The effects of neglecting users' costs on the spatial structure of public transport services

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Abstract.

It has been shown recently that the presence of a stringent financial constraint translates into an implicit reduction of users' time value in the design of public transport systems, inducing a less than optimal bus frequency and larger than optimal bus size. This conclusion was achieved with a microeconomic model for a single line only. When multiple lines are allowed in the context of an urban network, different spatial pattern of services can be envisioned, i.e. direct services (no transfers) or corridors (transfers are needed). The objective of this paper is to consider the spatial structure of services in the cost model, in order to study the impact of neglecting users' costs (i.e. their time). This is done through the analysis of four simple though illustrative networks. First, the optimal structure of lines is investigated, searching for the combination of lines, frequencies and vehicle sizes that minimize total costs for operator's and users'. Then the same problem is solved accounting for operators' costs only. The results show that, when all costs are accounted for, direct services are more likely to be the preferred outcome only when, for given time values, demand is sufficiently high. When only operators' costs are considered, the preferred outcome would be direct services under all circumstances, with lower frequencies. These results are explained in terms of fleet size requirements and both in-vehicle and waiting times associated to each objective.

1. INTRODUCTION

Imposing financial constraints in the design of a public transport service leads to a decrease in the relative weight of users' cost through the hidden reduction of their time values in the optimization problem, as shown by Jara-Díaz and Gschwender (2009) in their microeconomic analysis of a single transit line, as those time values get divided by one plus the multiplier of the constraint. This makes operators' cost weight more relative to users', causing lower frequencies and larger buses in comparison to the optimal values, as suggested by a recent case in Santiago de Chile. The extreme cases of zero and infinite values for the multiplier are equivalent to the minimization of total costs and operators' costs only (no users' time), respectively, which makes these limiting cases particularly interesting to study.

There is another important aspect of design that cannot be studied in a one-dimensional spatial environment and requires extension to a network: the spatial structure of the services. This can be done by extending the single line cost analysis to several lines in a network, introducing the choice between direct services – without transfers – and corridors where transshipments are necessary. This type of analysis was introduced by Jara-Diaz and Gschwender (2003b) as an extension of Mohring's (1972) and Jansson's (1984) single line models to find the frequency that minimizes social cost (users' plus operators'). The objective of this paper is to introduce the effect of the financial constraint into the analysis of the spatial structure of services, comparing the results of the two extreme cases identified above. The question is whether the optimal structure is or is not sensitive to the consideration of users' costs (time). To answer this we analyze four spatial demand structures on simple but representative networks, searching not only for frequencies but also for the lines structure and vehicle sizes that minimize a) total cost (users and operators) and b) operators' cost only. Results are comparatively presented, including service structures, fleet sizes needed, in-vehicle travel times and waiting times. It is shown that the best structure differs depending on the inclusion of users' costs in the objective and varies with the demand level.

The issue of service structure has been analyzed by Jara-Diaz and Basso (2003) in a three nodes network in relation with economies of spatial scope. Among other findings, they show that for the case of equal flows between each of the six origin-destination pairs and equal distances, direct services are less costly for an operator than a hub-and-spoke structure. Note that, although not strictly comparable, this type of discussion resembles that in air transport regarding the use of hubs (inducing transfers) versus fully connected networks (direct services; no transfers needed) for profit maximizing and socially optimal airlines. For example, Hendricks et al (1995) show that an unregulated airline might choose either structure depending on various elements including demand level. Using a simple network structure Brueckner (2004) shows that a monopolistic airline would be biased in favor of the hub-and-spoke structure and would choose lower than optimal frequencies and aircraft size. Pels et al (2000) conclude that "a fully connected network will be more profitable if the level of demand is relatively high, fixed costs are low and economies of density are low".

2. PROBLEM FORMULATION AND SOLUTION APROACH.

Following Jansson (1980, 1984), let us consider a circular public transport corridor of length *L* kilometers and operated at a frequency *f* with a fleet of *B* vehicles. The service is used by a total of *Y* passengers per hour homogeneously distributed over the corridor, all of them traveling a distance of *l* kilometers. Defining *T* as the time in motion of the vehicle in a cycle and *t* as the time that a passenger needs to board or alight, cycle time t_c is given by $t_c = T + 2t(Y/f)$. As frequency is the ratio between fleet size and cycle time (B/t_c), *B* can be written as

$$B = f\,T + 2tY. \tag{1}$$

The cost per vehicle-hour for the operator (*c*) can be written as $c = c_0 + c_1K$, where c_0 and c_1 are constants (Jansson, 1980, 1984; Jara-Díaz and Gschwender, 2003a). If the users' values of in-vehicle and waiting times are P_v and P_w respectively, then the total value of the resources consumed *VRC* per hour is

$$VRC_T = \left(fT + 2tY\right)\left(c_0 + c_1K\right) + P_w \frac{1}{2f}Y + P_v \frac{l}{L}\left(T + \frac{2tY}{f}\right)Y \ . \tag{2}$$

The first term in the right hand side corresponds to the expenditure of the operators; the second one is the total value of users' waiting time, assuming regular arrivals of users and vehicles; and the last term is the total value of users' in-vehicle time. As the service is assumed to have predetermined bus stops location, access time is not included in equation (2) as it can not be optimized. This expression shows that, *ceteris paribus*, increasing the frequency diminishes users expenditure, but at the same time increases operators' expenditure. Users' time decreases when frequency increases because both waiting and in-vehicle times decrease; this latter effect is due to fewer passengers boarding and alighting per vehicle.

Vehicle size *K* has to be enough to allow all passengers given by the load size *k* to travel inside the vehicle, i.e.

$$k(f) = \frac{Y}{f} \cdot \frac{l}{L} \leq K \ . \tag{3}$$

Optimal frequency f^* and optimal vehicle size K^* are obtained minimizing $VRC_T(f,k)$ in (2) subject to constraint (3), which will be always active as VRC_T does not decrease when K increases ($\partial VRC_T/\partial K$ is positive). Solving this problem we obtain

$$f^* = \sqrt{\frac{Y}{Tc_0}\left(\frac{1}{2}P_w + 2tY\frac{l}{L}\left(P_v + c_1\right)\right)} \ , \tag{4}$$

$$K^* = \frac{l}{L}\sqrt{Tc_0 Y\left(\frac{1}{2}P_w + 2tY\frac{l}{L}\left(P_v + c_1\right)\right)^{-1}} \ . \tag{5}$$

Optimal frequency follows the "square root formula" (Mohring, 1976; Jansson, 1980, 1984). Replacing (4) and (5) in (2) the total cost function C_T is obtained

$$C_T = 2tc_0Y + 2\sqrt{c_0TY\left(\frac{P_w}{2} + 2tY\frac{l}{L}\left(P_v + c_1\right)\right)} + TY\frac{l}{L}\left(P_v + c_1\right) \ . \tag{6}$$

This isolated-corridor framework can be extended to compare the total cost (operators and users) of different lines structures **for a given network and given passenger flows between specific origin-destination (OD) pairs**, noting that the total expenditure function can be optimized either on f or on the fleet size (B) equivalently. In the case of a network, it is necessary first to distribute the total vehicles among the lines with some optimality criterion. Afterwards B can be optimized in order to obtain the total cost function that allows the comparison. Therefore, the problem to be solved consists of **two stages, which have to be solved for each lines structure**. It is assumed that all line structures cover the same network and therefore do not affect access time, which is then irrelevant in the optimization. The two-stages framework that we present next is an extension of the one proposed by Jara-Díaz and Gschwender (2003b), which is now applied to a problem where operators cost depends on the vehicle size. Vehicle size is assumed to be the same for all vehicles in a line, but can vary among lines.

In the **fist stage** a given number of B vehicles has to be distributed in a vector $\vec{B}$ of fleets B_i among the different lines. To do so, waiting times (t_{wj}) and in-vehicle times (t_{vj}) associated to each OD pair j are expressed as functions of $\vec{B}$ and other parameters of the problem;

$$t_{wj} = t_{wj}\left(\vec{B}\right), \qquad t_{vj} = t_{vj}\left(\vec{B}\right) \ , \tag{7}$$

If Y_j is the passenger flow per hour in the pair j and Y their addition over all OD pairs, then the average waiting and in-vehicle times (over all OD pairs) are

$$\bar{t}_w = \frac{\sum_j t_{wj}\left(\vec{B}\right)Y_j}{Y} = \bar{t}_w\left(\vec{B}\right), \qquad \bar{t}_v = \frac{\sum_j t_{vj}\left(\vec{B}\right)Y_j}{Y} = \bar{t}_v\left(\vec{B}\right) \ , \tag{8}$$

Considering that $B = \sum_i B_i$ and $k_i \equiv \left(\lambda_i / f_i\right) \leq K_i$ (with λ_i the maximum flow in that line and f_i its frequency), the idea is to minimize on B_i the total value of the resources consumed

$$VRC_T = \sum_i B_i\left(c_0 + c_1K_i\right) + P_w\bar{t}_w\left(\vec{B}\right)Y + P_v\bar{t}_v\left(\vec{B}\right)Y \ , \tag{9}$$

