INTRODUCTION OF A CAPACITY SENSITIVE OD MATRIX ESTIMATION PROCESS APPLYING GENETIC ALGORITHM BASED CALIBRATION OF SCGE MODEL

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INTRODUCTION

Research and development procedures can be related to human demand and interests. Generally it is the same with transportation. Since activities originate from motivation, we aim to follow the mobility generating process back to its roots. Based on this methodology the relationship between basic human motivating factors and the orientation of transportation research or model development can be understood. This makes it possible to assign the suitable model development orientations in accordance with human needs.

Maslow's hierarchy of needs [1] helps us to define those kinds of basic human needs, which can result in mobility demand. The Maslow pyramid represents the hierarchy of human motivation. The two lower layers contain physiological and safety needs, while the upper layers are social, esteem and self-actualization needs. Our classification (lower and upper layers) is based on whether the investigated needs have a strong spatial mobility generating effect or not.

According to this approach, physiological needs have been definitely ordered to the class having a strong mobility generating effect. In these days subsistence activities - like production – generate clearly most of the freight and passenger traffic (e.g. raw material and labour transfer). Security level has been ordered to the mobility generating class as well, since a significant part of military actions causes mobility demand, hence transport activities.

Although the rest of the needs (social, esteem and self-actualization needs) can generate mobility demand – even so they are basically motivated and supplied by psychological stimulus and emotions, which certainly do not take place in the material world. Hence this introduction ignores the discussion of these upper kinds of needs.

The above mentioned principles can show us how mobility can be derived from human needs, but it may still not be clear how transportation research or model development orientations can be derived from human needs. To understand this relationship we need to look back in time to identify the first models and the root causes of the needs to estimate mobility demand.

Estimating and organizing procedures are related to solving problems which endeavour to satisfy human needs [2]. In ancient times the problem was also travel time, speed and capacity. Written records going back to ancient times tell us that needs to estimate or calculate mobility demand were related to troop movements in that time. It is well understandable that generally – referring to Maslow's needs – safety comparing to physiological needs could have produced much higher amount of mobility demand at one time in the earlier ages. For instance the army of the ancient Rome already had officers with the title "Logista" [3], who needed to organize supply, accommodation and transfer of the soldiers.

Generally physiological and subsistence needs (in economic term a significant part of these kinds of needs can be satisfied by production) induce demand for modelling transportation when the transportation system cannot support the improvement of economic competitiveness anymore, but causes bottlenecks and barriers to economic growth.

The first published models [4] applied by the "civil" sector appeared in the 19th century, just in the midst of the second industrial revolution. It is not by chance that they were published by that time. Bagwell wrote in "The Transportation Revolution" [5] that between the 18th and the 19th century the average travel time almost dropped to one tenth while mobility demand between the most important cities of the United Kingdom was multiplied by fifty.

The example above leads us to one of the key questions of the paper. Bagwell indirectly said that transport developments can result in a growth of mobility demand. As a result of this principle – mobility demand can appear thanks to the evolution of the transport system. Besides it is known – based on most of recently applied demand models [6] – that social and economic development generates mobility demand.

The assumption, according to which transport, social and economic development result changes in the mobility demand structure, leads us to the conclusion that mobility demand estimating methods should take into consideration the transport, the social and the economic environment simultaneously.

DEMAND AND SUPPLY INTERACTION IN TRANSPORT MODELS

In ancient times – as we have seen – travel time, speed and capacity problems already made people try to estimate mobility demand. During the industrial revolution the rapid development of the transport system caused a significant growth in mobility demand, which further triggered model development. This chapter continues to investigate models in the light of the above mentioned two aspects. The paper in this section tries to focus on how efficient methods satisfy the conditions below:

1. supporting the improvement of transport systems (travel time, speed, capacity);

2. describing the connection between mobility demand, social-economic environment and transportation system.

Traditional transport models

In traditional transport models mobility demand is represented by the origin-destination matrix and demand is assumed to be constant. However we know well that the choice of residence and production-place is strongly affected by travel time, speed and capacity of transport corridors between zones. Since residence and production-place determines mobility demand structure, it does not seem to be realistic to assume the origin-destination matrix to be constant.

