using *P*' instead of P. A complete repetition of this series is called a cycle.
- 9. When a cycle from step 2 to step 8 results in no replacement, the procedure is finished.

Notice that the final set of elements depends on the initial chosen set. It is convenient, then, to run the procedure several times, each time using a new starting set, choosing the best final site set out of these many operations. Each starting site set can be generated randomly or be selected by a specialist, by the administrator or by a person who knows the system very well.

The model was developed to allow either exhaustive search, when all possible locating options are tested or heuristic search, in which an optimal solution is sought from a initial location set.

For higher model efficiency, some restrictions were considered:

- a) The number of units or equipments to be allocated is settled previously, depending on the availability of these equipments and the maximum response time accepted by the system to suit the users' solicitations.
- b) Every district is supplied by, at least, one service unit. The software also indicates by order, considering the criterion of shortest distance, which units are closest to the solicitation site if the service unit allocated to this district is busy answering to another request.
- c) Every district is supplied by, at least, one service unit located within a distance inferior to an acceptable and previously established maximum value.
- d) It should be taken into account the hypothesis that, regardless of the objective function processing, some districts must forcefully set up one service unit. This is the case, for instance, of schools, military quarters or units already installed, where the administration may understand that the political drawback of removing the installation can be larger than the general benefit of relocating it.
- e) In opposition to the previous restriction, also is foreseen the possibility of restricting the equipment installation allocation in certain district. It could happen, for example, in the cases where neighborhoods do not contain an available site to build an outpost to shelter the serving unit, or where neighborhoods are too far away from the point that gathers the largest number of potential users.

The second part of the allocation model is the equipments distribution per se among the eligible districts. Once the distances file is opened, the model requires to be informed the number of installations that will be dispersed and if the processing will take place either by the means of exhaustion method or heuristic process.

It is also possible to work only with a part of the region of study. If there is interest in assigning the equipments only to a part of the city, the model allows that the districts that do not belong to the area of interest are excluded.

TEMPORAL ALLOCATION MODEL

The development of the temporal allocation model aims at figuring the time lapse after which the system should be reviewed, based upon the growth of the demand for the services in the several districts of the region of study.

Since the beginning of the 1960's many quantitative studies related to systems and expansions capacity of the installation have been carried out. Manne (1961) presented a productive article in the field, whose outlines were deepened by Srinivasan (1967), Freidenfelds (1980), Luss (1982), Li and Tirupati (1994) and others. In a general way, the capacity expansion problems consider a projection curve of the demand in time, which should be attended to within a certain pre-defined service level (response quality). In some moments along the horizon of the project, a new facility is added to the system with capacity x.

Every district has its own demand function which indicates the district's occurrence growth tendency. This function is obtained from the calibration of a curve that best describes the demand development for the service which is intended to be implemented in a certain district.

The possibility of the existence of a region where the growth in demand for emergency services presents a deterministic behavior is very unlikely. The most likely and natural way is for the demand to have a random behavior with a growth tendency. Then, in this instance, it is necessary to take into account this randomness, also considering that the region will be divided in m districts, supplied by several emergency units. Thus, according to Souza (1996), if the demand has a linear but probabilistic growth, the problem of determination the time of the first passage can be solved by the following relation:

$$E[t_{\tau}] = \frac{D_{j}^{*} - D_{j}^{(0)}}{\mu_{j}}$$
(10)

Where:

 D_{j}^{*} represents the maximum demand to which the service unit based in j is capable of responding (critical demand level);

 $D_{j}^{\left(0
ight)}$ represents the current demand level and

 μ_{j} represents the demand mean linear growth.

When the demand presents exponential growth, the equation is:

$$D_{j}^{*} = D_{j}^{(0)} \exp\left[\mu_{j}' E[t_{j}]\right]$$
(11)

Being $E[t_{ij}]$ the expected time for the demand to reach the critical level D_j^* , taking to:

$$E[t_{j}] = \frac{\ln D_{j}^{*} - \ln D_{j}^{(0)}}{\mu_{j}'}$$
(12)

With variance given by:

$$Var[t_j] = \frac{\sigma_j^2}{\mu_j^2} E[t_j]$$
(13)

It becomes necessary, then, to define what is demand level D_j^{r} which represents the maximum demand to which the service unit based in the districts defined by the spatial allocation process has reached its overburden point and, consequently, is not able to attend to its influence area successfully.

The time lapse for the system review is considered as the time in which the first demand function of all the districts reaches a maximum value in which the service installation based on this district is not able to supply the demand of this region with a minimum service level, in other words, has reached its overburden point.

The use of a calibrated function is necessary because the demand curves may vary a lot in short time gaps, disqualifying the system to be considered as run by a deterministic function. Out of the regression curve calibration are also obtained the means and the standard deviation of the data sequence.

The temporal allocation software processing sequence is the following:

Based on the demand expansion calibrated function and on the variation coefficient, represented by the standard deviation divided by the means of the data sequence regarding the occurrences, it is established, in each of the districts, the critical demand D^* , to which there is a pre-determined possibility of queuing.

From the demand of the month base Do, the critical demand D^* and the demand growth geometric rate, it is determined the time lapse after which each district will reach the critical demand.

The time lapse that prints the moment of system review is obtained when the first out of all districts reaches its critical demand, so the critical demand of the area of study is determined based on the shortest time lapse after which any of its districts can no longer serve its inhabitants successfully.

CONCLUSION

The spatial allocation model, even though applying the heuristic procedure, always provides results that, when do not reproduce the optimal value of the objective function obtained through the exhaustion process, are very close to it, proving to be a sturdy and trustworthy system.

One of the largest advantages of the proposed model is the simplicity of data introduction and results access and analysis. Considering the lack of statistics and the difficulties of data gathering in developing countries such as Brazil, the software allows that, with the information available from government organizations, it may be applied, attaining results which approaches very much to the optimal.

The lack of methodologies which aim at tuning the equipments spatial and temporal distribution systems leads universities to create mechanisms for the improvement of these systems and present them to society who, as user, will decide if the proposed solution is desired.

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