In (9), $c(K_i) = c_0 + c_1K_i$ represents the cost per vehicle-hour (for the operator) of having one additional vehicle of size K_i in service. Given that every K_i only increases the total value of the resources consumed to be minimized, the capacity constraint will always be active, yielding a relationship that allows writing K_i as a function of f_i. On the other hand, f_i can be written as a function of B_i from equation (1) – valid for each line – so that an expression $K_i(B_i)$ exists. So, the total value of the resources consumed (9) is minimized on every B_i, yielding $B_i^* = B_i^*(B)$ from which the optimal average waiting and in-vehicle times as functions of B are obtained

$$\bar{t}_w^* = \bar{t}_w\left[\vec{B}^*(B)\right] = \bar{t}_w^*(B) \qquad , \qquad \bar{t}_v^* = \bar{t}_v\left[\vec{B}^*(B)\right] = \bar{t}_v^*(B) \quad . \tag{10}$$

The **second stage** consists in the minimization on B and K_i of the following total expenditure function:

$$VRC_T = \sum_i B_i^*(B)\cdot(c_0 + c_1K_i) + P_w\bar{t}_w^*(B)Y + P_v\bar{t}_v^*(B)Y \quad . \tag{11}$$

Given that $K_i = K_i\left(B_i^*\right)$ and $B_i^*(B)$, the expenditure function can be written as a function of only one variable (B) yielding $B^*(c, P_e, P_v, Y)$. Replacing the optimal fleet in (11), the total cost function for each lines structure

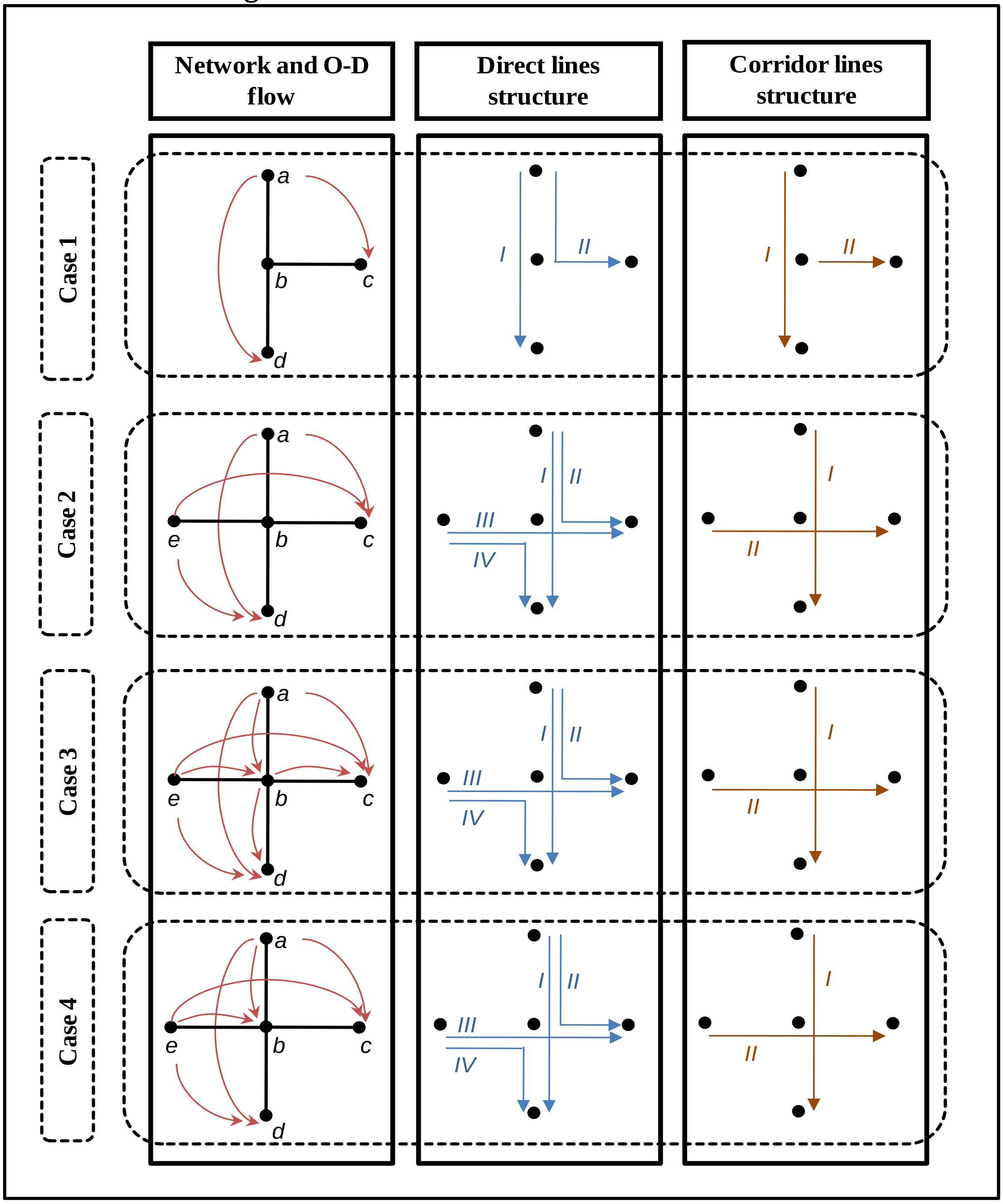
$$C_T = cB^*(c,P_e,P_v,Y) + P_e\bar{t}_e^*\left[B^*(c,P_e,P_v,Y)\right]Y + P_v\bar{t}_v^*\left[B^*(c,P_e,P_v,Y)\right]Y = C_T(c,P_e,P_v,Y), \tag{12}$$

is obtained, allowing the comparison to find out which structure requires a lower amount of resources. The optimal number of vehicles in each line is calculated using $B_i^* = B_i^*(B)$, which yields the optimal frequency (f_i*) from equation (1) and the optimal vehicle size (K_i*) from the maximum flow.

A similar approach can be followed to find the optimal spatial structure of lines omitting users' costs, the extreme effect of a financial constraint. In the next section we will apply this comparison method to the four simple networks shown in figure 1. In each case it is assumed that the total demand entering the system is Y passengers per hour distributed equally among the OD pairs, i.e. in case 1 there are Y/2 pas/h in each OD pair, Y/4 in case 2, Y/8 in case 3 and Y/6 in case 4. Two line structures that fulfill the demand requirements are presented for each network. The *direct lines* structure links every OD pair such that users need no transfers, whereas the *corridor lines* structure tries to minimize the total length of the lines, forcing transfers in some OD pairs.

Jara-Díaz and Gschwender (2003b) use the original version of this method minimizing operators' and users' cost with c and K fixed (c_1=0) for the first three cases of figure 1. They obtain a generic expression for the optimal fleet size that follows the square root formula with specific parameters for each case and structure. This parametric result is analytically obtained in all cases except the only asymmetric one (case 1, corridors, with line *I* longer than line *II*), where the distribution of the fleet among lines requires a numerical approximation. This approximation will also be needed in our analysis with c_1≠0 only in that same case, as it will still present asymmetry.

Figure 1. Direct and corridor lines structures.



3. OBTAINING OPTIMAL STRUCTURES: ANALYTICAL FRAMEWORK

3.1 Stage 1 for Case 4

We will apply the method described above to the new case 4. In the first stage, a given total fleet is distributed among the lines; as case 4 is symmetric for both direct and corridor lines, the fleet of each line as a function of the total fleet is $B_I = B_{II} = B_{III} = B_{IV} = B/4$ for direct lines and $B_I = B_{II} = B/2$ for corridors.

For the **direct lines structure in case 4,** $Y/6$ passengers per hour travel in each OD pair and have to be assigned to the lines. Line I receives $Y/6$ passengers traveling from a to d and $Y/12$

traveling from *a* to *b*. This second term corresponds to half of all passengers of the *a*-*b* pair, as they are assumed to be distributed between lines *I* and *II* proportional to their frequencies, and both frequencies are equal due to the symmetry of the problem. In total *Y*/4 passengers board line *I* per hour and, again because of symmetry, every line receives the same number of passengers. The number of passengers boarding and alighting from one vehicle in one cycle is the relation between *Y*/4 and the frequency of the line. Therefore, cycle time for each line *i* is

$$t_{ci} = 2T_0 + 2t \cdot \frac{Y/4}{f_i} \tag{13}$$

where $T_0/2$ is the vehicle travel time between two consecutive nodes of the network with no boarding or alighting, and *t* is the time that a user needs to board or alight. Frequency of line *i* is the ratio between the fleet of the line and its cycle time:

$$f_i = \frac{B/4}{2T_0 + 2t \cdot \frac{Y/4}{f_i}} \quad , \tag{14}$$

which yields

$$f_i = f_I = f_{II} = f_{III} = f_{IV} = \frac{B - 2tY}{8T_0} \quad . \tag{15}$$

Waiting time for passengers not traveling to *b* (superscript ~*b*), i.e. for OD pairs *a*-*d*, *a*-*c*, *e*-*c* and *e*-*d*, is:

$$t_w^{\sim b} = \frac{1}{2f_i} = \frac{4T_0}{B - 2tY} \tag{16}$$

For passengers with destination in *b* (superscript *b*), i.e. OD pairs *a*-*b* and *e*-*b*, frequency doubles as they can use two different lines in each case. Therefore, their waiting time is:

$$t_w^{b} = \frac{1}{2(2f_i)} = \frac{2T_0}{B - 2tY} \tag{17}$$

Using (15), (16), (17) and the corresponding amount of passengers, the average waiting time for direct lines as a function of *B* is

$$\overline{t_w} = \frac{1}{Y}\left(t_w^{\sim b} \cdot 4Y/6 + t_w^{b} \cdot 2Y/6\right) = \frac{5}{12f_i} = \frac{10/3 \cdot T_0}{B - 2tY} \tag{18}$$

Similarly, in-vehicle time for those who are not traveling to *b* is

$$t_v^{\sim b} = T_0 + t \cdot \frac{Y/12}{f_i} + \frac{t}{2} \cdot \frac{Y/6}{f_i} = T_0 + \frac{1}{6} \cdot \frac{tY}{f_i} \quad . \tag{19}$$

The expression in the middle has three terms. The first (defined earlier) corresponds to the vehicles in-motion travel time along two consecutive links; the second is the time passengers have to wait inside the vehicle while all passengers traveling to *b* (*Y*/12) alight; and the third term corresponds to the average time a passenger has to wait to get out while other passengers are descending (half of the time that all of them need to alight).