Namely if a network operates close to its capacity, then a realistic traffic assignment would modify the network's travel time matrix. That engenders changes in residence and production-place-choices and in long term this phenomenon would affect mobility demand structure and so the origin-destination matrix as well.

A part of the transport systems operate under their capacity limit. In these cases, according to our assumption, traffic does not affect the free-flow travel time. However it is still recommended to reconsider the applicability of traditional approach. Since the aim of transport modelling is the support of system developments these models are required to correctly estimate the future depending on the expected changes. Hence a suitable model needs to describe the changes in mobility demand structure and in traffic structure depending on the planned future interventions with adequate reliability. If the comparison of development scenarios is based on an inconsistent methodology it will lead to unreliable results. Thus it would be reasonable to estimate traffic – which depends on network characteristics – considering its effect on mobility demand. Since effects of social-economic and transport measures affect travel time what modifies decision preferences of residence- and production-place-choices it can be concluded again that the mobility demand – represented by the Origin Destination matrix – cannot be assumed to be constant.

Some of the traditional approaches – like gravity models – take into consideration some impedance-like variables (e.g. distance, travel time etc.) to generate traffic between zones (so demand depends on network characteristics and social-economic parameters as well).

$$T_{ij} = K \frac{M_i^{\alpha} M_j^{\alpha}}{D_{ij}^{\gamma}} \quad (1)$$

"Where K is a constant coefficient and the fitnesses of i and j are represented by M_i and M_j. D_{ij} is usually defined as the Euclidean length; however, it can also have other meanings such as time. The two exponents, α and γ, represent the dependence of the system on fitness and spatial constraints. When describing the interaction between cities, it was theoretically proved that α=1, while γ was demonstrated to range from 0.2 to 2.7 by a variety of real data [7]."

However this approach does still not suit adequately the requirements in as much as describing realistically the connection between transport and social-economic environment. Gravity models generate traffic based directly on the impedance-like variable and the weight parameters – depending on social-economic characteristics – of the investigated zone-pair. So consequently social-economic parameters of the investigated zone-pair, like population or number of employees, do not change when distance, capacity, speed or travel time changes. Though it is evident that travel time reduction affects the families' and companies' long-term-place-choices. Consequently this kind of model is still hardly applicable to suit our requirements in describing realistically the connection of mobility demand, social-economic environment and transport system.

Spatial Computable General Equilibrium models

As Tavasszy mentioned, in the common used transport models production and attraction rates are not elastic. Later he pointed out, that "*these elasticities are endogenous in SCGE models"* [8].
SCGE (Spatial Computable General Equilibrium) models originate from the Dixit-Stiglitz framework [9], which describes market equilibrium based on the decisions of the firms and the individuals, assuming monopolistic competition, but without the representation of spatial processes. Krugman introduced the spatial representation of market processes as a part of the new economic geography school [10].
Anas [11] used SCGE models with detailed transport module to investigate urban areas. Basically Anas applied the traditional form of the congestion function to describe the effect of traffic on demand.

$$g_i = a_i[1 + b(F_i/K_i)^c] \qquad (2)$$

Where g_i is the *travel time*, which is needed to cross zone i. F_i is the *traffic*, which goes through zone i. K_i is the transport *capacity* of zone i, *a and b is a constant* bigger than 0 and *c* is also a *constant* bigger or equal to one. This approach can enable in the future to cover the whole transport process in a fully closed methodological frame from the appearance of the mobility demand until its satisfaction.

