

WELFARE IMPLICATIONS OF PRICE CAP REGULATION COMBINED WITH TOTAL REVENUE CONSTRAINT

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ABSTRACT

This paper assumes the introduction of price cap regulation under the constraints of policy requirements; it highlights the price cap regulation under the condition that the operator's revenue remains constant. A government may impose this when it expects the operators to earn pre-fixed amounts of revenue for repaying their debt. This paper analyzes the mechanism of a proposed pricing system after formulating a simple model. Two prices of a small vehicle and a large vehicle in the expressway service are assumed in the analysis. The theoretical analysis shows that the proposed pricing system leads to the maximization of consumer surplus under the constant-revenue constraint. Then, simple numerical simulations based on the proposed model are presented. The results show that the proposed model works well.

Keywords: *Price cap regulation, constant-revenue constraint, Ramsey pricing*

1. INTRODUCTION

In October 2005, the Japan Highway Public Corporation was privatized and split into three expressway companies (Mizutani and Uranishi, 2008). This division aimed to correct the X-inefficiency and improve service quality by utilizing the skills and knowledge of private companies. Although the inter-urban expressway service was deregulated, some type of price regulation is necessary, because each expressway company has monopolistic power in its region. On the other hand, expressway companies are required to repay their debt to the government within 45 years following privatization. This implies expressway companies should designate certain amount of revenues for repayment in those 45 years. The other policy target of controlling the monopolistic power of the expressway service is probably due to public pressure or policy discussions.

To control the monopolistic power of the expressway company, it is necessary to introduce some form of regulation. Price cap regulation is one such appropriate regulation system. Price cap regulation, under which prices are adjusted according to exogenous input price and performance benchmarks, is one of the incentive regulatory mechanisms available (Train, 1991). Kato *et al.* (2010) empirically analyze the introduction of price cap regulation for the expressway service in Japan, with the constraint that the expressway operator's revenue should remain constant. They formulate a simple model with a two-good case and numerically test the feasibility of the solution. Their results show that the existence of a solution depends upon a combination of price elasticities. Although they check the existence of the solution and the tendency of the price level, it needs to be explained how the price cap regulation improves social welfare.

This paper examines the impact of price cap regulation under policy constraints on social welfare. It highlights price cap regulation with the condition that the expressway operator's revenue remains constant and analyzes the mechanism of a proposed pricing system that leads to the maximization of consumer surplus.

The paper is organized as follows. Section 1 shows the motivation and goal of this paper. Next, literature reviews on price regulation and the monopolist's pricing theory are

presented in Section 2. A simple price regulation model with the constraint of constant revenue is formulated, and the mechanism of a pricing system that leads to the maximization of consumer surplus is presented in Section 3. Section 4 provides numerical simulations of toll prices in simple case studies. Finally, the achievements of the paper are summarized.

2. LITERATURE REVIEWS

Price cap regulation requires the regulator to set the upper bound for the increased price rate. As long as the price does not exceed that upper bound, firms may change the price of each service as they like. A typical price cap regulation is shown as follows:

$$p^t \leq (1+I-X)p^{t-1}, \quad (1)$$

where p^t denotes the upper-bound price level at time t , p^{t-1} the price set by the regulated firms at $t-1$, I the price index (the Retail Price Index or Consumer Price Index is often used), and X the productivity improvement rate required by the regulator. This implies that the regulated firms are forced to lower their price by X . They are required to improve their productivity by reducing expenses and/or providing more attractive services to comply with the price regulation.