In-vehicle time for passengers traveling to *b*, i.e. OD pairs *a*-*b* and *e*-*b*, is

$$t_v^b = \frac{T_0}{2} + \frac{t}{2} \cdot \frac{Y/12}{f_i} = \frac{T_0}{2} + \frac{1}{24} \cdot \frac{tY}{f_i} \tag{20}$$

As there are 2*Y*/3 passengers (4 OD pairs) that do not have *b* as their destination and *Y*/3 (2 OD pairs) that are traveling to *b*, average in-vehicle time is

$$\overline{t_v} = \frac{1}{Y}\left(\frac{2Y}{3} t_v^{\sim b} + \frac{Y}{3} t_v^b\right) = \frac{5}{6} T_0 + \frac{1}{8} \cdot \frac{tY}{f_i} \tag{21}$$

Finally, using (15), average in-vehicle time for direct lines as a function of *B* is

$$\overline{t_v} = \frac{5}{6} T_0 + \frac{T_0 tY}{B - 2tY} \tag{22}$$

For the **corridor lines in case 4** cycle time is

$$t_{ci} = 2T_0 + 2t \cdot \frac{4Y/6}{f_i} \tag{23}$$

where 4*Y*/6 corresponds to the passengers that board every line in an hour. For example, passengers boarding line *I* are those from OD pairs *a-d*, *a-b*, *a-c* and *e-d*. Frequency is the ratio between the corresponding fleet and cycle time, i.e.

$$f_i = \frac{B/2}{2T_0 + 2t \cdot \dfrac{4Y/6}{f_i}} \tag{24}$$

which yields

$$f_i = f_I = f_{II} = \frac{3/4 \cdot B - 2tY}{3T_0} \tag{25}$$

Proceeding as in the case of direct lines, average waiting and in-vehicle times for corridor lines as functions of *B* are

$$\overline{t_w}^* = \frac{2T_0}{\dfrac{3}{4} B - 2tY} \tag{26}$$

$$\overline{t_v}^* = \frac{5}{6} T_0 + \frac{\dfrac{7}{6} T_0 tY}{\dfrac{3}{4} B - 2tY} \tag{27}$$

Note that the expressions for average waiting and in-vehicle times in equations (18), (22), (26) and (27) follow the same general form as those found in Jara-Díaz and Gschwender (2003b).

Let us now obtain an expression for the value of the resources consumed by the operators as a function of *B*. In the case of the **direct lines in case 4**, *Y*/4 passengers board each line per hour, which yields simultaneously the maximum load of each line. Due to the symmetry of the problem vehicles of all lines have the same size

$$K_I = K_{II} = K_{III} = K_{IV} = \frac{2T_0 Y}{B - 2tY} \tag{28}$$

Given that the total fleet is equally distributed among lines, the value of the resources consumed by the operators is

$$VRC_O = \sum_{i \in R} B_i \left(c_0 + c_1 K_i\right) = \sum_{i=I}^{IV} B/4 \cdot \left(c_0 + c_1 \cdot \frac{2T_0 Y}{B - 2tY}\right) = Bc_0 + \frac{2Bc_1 T_0 Y}{B - 2tY} \qquad (29)$$

In the case of **corridor lines**, the number of passengers boarding each line is 2*Y*/3. However, the maximum load of each line is *Y*/2. This maximum occurs in the link *a-b* for line *I* and *e-b* for line *II*. Using (25) and the maximum loads, an expression for the vehicle size of each line can be obtained, which again is the same for all lines:

$$K_I = K_{II} = \frac{\frac{3}{2} T_0 Y}{\frac{3}{4} B - 2tY} \qquad (30)$$

Proceeding in a similar way as in the case of direct lines, operators' expenditure as a function of *B* can be shown to be

$$VRC_O = Bc_0 + \frac{\frac{3}{2} Bc_1 T_0 Y}{\frac{3}{4} B - 2tY} \qquad (31)$$

which has the same form as equation (29).

Now we are able to obtain the total value of the resources consumed for case 4, using (29), (18) and (22) for direct lines and (31), (26) and (27) for corridor lines:

- **Total expenditure direct lines**

$$VRC_T^D = Bc_0 + \frac{2Bc_1 T_0 Y}{B - 2tY} + P_w \cdot \frac{10/3 \cdot T_0}{B - 2tY} \cdot Y + P_v \cdot \left(\frac{5}{6} \cdot T_0 + \frac{T_0 tY}{B - 2tY}\right) \cdot Y \qquad (32)$$

- **Total expenditure corridors**

$$VRC_T^C = Bc_0 + \frac{\frac{3}{2} Bc_1 T_0 Y}{\frac{3}{4} \cdot B - 2tY} + P_w \cdot \frac{2T_0}{\frac{3}{4} \cdot B - 2tY} \cdot Y + P_v \cdot \left(\frac{5}{6} \cdot T_0 + \frac{\frac{7}{6} \cdot T_0 tY}{\frac{3}{4} \cdot B - 2tY}\right) \cdot Y \qquad (33)$$

3.2 Generalization of Stage 1

Solving the first stage for the other three cases, the results are such that they can always be written as a general expression with parameters that are specific for each case, as occurred in Jara-Díaz and Gschwender (2003b). These general expressions for average waiting and in-vehicle times, resources consumed by the operators and total resources consumed are, respectively:

$$\overline{t_w}^* = \frac{\varphi_w T_0}{\delta B - 2tY} \qquad (34)$$

$$\overline{t_v}^* = \psi T_0 + \frac{\varphi_v T_0 tY}{\delta B - 2tY} \qquad (35)$$

$$VRC_O = B\left(c_0 + c_1\varphi_c \frac{\delta T_0 Y}{\delta B - 2tY}\right) \tag{36}$$

$$VRC_T = Bc_0 + \varphi_c c_1 \frac{\delta B T_0 Y}{\delta B - 2tY} + P_w \cdot \frac{\varphi_w T_0}{\delta B - 2tY} \cdot Y + P_v \cdot \left(\psi T_0 + \frac{\varphi_v T_0 tY}{\delta B - 2tY}\right) \cdot Y \tag{37}$$

The values of parameters ψ, δ, φ_w and φ_v corresponding to each case coincide with the ones found by Jara-Díaz and Gschwender (2003b) for the cases they studied (1, 2 y 3). This happens because the more detailed expression for the operators' expenditure does not affect waiting and in-vehicle times at this stage where B is given. In other words, parameters ψ, δ, φ_w and φ_v defined here exist only because of users' cost. In addition, the general form for the operators' expenditure adds a new parameter φ_c as presented in equations (36) and (37). The parameters that characterize each case and each lines structure are summarized in table 1.

Table 1: Value of the parameters for each case and line structure

Case	Structure	ψ	δ	φ_w	φ_v	φ_c	$1/\delta$	φ_w/δ	φ_v/δ	φ_c/δ
1	Direct	1	1	2	1	2	1	2	1	2
1	Corridor	1	2/3	3/2	5/4	9/4	3/2	9/4	1,875	3,375
2	Direct	1	1	4	1	2	1	4	1	2
2	Corridor	1	2/3	2	3/2	2	3/2	3	2,25	3
3	Direct	3/4	1	3	9/8	3/2	1	3	1,125	1,5
3	Corridor	3/4	4/5	2	3/2	3/2	5/4	5/2	1,875	1,875
4	Direct	5/6	1	10/3	1	2	1	10/3	1	2
4	Corridor	5/6	3/4	2	7/6	2	4/3	8/3	1,556	2,667

Finally, for a given fleet size B (first stage) the general equation for K_i (i.e. the generalized version of equations like 28 and 30) is

$$K_i = \frac{\eta_i T_0 Y}{\delta B - 2tY} \tag{38}$$

The values of η_i are shown in table 2. With only one exception, in every line structure and case all lines have the same vehicle size. The exception is the corridor lines structure in case 1, which is the only asymmetrical situation with lines of different lengths. The general expressions found are valid both for the new case 4 as for the other three cases originally developed in Jara-Díaz and Gschwender (2003b), which were now modified considering $c = c(K)$. As announced at the end of the previous section, only in case 1 with corridors an approximation for the fleet distribution is needed (see appendix 1).

