Anticipating behavior of public transport users to travel time reliability[1]

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Abstract

We analyze the behavior of public transport users when confronted with an expected delay. Some travelers may reschedule their trips and hence avoid an increase in their schedule delay costs. We build a general model for travelers anticipating to delays in the case of discrete departure time choice, common in public transport. We solve the model for the case of exponentially distributed delays. Public transport travelers fully offset the incurred schedule delay costs in the case of a deterministic delay and largely offset these costs if delays are stochastic. We use empirical route level delay data from ten busy train routes in the Dutch Randstad area to illustrate our results. The numerical results suggest that ignoring anticipating behavior would lead to an overestimation of the welfare costs of delays of 2.4 to 5.4 percent, despite the fact that delays are relatively low in our study area. The overestimation of welfare costs of unreliability may lead to an inefficiently high level of investments in projects and measures aimed at preventing delays.

KEYWORDS: reliability, punctuality, public transport, anticipating behavior

1. Introduction

Transport reliability has been the subject of growing attention from researchers and policy makers in recent years. A stream of theoretical and empirical research (see Noland and Polak, 2002, for an overview) has substantially increased our understanding of travelers' reactions to delays and uncertainty in transport. Travelers realize that delays may occur and act on that. If they value arriving late sufficiently high or if connecting times are tight, they may change their departure time.

We focus on behavioral responses to expected delays in public transport, zooming in on the stochastic nature of these delays. Noland and Small (1995) have analyzed the effect on stochastic delays on car users, but the case of public transport is different, as the choice of departure times is discrete and linked to the schedule. Fosgerau (2009) assumes that that travelers in high frequency systems do not plan their trips, as their expectation of delays relative to the headway is small. Bates *et al.* (2001) show how travelers react to deterministic delays in public transport and Tseng (2008) analyzed anticipating behavior for public transport, using two mass-points travel time distributions. We build a more general

[1] This research took place in the context of the Transumo programme

model, allowing for all types of distributions and solve the model for the exponential distribution of delays, which reflects the fact that the probability of short delays is generally larger than the probability of longer delays. The main contribution of our paper to the literature is that it formulates a general model for anticipating behavior in the case of discrete departure and arrival time choices and combines that general model with a realistic distribution of delays.

Anticipating behavior by public transport travelers is not only interesting from a behavioral point of view. It also affects the welfare loss associated with delays. If travelers anticipate delays, they reduce welfare losses coming from these delays. Ignoring anticipating behavior will therefore lead to overestimating these welfare losses, and hence to overestimating welfare benefits of transport projects or policies aimed at increasing the reliability. As Noland and Polak (2002, p.52) phrase it: “A clearer understanding of the behavioural response to reliability changes is essential for improving cost benefit assessment of transport projects.” This paper adds to the understanding of such response by explicitly modeling travelers’ reactions to expected delays.

The remainder of this paper is organized as follows. Section 2 lines out the general framework of our analysis. We then look at anticipating behavior by travelers and to the welfare effects with and without anticipating behavior in section 3. Section 4 applies the model and its findings to 10 real life cases using data from the Dutch Randstad area. Section 5 concludes and looks at possible directions for further research.

2. Framework

This section describes the general framework used in the analysis. It loosely follows the framework used in Bates *et al.* (2001), defining disutility (U) from a trip as:

$$U = \alpha\left(T - t_h\right) + \beta SDE + \gamma SDL \qquad (1)$$

Where SDE and SDL denote the early and late schedule delay respectively, T reflects the arrival time and t_h denotes the time of departure, hence total travel time is given by $T - t_h$. Parameter values are bounded by $\beta < \alpha < \gamma$.[2] With scheduled services, the choice of departure times is a discrete choice, whereas it is continuous in private modes. We can therefore model the choice for the departure time as the choice between service 1 and service 2. A traveler will be indifferent between these services if the expected

[2] Small (1982) and Bates *et al.* (2001) add a dummy variable to the utility function for all trips that arrive after the desired time. This implies that arriving late in itself yields disutility, not matter how large the delay is. Mahmassani and Chang (1986), apply a threshold value for delays to matter in the first place. Some travelers do not have strict preferences about arrival times and may accept a delay (in either direction) of a couple of minutes. The question which of these specifications is correct is an empirical one. Recent results by Tseng (2008) suggest that neither of the alternative approaches should be preferred over the one in equation (1).

utility of both services is equal. We assume that both services have equal travel times and equal expected delays, allowing us to ignore travel times when choosing between travel options.

