AN IMPLICIT C-LOGIT ASSIGNMENT MODELS: THE CAPACITY OF FLOW REPRODUCTION ON REAL SYSTEMS

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ABSTRACT

The route choice model on a transportation system can be separated into two sub-models: choice set generation and choice of the alternatives given the choice sets. For choice set generation several models can be used; the more common mono- or the multi-set approach. For route choice the more common models are Logit and Probit. This paper proposes the specification of the general problem of route choice and reports the D-C-Logit model proposed recently for implicit assignment. The problem of choice set generation in the literature is solved with a fixed choice set or considering all the loop-less routes. In this paper a general model is proposed and numerical results are reported in order to ascertain how this sub-model affects the final results for user choice. In relation to the route choice the D-C-Logit model is reported. It combines several positive features found in the literature for choice set generation and choices from a given choice set: generation of a set of alternatives with a selective approach; calculation of the route choice probability in a closed form; simulation of the overlapping effect among alternative routes; computation of just one tree for each origin avoiding explicit route enumeration.
Keywords: Route Choice, Assignment

1. INTRODUCTION

One of the main components of traffic assignment models is the route choice model. The quality of the model increases if the generation of route choice sets is subject to behavioural rules. Such models are used both for congested networks within equilibrium models or dynamic models. The problem can be solved with *implicit* or *explicit* route enumeration algorithms and with selective (a subset of feasible routes are admissible) or exhaustive (all routes without loops are admissible) approaches.

Implicit route enumeration avoids explicit route generation and allows efficient algorithms to be used. The benefits of implicit route enumeration outweigh the benefits of explicit enumeration if hypotheses can be made on route choice generation and route choice. In the

field of the implicit route enumeration with a selective approach, Dial (1971) proposed a probabilistic choice model that does not consider route overlapping. In the field of the implicit route enumeration with exhaustive approach the Probit model proposed by Sheffi and Powell (1981) overcomes the route overlapping problem by introducing a covariance proportional to the degree of route overlapping. Models in the Probit family cannot be expressed in closed analytical form and hence, to be solved, require Montecarlo techniques which, though based on implicit route enumeration algorithms, generally imply a large number of iterations to reduce the error of the convergence and, in a general case, if repeated do not give exactly the same numerical results. These models are inserted also in design models (Cantarella and Vitetta, 2006; Cantarella *et al.*, 2006, Russo and Vitetta, 2006a) and in emergency conditions Russo and Vitetta, 2006b).
With the development of research and the increasingly of the powers of elaboration, global assignment models are developed where the problems of generation of perceived alternatives and the choice of the alternatives are studied in two separate models: set choice set and route choice. The general model was first formulated by Manski (1977).

In this field of *explicit* route enumeration with a *selective* approach some of the routes which are topologically admissible on the basis of satisfying certain rules with descriptive (Ben Akiva *et al*. 1984, Russo and Vitetta 1996) or behavioural probabilistic (Morikawa 1996) approaches are considered for the route generation. In this field two conceptual steps are considered in the literature:

- the set choice which consists in the construction of sets perceived by users (*formation level*) and in obtaining the evaluation probability for each choice set perceived (*extraction level*);
- the *route choice* which consists in the route choice evaluation in each alternative contained in the extracted sets.

The *formation level* consists in the construction of the sets perceived in which the user chooses the alternative.
The *extraction level* consists into obtain the probability evaluation for each choice set perceived by users and defined in the formation level.
The *route choice* model consists in the probability evaluation in each alternative contained in the extracted sets.
In the field of explicit route enumeration with a selective approach, Cascetta *et al*. (1996) proposed a C-Logit model and it was switched in terms of model and algorithm from explicit to implicit in Russo and Vitetta (2003) called D-C-Logit for short (D in D-C-Logit stands for Dial).
This work has three main objectives: (i) to specify a general model for route choice; (ii) to specify the D-C-Logit model inside the general model for route choice; (iii) to verify models and algorithms in terms of numerical application.
The work is structured as follows: (i) in section 2 route choice models in terms of generating the choice set and choosing the alternative are reported; (ii) in section 3 the main features of the D-C-Logit are described; (iii) in section 4 the numerical applications are presented. Lastly, conclusions are drawn in section 5.

2. ROUTE CHOICE: STATE OF THE ART IN MODELS

A general choice model can be specified considering the Manski (1977) formulation. Defining with M_n the set of all the feasible alternatives, which in route choice models are all the loopless routes, for user n, the probability of choosing alternative k can be expressed considering two models, generation of the choice set and choice of the alternative, as:

$$p_n(k/G_n \neq \emptyset) = \sum_{\forall\, C \in G_n} p_n(k/C)\, p_n(C/G_n \neq \emptyset)$$

where

- G_n is the set containing all the sub-set that can be extracted from M_n, considered by the users in the model;
- C is the generic set perceived by user n (C belongs to G_n);
- $p_n(k/C)$ is the probability of user n choosing alternative k conditional on choosing from set C;
- $p_n(C/G_n \neq \emptyset)$ is the probability of user n choosing set C provided that G_n is not empty;
- $p_n(k/G_n \neq \emptyset)$ is the probability of user n choosing alternative k provided that G_n is not empty.