If the firms have two or more outputs, there are two methods of price cap regulation: the tariff basket regulation and the average revenue regulation. The tariff basket regulation is formulated as

$$\sum_{i=1}^n p_i^t q_i^{t-1} \leq (1+I-X) \sum_{i=1}^n p_i^{t-1} q_i^{t-1} \quad (2)$$

where p_i^t indicates the price of good i at time t and q_i^t the output of good i at time t . This implies that the regulator sets the prices so that the pseudo-revenue obtained with the current price p_i^t and the output of the previous period q_i^{t-1} should not exceed the amount of the revenue of the previous period multiplied with $(1+I-X)$. Although tariff basket regulation is desirable for the traditional rate-of-return regulation, it does not guarantee Ramsey prices (Ramsey, 1927). Vogelsang and Finsinger (1979) propose a similar dynamic regulation mechanism. This mechanism requires the regulator to set the prices as

$$\sum_{i=1}^n p_i^t q_i^{t-1} \leq C(\mathbf{q}^{t-1}) \quad (3)$$

where $C(\mathbf{q}^{t-1})$ denotes the cost function at the previous period. Note that it is assumed the regulator can observe the cost function. Vogelsang and Finsinger (1979) show that this regulation gives the Ramsey price in a long-run equilibrium. If X , in the tariff basket regulation, is set to deprive firms of all excess profit at the previous period and the price index I is not considered, the price mechanism presented in Equation (2) would be equal to Equation (3).

Next, the average revenue regulation is simply expressed as

$$\sum_{i=1}^n \frac{p_i^t q_i^t}{q_i^t} \leq (1 + I - X) \sum_{i=1}^n \frac{p_i^{t-1} q_i^{t-1}}{q_i^{t-1}} \quad (4)$$

This requires the firms to set the price so that the average revenue, at the current period, does not exceed that of the previous period. Although this regulatory system is widely used, it presents two major problems. First, unlike with tariff basket regulation, the regulator needs to forecast the output at the current period. Second, the average revenue regulation theoretically does not lead to Ramsey prices (Armstrong and Vickers, 1991).

3. MODEL

3.1 Model Formulation

This paper assumes a simple model to analyze the toll prices for two vehicle types, under the condition that the expressway operator's revenue should remain constant. The toll is assumed to be in proportion to the running kilometer. Thus, in this paper, the price refers to the toll charge for a vehicle running 1 km. It is also assumed that the vehicles are categorized into large-vehicle and small-vehicle groups. Large vehicles include trailers, large-scale trucks with more than three axles, and buses. Small vehicles include private automobiles, small-scale trucks, and taxis. This is because the price elasticity of demand varies between vehicle types.

Suppose the original price in the initial year (t) is given. Then, a new price cap regulation is introduced. This regulation forces the expressway operators to decrease the price gradually by a required constant price reduction rate (X) while the revenue of the operators should remain the same as at the initial level. This regulation continues for T years. It allows different prices to be charged for large and small vehicles. The demand function is assumed to be the same among the routes, whereas the demand functions of large and small vehicles can be different. The general consumption price is assumed to be constant throughout the regulated years. Additionally, we assume that the capped price is controlled according to the tariff basket method (Train, 1991).

Thus, the prices of the large vehicle and the small vehicle should satisfy the following two conditions under the above-mentioned regulation:

$$p_s^{t+1} \cdot q_s^t(p_s^t, p_l^t) + p_l^{t+1} \cdot q_l^t(p_s^t, p_l^t) \leq (1 - X)(p_s^t \cdot q_s^t(p_s^t, p_l^t) + p_l^t \cdot q_l^t(p_s^t, p_l^t)) \quad \text{and} \quad (5)$$

$$p_s^{t+1} \cdot q_s^{t+1}(p_s^{t+1}, p_l^{t+1}) + p_l^{t+1} \cdot q_l^{t+1}(p_s^{t+1}, p_l^{t+1}) = p_s^t \cdot q_s^t(p_s^t, p_l^t) + p_l^t \cdot q_l^t(p_s^t, p_l^t) \quad (6)$$