The choice between two services is therefore ultimately determined by schedule delay, i.e. the amount by which the arrival time of a scheduled service deviates from the preferred arrival time of the traveler. Following the common definition of schedule delay (e.g. Small, 1982; Noland and Small, 1995; Bates *et al.,* 2001), we define:

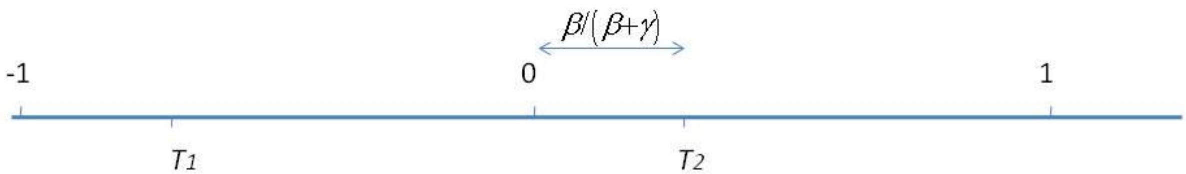
$$SDL = \begin{cases} T - PAT_i & if \quad T > PAT_i \\ 0 & otherwise \end{cases} \tag{2a}$$

$$SDE = \begin{cases} PAT_i - T & if \quad T < PAT_i \\ 0 & otherwise \end{cases} \tag{2b}$$

Where PAT_i is the preferred arrival time of individual *i*.

We focus on the choice between service 1 and 2, with arrival times T_1 and T_2 respectively. Without loss of generality, we assume *that* $T_2>T_1$ and we normalize the headway between the services to unity, so that $T_2=T_1+1$. Furthermore, we normalize the preferred arrival time of the traveler that is indifferent between both services (without delays) to zero. We assume that preferred arrival times are distributed uniformly along the range [-1,1]. Figure 1 graphically depicts the resulting framework.

Figure 1 *Framework of the analysis*

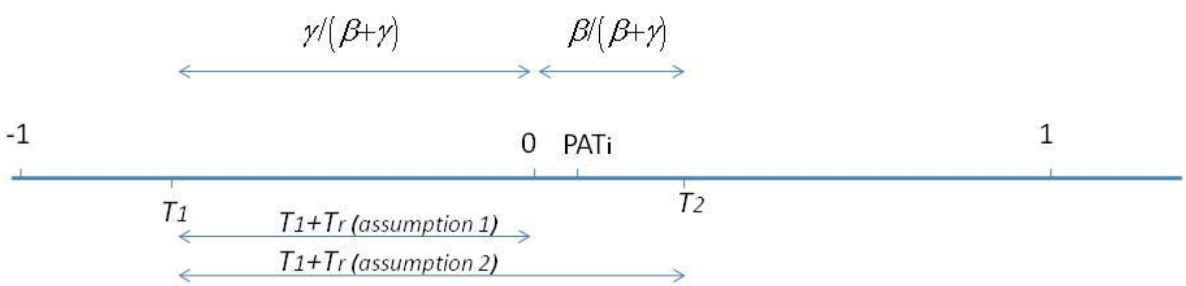


Travelers with a preferred arrival time between -1 and 0 choose service 1, whereas travelers with a preferred arrival time between 0 and 1 choose service 2. Bates et al (2001) analyze a similar choice problem without delays, and show that $\gamma/(\gamma+\beta)$ of the travelers choose the service that arrives before their preferred arrival time. See figure 1 in Fosgerau (2009) for an intuitive graphical representation of this finding. This implies that in each train, $\beta/(\gamma+\beta)$ of the travelers will arrive after their preferred arrival time and loose utility from being late. The remaining $\gamma/(\gamma+\beta)$ of the travelers will arrive before their preferred arrival time and loose utility from being early.