In relation to the size of G_n there are two types of choice set:

- *mono-set*, where the size of G_n is equal to 1;
- *multi-sets*, where the size of G_n is greater than 1.

In each type the exhaustive or selective approach can be considered. Choice sets in demand models consist of a certain number of alternatives, which are usually physically different from one another. The *exhaustive approach* considers as admissible all routes, without loops, found on the network in question. By contrast, the *selective approach* identifies only some of the routes which are topologically admissible on the basis of satisfying certain rules.
From this it emerges that the mono-set type is used widely in theoretical and applicative developments, in the exhaustive and selective approach; multi-sets find an elegant mathematical formulation in the expression proposed by Manski (1977) but its use is not found within the literature in the field of the assignment.
For the general structure of the model, three levels are defined: (a) Formation, (b) Extraction, (c) Choice.

(a)
The *formation* model consists in the construction of the sets G_n perceived in which the user n chooses the alternative. In the mono-set exhaustive approach only one set is hypothesized considered by the user equal to M_n. In the mono-set selective approach only one set is considered, which is a subset of M_n. In the multi-set exhaustive approach all the possible combinations of routes extracted from M_n is considered. In the multi-set selective approach a subset of all the possible combinations of routes extracted from M_n is considered.
This model is connected with the decisional power of flexible and efficient algorithms that concur to generate routes attractive for users to generate one (mono-set) or many (multi-sets perceived sets. A detailed analysis of such algorithms is contained in Russo and Vitetta,

2006c and moreover a transferability test for route generation algorithms that help to identify also cost-effective techniques is reported in Bekhor and Prato (2009).

(b)
The *extraction* model consists in obtaining the evaluation probability for each choice set perceived by users and defined in the formation model ($p(C/G_n)$ that is equal to 1 for mono-set and $p(C_p/G_n)$ for multi-sets). Considering the mono-set approach: in Probit model proposed in Sheffi (1985) the exhaustive approach is proposed with one criterion for the formation level. In Dial (1971) the selective approach is proposed with one criterion for the formation level; in Ben-Akiva *et al*. (1984), Russo and Vitetta (1996, 2003), Cascetta *et al*. (2002), the selective approach is considered with multi criteria for the formation level. In all these approaches the probability associated to the set extraction is equal to 1. Considering the multi-set approach in Swait and Ben Akiva (1987), Ben Akiva and Boccara (1995), and Russo *et al*. (2007), Quattrone and Vitetta (2008), the exhaustive approach is considered and the structure for generation and choice models is different. The formation and extraction level very often in literature are considered simultaneously, considering just one set of alternatives (mono-set approach).
In this paper the influence of the structure of the model due to multi-set exhaustive and selective approaches is considered.

(c)
The *route choice* model consists in the route choice evaluation in each alternative contained in the extracted sets.
The choice models most widely used in the literature belong to the family of well established Random Utility Models (RUM), namely: Multinomial Logit (Dial, 1971; Ben Akiva *et al.*, 1984); Nested Logit (Domencich and McFadden, 1975; Vovsha, 1997); Cross Nested Logit and Generalized-Nested Logit (Vovsha and Bekhor, 1998; Prashker and Bekhor, 1998; Bekhor and Prashker, 2001, Bekhor *et al*. 2001), Probit (Sheffi, 1985) and modified Multinomial Logit as C-Logit (Cascetta *et al.*, 1996), DC-Logit (Russo and Vitetta, 2003), Route Size (Ben Akiva and Lerman, 1985; Ben Akiva and Bierlaire,1999; Frejinger and Bierlaire, 2007) and Mixed (Cascetta and Papola, 2001). Route generation and choice models at inter-urban level have been proposed in several papers (Leurent 1993; De La Barra *et al*. 1993 Ben-Akiva *et al.,* 1984; Russo and Vitetta, 1996 Cascetta *et al.,* 1996).
In the field of explicit route enumeration with a selective approach, Cascetta *et al*. (1996) proposed a C-Logit model which, though retaining a closed analytical form, allows us to take account of route overlapping problems. The C-Logit route choice model keeps the simple mathematical structure of the multinomial Logit model with a modified systematic utility, inserting an attribute named Commonality factor (C stands for Commonality). The C-Logit model was switched in terms of algorithm from explicit to implicit in Russo and Vitetta (2003). The model and its solution algorithm (implicit), called D-C-Logit for short, based on a Dial structure, combine several positive features from models and algorithms found in the literature: a selective approach with behavioural rules of routes; closing form for route choice; consideration of route overlap; efficient like Dial's algorithm (D in D-C-Logit stands for Dial).