where p_s^t is the price (yen/vehicle km) charged for the small vehicle (s) in the current year (t), p_l^t the price (yen/vehicle km) charged for the large vehicle (l) in the current year (t), p_s^{t+1} the price (yen/vehicle km) charged for the small vehicle (s) in the next year ($t+1$), p_l^{t+1} the price (yen/vehicle km) charged for the large vehicle (l) in the next year ($t+1$), $q_s^t(\cdot)$ the demand (vehicle km) of the small vehicle (s) to consume expressway service in the current year (t), $q_l^t(\cdot)$ the demand (vehicle km) of the large vehicle (l) to consume expressway service in the current year (t), $q_s^{t+1}(\cdot)$ the demand (vehicle km) of the small vehicle (s) to consume expressway service in the next year ($t+1$), $q_l^{t+1}(\cdot)$ the demand (vehicle km) of the large vehicle (l) to consume expressway service in the next year ($t+1$), and X the required annual reduction rate of the price ($0 \leq X \leq 1$). Equation (5) shows that the price should be less than or equal to the maximum price under price cap regulation. This is called a “price constraint” in this paper. Equation (6) shows that the current year’s revenue should be the same as that for the next year. This is called a “revenue constraint” in this paper. Note that although a revenue constraint is introduced, no constraint has been imposed on the operator’s profit. We call the above model a Kato-Tanabe-Ohta model.

3.2 Mechanism of Proposed Pricing System

The Kato-Tanabe-Ohta model formulated above leads to maximization of social benefit under a constant-revenue constraint. A rough sketch of this mechanism based on Train (1991) is presented below.

First, it is assumed that the expressway company maximizes its profit with respect to the prices. The profit maximization of the operator is formulated as

$$\text{Max}_{p_s^{t+1}, p_l^{t+1}} \pi^{t+1} = p_s^{t+1} \cdot q_s^{t+1} + p_l^{t+1} \cdot q_l^{t+1} - C^{t+1}(q_s^{t+1}, q_l^{t+1}) \quad (7)$$

where π^{t+1} denotes the operator's profit at year $t+1$ and $C^{t+1}(q_s^{t+1}, q_l^{t+1})$ denotes the operating cost at year $t+1$. For analytical simplicity, it is assumed that the operating cost is zero. Even if variable operating costs exist, the following discussions hold true. Note that the operator may have an incentive to reduce operating costs if such costs exist.

The profit maximization shown in Equation (7) with the constraints of Equations (5) and (6) can be presented illustratively as shown in **Figure 1**. **Figure 1** presents the revenue of the operator and the social benefit, which is measured by total surplus, on a $p_s - p_l$ plane. Let the initial revenue of the operator $R^t = p_s^t \cdot q_s^t + p_l^t \cdot q_l^t$. Then, Equation (5) is rewritten as

$$p_l^{t+1} \leq -\frac{q_s^t}{q_l^t} p_s^{t+1} + (1-X) \left(\frac{R^t}{q_l^t} \right) \quad (8).$$

Suppose that a set of initial prices is located at $A(p_s^t, p_l^t)$. An iso-revenue contour, on which any point realizes the same revenue as the initial revenue, is depicted as a circle in **Figure 1**. The given iso-revenue contour should run on $A(p_s^t, p_l^t)$. The operator service maximizes its revenue at point M. The operator's revenue in the area outside the iso-revenue contour is lower than the initial revenue. Next, a frontier of the price constraint at year $t+1$, shown in Equation (8), is depicted as a line with a slope of $-q_s^t/q_l^t$, which crosses the iso-revenue contour at B_1 and B_2 and the vertical axis at