SCGE model calibration

Above we have seen that SCGE models apply a general economic approach which additionally makes it possible to analyse the transport system as well. The mentioned SCGE models have already tried to handle the problem of transportation modelling but there are still problems that need to be solved. The calibration process of the SCGE models is quite a complex problem since a lot of required input data of the model are not available. Considering the mentioned calibration difficulties alternative calibration methods would need to be developed to make this model approach better applicable.
Traditionally CGE (Computable General Equilibrium) models may be represented as a system of non-linear net demand equation for sectors of the market (SCGE models originate from CGE models – extended with spatial representation of market processes). Social Account Matrices (SAM) are applied to connect the theoretical model to the observed model by [12]. There are two traditionally proposed data adjustment procedures to calibrate the CGE (or SCGE) models' utility and production functions. Bacharach [13] proposed Row and Column Scaling (RCS), Stone and Byron used a weighted constraint quadratic minimization algorithm. In many cases the input database is not fully completed, hence other calibration alternatives are required to attain higher estimation efficieny.

DIRECTION IN DEVELOPMENT

Due to the above mentioned problems our aim was to find alternative calibration techniques. Moreover to create a possible implementation of the traditional congestion function in SCGE models.

The basics of the methodology originate from the well known 4R equalities of Fujita, Krugman and Venables [14]. Although we accepted the model assumptions, some additional extensions were applied to make the model more realistic.

Firstly the simplifications suggested by the authors (Fujita, Krugman and Venables, 1999, Chapter 4.4 Some Normalizations) regarding the 4R equations are ignored, since unnormalized values make it possible to describe economical processes more realistically.

Besides – contrary to the basic 4R model – share parameters of the CES utility function were applied [15]. This extension can be explained with the same reason: a CES utility function with share parameters can explain customers' decision processes more realistically. Applying the above mentioned model extensions we get the equation system below:

$$G_s = \left[\sum \beta_r^{\sigma} \cdot (p_{rM} \cdot T_{rs})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{3}$$

$$w_{rM} = \beta_r \frac{\sigma-1}{\sigma \cdot c_M}\left[\frac{\mu}{q_r}\sum Y_s (T_{rs})^{1-\sigma}\,(G_s)^{\sigma-1}\right]^{\frac{1}{\sigma}} \tag{4}$$

$$Y_r = l \cdot \mu \cdot \lambda_r \cdot w_{rM} + l \cdot (1-\mu) \cdot \lambda_r \cdot w_{rA} \tag{5}$$

$$p_{rM} = \beta_r \cdot \left[\frac{\mu}{q_r} \cdot \sum Y_s (T_{rs})^{1-\sigma}\,(G_s)^{\sigma-1}\right]^{\frac{1}{\sigma}} \tag{6}$$

Where G is the *price index, r and s* are the *number of regions*, β is *the share parameter*, σ is the *elasticity of substitution*, M and A are the *manufacturing* and the *agriculture* sector, T_{rs} is the *iceberg transport cost*, w is the *wage*, c_M is *the marginal input requirement*, μ is the *share of manufactured goods*, q is the *produced goods*, Y is the *income of the given region*, l is the *number of workers*, λ is the share of the given region's worker supply and p is the *unit price*.

The 4R model applied the *iceberg transport cost* as a constant. However constant transport cost would contradict our main idea because one of our basic assumptions was that the transport system, which includes transport cost as well, is in interaction with mobility demand, thus it cannot be invariable.

So – similarly to Anas [11] – the traditional congestion function was introduced in the model to describe the effect of capacity on mobility demand.

$$g_{rs} = a_{rs}\left[1 + b\left(\frac{q_{rs}}{U \cdot K_{rs}}\right)^{c}\right] \tag{2}$$

Where g_{rs} represents the *travel time*, q_{rs} represents the *shifted goods, U* represents goods vehicle *capacity*, K_{rs} represents the aggregate *capacity* between the rth and the sth model

zone, and a_{rs}, b_{rs} and c_{rs} are *parameters*. Traffic is described directly by the $\frac{q_{rs}}{U}$ fraction, which theoretically connects goods flow and freight traffic.
To involve transport cost endogenously in the model, iceberg transport [16] cost has been expressed depending on other model variables. The simplification below assumes that the overproduction that needs to be done to offset transport cost is equal to vehicle operation cost plus the salary of the driver.

$$T_{rs} = \frac{p_{rM} + C_{Ors} + t \cdot w_{rs} \cdot g_{rs}}{p_{rM}} \qquad (2)$$