In the mono-set approach, only one perceived set exists and the probability of being chosen is equal to one. In this context the probability of choosing the alternative is not dependent upon the perceived choice set:

$$p_n(k/G_n \neq \emptyset) = p_n(k/C).$$

In the mono-set approach different perceived sets exist and the two probabilities are different.

The route choice, in the context of random utility models, assumes that a generic user, travelling between an origin-destination pair (*o, d*), associates to each route *k* belonging to the set C_{od}, named for simplicity C, of available routes connecting that (*o, d*) pair, a perceived utility U_k which may be expressed as:

$$U_k = V_k + \varepsilon_k \qquad \forall\, k \in C_{od}$$

The variable V_k denotes the average, or systematic, utility of route *k* while the random residual ε_k is usually assumed to include perception errors of the decision maker as well as the modelling approximation of the analyst.

Under the assumption of random utility models, route choice probabilities of a generic user can be expressed as:

$$p(k/C_{od}) = \text{prob}[V_k + \varepsilon_k \geq V_h + \varepsilon_h] \qquad \forall\, k \neq h;\ k, h \in C$$

If the ε_k are assumed to be independent and identical Gumbel variates of zero mean and parameter θ, then the well-known Multinomial Logit model of route choice results.

If the residuals are assumed to be jointly distributed as a Multivariate Normal of zero mean, the Probit route choice model is obtained.

The C-Logit formulation, described in Cascetta *et al* (1996), has the simple mathematical structure of the multinomial Logit model, but with a modified systematic utility as:

$$V^*_k = V_k - CF'_k \qquad \forall\, k \in C$$

and hence

$$p(k/C) = \exp(V_k : \theta - CF'_k : \theta) : \Sigma_{h \in Cod} \exp(V_h : \theta - CF'_h : \theta)$$

The term CF'_k denoted as "commonality factor" of the route *k*, is directly proportional to the degree of similarity (or overlapping) of the route *k* with the other routes belonging to C_{od}. The role played by CF'_k is clear; heavily overlapping routes have larger commonality factors and thus a smaller systematic utility with respect to similar but independent routes. A general expression for CF'_k is:

$$CF'_k = \theta\, \beta'_0\ \phi(\mathbf{q}_k) \qquad (1)$$

where

- β'_0 is a parameter to calibrate;

- ϕ is a functional form that should be monotonically increasing with increasing overlap;
- $\boldsymbol{q}_k$ is a vector of attributes defining the systematic utility of overlapping between route *k* and the other routes belonging to the choice set.

It is assumed that the CF'_k term is independent of θ.
For simplicity's sake a parameter β_0 to calibrate is used instead of $\theta\,\beta'_0$, and the new C-Logit route choice model is:

$$p(k/C) = \exp(V_k : \theta - CF_k) : \Sigma_{h\in Cod} \exp(V_h : \theta - CF_h)$$

Because of this we express in a generic (*o, d*) pair the overall route utility in term of "cost" for a route *k* as:

$$g_k = - V_k : \theta + CF_k \qquad (2)$$

The cost is a disutility and for simplicity it is considered with opposite sign of the modified utility and containing the value of parameter θ.
With reference to the mono-set approach an implicit approach is also considered (for the selective approach see Cascetta and Papola 1997, Cascetta *et al*. 2002; for the exhaustive approach Morikawa 1996, for the modal split). The perception of each alternative consists in simulating through an attribute or model what is inside the utility specification of the route choice model.

3. ROUTE CHOICE: MONO AND MULTI-SETS D-C-LOGIT

A new classification of the model and the main references relative to the route choice are reported in Table I. The D-C-Logit model can be specified for the mono-set and multi-set selective approaches.

Table I – Specification for route choice formulation in the different cases

		Mono-set		Multi-sets	
		Exhaustive	Selective	Exhaustive	Selective
Generation	G_n *Formation*	$C = M_n$ $G_n = \{C\}$	$C \subset M_n$ $G_n = \{C\}$	$C_p = Comb(M_n)$ $\cup_p C_p = All\ Comb(M_n)$ $G_n = \{.., C_p, ..\}$	$C_p = Comb(M_n)$ $\cup_p C_p \subset All\ Comb(M_n)$ $G_n = \{.., C_p, ..\}$
	C *Extraction*	$p(C/G_n) = 1$	$p(C/G_n) = 1$	$0 < p(C_p/G_n) < 1$ $\Sigma_p\, p(C_p/G_n) = 1$	$0 < p(C_p/G_n) < 1$ $\Sigma_p\, p(C_p/G_n) = 1$
Choice alternative k		$0 \le p(k/C) \le 1$ $\Sigma_{h\in C}\, p(k/C) = 1$	$0 \le p(k/C) \le 1$ $\Sigma_{h\in C}\, p(k/C) = 1$	$0 \le p(k/C_p) \le 1$ $\Sigma_{h\in Cp}\, p(k/C_p) = 1$	$0 \le p(k/C_p) \le 1$ $\Sigma_{h\in Cp}\, p(k/C_p) = 1$
Main References		Sheffi, 1985	Dial, 1971 Ben-Akiva *et al.*, 1984 Russo and Vitetta, 1996 Cascetta *et al.*, 2002 Russo and Vitetta, 2003	Swait and Ben-Akiva, 1987 Ben-Akiva and Boccara, 1995 Russo *et al.*, 2007	Discussed in this paper