$$(1-X) \left(\frac{R^t}{q_l^t} \right).$$

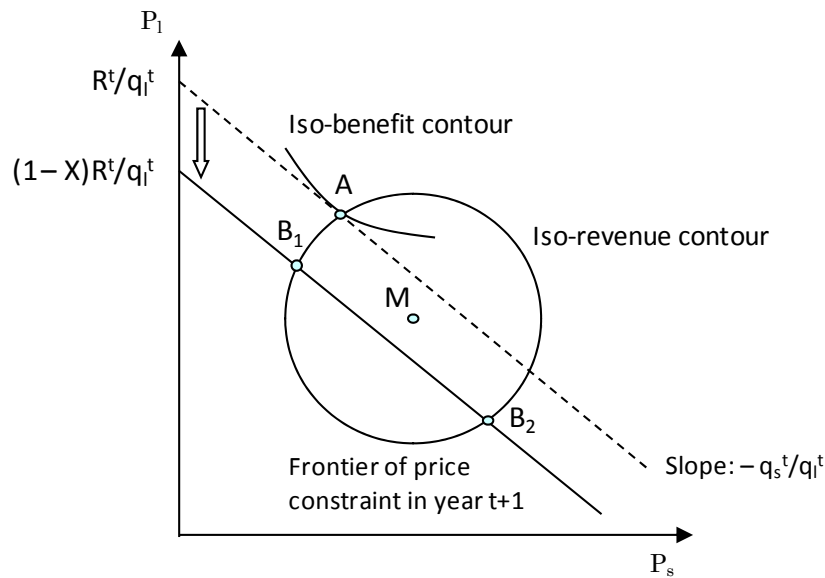


Figure 1 – Shift of price-constraint frontier from year t to $t+1$ in the Kato-Tanabe-Ohta model

Then, the operator may choose a new price set at B_1 in year $t+1$. First, as Train (1991) shows, the tangent to the iso-benefit contour is equal to $-q_s/q_l$ at any price set¹. Social benefit is defined as the consumer surplus, because we assume the regulator has imposed a revenue constraint on the operator and that the operating cost is zero. An iso-benefit contour is depicted in **Figure 1**. The level of benefit is higher as the iso-benefit contour approaches the origin $(0,0)$ of the $p_s - p_l$ plane. The tangential line of the given iso-benefit at $A(p_s^t, p_l^t)$ is depicted as a dotted line in **Figure 1**. It should be noted that the dotted line crosses the vertical axis at $\frac{R^t}{q_l^t}$. The price constraint is indicated by the shift from the original frontier of the price constraint in the initial year t to the new frontier of price constraint in the next year $t+1$ down by X . Note that the iso-revenue contour does not change since the revenue constraint holds.

The operator can choose any price in the price set on the southwest side of the given iso-revenue contour between B_1 and B_2 . As the revenue is constant on the given

¹ This is derived as follows. Suppose the consumer surplus $B(p_s, p_l)$ is defined as $B(p_s, p_l) \equiv \int_{p_s}^{\infty} q_s(p_s, p_l) dp_s + \int_{p_l}^{\infty} q_l(p_s, p_l) dp_l$. Then, the total differential of $B(p_s, p_l)$ is derived as $dB(p_s, p_l) = \partial B / \partial p_s \cdot dp_s + \partial B / \partial p_l \cdot dp_l$. As $dB(p_s, p_l)$ should be equal to zero at any point on the iso-benefit contour, $dB(p_s, p_l) = 0$. Then, $dp_l / dp_s = -\frac{\partial B / \partial p_s}{\partial B / \partial p_l} = -\frac{q_s}{q_l}$.

iso-revenue contour, any set of prices on the iso-revenue contour is indifferent from the viewpoint of the revenue constraint. Which set of prices is selected by the operators? To determine a unique solution, the following assumption is additionally introduced into our model framework: “the price set causing smaller changes in prices is preferred.” This small-price-change assumption may be quite reasonable because the transaction costs of the price change, including the political fallout, may be lower as the price change is relatively smaller. If this assumption is accepted, the new price set results in B_1 . It should be noted that the change of price set from A to B_1 increases the consumer surplus, that is, the social benefit.

In **Figure 1**, the change from A to B_1 reduces both prices, while the total revenue is unchanged. It means that the price elasticity of demand of one vehicle is less than 1 and that of the other vehicle is more than 1. This is consistent with the following first numerical simulation.