3. Anticipating behavior

We will now take into account anticipating behavior by travelers expecting a delay T_r.[3] We repeat the analysis, taking into account behavioral responses to expected delays. We define traveler i, with preferred arrival time PAT_i, who is indifferent between services 1 and 2 after taking into account the expected delay. Note that $0 \leq PAT_i < T_2$, as switching to a later service in response to a delay does not make sense and $PAT_i \geq T_2$ requires an unrealistically high level of unreliability or a very low value of schedule delay early. We adjust figure 1 in the following way:

Figure 2 *Assumptions in the model*



As figure 2 suggests, we have the choice of proceeding under one of two different assumptions:

[3] Note that normalizing the headway to one implies that delays are now expressed as a fraction of the headway.

1. Delays larger than $\gamma/(\gamma+\beta)$ are not taken into account by passengers
2. Delays larger than the headway are not taken into account by passengers

The first assumption is obviously more restrictive, but simplifies the analysis considerably. It implies that every traveler with a preferred arrival time between 0 and $\beta/(\gamma+\beta)$, expects to be on time using the first service and late using the second. The second assumption has a more intuitive implication: A traveler does not take into account delays larger than the headway, as he will simply take the next train if the delay exceeds the headway. Formally, we would also have to take into account the expected delay of the next service, but travelers may use heuristics rather than exact calculations. Based on empirical observation (see section 5) and the desire to present tractable results, we will adopt the first assumption here and present the results of assumption 2 in Appendix B.

Under assumption 1, traveler i is indifferent between services 1 and 2, if:

$$\begin{aligned} &\alpha \int_0^{\gamma/\beta+\gamma} \left(T_1 - t_{h,1} + T_{r,1}\right) dT_{r,1} + \beta \int_0^{\gamma/\beta+\gamma} \left(PAT_i + T_{r,1} - T_1\right) dT_{r,1} \\ &= \alpha \int_0^{\gamma/\beta+\gamma} \left(T_2 - t_{h,2} + T_{r,2}\right) dT_{r,2} + \gamma \int_0^{\gamma/\beta+\gamma} \left(T_2 - PAT_i + T_{r,2}\right) dT_{r,2} \end{aligned} \tag{3}$$

We assume that the expected delays for both services are independent and follow the same distributions with equal means and standard deviations, so that the first terms of both sides cancel out. Furthermore, we substitute $T_1 = T_2 - 1$ and $T_2 = \beta/(\gamma+\beta)$, to find:

$$\beta \int_0^{\gamma/\beta+\gamma} \left(PAT_i - T_r + \gamma/\beta+\gamma\right) dT_r = \gamma \int_0^{\gamma/\beta+\gamma} \left(\beta/\beta+\gamma - PAT_i + T_r\right) dT_r \tag{4}$$

We may simplify a little further to find:

$$\beta \int_0^{\gamma/\beta+\gamma} \left(PAT_i - T_r\right) dT_r = \gamma \int_0^{\gamma/\beta+\gamma} \left(-PAT_i + T_r\right) dT_r \tag{5}$$

In the deterministic case, where T_r has a fixed value rather than a distribution of possible outcomes, equation (5) simply reduces to $PAT_i = T_r$, i.e. travelers take into account the expected delay and adjust their departure time by the same amount, thus fully offsetting the delay. The intuition behind this adjustment is that passengers interpret the delayed arrival time as actual arrival time. Since the delayed arrival time is deterministic and known to the passenger, this boils down to a mere shift in time tables.

In real life however, passengers do not know the delays exactly. They have an expectation based on experience. The expectation follows some distribution with positive mean and zero minimum. We evaluate equation (5), using an exponential distribution,[4] which is defined by probability density function $f\left(T_r\right)=\lambda e^{-\lambda T_r}$. The advantage of the exponential distribution in this analysis is that it represents the fact that small delays are more likely to occur than larger ones. As we will see in section 5, the exponential distribution is a fairly good representation of train delays as they occur in reality. Parameter *1/λ* in the exponential distribution represents both the mean and the standard deviation of the delay as a fraction of the headway. Substitution into equation (5), integration and some rewriting yields:

$$PAT_i=\frac{1}{\lambda}-\frac{\gamma}{\left(\beta+\gamma\right)\left(e^{\frac{\gamma\lambda}{\beta+\gamma}}-1\right)} \tag{6}$$