Comb(A) = Combination of the elements of the set A

3.1 Formation

The choice set formation is modelled following a selective approach, in which we suppose that the user identifies only a few routes from those which are topologically admissible, according to whether they satisfy some behavioural hypothesis in descriptive mono-set or multi-set approaches.

In order to generate the sets C for mono-sets and all C_p for multi-sets, it is required that only efficient routes, with regard to the origin and/or destination, are generated. In the mono-set approach, the routes are generated considering a specific criterion; in the multi-set approach, for each set C_p the routes are generated considering a specific criterion p. Considering one of these sets there is a criterion that identify the set and a cost associated to each link and relative to the criteria named criterion cost. Each route has links with an initial node that leads away from the origin with regard to the cost criterion minimum route, and a final node that tends to the destination with regard to the cost criterion minimum route. In other words, a link $l=(i,j)$ belongs to an efficient route only if the cost criterion minimum route to reach the initial node from the origin is less than the cost criterion minimum route to reach the final node, and the cost criterion minimum route to reach the destination from the final node is less than the cost criterion minimum route to reach it from the initial node (Dial, 1971; Sheffi, 1985).

The D-C-Logit model considers in the choice set only efficient routes, consisting of links belonging to routes which “move away” from the origin and "move towards" the destination in relation to the defined choice set generation model in the same way proposed by Dial (1971).

Let:

- *o* be the origin in question;
- *ij* be the link in question from node i to node j;
- $g_{(ab)}$ the cost criterion minimum route of reaching node *b* starting from *a*.

Link *ij* is considered efficient if:

$$g_{(oj)} \geq g_{(oi)} \text{ and } g_{(id)} \geq g_{(jd)}$$

The choice set generated with the criterion thus defined, satisfies some of the main elements which arise in the literature and which are transferred to the various proposed models. These main features may be summarized as follows:

- a finite number of routes is considered;
- routes are considered, using a function based on the criterion cost;
- subsequent to the first route, other routes are generated according to an exact (non-heuristic) algorithm;
- routes which contradict basic behavioural hypotheses, though topologically feasible, are not considered; hence the sets are definitely limited in size by the number of loop-less routes;
- multi-criteria and multi-set approach are considered.

An alternative choice set formation model can be proposed which is a variation of the previous one in the same context of the selective mono-criterion approach. In this alternative way, one of the conditions for route efficiency was discarded: all routes composed of links whose initial node is nearer the origin node than is its final node are considered. A link *ij* is considered efficient if and only if the link has its initial node *i* closer to the origin node than its final node *j*:

$$g_{(oj)} \geq g_{(oi)}$$

In this condition a larger number of efficient routes can be present in the choice set than in previous specifications.

3.2 Extraction

The choice set extraction is evaluated inside the mono-set or multi-set approach and it is supposed that users perceive with a different probability the different sets. The probability of user n choosing the set C on condition that G_n is:

- in a mono-set approach (there is only one set named C)

$$p_n(C/G_n \neq \emptyset) \text{ with } p_n(C/G_n \neq \emptyset) = 1$$

- in a multi-set approach (there is more than one set, each called C_p)

$$p_n(C_p/G_n \neq \emptyset) \text{ with } \Sigma_p\, p_n(C_p/G_n \neq \emptyset) = 1$$

For the $p_n(C_p/G_n \neq \emptyset)$ evaluation several models can be used with the extension to this level of the models generally considered for the level relative to the choice of the alternative. Random utility (Domencich and McFadden 1975; Ben Akiva and Lerman, 1985) and fuzzy utility (Zadeh, 1965) models are the most common
The attributes in the utility specification are relative to the criterion considered for generating the set. Some of the possible attributes are label, size, crisp (for fuzzy), the distance between attributes in the routes of C_p and the best value in the routes of C_p.
The probability for user n to choose set C conditional upon G_n with this specification can be evaluated with attributes differing from that considered for the choice of the alternative.