Figure 2 shows the process of determining the new price set in year $t+2$. As the price set in year $t+1$ has changed from that in year t , the tangent to the iso-benefit contour in year $t+1$ at B_1 also changes to $-q_s^{t+1}/q_l^{t+1}$. The new frontier of the price constraint in the next year, $t+2$, is defined by applying the price constraint again. Then, the price set moves from B_1 to C . This means that the consumer surplus increases again. In the same way, the price set keeps moving to the southwest direction along the

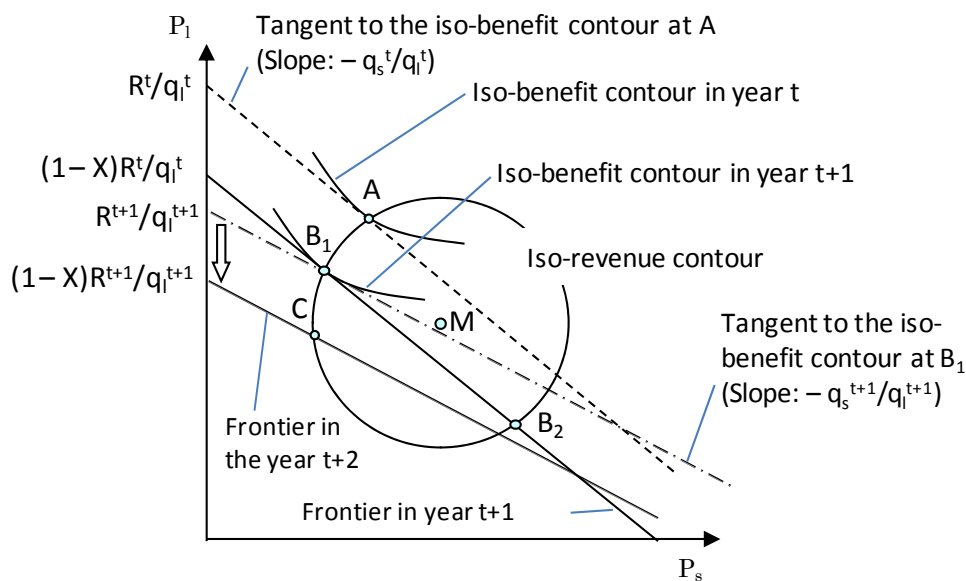


Figure 2 – Shift of price-constraint frontier from year $t+1$ to $t+2$ in the Kato-Tanabe-Ohta model

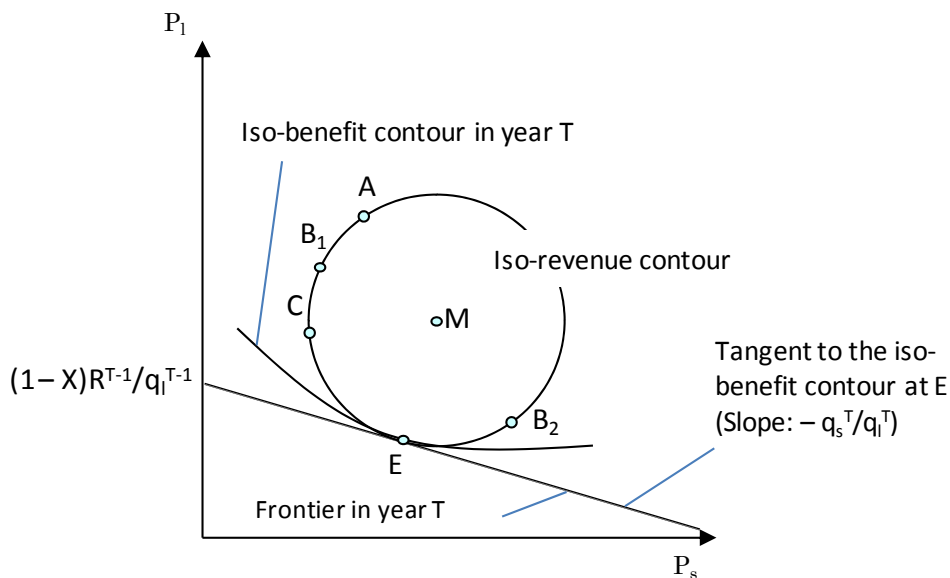


Figure 3 – The equilibrium point in the final year T in the Kato-Tanabe-Ohta model

iso-revenue contour by following the frontier of the price constraint while this frontier crosses the iso-revenue contour.