Table 2: Values of parameter η

Case	Structure	η_I	η_{II}	η_{III}	η_{IV}
1	Direct	2	2	-	-
1	Corridor	2	1	-	-
2	Direct	2	2	2	2
2	Corridor	4/3	4/3	-	-
3	Direct	3/2	3/2	3/2	3/2
3	Corridor	6/5	6/5	-	-
4	Direct	2	2	2	2
4	Corridor	3/2	3/2	-	-

3.3 Stage 2 for all cases and two objectives.

In the second stage the total expenditure (37) is minimized on *B*, in order to compare total costs among both line structures. *B** is

$$B^* = 2tY\frac{1}{\delta} + \sqrt{\frac{T_0 Y}{c_0}\left(P_w\frac{\varphi_w}{\delta} + P_v\frac{\varphi_v}{\delta}tY + 2tYc_1\frac{\varphi_c}{\delta}\right)} \qquad (39)$$

By replacing (39) in (37) the total cost function for each lines structure *l* is obtained:

$$C_T^l = 2tYc_0\frac{1}{\delta^l} + 2\sqrt{T_0 Y c_0\left(P_w\frac{\varphi_w^l}{\delta^l} + P_v\frac{\varphi_v^l}{\delta^l}tY + 2tYc_1\frac{\varphi_c^l}{\delta^l}\right)} + T_0 Y\left(P_v\psi + c_1\varphi_c^l\right) \qquad (40)$$

As expected, this result generalizes the one obtained by Jara-Díaz and Gschwender (2003b) considering a fixed *c*, which is recovered when c_1=0 and $c_0 = c$.

The optimal vehicle size for line *i* in structure *l* is

$$K_{li}^* = \frac{\eta_i^l}{\delta^l}\cdot\sqrt{T_0 Y c_0\left(P_w\frac{\varphi_w^l}{\delta^l} + P_v\frac{\varphi_v^l}{\delta^l}tY + 2tYc_1\frac{\varphi_c^l}{\delta^l}\right)^{-1}} \qquad (41)$$

In order to compare the impacts of neglecting user time in the problem, expressions for the cost function and service variables have to be found minimizing only operators' expenses. This procedure yields the same results as imposing zero time values in the general results ($P_w = P_v = 0$). The fleet and vehicle size that minimize operators' expenses are

$$B^\bullet = Y\cdot\left[2t\frac{1}{\delta^l} + \sqrt{\frac{2T_0 tc_1}{c_0}\cdot\frac{\varphi_c{}^l}{\delta^l}}\right] \qquad (42)$$

$$K_{li}^\bullet = \eta_i{}^l\cdot\sqrt{\frac{T_0 c_0}{\varphi_c{}^l\delta^l\, 2tc_1}} \qquad (43)$$

The minimum operators' expense for each line structure *l* is

$$C_l^\bullet = Y\cdot\left(2tc_0\frac{1}{\delta^l} + 2\sqrt{2T_0 tc_0 c_1\frac{\varphi_c{}^l}{\delta^l}} + T_0 c_1\varphi_c{}^l\right) \qquad (44)$$

With these results we can find the best line structure under each objective - minimization of users' and operators' costs and minimization of operators' cost only – by comparing the analytical expressions found for direct and corridor lines for each case.

4. BEST STRUCTURE FOR DIFFERENT OBJECTIVES

4.1 Optimal structure considering total cost.

Using equation (40), i.e. the total cost function for each line structure considering both users' and operators' costs, the optimal structure can be found. As in Jara-Díaz and Gschwender (2003b), the numerical evaluation of (40) shows that the first term is negligible (see appendix 2) and can be ignored in the comparison. On the other hand, the last term of equation (40) in cases 2, 3 and 4 is equal for both direct and corridor lines. Therefore, in these cases the only relevant term for comparison is the square root. In case 1, however, the last term of (40) is always larger for corridor lines, implying that if the square root were also larger for corridor lines (as we will see it happens), direct lines would always have the lower total cost. So, the square root is the key relevant term in order to determine the optimal structure.

The comparison of the square root of (40) for direct (*D*) and corridor (*C*) lines yields that **the total cost of direct lines is lower when one of the following conditions hold**:

$$\frac{1}{tY} < \frac{P_v}{P_w} \cdot \frac{\left(\frac{\varphi_v^C}{\delta^C} - \frac{\varphi_v^D}{\delta^D}\right)}{\left(\frac{\varphi_w^D}{\delta^D} - \frac{\varphi_w^C}{\delta^C}\right)} + 2\frac{c_1}{P_w} \cdot \frac{\left(\frac{\varphi_c^C}{\delta^C} - \frac{\varphi_c^D}{\delta^D}\right)}{\left(\frac{\varphi_w^D}{\delta^D} - \frac{\varphi_w^C}{\delta^C}\right)} \quad \text{if} \quad \frac{\varphi_w^D}{\delta^D} > \frac{\varphi_w^C}{\delta^C} \tag{45}$$

$$\frac{1}{tY} > \frac{P_v}{P_w} \cdot \frac{\left(\frac{\varphi_v^C}{\delta^C} - \frac{\varphi_v^D}{\delta^D}\right)}{\left(\frac{\varphi_w^D}{\delta^D} - \frac{\varphi_w^C}{\delta^C}\right)} + 2\frac{c_1}{P_w} \cdot \frac{\left(\frac{\varphi_c^C}{\delta^C} - \frac{\varphi_c^D}{\delta^D}\right)}{\left(\frac{\varphi_w^D}{\delta^D} - \frac{\varphi_w^C}{\delta^C}\right)} \quad \text{if} \quad \frac{\varphi_w^D}{\delta^D} < \frac{\varphi_w^C}{\delta^C} \tag{46}$$

Note that conditions (45) and (46) collapse into those obtained by Jara-Díaz and Gschwender (2003b) when there is no consideration of the effect of vehicle size on the operators' cost, i.e. c_1=0.

From the values of the parameters in Table 1, the relevant condition in case 1 is (46). Given that the right hand of (46) is negative and the left hand is positive in this case, the condition is always fulfilled. In addition, the last term of (40) is always larger for corridor lines in this case (larger φ_c), as explained before. This implies that direct lines always have the lowest total cost in case 1, result that is consistent with what Jara-Díaz and Gschwender (2003b) found. In cases 2, 3 and 4 the ratio between φ_e and δ is larger for direct lines and the relevant condition is (45). Now the right hand is positive, and the conclusion is again consistent to what Jara-Díaz and Gschwender (2003b) found: as *tY* becomes larger or P_e/P_v or P_e/c_1 becomes lower, the probability of direct lines being the more convenient structure becomes larger.

Comparison of waiting and in-vehicle times

The intuition behind these results is related with waiting and in-vehicle times in each structure. Larger P_w values increase the probability of corridor lines being the best ones. This suggests that waiting time should be lower in corridor lines than in direct lines, in spite of the necessary transfers in the corridor structure. On the other hand, a larger P_v increases the probability of direct lines being the best ones, suggesting that in-vehicle time should be lower for that structure. Let us examine this.

Replacing the optimal fleet (39) in the general expressions for the waiting time (34) and in-vehicle time (35) yields:

$$\overline{t_w^*} = \frac{\varphi_w}{\delta}\sqrt{\frac{T_0 c_0}{Y}\left(P_w \frac{\varphi_w}{\delta} + P_v \frac{\varphi_v}{\delta} tY + 2tYc_1 \frac{\varphi_c}{\delta}\right)^{-1}} \tag{47}$$

$$\overline{t_v^*} = T_0\psi + \frac{\varphi_v t}{\delta}\sqrt{T_0 c_0 Y\left(P_w \frac{\varphi_w}{\delta} + P_v \frac{\varphi_v}{\delta} tY + 2tYc_1 \frac{\varphi_c}{\delta}\right)^{-1}} \tag{48}$$

Comparing waiting times (47) for direct and corridor lines, the following **condition has to be met for corridor lines to have the lowest waiting time**:

$$\frac{1}{tY} > \frac{P_v}{P_w}\cdot\frac{\left(\frac{\delta^D\varphi_v^D}{\left(\varphi_w^D\right)^2} - \frac{\delta^C\varphi_v^C}{\left(\varphi_w^C\right)^2}\right)}{\left(\frac{\delta^C}{\varphi_w^C} - \frac{\delta^D}{\varphi_w^D}\right)} + 2\frac{c_1}{P_w}\cdot\frac{\left(\frac{\delta^D\varphi_c^D}{\left(\varphi_w^D\right)^2} - \frac{\delta^C\varphi_c^C}{\left(\varphi_w^C\right)^2}\right)}{\left(\frac{\delta^C}{\varphi_w^C} - \frac{\delta^D}{\varphi_w^D}\right)} \quad \text{if} \quad \frac{\delta^C}{\varphi_w^C} > \frac{\delta^D}{\varphi_w^D} \tag{49}$$

$$\frac{1}{tY} < \frac{P_v}{P_w}\cdot\frac{\left(\frac{\delta^D\varphi_v^D}{\left(\varphi_w^D\right)^2} - \frac{\delta^C\varphi_v^C}{\left(\varphi_w^C\right)^2}\right)}{\left(\frac{\delta^C}{\varphi_w^C} - \frac{\delta^D}{\varphi_w^D}\right)} + 2\frac{c_1}{P_w}\cdot\frac{\left(\frac{\delta^D\varphi_c^D}{\left(\varphi_w^D\right)^2} - \frac{\delta^C\varphi_c^C}{\left(\varphi_w^C\right)^2}\right)}{\left(\frac{\delta^C}{\varphi_w^C} - \frac{\delta^D}{\varphi_w^D}\right)} \quad \text{if} \quad \frac{\delta^C}{\varphi_w^C} < \frac{\delta^D}{\varphi_w^D} \tag{50}$$

In cases 2, 3 and 4 condition (49) applies and, as the right hand side is always negative, the inequality holds always, which implies that **waiting time is always lower for corridor lines in cases 2, 3 and 4**. Therefore, in spite of the transfers needed in this structure, total waiting time is lower than in direct lines because the frequencies that passengers observe are higher.[1] This explains why an increase in the ratio P_w/P_v reduces the probability of direct lines being the best ones.