Equation (6) represents the difference between the preferred arrival time of the marginal passenger with delays and the preferred arrival time of the marginal passenger without delays (which we normalized to zero). It can be checked that PAT_i goes to zero as λ goes to infinity, which makes sense, as this represents the indifferent traveler in the case without delays. Under the assumption of uniformly distributed preferred arrival times, and for *1/λ*>0, we may define the fraction of the average delay that is offset by anticipating behavior by:

$$PAT_i=1-\frac{\lambda\gamma}{\left(\beta+\gamma\right)\left(e^{\frac{\gamma\lambda}{\beta+\gamma}}-1\right)} \tag{7}$$

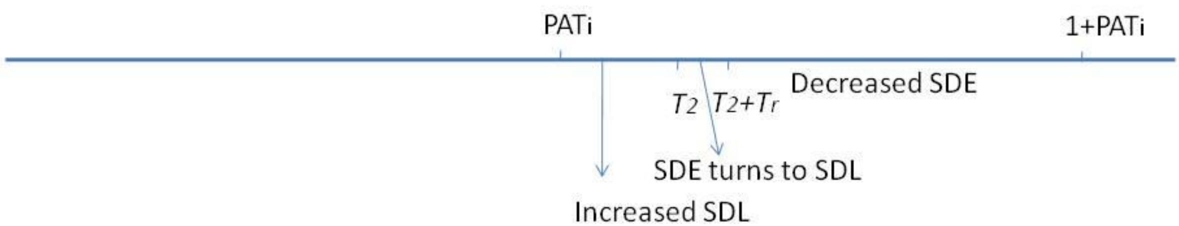
It can be checked from equation (7) that $\partial\lambda PAT_i/\partial\lambda>0$ and that λPAT_i converges to unity for smaller expected delays (i.e. larger values of λ). Also note that $\partial\lambda PAT_i/\partial\gamma>0$ and $\partial\lambda PAT_i/\partial\beta<0$, implying that high valuations of schedule delay late (early) lead to an increase (decrease) in the amount of anticipating behavior. The intuition behind this finding is fairly straightforward. When anticipating to delays, one decreases the risk of being late at the expense of being early more often. Travelers that dislike being late more are therefore more likely to adjust their behavior to expected delays.

[4] The exponential distribution is also used by Fosgerau and Karlström (2009) and by Noland and Small (1995) in their analysis for a scheduled service.

4. Welfare effects

To illustrate the welfare effects of delays with and without anticipating behavior, we graphically represent the effect of service 2 arriving late by a fixed delay T_r within a framework similar to the one depicted before.

Figure 3 *Welfare effects of a delay*



All passengers undergo an increase in their travel time (not reflected in figure 2)[5]. Passengers that were already late, $\beta/(\beta+\gamma)-PAT_i$ of all passengers, undergo an increase in their schedule delay late. Passengers with a *PAT* between T_2 and T_2+T_r now change from being early to being late. Finally, passengers with a *PAT* after between T_2+T_r and *1+*PAT_i experience a decrease in their schedule delay early.

[5] We do not distinguish between delays occurring prior to departure on the one hand and delays occurring during the trip, although in practice the valuation of waiting time and in-vehicle time may differ.

From the illustration above and expressing delays as a distribution, we may define welfare costs (*dU*) of a delay as:[6]

$$dU = \alpha \int T_r dT_r + \int \left(\frac{\beta}{\beta+\gamma} - PAT_i \right) \gamma T_r dT_r + \frac{1}{2}(\gamma-\beta) \int T_r^2 dT_r - \int \left(PAT_i + \frac{\gamma}{\beta+\gamma}_r - T_r \right) \beta T_r dT_r \quad (8)$$