If the reduction rate of the price, X , is discrete and constant, the price may not converge into a unique solution. Thus, if a new frontier of the price constraint is located under the iso-revenue contour, the operator is required to report that no further change in price is feasible. It should be noted that the regulator cannot judge if the operator's report is true only at the final step. In other words, it is possible that the operator may report false results to the regulator. However, as the operator has no incentive to report a wrong result because constant revenue is guaranteed, the regulator expects the operator to report the result honestly. If the above discussions hold true, the unique solution may be found by fine-tuning the value of X at the final step. This unique solution realizes the maximum consumer surplus under the constant-revenue constraint.

Figure 3 illustrates the final equilibrium status in the proposed model. E indicates the equilibrium price set. At this point, the iso-revenue contour and the highest iso-benefit contour come in contact with each other. Therefore, social benefit is maximized under the constraint of total revenue. Consequently, the equilibrium price set must be the Ramsey price, although the operator can earn a positive profit.

3.2 Comparison of Proposed and Existing Pricing Models

Although the above-mentioned pricing algorithm is quite different from the regulation algorithm developed by Vogelsang and Finsinger (1979), the expected results of our model seem similar to the results of the Vogelsang and Finsinger model. Why does the former price become similar to the latter?

The Vogelsang and Finsinger model assumes the following price constraint:

$$p_s^{t+1} \cdot q_s^t + p_l^{t+1} \cdot q_l^t \leq C^t \tag{9}$$

The operator service is assumed to maximize its profit under this price constraint. The pricing mechanism of the Vogelsang and Finsinger model is depicted in **Figure 4**. Consider the price set in the initial year t . Profit maximization under a price constraint implies that in the next year $t+1$ the operator chooses a price set, on the frontier of the price constraint, that maximizes the profit. The new price set is shown as W in **Figure 4**. After the iterative pricing process based on the above rule, the operator will finally choose the price set at which the tangential line of the zero-profit contour is equal to the frontier of the price constraint. Vogelsang and Finsinger (1979) show that this price set is equal to the Ramsey price. This is the second-best price which maximizes the consumer surplus under the price constraint.