In case 1 condition (50) applies and the right hand is positive, implying that if tY increases or the ratios P_w/P_v or P_w/c_1 decrease, it is more likely for the waiting time to be lower in corridors. Nevertheless, this is irrelevant for the best structure, given that in case 1 direct lines have always the lower total cost, even if waiting time was higher. As we will discuss later, this happens because of a smaller and better used fleet, and a lower in-vehicle time in direct lines.

On the other hand, in-vehicle time is expected to be larger for corridor lines, because transfers imply a larger number of passengers boarding and alighting, increasing stop and cycle times. This can be analyzed comparing expression (48) for both line structures; as the first term $T_0\Psi$ is the same for both, only the second one is relevant for the comparison. **The condition that has to be met for in-vehicle time to be larger in corridors than in direct lines is**:

$$\frac{P_e}{tY} > \frac{P_v}{P_w}\cdot\frac{\left(\frac{\delta^D}{\varphi_v^D} - \frac{\delta^C}{\varphi_v^C}\right)}{\left(\frac{\delta^C\varphi_w^C}{\left(\varphi_v^C\right)^2} - \frac{\delta^D\varphi_w^D}{\left(\varphi_v^D\right)^2}\right)} + 2\frac{c_1}{P_w}\cdot\frac{\left(\frac{\delta^D\varphi_c^D}{\left(\varphi_v^D\right)^2} - \frac{\delta^C\varphi_c^C}{\left(\varphi_v^C\right)^2}\right)}{\left(\frac{\delta^C\varphi_w^C}{\left(\varphi_v^C\right)^2} - \frac{\delta^D\varphi_w^D}{\left(\varphi_v^D\right)^2}\right)} \tag{51}$$

[1] It is worth noting that in our model transfers only produce additional waiting time. Neither the negative perception of transfers nor additional walking time is considered.

Using the adequate parameter values, the right hand of (51) results negative in all cases, implying that the condition is always met. This confirms that **in-vehicle time is always larger in corridor lines** for the four cases. This analysis shows that as the product tY gets larger (implying more boarding and alighting times) in-vehicle time increases in corridor lines and the probability of direct lines being the best ones increases in cases 2, 3 and 4.

<u>Comparison of fleet and vehicle size</u>

Conditions represented by equations (45) and (46) show that the value of c_1 is only relevant when the former applies, i.e. in cases 2, 3 and 4 (remember that in case 1 the latter applies and the right hand is always negative). As c_1 increases, it becomes more likely for direct lines to be the best structure. Nevertheless, as we will se later, while the fleet of direct lines increases with c_1, the vehicle size for that structure diminishes. Thus, the behavior of $\Sigma B_i \cdot K_i$ as c_1 increases is not clear, which does not allow to obtain a precise conclusion about the role of c_1 in the selection of the best lines structure.

The analysis of the **fleet size** in each structure is similar to the one made in the comparison of the total cost. In equation (39) the first term is negligible with respect to the second one and therefore only the square root is relevant in the comparison, yielding the same conditions described in (45) and (46). So, **the conclusion in the fleet size comparison are the same obtained in the total cost comparison**, i.e. in cases 2, 3 and 4 as tY becomes larger or P_w/P_v or P_w/c_1 becomes lower, the probability of direct lines having the smallest fleet size increases. In case 1 direct lines will always have the smallest fleet size.

In cases 2, 3 and 4 all lines within a given structure have the same **vehicle size**, which permits a clean comparison. From equation (41) the following **condition for direct lines having larger vehicles than corridors in cases 2, 3 and 4** is obtained:

$$\frac{1}{tY} < \frac{P_v}{P_w} \cdot \frac{\left(\frac{\varphi_v^D \delta^D}{\left(\eta_i^D\right)^2} - \frac{\varphi_v^C \delta^C}{\left(\eta_i^C\right)^2} \right)}{\left(\frac{\varphi_w^C \delta^C}{\left(\eta_i^C\right)^2} - \frac{\varphi_w^D \delta^D}{\left(\eta_i^D\right)^2} \right)} + 2\frac{c_1}{P_w} \cdot \frac{\left(\frac{\varphi_c^D \delta^D}{\left(\eta_i^D\right)^2} - \frac{\varphi_c^C \delta^C}{\left(\eta_i^C\right)^2} \right)}{\left(\frac{\varphi_w^C \delta^C}{\left(\eta_i^C\right)^2} - \frac{\varphi_w^D \delta^D}{\left(\eta_i^D\right)^2} \right)} \qquad (52)$$

As both sides of the inequality are positive, it can be concluded that in cases 2, 3 and 4 increasing tY or decreasing P_w/P_v or P_w/c_1, increases the probability of direct lines having larger vehicles than corridor lines.

In case 1 all direct lines have the same vehicle size, but there are different vehicle sizes in lines *I* and *II* of the corridors structure. The **conditions for direct lines having larger vehicles than lines *I* and *II* of corridors structure in case 1, are (53) and (54) respectively**.

$$\frac{1}{tY} < \frac{P_v}{P_w} \cdot \frac{\left(\frac{\varphi_v^D \delta^D}{\left(\eta_i^D\right)^2} - \frac{\varphi_v^C \delta^C}{\left(\eta_I^C\right)^2} \right)}{\left(\frac{\varphi_w^C \delta^C}{\left(\eta_I^C\right)^2} - \frac{\varphi_w^D \delta^D}{\left(\eta_i^D\right)^2} \right)} + 2\frac{c_1}{P_w} \cdot \frac{\left(\frac{\varphi_c^D \delta^D}{\left(\eta_i^D\right)^2} - \frac{\varphi_c^C \delta^C}{\left(\eta_I^C\right)^2} \right)}{\left(\frac{\varphi_w^C \delta^C}{\left(\eta_I^C\right)^2} - \frac{\varphi_w^D \delta^D}{\left(\eta_i^D\right)^2} \right)} \qquad (53)$$

$$\frac{1}{tY} > \frac{P_v}{P_w} \cdot \frac{\left(\frac{\varphi_v^D \delta^D}{\left(\eta_i^D\right)^2} - \frac{\varphi_v^C \delta^C}{\left(\eta_{II}^C\right)^2} \right)}{\left(\frac{\varphi_w^C \delta^C}{\left(\eta_{II}^C\right)^2} - \frac{\varphi_w^D \delta^D}{\left(\eta_i^D\right)^2} \right)} + 2\frac{c_1}{P_w} \cdot \frac{\left(\frac{\varphi_c^D \delta^D}{\left(\eta_i^D\right)^2} - \frac{\varphi_c^C \delta^C}{\left(\eta_{II}^C\right)^2} \right)}{\left(\frac{\varphi_w^C \delta^C}{\left(\eta_{II}^C\right)^2} - \frac{\varphi_w^D \delta^D}{\left(\eta_i^D\right)^2} \right)} \quad (54)$$

Expression (53) corresponds to the condition for direct lines having larger vehicles than line *I* of corridor lines. As the right hand is negative, the condition is never fulfilled, i.e. direct lines have always smaller vehicles than line *I* of the corridors structure. On the other hand, expression (54) is the condition for direct lines having larger vehicles than line *II* of corridor lines. As the right hand is negative, the condition is always fulfilled, i.e. direct lines have always larger vehicles than line *II* of the corridors structure, exactly the opposite that occurred with line *I*.

Summary of the comparisons

Table 3 summarizes the results obtained in the comparisons.

Table 3: Summary of the analysis with operators' and users' cost

Case	**Result**
1	$C_T^D < C_T^C$
	$B_D^* < B_C^*$
	$K_D^* < K_{IC}^*$
	$K_D^* > K_{IIC}^*$
	$t_{wD}^* > t_{wC}^*$
	$t_{vD}^* < t_{vC}^*$
2, 3 and 4	C_T, B and K depend on tY, P_w/P_v and P_w/c_1 [a]
	$t_{wD}^* > t_{wC}^*$
	$t_{vD}^* < t_{vC}^*$

(a) The probability of $B_D < B_C$, $C_T^D < C_T^C$ and $K_D > K_C$ increases with tY and decreases with P_w/P_v and P_w/c_1.

4.2 Best structure considering operators' costs only.

As seen in the previous section, the omission of users' cost (the effect of a stringent financial constraint) yields a change in the value of the design variables and in the best structure. The structure that minimizes only operators' cost can be found comparing expression (44) for both line structures. This comparison shows that **direct lines are always better for operators than corridors**: as can be seen in table 1, the values of $1/\delta$, φ_c/δ y φ_c are lower or equal for direct lines in all cases.