Following our results from the previous section, we have $\int T_r dT_r \geq PAT_i > 0$. To isolate the welfare effect of anticipating behavior, we substitute $\int T_r dT_r > 0, PAT_i = 0$ into equation (8), yielding:

$$dU = \alpha \int T_r dT_r + \int \left(\frac{\beta}{\beta+\gamma}_i \right) \gamma T_r dT_r + \frac{1}{2}(\gamma-\beta) \int T_r^2 dT_r - \int \left(\frac{\gamma}{\beta+\gamma}_r - T_r \right) \beta T_r dT_r \quad (8')$$

The difference between (8) and (8') denotes the welfare gain due to anticipating behavior, which is positive by definition. It also denotes the mistake we are making when we try to calculate welfare effects of a delay without taking anticipating behavior into account. Subtracting (8) from (8') yields $\int PAT_i (\beta+\gamma) T_r dT_r$ i.e. the amount by which a welfare loss from a delay was prevented by anticipating behavior. Apart from that, all travelers that have changed their behavior, experience a cost from rescheduling, regardless whether a delay occurs or not. For the median traveler of all travelers switching, these costs amount to $\frac{1}{2}(\beta+\gamma) PAT_i$. With the share of passengers switching given by PAT_i, the net benefits of anticipating behavior amount to $\int PAT_i (\beta+\gamma) T_r dT_r - \frac{1}{2}(\beta+\gamma) PAT_i^2$. Note that in the limiting case of $PAT_i = \int T_r dT_r$ (which holds for very small expected delays) the costs of rescheduling equal half of the benefits, which follows from the uniform distribution of preferred arrival times.

[6] We omit the range of integration for notational ease, while maintaining assumption 1.

5 Numerical illustrations

This section provides some numerical illustrations to give the reader a feel of the real world consequences of anticipating behavior of public transport travelers. Using values provided in Tseng (2008) and assuming a headway of fifteen minutes, we plot the outcomes of equations (7), (8) and (8') as a function of the expected delay/headway (*1/λ*) in figure4 below.[7]