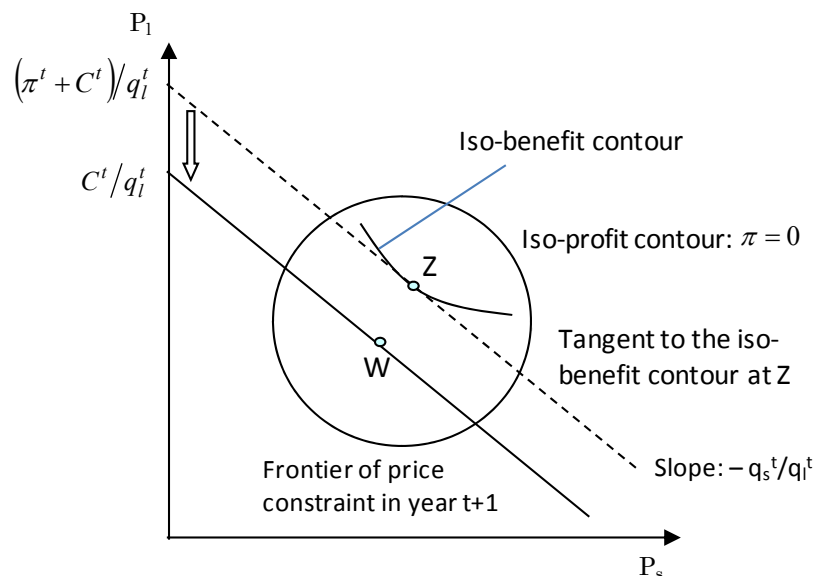


Figure 4 – Acceptable prices in the Vogelsang and Finsinger model

The price constraint of our model uses $(1-X)R^t$, and not C^t , as shown in Equations (5) and (9). However, if $(1-X)R^t$ is substituted for C^t , the required price reduction rate is derived as $X = \pi^t/R^t$. Then, the price constraint of our model, shown in Equation (5), can be rewritten into the price constraint of the Vogelsang and Finsinger model shown in Equation (9). This means that the price constraint of our model is the same as that of the Vogelsang and Finsinger model.

One of the differences of our model, from the Vogelsang and Finsinger model, is that our model adds the revenue constraint shown in Equation (6), whereas the Vogelsang and Finsinger model does not. The operator can choose a price set to maximize profits in the Vogelsang and Finsinger model. In our model, although the operators can maximize their profit, they are not allowed to earn excess revenue above a pre-fixed level. Instead, the price set is determined by the revenue constraint in our model. However, as the price constraint works, the price set moves gradually to the second-best price, the Ramsey price, under the small-price-change assumption.

Another difference in our model, compared to the Vogelsang and Finsinger model, concerns the conditions for stopping price changes. In the Vogelsang and Finsinger model, the operator will stop changing the prices when the price set reaches the zero-profit contour, whereas in our model the operator will stop changing the prices when no solution is found. Thus, the solution in the final year based on our model may be different from that based on the Vogelsang and Finsinger model. Both solutions would be equal only if the revenue constraint in our model is defined to be equal to the zero-profit constraint. This depends on the relationship between the pre-fixed revenue and the total cost.

4. NUMERICAL SIMULATIONS

A numerical simulation will be presented based on the Kato-Tanabe-Ohta model. First, let the demand functions of the small and large vehicles be $q_s = 10 - p_s$ and $q_l = 10 - p_l$, respectively. For analytical simplicity, we set the same demand function for both vehicle types. Also, assume the operating cost of a monopolistic operator is zero, which means

that the marginal cost of the operator is zero. Under these assumptions, both types of vehicles are expected to have the same equilibrium price at the final stage.

Suppose the initial prices of small and large vehicles are 4 and 6, respectively. This assumption at the initial period means that the price elasticity of demand of the small vehicle is less than 1 while that of the large vehicle is more than 1. Also, this means that the initial point is (4, 6) on the $p_s - p_l$ plane. Then, the iso-revenue contour or circle is defined as in **Figure 1**. Note that no iso-revenue contour can be defined if the initial price set is (5, 5). This is because the iso-revenue contour is equal to the center of the iso-revenue circles. Note also that the revenue by its constraint is equal to 48. The prices of small and large vehicles with consumer surplus are simulated under the assumption that the price reduction rate is 0.02. As the demand functions are linear, the revenue constraint functions are quadratic, and thus there are two solutions.

Based on the small-price-change assumption, the solution that includes the prices closer to those of the previous year is selected. **Table 1** shows the results of the simulation. No solution is found in the 11th year. The second-best price indicates the Ramsey price, which is achieved theoretically. The results show that the price of the small vehicle decreases from the initial year up to the 8th year while it increases from the 8th year up to 10th year. The price of the large vehicle keeps decreasing through both periods; the consumer surplus keeps increasing and approaches 36, which is the consumer surplus at the second-best price. The consumer surplus increases from the initial year to the final year by 37.8 percent. The consumer surplus in the 10th year accounts for 99.5 percent of the consumer surplus at the second-best price.