Expression (42) gives the fleet size that minimizes operators' cost; there, $1/\delta$ and φ_c/δ are the only parameters that depend on the line structure and these are always lower for direct lines. This implies that **the fleet size is always lower for direct lines when only operators' cost is taken into account**. Regarding vehicle size, the expression to analyze is (43), where the relevant parameters are η_i, φ_c and δ. **In cases 2, 3 and 4 the result is that vehicle size is always larger for direct lines when only operators' cost is considered**. In case 1 vehicle size in line *II* of corridors is the lowest and that in line *I* is the largest, while the single vehicle size in direct lines is in between, which replicates the result obtained for the total cost minimization analysis.

Replacing the fleet size expression (42) –for operators' cost minimization – in the general waiting and in-vehicle time equations (34) and (35), the following expressions are obtained:

$$\overline{t_w^\bullet} = \frac{1}{Y}\sqrt{\frac{T_0 c_0}{2tc_1}\cdot\frac{\varphi_w^2}{\delta\varphi_c}} \tag{55}$$

$$\overline{t_v^\bullet} = T_0\psi + \sqrt{\frac{T_0 c_0 t}{2c_1}\cdot\frac{\varphi_v^2}{\delta\varphi_c}} \tag{56}$$

Comparing the relevant parameters we can conclude that **when only operators' cost are minimized, waiting time is always lower in corridors, whereas in-vehicle time is always lower in direct lines**. All previous results are summarized in table 4.

$$C_l^\bullet = Y\cdot\left(2tc_0\frac{1}{\delta^l} + 2\sqrt{2T_0 tc_0 c_1\frac{\varphi_c^{\ l}}{\delta^l}} + T_0 c_1 \varphi_c^{\ l}\right)$$

Table 4: Summary of the analysis with operators' cost only

Case	Result
1, 2, 3 and 4	$C_D^\bullet < C_C^\bullet$
	$B_D^\bullet < B_C^\bullet$
	$t_{wD}^\bullet > t_{wC}^\bullet$
	$t_{vD}^\bullet < t_{vC}^\bullet$
1	$K_D^\bullet < K_{IC}^\bullet$
	$K_D^\bullet > K_{IIC}^\bullet$
2, 3 and 4	$K_D^\bullet > K_C^\bullet$

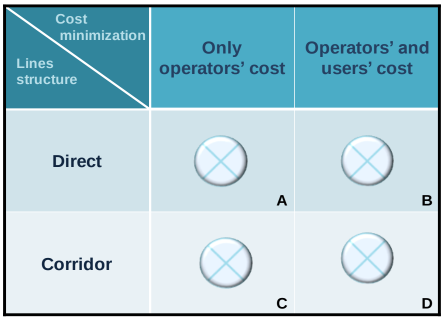
5. THE IMPACT ON THE DESIGN OF NEGLECTING USERS' COST

5.1 Identification of the scenarios to be compared.

In order to analyze the impact of neglecting users' cost we show in figure 2 the four scenarios that result from the combination of the two objective functions and the choice of lines structures. In the previous section only “vertical” comparisons between scenarios were made, i.e. the comparison between direct and corridor lines for the same type of cost minimization (scenario **B** against **D** for users' and operators' cost minimization, and scenario **A** against **C** for the minimization that only considers operators' cost).

We will now analyze the impact on costs and on the service variables that occurs when moving in each case from the optimal scenario – corresponding to the best lines structure when users' and operators' cost is minimized - to the relevant scenarios in which only operators' cost is minimized. As the optimal scenario depends in general on the demand level, it is necessary to identify ranges for which it is relevant to make the comparison. This will be easier after we define a *critical demand* in the next paragraph.

Figure 2: Scenarios for the comparison



In the previous section we saw that when operators' and users' cost is minimized, in case 1 the best structure is always direct lines, while in cases 2, 3 and 4 the best structure depends on the demand, boarding/alighting time, the values of time and the marginal cost of the vehicle size. Essentially, the larger the product tY, the higher the probability of direct lines being the best ones. The same happens the lower the reason between the values of waiting and in-vehicle times, and the lower the ratio between the value of waiting time and the marginal cost of the vehicle size. Therefore, there is a **critical demand (Y_c)** above which direct lines are the best structure. Using the data in table 5 the values of Y_c shown in table 6 were obtained. For demands larger than 3,200 pas/hr, direct lines are always the best. On the other hand, if demand is lower than 2,100 pas/hr, corridors will be the best structure.

Table 5: Values of the parameters for the simulation

Parameter	Value	Unit
c_0	10.65	US$/hr
c_1	0.203	US$/hr
t	2.5	Sec
T_0	2.72	Hr
P_w	4.44	US$/hr
P_v	1.48	US$/hr

Table 6: Critical demand

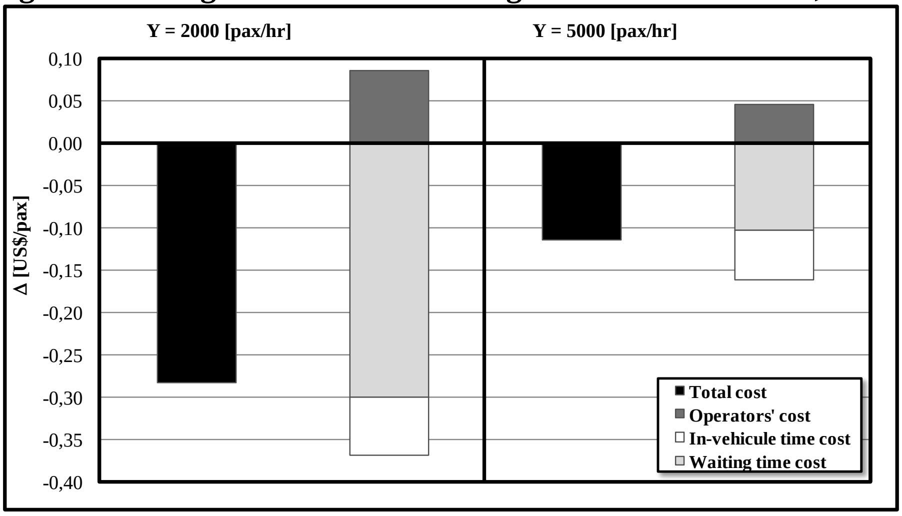
Case	Y_c (pas/hr)
2	2,374
3	2,184
4	3,195

The critical demand depends on the values of time. For example, if a unique value of 1.48 US$/hr were used for both waiting and in-vehicle time, the critical demand – over which direct lines are more convenient – would decrease to 791 and 1,064 pas/hr for cases 2 and 4 respectively.

5.2 Comparison of costs in Case 1.

In case 1 direct lines are more convenient in both cost minimizations (see tables 3 and 4). Therefore, the optimal scenario in this case is **B** and the only comparison that makes sense is with scenario **A** (horizontal comparison in figure 2). Figure 3 – constructed using the values of table 5 – shows the savings per passenger and the losses that occur when moving from scenario **B** to **A**, for two values of *Y*. Users lose both in waiting and in-vehicle times, whereas operators have a slight gain, yielding an important total loss, as expected given the objective functions used in both scenarios.

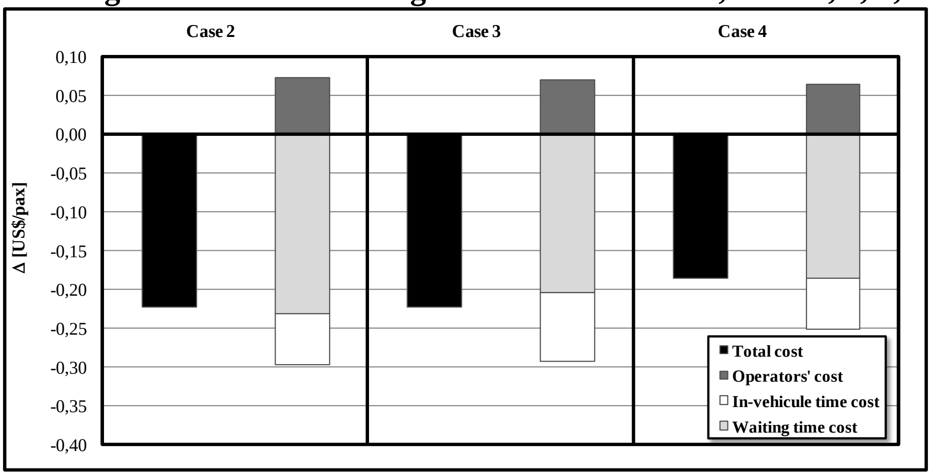
Figure 3: Savings - losses when moving from scenario B to A, case 1



5.3 Comparison of costs in Cases 2, 3 and 4 with a demand larger than the critical one.

In table 6 the critical demands for cases 2, 3 and 4 were calculated, over which direct lines are the best ones for users' and operators' cost minimization, i.e. scenario **B**. Given that operators' cost minimization yields that direct lines are always more convenient (see table 4), the only comparison that makes sense is with scenario **A** (horizontal comparison in figure 2). Figure 4 shows the savings and losses that occur when moving from scenario **B** to **A** in cases 2, 3 and 4, for *Y* = 5,000 pas/hr, using the values of table 5. Similarly to case 1, users suffer an important loss in comparison to the small gain obtained by the operators, yielding a considerable total loss mainly explained in all cases by the increase in waiting times.