Figure 4 *Welfare loss as a function of expected delays*

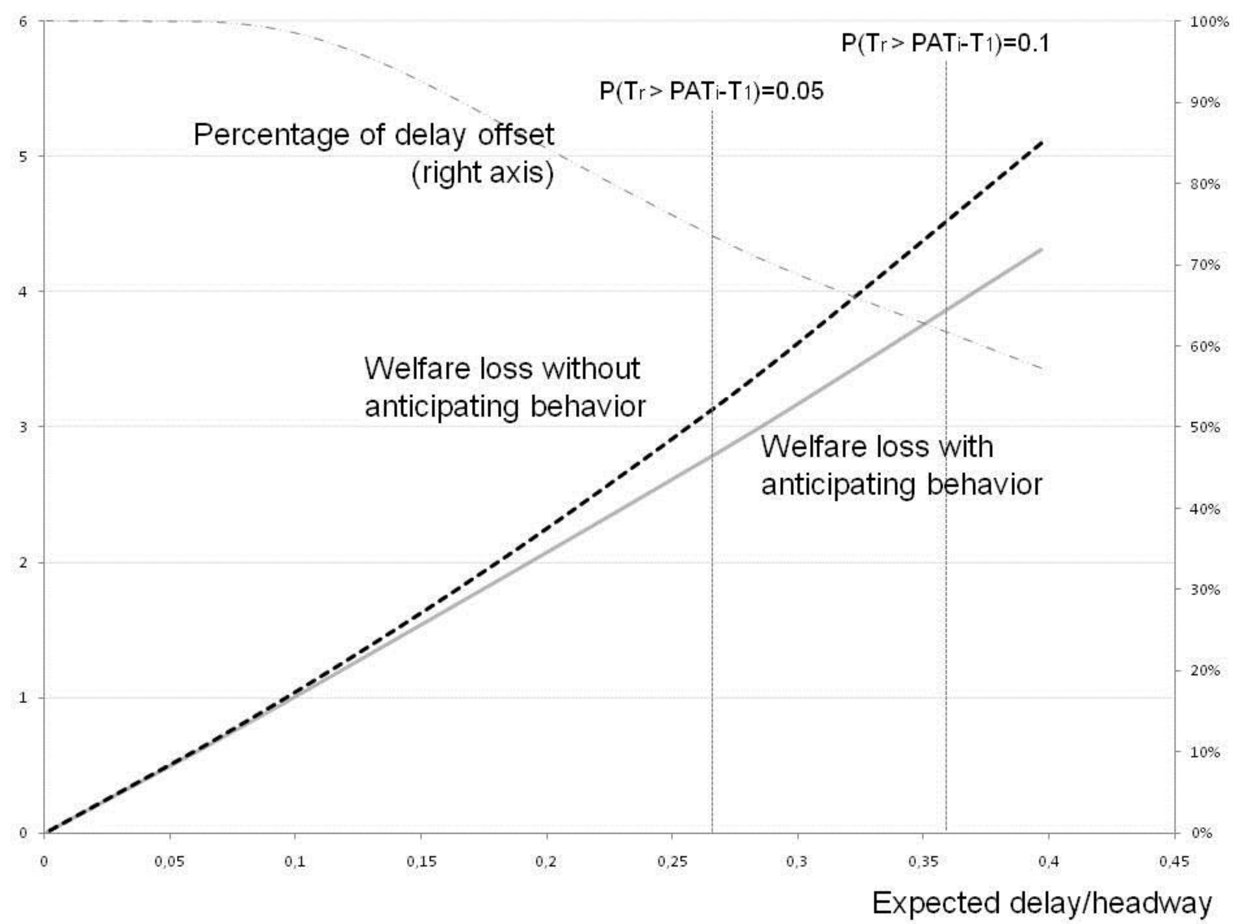


Figure 4 shows that small delays (up to 9 per cent of the headway) are fully offset by anticipating behavior. As delays and their standard deviations increase, anticipating behavior, i.e. taking an earlier service, becomes too costly, because travelers have to accept too large an amount of schedule delay early to account for delays.

[7] Tseng, 2008, table 4.9, page 79, model 3. The value used are *α=9.66, β=6.43 and γ=9.69*, all expressed in Euro's per hour.

Welfare losses are lower due to anticipating behavior as was already clear from the model. Welfare losses obviously increase with the expected delay. The two vertical lines in the graph reflect the 5 and 10-percent probabilities of assumption 1 being violated. As we will show below, real life values for λ are more likely to be in the range where assumption 1 holds.

We use delay data provided by the Dutch railway company (NS) to provide a real life order of magnitude for our results. The data list the delay of every single train for the second Quarter of 2008. We select 10 heavily travelled routes in the Dutch Randstad Area, each of which has a headway of 15 minutes. Negative delays and delays larger than the headway are ignored to provide consistency with our theoretic framework.[8] Table 1 provides an overview of the delay data for these routes.

Table 1 *Delays in minutes at selected routes*

	Mean	sd	#obs
Utrecht - Schiphol Airport	1.15	1.864	2674
Schiphol Airport – Utrecht	1.28	1.989	2928
Utrecht - Amsterdam (coming from South)	1.41	2.145	2489
Amsterdam - Utrecht (heading South)	1.48	1.921	3067
Utrecht - Amsterdam (coming from East)	1.11	1.852	2185
Amsterdam - Utrecht (heading East)	1.60	1.981	2907
Rotterdam - The Hague Central Station	1.50	2.384	2724
The Hague Central Station – Rotterdam	0.82	1.287	3041
Leiden - Schiphol Airport	0.72	1.101	2897
Schiphol Airport – Leiden	0.91	1.669	3274

It is clear from the table that the delay data do not follow the exponential distribution exactly. This would have required that the standard deviation of delays equals the mean delay, which it does not. Even at the level of individual trains (with the same departure time every day), the standard deviation is generally larger than the mean. Based on visual inspection of the histograms and the figures presented in the table, we feel confident that the actual distribution of delays lies close enough to the exponential distribution to apply the data to our framework. For our numerical illustrations, we will use the inverse

[8] When testing assumption 1 (see table 2), delays larger than the headway were included in the data.

of the means presented in table 1 divided by the headway to represent the values of λ in our model.[9] This yields values for of λ in the range of 6 to 8. It is clear from figure 2that these values meet the condition $PAT_i < T_2$, so that we can proceed with equations (6) and (7) from our framework.