Next, numerical simulation based on a model with only a price constraint is carried out in order to compare our model results. This means that the price constraint shown in Equation (5) is applied but the revenue constraint shown in Equation (6) is not used in this simulation. The conditions of numerical simulation, including the initial prices and the price discount rate, are the same as the earlier numerical simulation based on our model. Note that the operating cost is assumed to be zero. Thus, the profit maximization is equal to the revenue maximization, and it is obvious that the operator's profit is positive in the initial year. The results are shown in **Table 2**.

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Table 1 – Prices and consumer surplus in numerical simulation based on the Kato-Tanabe-Ohta model ($X = 0.02$)

Year	0	1	2	3	4	5	6	7	8	9	10	Second-best price
Small vehicle (P_s)	4.00	3.90	3.83	3.75	3.69	3.64	3.60	3.59	3.59	3.64	3.83	4.00
Large vehicle (P_l)	6.00	5.90	5.79	5.67	5.53	5.39	5.23	5.05	4.85	4.60	4.20	4.00
Consumer surplus	26.0	27.0	27.9	28.9	29.9	30.9	31.8	32.8	33.8	34.8	35.8	36.0

Table 2 – Prices and consumer surplus in numerical simulation based on a model with only price constraint ($X = 0.02$)

Year	0	1	2	3	4	5	6	7	8	9	10	...
Small vehicle (P_s)	4.00	4.66	4.62	4.53	4.44	4.35	4.26	4.18	4.09	4.01	3.93	
Large vehicle (P_l)	6.00	4.77	4.62	4.53	4.44	4.35	4.26	4.18	4.09	4.01	3.93	
Consumer surplus	26.0	27.9	28.9	29.9	30.9	31.9	32.9	33.9	34.9	35.9	36.8	
Revenue	48	49.8	49.7	49.6	49.4	49.2	48.9	48.6	48.4	48.0	47.7	

The results show that prices and consumer surplus are changing more quickly than the earlier simulation based on our model. As no revenue constraint is given, the operator's revenue varies across the years. The revenue in any year after the second is larger than that of the initial year, but it keeps decreasing after the second year. The revenue in the 10th year becomes less than the initial revenue.

5. CONCLUSIONS

This paper proposes a new pricing scheme in which the price is regulated under the constraint that the operator's revenue remains constant, and examines its impact on social welfare with a simple two-good model based on the proposed scheme. The model is formulated and the mechanism of pricing is analyzed in an illustrative fashion. Then, the proposed model is compared with an existing numerical simulation model.

The proposed pricing model, the Kato-Tanabe-Ohta model, consists of a constant-revenue constraint and a tariff basket price cap regulation with the regulated operator showing profit maximization behavior. Further, the Kato-Tanabe-Ohta model leads the operator to set the Ramsey price. The Kato-Tanabe-Ohta model can be useful for a price regulation policy for monopolies such as toll road operators in order to achieve the second-best solution.

REFERENCES

- Armstrong, M. and Vickers, J. (1991) Welfare effects of price discrimination by a regulated monopolist. *RAND Journal of Economics* 22(4), 571 – 580.
- Kato, H., Tanabe, K., and Ohta, K. (2010) Toll price of inter-urban expressway service under price cap regulation with an additional policy target: An empirical case study in Japan. A paper submitted to the 15th International Conference of Hong Kong Society for Transportation Studies (in press).
- Mizutani, F. and Uranishi, S. (2008) Privatization of the Japan Highway Public Corporation: focusing on organization structural change. *Transport Reviews* 28(4), 469–493.
- Ramsey, F. (1927) A contribution to the theory of taxation. *Economic Journal* 37, 47–61.
- Train, K. (1991) *Optimal Regulation: The Economic Theory of Natural Monopoly*. Cambridge, MA: MIT.
- Vogelsang, I. and Finsinger, J. (1979) A regulatory adjustment process for optimal pricing by multiproduct monopoly firm. *Bell Journal of Economics* 10(1), 157–171.