Where T_{rs} is the *iceberg transport* cot between the rth and the sth model zone, g_{rs} is the *travel time*, w_{rs} is the *driver's salary, t* is the *parameter* to convert the value of the salary to become minute specific and p_{rM} is the unit *price* of a product manufactured in r.
When the model equalities have been compiled calibration is necessary before applying it in practice. Sometimes the traditional calibration process is quite complicated since many parameters are not available.
Hence at the beginning of the calibration all the parameters and variables should be handled together on the same level. So there are no parameters and variables. Firstly we should concentrate on the availability of the data and select that, which is not too difficult to gain access to. The aim of the calibration is to determine the unknown parameters so already in the beginning it is possible to approach the problem as an equation system, where the later parameters can be variables as well. Now beyond the availability of the data, the solvability of the equation system has to be considered as well, so the number of the variables shall be equal to the number of equations.
After the calibration, based on social-economic input data, the model can be used to estimate mobility demand between the model zones. However, beyond the additional possibility to utilize this advantageous part of the model, we shall keep in mind that the model was basically developed to produce economic output data! This property means that transport investigations can be evaluated in one closed and coherent model which gives us a much more detailed picture about the effects of a given investment.
However, a closed and coherent equation system can be easily optimized with an object function. In this case, the importance from estimating the effects of a certain measure would shift to define the best measure. So it would became possible to optimize spatial networks and urban areas based on infrastructure and land-use measures with a predefined object function which can contain technical, social and economic data as well.

Notes on the introduced model

Before discussing the applied development environment and the results, we should outline the deficiencies of the model. Beyond the assumptions and simplifications applied by Fujita, Krugman and Venables in the 4R equations, one has to mention that the introduced extensions need to be improved, since they are rather just thought-provoking, intermediate results than final solutions.

Although the relation-pairs of the origin-destination matrix can be described with a kind of aggregate capacity index, travel time, speed or distance, it is still not solved methodically to involve the whole transportation network in the model as a fully closed endogenous module (without applying an exogenous traffic model). The necessity of implementing route-choice decisions, transport-mode decisions justifies the further research process. Besides, other questions – like, whether a system or an individual equilibrium would be more realistic, applicable and implementable in the SCGE models' transport modules – have to be discussed.

The secondly introduced function determines iceberg cost. Since iceberg cost theory has been criticized many times [11], it is not necessary to emphasize its weakness. However we have to emphasize, that the "selling-price" – the denominator of the iceberg cost – does not contain all the required cost elements to describe transport activities realistically. This form of the function only presents a methodological scheme.

The third deficiency of the model originates from the 4R model. Since the early model was concentrated on freight transport, it is still not involved in our model. Although a lot of intraregional models [16] have already discussed individual mobility as well – there are still a lot of things to do to merge the two conceptualities into one fully closed model.

Results

The solution of the non-linear equation system was carried out with the MATHLAB software. A non-classical solving approach was investigated as well. Since evolutionary algorithms are getting more and more widespread in this research area [17], a basic genetic algorithm (genetic-algorithm toolbox) framework was compared to a traditional non-linear equation solver method (fsolve module). Since the aim of the analysis is not to prepare a "ready-to-use" model but to compare the mentioned approaches, in both cases the default options were applied with a low iteration number. Hence the results do not let us to evaluate the two models of their own, only a comparison is possible.

First a symmetric case was assumed which includes two regions. The regions and the data are realistic, similar Hungarian counties could be found. The table below presents the list of parameters and results.

1. Table: constants of the solution process

Constant	Value
β, share parameter:	*0.5*
σ, elasticity of substitution:	*3*
T_{rs}, iceberg transport cost:	*1.7*
w, wage:	*220000*
c_M, marginal input requirement:	*0.45*
μ, share of manufactured goods:	*0.5*
l, number of workers:	*13600*
λ, share of the given region of worker supply:	*0.5*

2. Table: result of the solution process, 442 iterations

Variable	Value
G_1, price index:	*106078.26*
G_2, price index:	*106078.26*
q_1, produced goods:	*36633.00*
q_2, produced goods:	*36633.00*
p_1, production price:	*148500.00*
p_2, production price:	*148500.00*
Y_1, regional income:	*10880000000.00*
Y_2, regional income:	*10880000000.00*