Table 2 lists the preferred arrival time of marginal passengers by route, as well as the fraction of delays offset by anticipating behavior. The right hand column tests the probability of assumption 1 being violated, defined as $1 - \int_0^{\gamma/\beta+\gamma} T_r dT_r$.

Table 2 *Preferred arrival time of marginal passenger, fraction of delays offset and probability of violating assumption 1*

	PATi	λPAT_i	P(ass. 1 violated)
Utrecht - Schiphol Airport	0.077	0.999	0.002
Schiphol Airport – Utrecht	0.085	0.997	0.003
Utrecht - Amsterdam (coming from South)	0.093	0.994	0.008
Amsterdam - Utrecht (heading South)	0.098	0.992	0.005
Utrecht - Amsterdam (coming from East)	0.074	0.999	0.001
Amsterdam - Utrecht (heading East)	0.105	0.988	0.006
Rotterdam - The Hague Central Station	0.099	0.992	0.013
The Hague Central Station – Rotterdam	0.055	1.000	0.000
Leiden - Schiphol Airport	0.048	1.000	0.000
Schiphol Airport – Leiden	0.061	1.000	0.000

Table 2 clearly shows that, for realistic values of expected delays and their standard deviations, the delays are almost totally offset by anticipating behavior. It is also clear from the last columns in the table that the crucial assumption in our model, that delays larger than $\gamma/(\gamma+\beta)$ are not taken into account by passengers, is likely to hold in real life situations, with probabilities smaller than 1 percent for most of the case studies.

[9] It can be checked that using the (slightly higher) standard deviations rather than the means does not change the conclusions from this section.

Using the valuations mentioned earlier, we may calculate the results from equations (8) and (8') to illustrate how welfare costs may be overestimated by ignoring anticipating behavior by travelers. Table 3 below provides this illustration for our case studies.

Table 3 *Welfare costs (€ per passenger per trip)and percentage overestimation when anticipating behavior is ignored*

	αTr	Total welfare costs		Overestimation
		Eq(8')	(eq8)+fixed costs	
Utrecht - Schiphol Airport	0.19	0.22	0.21	3.9%
Schiphol Airport – Utrecht	0.21	0.25	0.24	4.3%
Utrecht - Amsterdam (coming from South)	0.23	0.27	0.26	4.8%
Amsterdam - Utrecht (heading South)	0.24	0.29	0.27	5.0%
Utrecht - Amsterdam (coming from East)	0.18	0.21	0.21	3.8%
Amsterdam - Utrecht (heading East)	0.26	0.31	0.30	5.4%
Rotterdam - The Hague Central Station	0.24	0.29	0.28	5.1%
The Hague Central Station – Rotterdam	0.13	0.16	0.15	2.8%
Leiden - Schiphol Airport	0.12	0.14	0.13	2.4%
Schiphol Airport – Leiden	0.15	0.17	0.17	3.1%

Table 3 illustrates that ignoring anticipating behavior by travelers would lead to a slight overestimation of the welfare costs of delays. Depending on the actual delays, the overestimation of welfare costs ranges from 2.4% to 5.4%. This number is fairly low, since a large part of the delay costs, i.e. the cost of extra travel time as given in the first column, is not offset by anticipating behavior. However, in terms of total welfare loss, or in terms of the calculated benefits of a project aimed at increasing reliability, the overestimation is likely to be substantial. It should also be noted that the overestimation is larger in systems that have larger average delays than the highly reliable Dutch system.

6 Conclusions

This paper provides a model in which public transport travelers have expectations about delays and anticipate to these expectations by adjusting their travel behavior. Public transport travelers are different from road users in the sense that their choice of departure (and hence arrival) time is discrete rather than continuous. This requires quite a different approach to travel behavior, where travelers choose between two services with fixed departure and arrival times, rather than choosing their departure time freely. This implies that anticipating to a delay does not simply imply choosing an earlier departure time but a choice whether to take an earlier service or not.

We model this choice and solve the model, showing that travelers fully offset the scheduling costs caused by delays if the delay is deterministic. For stochastic delays, this is not by definition the case. We solve the model for the case of exponentially distributed delays and show that anticipating behavior prevents welfare losses due to delays.