3.3 Choice

In this section the model has to be applied considering a fixed choice set that derives from the formation level (C for mono-set or C_p for multi-set). In the case of mono-set the index must not be considered. In the case of multi-set the index has to be considered but it is not reported in order to simplify the variable notation.
The commonality factor specification proposed in this paper and useful for the proposed model is as follows:

$$CF_k = \beta_0\, \Sigma_{ij \in k}\, q_{ij\,od} \log (N_{ij,od}) \tag{3}$$

where

- β_0 is a parameter to be calibrated;

- $q_{ij,od}$ is the ratio between the cost of the link *ij* and the minimum route cost $g_{(od)}$ on the (*o, d*) pair;
- $N_{ij,od}$ is the link multiplicity (number of routes which join the (*o, d*) pair, containing the link ij).

The cost c_{ij} of the link *ij*, considering the C-Logit term $cf_{ij,od}$, is related to each O/D pair and is expressed as:

$$c_{ij,od} = c_{ij} + cf_{ij,od} = c_{ij} + \beta_0\, q_{ij,od} \log(N_{ij,od}) = c_{ij}\, [1 + \beta_0 \log(N_{ij,od}) : g_{(od)}]$$

The cost $c_{ij,od}$ is an attribute or a combination of attributes relative to the criteria considered for the set formation. For example: if the criterion minimum time is considered for the extraction, the label time can be considered and for the choice of the alternative the travel time (and/or the cost) can be considered; if the criterion maximum scenic is considered, for the extraction the label scenic can be considered and for the choice of the alternative the travel time on the scenic link and/or the travel time on the high quality road can be considered.
The overall route cost for route k is:

$$g_k = \Sigma_{ij\in k}\, c_{ij,od} = \Sigma_{ij\in k}\, (c_{ij}+ cf_{ij,od}) = \Sigma_{ij\in k}\, c_{ij}+ \beta_0\, \Sigma_{ij\in k}\, q_{ij,od}\log(N_{ij,od}) \qquad \forall\, k\in C_{od} \qquad (4)$$

The probability of using route *k*, considering the new term, between *o* and *d*, given a choice set *C* of efficient routes, p(*k/C*), is:

$$p(k/C) = \exp(-g_k) : \Sigma_{h\in C} \exp(-g_h) = Z_{od}\, \Pi_{ij\in k}\, e_{ij,od} \qquad \forall\, k\in C_{od} \qquad (5)$$

with
$Z_{od} = 1 : \Sigma_{h\in C} \exp(-g_h)$

$$\begin{cases} e_{ij,od} = \exp(-c_{ij,od}) & \text{if on link } ij \text{ efficiency conditions (section 3.2) occur} \\ e_{ij,od} = 0 & \text{otherwise} \end{cases}$$

Dial (1971) demonstrated that specification (5) is satisfied if the link flow x_{ij} relative to the (*o, d*) pair is obtained as:

$$\begin{cases} x_{ij} = p(ij/j)\, d_{od} & \text{for } j=d \\ x_{ij} = p(ij/j)\, \Sigma_{m\in F(j)}\, x_{jm} & \text{for the other links} \end{cases} \qquad (6)$$

where *F(j)* is the forward star of outgoing links from node *j* and d_{od} is the total demand flow from the origin *o* to the destination *d*. The probability *p(ij/j)* is obtained as follows:

$$p(ij/j) = w_{ij,od} / \Sigma_{m\in B(j)}\, w_{mj,od}$$

where $w_{ij,od}$ is the "link weight", depending on the different (*o, d*) pairs considered.
In the case of a multi-set considering the set C_p, the link flow can be named x_{ij}^p.
With the use of the CF factor, the link weight has the structure:

$$\begin{cases} w_{ij,od}=N_{ij,od}^{\,(-\beta_0\, c_{ij}\,:\, g_{(od)})}\exp(-c_{ij})\Sigma_{m\in B(i)}N_{mi,od}^{(-\beta_0\, q_{ij,od})}\exp(-c_{mi}) & \text{if } ij \text{ is efficient} \\ w_{ij,od} = 0 & \text{otherwise} \end{cases} \quad (7)$$

where *B(i)* is the backward star of the incoming links in node *i*.
Having identified a link *ij* and an (*o, d*) pair, the multiplicity $N_{ij,od}$ of the link *ij* with respect to the (*o, d*) pair can be calculated. Such multiplicity indicates the number of routes which join the (*o, d*) pair and use the link *ij*.
For the D-C-Logit model, a route *k* joining an (*o, d*) pair is subdivided into three parts: one which precedes node *i*, one coinciding with link *ij* and one which follows node *j*. Hence, having considered all the efficient routes joining the (*o, d*) pair and using link *ij*, we may define two sets containing parts of routes which, respectively, precede node *i* and follow node *j*. Let $NA_{ij,o}$ and $NB_{ij,d}$ denote, respectively, the cardinality of the two sets. The multiplicity of link *ij* may be obtained as:

$$N_{ij,od} = NA_{ij,o} \quad NB_{ij,d} \quad (8)$$

This specification can be considered similar to that proposed by Van Vliet (1981), who proposed a modified Dial algorithm in order to evaluate the number of routes from a fixed (*o, d*) pair passing between a set sequence of nodes.
Once an (*o, d*) pair has been identified, results may be carried out to calculate the single terms of the above relations. Calculation of $NA_{ij,o}$ and $NB_{ij,d}$ of the two sets is based on a recursive principle.
The values:

- $NA_{ij,o}$ is given by summing the $NA_{bi,o}$ of links *bi* which are incident with *i* and efficient;
- $NB_{ij,d}$ is given by summing the $NB_{jp,d}$ of links *jp* which depart from *j* and are efficient.

Initialization consists in setting:

- $NA_{om,o}$ equal to 1 of links *om* departing from *o* ;
- $NB_{nd,d}$ equal to 1 of links *nd* arriving at *d*.

Moreover, the network nodes must be visited in ascending (descending) order of minimum cost of reaching them from the origin (destination).
For the algorithm, respect the STOCH (Sheffi, 1985) algorithm, the multiplicity of each link have to be calculated.

3.4 Link flow

At the end, the link flow x~~f~~$_{ij}$ on the link ij must be obtained from the general model. We can have two cases:

- in a mono-set approach there is only one set extracted and the probability of being extracted is 1; the flow x_{ij} is evaluated as reported in section 3.3; the final flow is

$$\sout{f_{ij} =} x_{ij} \quad \forall\, ij$$

- in a multi-set approach there is more than one set, one for each criterion p considered named C_p, and for each of them the probability $p_n(C_p/G_n\neq\varnothing)$ is evaluated with a model reported in section 3.2; for each criterion p the link flow x_{ij}^{p} is evaluated as reported in section 3.3; the final flow is

$$xf_{ij} = \sum_{Cp} x_{ij}^{p} p_n(C_p/G_n \neq \emptyset) \qquad \forall ij$$

4. NUMERICAL APPLICATIONS

There are two objectives in the numerical applications:

- to evaluate the influence of the choice set on user modelling;
- to evaluate the applicability of D-C-Logit model on a real system.

The first objective is evaluated on a small test system in order to verify how the results change in relation to the choice set with a fixed model for the choice of the alternative.
The second objective is evaluated on a real system in order to verify how the D-C-Logit model can be applied and give results comparable with the best consolidated models.
To conduct the test, various approaches are identified for determining the parameter specifications of the Logit and Probit route choice models. The two models depend on the distribution considered for the residual part of the utility associated to the alternatives of each user in route choice model.
If the residual parts are assumed to be independent and identical Gumbel variates with θ parameter the well-known Multinomial Logit or C-Logit model is considered. If the residuals are assumed to be jointly distributed as a Multivariate Normal of zero mean and coefficient of variation *cv*, the Probit route choice model is obtained (Daganzo, 1979). In this paper, the Powell and Sheffi approach (1982) for variance and covariance specification is considered.
Values of θ (for the Logit and C-Logit models) and *cv* (for the Probit model) used in the model were obtained by fixing exogenous values for the coefficient *cv* and setting the same variance for the two models for a given (*o, d*) pair:

$$\pi^2 \theta^2 : 6 = (cv\, g)^2$$

where θ is the Logit parameter for pair (*o, d*).
The parameter θ for the Logit model can be obtained as follows:

$$\theta = 1 : \beta = cv\, g\, 6^{0.5} : \pi$$

4.1 Test system

In order to evaluate the difference in results with mono-set and multi-set exhaustive models, an application on a test system is developed. The non-exhaustive case is only a particular case of the exhaustive case and is not reported.
The network considered for the simulation has 4 nodes and one O/D pair (Fig. 1).

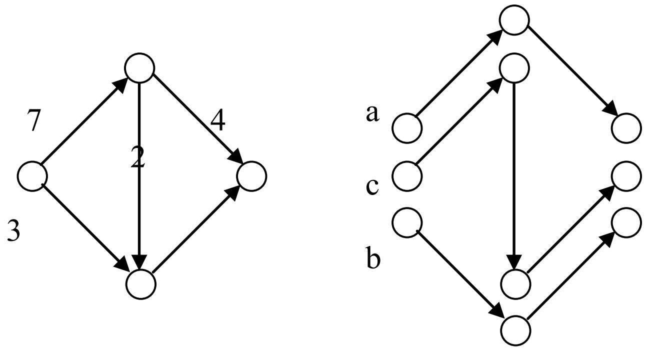

Figure 1 – The test system

In the test system there are three possible routes reported in Fig. 1 with the letter a, b and c. Considering the Manski (1977) formulation, with the definition reported in the section 2, M_n is the set of the feasible alternatives and in the test system is:

$$M_n = \{a, b, c\}$$

In relation to the size of G_n, the set containing all the sub-set that can be extracted from M_n, there are two type of choice set: mono-set and multi-sets.