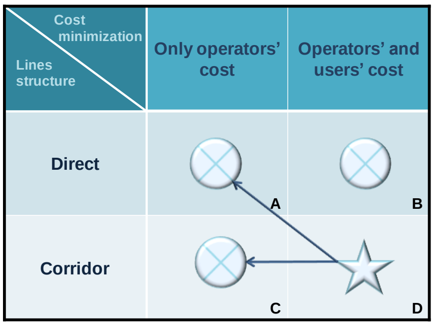
Figure 4: Savings - losses when moving from scenario B to A, cases 2, 3, 4; *Y* = 5,000



5.4 Comparison of costs in Cases 2, 3 and 4 below the critical demand.

For a demand level lower than the critical demand (e.g. $Y = 2{,}000$ pas/hr, according to table 6), corridors are the best structure when minimizing users' and operators' cost, i.e. scenario **D**. Given that in the operators' cost minimization direct lines always win (see table 4), there are two scenarios that are of interest to be compared with **D**: scenario **C** – in which the structure is maintained but only operators' cost is minimized – and scenario **A** – in which the structure is changed into the one that minimizes operators' cost, i.e. direct lines. The comparisons are shown in figure 5.

Figure 5: Comparison of scenarios in cases 2, 3 and 4 with low demand



Figures 6 and 7 show the savings and losses that occur when moving from scenario **D** to **A** and from **D** to **C**, respectively, in cases 2, 3, and 4 for $Y = 2{,}000$ pas/hr using the values of table 5. Figure 6 shows that when moving from **D** to **A** users suffer a considerable loss mainly in the waiting time. This is consistent with the fact that – as we already showed – waiting time is larger in direct lines (**A**) than in corridors (**D**). Again, operators' gains are lower than users' losses, yielding a total loss.

Figure 6: Savings - losses when moving from scenario D to A, cases 2, 3, 4; *Y* = 2,000

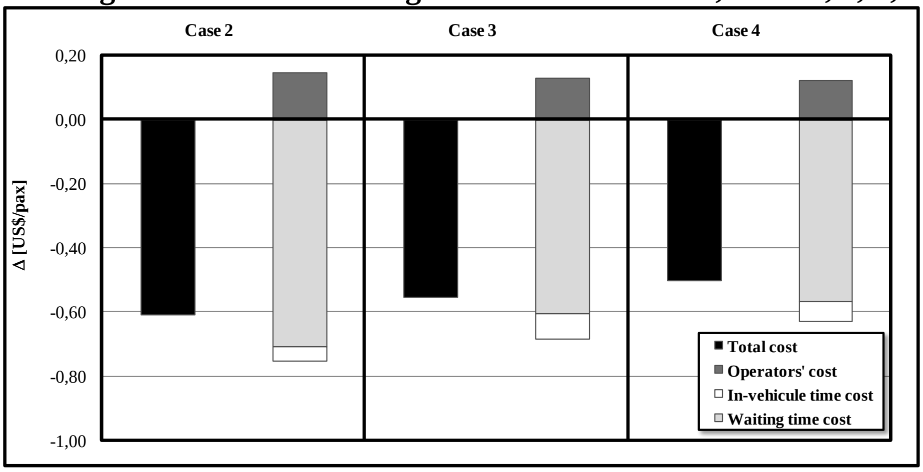
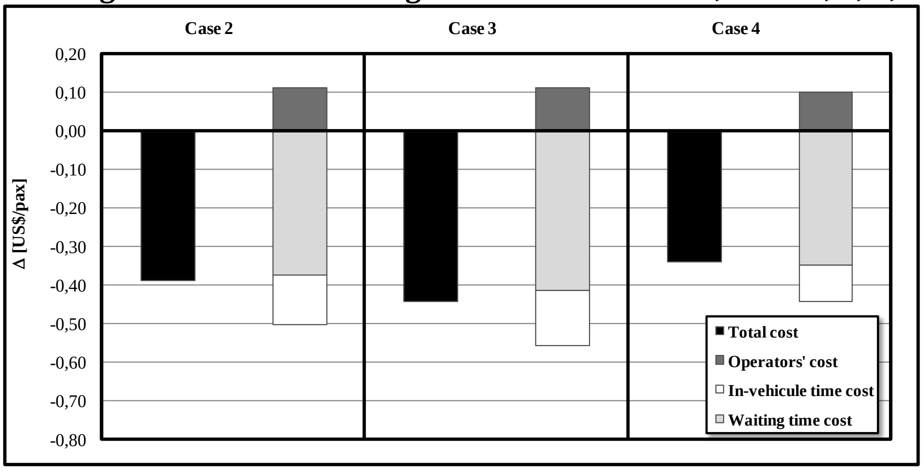


Figure 7 shows a qualitatively similar situation to the one analyzed in section 5.3. When moving from scenario **D** to **C**, users suffer a loss in waiting and in-vehicle times, whereas operators obtain a gain. It is worth noting that although operators do not change to their preferred scenario (**A**), they do have a gain because of the objective function that is optimized.

Figure 7: Savings - losses when moving from scenario D to C, cases 2, 3, 4; *Y* = 2,000



5.5 Comparison of service variables.

There are large differences between the **fleet size** obtained minimizing total cost and the one obtained minimizing operators' cost only. In the former case the large fleet obtained produces high frequencies and low waiting times, benefiting users. In the latter case, the fleet is considerable smaller, serving the demand with lower frequencies given that waiting and in-vehicle times do no matter. On the other hand, the **vehicles size** obtained minimizing operators' and users' cost is much smaller than the one obtained when users' cost is ignored. To illustrate this, figures 8 and 9 show the fleet and vehicle size for different demand levels for case 4 with direct lines, using the values of table 5. Important differences were also obtained when comparing **waiting and in-vehicle times** between both objective functions, as shown in figures 3 to 7.

Figure 8: Fleet size in case 4 – Direct lines structure

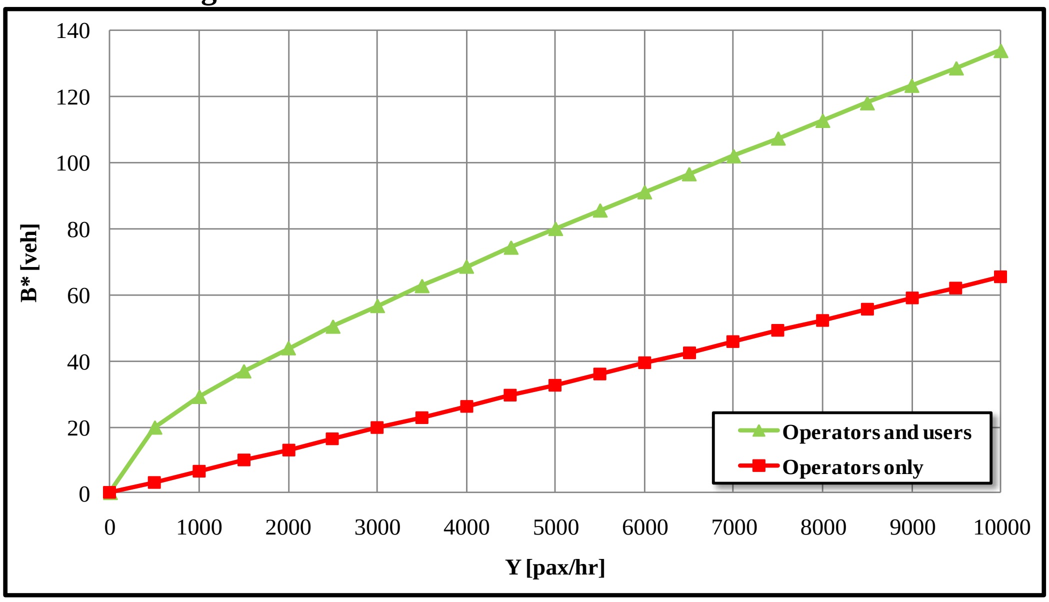
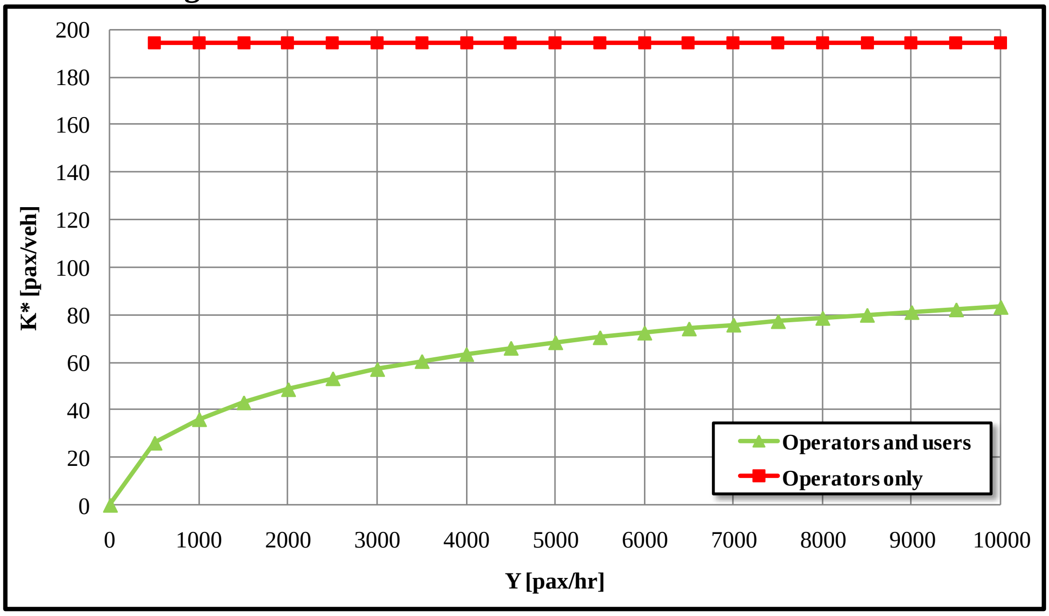


Figure 9: Vehicle size in case 4 – Direct lines structure



6. CONCLUSIONS

In this paper we investigated the effect on the design of a public transport service of the suppression of users' cost that tends to happen when financial constraints exist (Jara-Díaz and Gschwender, 2009). To do this, we adapted the approach for the comparison of lines structures – direct or corridors – developed by Jara-Díaz and Gschwender (2003b) but now including the effect of vehicle size on operators' costs. Three design variables were included in the analysis: frequency, vehicle size and, most important, the spatial structure of the services. These variables were obtained for simple networks minimizing two objective functions, namely total cost (users and operators) and operators' costs only as extreme cases under a financial constraint. Comparing the resulting structures, the effect of neglecting users' costs was found.