The amount by which travelers anticipate to unreliability depends on the level of unreliability (i.e. the expected delay) and the valuation of schedule delay early and late. In fairly reliable systems, travelers offset the scheduling costs caused by delays almost entirely by choosing an earlier service. As reliability decreases, the (relative) amount of anticipating behavior decreases. Travelers with a high value of schedule delay late relative to schedule delay early are more likely to respond to delays by taking an earlier service than travelers with a relatively low value of schedule delay late.

Using real life data from 10 busy rail routes in the Dutch Randstad area, we show that almost all the expected delay is offset by travelers' anticipating behavior. Ignoring this type of behavior leads to overestimating the welfare costs of delays by 2.4 to 5.4 percent. We also show that the crucial assumption in our model holds for real life values.

Our analysis has several policy implications. First of all, overestimating the welfare costs of delays would lead to an overestimation of the benefits of projects and measures aimed at increasing reliability. Such an overestimation would lead to an inefficiently high level of investments in such projects and measures. Secondly, knowing the mechanism behind anticipating behavior to delays may also increase our understanding of how people plan their trips in high frequency public transport systems. Rather than not planning their trips at all, as Fosgerau (2009) assumes, travelers may use the lowered costs of rescheduling to account for delays. This would require a modification of our model.

Our analysis provides new insight into the response of travelers to delays. It also brings up new questions however. We already mentioned the issue of trip planning in high frequency public transport systems. Other questions include: How would anticipating behavior change in the case of connecting trains? What would be the impact of changing the assumption that preferred arrival times are distributed uniformly over travelers? How would our results be affected by imperfect information on the side of (some of) the travelers? These questions provide ample room for further research. Furthermore, our increased understanding of how public transport travelers react to expected delays may help us further develop our understanding into the effects of reliability on modal choice.

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Appendix

In this appendix, we explore how replacing assumption (1) by assumption (2) affects the outcome of our model. Similar to equation (3) in the main text, traveler i is indifferent between services 1 and 2, hence:

$$\alpha\int_0^1\left(T_1-t_{h,1}+T_{r,1}\right)dT_{r,1}+\beta\int_0^{PAT_i-T_1}\left(PAT_i+T_{r,1}-T_1\right)dT_{r,1}+\gamma\int_{PAT_i-T_1}^{1}\left(T_1-PAT_i+T_{r,1}\right)dT_{r,1}$$
$$=\alpha\int_0^1\left(T_2-t_{h,2}+T_{r,2}\right)dT_{r,2}+\gamma\int_0^1\left(T_2-PAT_i+T_{r,2}\right)dT_{r,2} \tag{A1}$$

Again, we assume that the expected delays for both services are independent and follow the same distributions with equal means and standard deviations, so that the first terms of both sides cancel out. Furthermore, we substitute *T_2=T_1+1* and $T_2=\beta/(\gamma+\beta)$, to find:

$$\beta\int_0^{PAT_i+\frac{\gamma}{\beta+\gamma}}\left(PAT_i+T_{r,1}+\frac{\gamma}{\beta+\gamma}\right)dT_{r,1}+\gamma\int_{PAT_i+\frac{\gamma}{\beta+\gamma}}^{1}\left(-\frac{\gamma}{\beta+\gamma}-PAT_i+T_{r,1}\right)dT_{r,1}$$
$$=\gamma\int_0^1\left(\frac{\beta}{\beta+\gamma}-PAT_i+T_{r,2}\right)dT_{r,2} \tag{A2}$$

Integration and some rewriting leads to a more complicated solution than before, i.e.:

$$PAT_i=\frac{1+\mathrm{Pr}\,oductLog\left(-e^{-\frac{\beta+\gamma\left(1+\lambda-\lambda e^{-\lambda}\right)}{\beta+\gamma}}\right)}{\lambda}-\frac{\gamma}{(\beta+\gamma)e^{\lambda}} \tag{A3}$$

The solution has roughly the same properties as the one in equation (6) in the main body of the text but has to be evaluated numerically due to the productlog in the e quation.