In the mono-set exhaustive approach the set M_n contain only one set perceived by users, named C

$$G_n = \{C\}$$

C contains the three routes a, b, and c. In this case:

$$C = \{a, b, c\}$$

In the mono-set approach, the probability that C is perceived by users is equal to 1:

$$p_n(C/G_n \neq \emptyset) = 1$$

In the multi-set exhaustive approach the set M_n contain seven sets perceived by users, named C_1, C_2, C_3, C_4, C_5, C_6, C_7:

$$G_n = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$$

The sets C_i are:

$$C_1 = \{a\},\ C_2 = \{b\},\ C_3 = \{c\},\ C_4 = \{a, b\},\ C_5 = \{a, c\},\ C_6 = \{b, c\},\ C_7 = \{a, b, c\}$$

In the multi-set selective approach some of the previously reported sets perceived by users are considered (in this case G_n has a cardinality less than 7). In every case, in the multi-set approach the probability $p_n(C/G_n \neq \emptyset)$ has to be evaluated. The probability depends by the model considered. We could consider different hypotheses on the model for evaluating the probability. Some possibilities are:

- uniform probability between the sets;
- probability proportional to the size of each set;
- Probit: considering a Probit choice model and in each set a linear systematic utility with attributes: the average route time of the set, variance proportional to the average route time of the set, covariance proportional to the common routes (proportional coefficient equal to 0.1);
- C-Logit: considering a Logit choice model and in each set a linear systematic utility with attributes: the average route time of the set time and the C-Logit term depending on the common route between sets;
- CRVV considering a perception model for each route reported in Cascetta, Russo, Viola, Vitetta, 2002; it is supposed a binomial model for the route perception associated to each route.

The results of different cases for the probability of choice set extraction are reported in Table II. In this context it is evident that the different models for choice set perception give results which are not comparable. Route choice probability is influenced by the choice set extraction model considered.

Table II – Probability of choice set extraction (formation, extraction) in the case of mono and multi-set exhaustive approaches in relation to the extraction model

		Extraction						
	Extraction model	*C1*	*C2*	*C3*	*C4*	*C5*	*C6*	*C7*
Mono-set	---	100						
	Uniform	14,3	14,3	14,3	14,3	14,3	14,3	14,3
	Proportional	8,3	8,3	8,3	16,7	16,7	16,7	25,0
Multi-sets	*Probit*	21,6	21,6	5,1	20,7	9,8	9,8	11,4
	C-Logit	14,9	14,9	3,5	24,2	11,7	11,7	19,3
	CRVV	7,6	8,4	2,7	28,0	9,2	10,2	34,0

In order to compare the results at route choice level in multi-sets approach, with the consolidated models, mono-set approach, the extraction models are considered for evaluate also the route choice probability with the Probit proposed in Sheffi (1985) and Powell and Sheffi (1982). The results are reported in the Table III. In the first raw, considering that is a mono-set, the results are relative to the Probit model proposed by Sheffi (1985) and Powell and Sheffi (1982). The application confirms that the numerical results are influenced by the model and the traditional mono-set approach is not necessary the best.

Table III – Route choice probability in the case of mono and multi-set exhaustive approaches in relation to the extraction model and with Probit model for the choice routes level

		Route choice		
	Extraction model	*a*	*b*	*c*
Mono-set	---	43,2	43,2	13,6
	Uniform	37,2	37,2	25,6
	Proportional	38,7	38,7	22,6
Multi-sets	*Probit*	43,5	43,5	13,0
	C-Logit	43,2	43,2	13,6
	CRVV	43,1	43,1	13,8

Some preliminary results on real networks confirm the validity of multi-sets approach. The models are applied in two small real Italian urban networks:

- CBD of Crotone, total demand 5200 vehicles/hours, 352 links;
- CBD of Reggio Calabria, total demand 8700 vehicles/hours, 883 links.

In the two cases the probability of choice extraction is evaluated with a C-logit model and the route choice probability is evaluated with a Probit model.
The results in term of RMSE% between simulated flows and counted flows are:

- CBD of Crotone, 24.41% in the mono-set approach, 21,36% in the multi-sets approach;
- CBD of Reggio Calabria, 13,39% in the mono-set approach, 10,22% in the multi-sets approach.