The networks analyzed include one central and several peripheral nodes. When users' and operators' cost is minimized, only in the most simple network (number 1 with two OD pairs) the direct lines structure is always the most convenient. In the more complex cases (2, 3 and 4), as demand increases or the ratios P_w/P_v and P_w/c_1 diminish (everything else constant) it becomes more likely that direct lines are the most convenient. This happens mainly because waiting times decrease in the corridors as a result of higher frequencies. When only operators' cost is minimized, direct lines are always more convenient because they avoid transfers, diminishing boarding and alighting time, thus reducing cycle times and operators' cost. Nevertheless, when demands are low, each direct line (specialized in one OD pair) may result in low frequencies yielding large waiting times. This is the reason why the inclusion of users' cost (time) in the optimization changes the optimal structure towards corridor lines for low levels of demand. It was found that for both objectives and in all the analyzed networks, corridor lines yield always lower total waiting times than the direct lines. Although the opposite happens with in-vehicle times, it is the former effect which dominates. The fact that total in-vehicle time is larger in corridors than in direct lines is explained by the transfers, which imply higher in-vehicle times for some passengers.

In summary, for a system with given technical characteristics direct lines are the best structure for the operators for all demand levels[2]. Interestingly, direct lines are also the optimal structure for users and operators when demand is above a critical level[3]. However, the fleet size is lower in the first case (with larger vehicles) negatively affecting users through the waiting time. It is worth noting that the optimal structure is influenced by the term *tY*, i.e. demand acts through the boarding and alighting time of passengers. Therefore, the demand effect is reduced when boarding and alighting is made easier for large groups of passengers, for example using several doors simultaneously (as in metro systems), favoring the corridors structure. On the other hand, if a transfer penalty was considered, the probability of direct lines being the best ones would increase. Table 7 summarizes the optimal values for the design variables and levels of service for the different networks and objectives analyzed.

Table 7: Summary of results for different cases and objectives

Case	Objective	Best structure	Fleet size	Vehicle size	Average waiting time	Average in-vehicle time
1	$\min \mathbf{C_U} + \mathbf{C_O}$	direct	Lower for best structure	Line dependent	Lower in corridors	Lower in direct
	$\min \mathbf{C_O}$					
2-3-4	$\min \mathbf{C_U} + \mathbf{C_O}$	Direct for *tY* large		Larger for best structure		
		Corridors for *tY* low				
	$\min \mathbf{C_O}$	direct				

This analysis could be extended to study more complex networks or demand structures with more unbalanced demands. From an analytical viewpoint, it would be interesting to include crowding, expressed as the ratio between load size *k* and vehicle size *K*. There are at least two ways to do this: writing the waiting time as an increasing function of *k*/*K* (probability of not being able to board the vehicle) or writing the in-vehicle time value as a function of that ratio representing discomfort. In both cases the results could yield an optimal vehicle size larger than the maximum load (capacity constraint inactive), as obtained by Jara-Díaz and Gschwender (2003a) for one line.

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[2] This coincides with the result obtained by Jara-Diaz and Basso (2003) for their simplest case (equal distances, equal flows) in a three nodes network.
[3] This resembles the results obtained in the air transport literature for a socially optimal service structure that depends on demand (Brueckner, 2004), if one associates hub and spoke with corridors (both have transfers) and fully connected with direct lines (no transfers).

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Appendix 1: Numerical approximation of the fleet distribution in corridors of case 1.

Following figure 1, case 1, in order to distribute the total fleet between lines *I* and *II* it has to be pointed out that the average waiting and in-vehicle times are the same deducted in Jara-Díaz and Gschwender (2003b), because the difference introduced here is the dependence *c(K),* which only affects operators' cost while users' cost remain equal.

$$\bar{t}_w = \frac{1}{2}\left[\frac{2T_0}{B_I - 2tY} + \frac{T_0}{2(B - B_I - tY)}\right] \tag{A1-1}$$

$$\bar{t}_v = \frac{1}{2}\left[2T_0 + \frac{2T_0 tY}{B_I - 2tY} + \frac{T_0 tY}{4(B - B_I - tY)}\right] \tag{A1-2}$$

As *Y* passengers board line *I* and *Y*/2 line *II*, amounts that also correspond to the maximum load of each line, operators' expenditure is

$$VRC_O = Bc_0 + \frac{2c_1 YT_0 B_I}{B_I - 2tY} + \frac{c_1 YT_0 (B - B_I)}{2(B - B_I - tY)} \tag{A1-3}$$

Using expressions (A1-1), (A1-2) and (A1-3) the total value of the resources consumed as a function of B_I is

$$VRC_T = Bc_0 + \frac{c_1 YT_0 B_I}{B_I - 2tY} + \frac{c_1 YT_0 (B - B_I)}{2(B - B_I - tY)} + \frac{P_w T_0 Y}{2}\left[\frac{2}{B_I - 2tY} + \frac{1}{2(B - B_I - tY)}\right] + \frac{P_v T_0 Y}{2}\left[2 + \frac{2tY}{B_I - 2tY} + \frac{tY}{4(B - B_I - tY)}\right] \tag{A1-4}$$

Minimizing on B_I the optimal fleet distribution is obtained

$$B_I^* = \frac{\sqrt{P_w + P_v tY + 4c_1 tY}}{\sqrt{P_w + P_v tY + 4c_1 tY} + \frac{1}{2}\sqrt{P_w + P_v tY/2 + 2c_1 tY}} \cdot B +$$

$$\frac{tY\left[\sqrt{P_w + P_v tY/2 + 2c_1 tY} - \sqrt{P_w + P_v tY + 4c_1 tY}\right]}{\sqrt{P_w + P_v tY + 4c_1 tY} + \frac{1}{2}\sqrt{P_w + P_v tY/2 + 2c_1 tY}} \qquad \text{(A1-5)}$$

$$= \alpha \cdot B + \gamma$$

Using the parameters of table A1-1 the values of α and γ in table A1-2 are obtained for different demand levels and values of time.

Table A1-1: Values of the parameters

Parameter	Value	Unit
c_0	10.65	US$/hr
c_1	0.203	US$/hr
t	2.5	Sec

Table A1-2: Exact values of α and γ

Y **(pas/hr)**	$\boldsymbol{P_w}$ **(US$/hr)**	$\boldsymbol{P_v}$ **(US$/hr)**	α	γ
2,000	1.48	1.48	0.711	-0.186
2,000	4.44	1.48	0.692	0.106
2,000	8.11	2.70	0.689	0.095
1,000	1.48	1.48	0.699	0.068
1,000	4.44	1.48	0.682	0.032
1,000	8.11	2.70	0.680	0.028
200	1.48	1.48	0.677	0.004
200	4.44	1.48	0.670	0.002
200	8.11	2.70	0.670	0.001

The values of γ found are negligible in comparison to the first term (αB) and the values of α are close to 2/3, similarly to what Jara-Díaz and Gschwender (2003b) found. Therefore, the optimal fleet distribution for corridors in case 1 can be approximated to:

$$B_I^* \approx \frac{2}{3}B \quad B_{II}^* \approx \frac{1}{3}B \qquad \text{(A1-6)}$$

Appendix 2: Relative analysis of the terms in the total cost equation

In the cost comparison between direct and corridor lines (equation 40) the first term of the cost function is neglected. The cost function analyzed is:

$$C_T^l = 2tYc_0\frac{1}{\delta^l} + 2\sqrt{T_0 Y c_0\left(P_w\frac{\varphi_w^l}{\delta^l} + P_v\frac{\varphi_v^l}{\delta^l}tY + 2tYc_1\frac{\varphi_c^l}{\delta^l}\right)} + T_0 Y\left(P_v\psi + c_1\varphi_c^l\right) \qquad \text{(A2-1)}$$

In table A2-1 values for the first term (**X**) and the addition of the other two terms (**Z**) are shown for several demand levels in the network of case 4, using the parameters of table 5. **X** results negligible with respect to **Z** (0.007<**X**/**Z**<0.013).

Table A2-1: Numerical comparison of the cost function terms

***Y* (pas/hr)**	**X (US$/hr)**	**Z (US$/hr)**
5,000	74	5,657
2,000	30	2,515
1,000	15	1,411
500	7	818
250	4	489
200	3	418