It was possible to test the calibration process based on our previous calculation, since we knew which results to expect. Considering the selection criteria above, the *price index (G), the share parameters(β)*, the *elasticity of substitution (σ)* and *the marginal input requirement* ($c_{M)}$) and *the iceberg cost (T_{rs})* were assigned to calibrating variables, assuming their availability to be weak. The other parameters were constant, as presented below. During the comparison our aim was to generate similar optimizing conditions for the two methods so as to be able to analyse which approach could be better applicable in SCHE model calibration. So we tried to harmonise starting values and iteration numbers.

3. Table: constants of the solution process

Constant	Value
q, produced goods:	*36633.00*
p, production price:	*148500.00*
Y_1, regional income:	*10880000000.00*
w, wage:	*220000*
μ, share of manufactured goods:	*0.5*
l, number of workers:	*13600*
λ, share of the given region of worker supply:	*0.5*

4. Table: result of the solution process, traditional method, 1503 iterations

Variable	Starting point	Value	Deviation
G_1, price index:	*100000*	*100000*	*6%*
G_2, price index:	*100000*	*100000*	*6%*
$β_1$, share parameter:	*0.5*	*1.2055*	*58%*
$β_2$, share parameter:	*0.5*	*1.2055*	*58%*
σ, elasticity of substitution:	*5*	*5*	*40%*
c_M, marginal input requirement:	*0.1*	*0.1*	*77%*
T_{12}, iceberg cost:	*2*	*1.96*	*13%*
T_{21}, iceberg cost:	*2*	*1.96*	*13%*

5. Table: result of the solution process, genetic algorithm method, 1500 iterations

Variable	Bounds	Value	Deviation
G_1, price index:	*[90000 ; 110000]*	*90000*	*15%*
G_2, price index:	*[90000 ; 110000]*	*90000*	*15%*
β_1, share parameter:	*[0.3; 0.7]*	*0.3*	*58%*
β_2, share parameter:	*[0.3; 0.7]*	*0.7*	*58%*
σ, elasticity of substitution:	*[2; 7]*	*2*	*33%*
c_M, marginal input requirement:	*[0.1; 0.7]*	*0.1*	*77%*
T_{12}, iceberg cost:	*[1.1; 2]*	*1.37*	*19%*
T_{21}, iceberg cost:	*[1.1; 2]*	*1.1*	*35%*

Although we have attained better results with the traditional method, it has to be mentioned that generally non-classical methods need much more preparation to reach a suitable efficiency, hence further research is needed to reach the final results.

CONCLUSION

In earlier times – as we have seen – travel time, speed and capacity problems already made people try to estimate mobility demand. OD matrices describing mobility demand between zones are traditionally assumed to be constant. However, when a network operates close to its capacity, then a realistic traffic assignment would modify the network's travel time matrix. That engenders changes in residence and production-place-choices and in long term this phenomenon would affect mobility demand structure and so the origin-destination matrix as well. Hence the assumption of constant demand does not prove to be realistic.
The continuous development of SCGE (spatial computable general equilibrium) models has made it possible to describe the behaviour of actors playing various roles in geographically closed economic space. Nowadays SCGE models can be applied at acceptable estimation efficiency to evaluate the expected spatial economic structure and the development of a given region.
Beside the traditionally produced output variables of general equilibrium models (e.g.: process, wages, rents) the objective function of the genetic algorithm included also transportation demand so as to be able to calibrate the model with traffic parameters as well. This approach makes it possible to enhance the traffic flow estimation efficiency of SCGE models and extends the traditional engineering approach by involving demand matrices in modelling process endogenously, furthermore it can help increase the sensitivity of OD matrices to capacity bottlenecks caused by congestion.
Thus the paper has introduced a new calibration possibility of SCGE models, which makes it possible to enhance the traffic modelling ability of SCGE models. In this way it is possible to optimise our interventions on the investigated system through modifying some of the exogenous parameters of the equilibrium model.

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