4.2 Real system

In the real system the computations were carried out by comparing flows obtained with the SNL Probit and flows obtained with Dial Logit and proposed D-C-Logit. The comparison was carried out by using the RMSE% and MSE.
The model is applied on a real urban road network in the city of Salerno (Italy). The city has about 200,000 inhabitants. The city is subdivided into 53 homogeneous internal zones and 9 external ones, the network thus consisting of 62 centroids, 466 real nodes and 1127 links. The network at morning peak hour is congested. The average flow/capacity ratio is 0.8 and 8% of links have a flow/capacity ratio greater than 0.9. The link travel time flow function is considered as depending on the flow/capacity ratio through the BPR link cost function. The (*o, d*) demand for the a.m. peak hour is derived from a sample of users interviewed both at home and at cordon sections. The resulting (*o, d*) demand matrix is "corrected" for level using traffic counts on cordons by means of an assignment matrix derived from a DUE (Cascetta, 2001).
All the analyses are carried out by comparing flow counts on 69 sections with flows predicted by an evaluation method on the same links. The evaluation method is developed with the following steps: the congested travel time on the network is obtained with a DUE (Deterministic User Equilibrium); SNL Logit (Dial, 1971), D-C-Logit and SNL Probit (Sheffi, 1985), using link times obtained with DUE without capacity restraint, have been performed; the comparison between measured flows and flows obtained from models is carried out by using an RMSE% indicator.
The RMSE% obtained with DUE without capacity restraint flows is 28.7%. The RMSE% obtained with AON (All Or Nothing) network loading is 41.8%.
The numerical application is executed with different values of *cv* and parameters β_0 considered for the C-Logit terms. In the figure 2 the results obtained with implicit Probit, Logit (β_0=0) and proposed D-C-Logit model are compared in terms of RMSE% in relation to traffic counts on a set of links. Different levels of variance for user-perceived costs are tested and in order to calibrate the β_0 parameter for commonality factor, different values are considered.
As in the test network, and in previous experiments with the explicit method, also in the used real network the D-C-Logit assignment reproduces observed flows better than Logit assignment and Probit.

On the real network with implicit SNL D-C-Logit with *cv*=0.1 are reported. The best values of RMSE% are obtained with parameter β_0 just greater than 1.

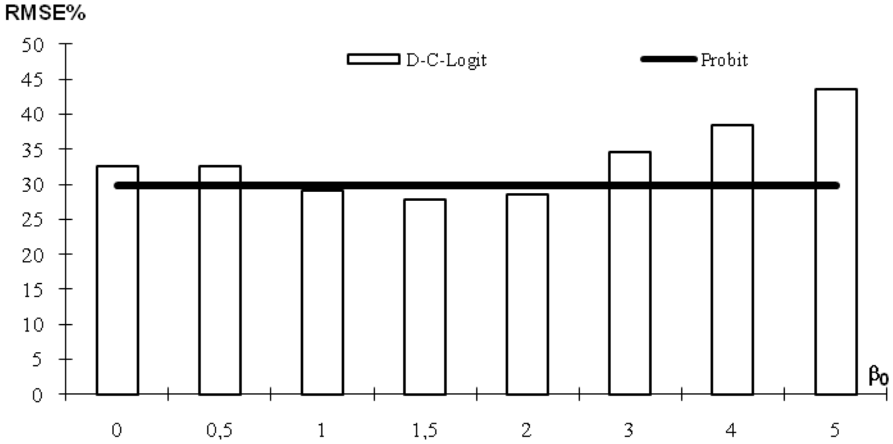

Figure 2 – The results in terms of RMSE% on the real network with implicit SNL and cv = 0.1

The results obtained confirm that:

- Probit and D-C-Logit are better than Logit;
- with β_0=1 (theoretical value, Cascetta *et al.*, 1996), D-C-Logit results are better than Probit;
- with β_0 appropriately calibrated for the real situation, the D-C-Logit is better than Probit due to the goodness of the D-C-Logit model and to the greater number of calibrated parameters than in the Probit model.

The modest increase in computational time for D-C-Logit compared with the classical Dial, about 5%÷6%, confirm also that the use of the proposed model can be preferable.

5. CONCLUSION

In this paper the problem of route choice in transportation systems is reported. Route choice modelling can be divided into two sub-problems: choice set generation and choice of the alternative.
In relation to the first problem, choice set generation, very often in the literature a fixed choice set is considered and all routes are considered with the same probability of being considered. In this paper different hypotheses on choice set perception are supposed and are applied on a small system. The application confirms that the numerical results are influenced by the model. Some preliminary results on real networks confirm that the multi-sets approach give better results than the mono-set-approach. User behaviour for choice set perception is an area where further research is required.
A route choice model named D-C-Logit which obviates enumeration and overlapping problems is presented and applied. The D-C-Logit model gives results which are very close to the Probit when all routes have similar costs, while in other cases an accurate parameter calibration gives better results. The calculation of the route choice probability in closed form guarantees result reproducibility independently of solution algorithm parameters (such as the

number of iterations or halting test parameter). This is particularly relevant in dynamic process assignment and network design where small differences in results would multiply the effect during application both in the demand update and in reverse assignment.

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