Travel-time Reliability Impacts on Railway Passenger Demand: A Revealed Preference Analysis

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Abstract:

Travel-time reliability is an important attribute of a transportation system and has been studied in many situations. In this paper we study the impact of travel time reliability on trips made by railway passengers. Unlike most of the studies in this area, which make use of stated preference survey data, we make use of a revealed preference dataset obtained by measuring the railway reliability and the number of season-ticket holders on the Dutch railway network. We make use of six travel time reliability indicators, including the standard deviation and the $80th$ minus the $50th$ percentile of travel time. Our results indicate that the $80th$ minus the $50th$ percentile indicator best explains the fluctuations in the number of season-ticket holders. A 10 per cent improvement of the indicator results in a 1.47 per cent increase in the number of season-ticket holders.

Keywords: Reliability, Value of travel time, Railway reliability, Revealed preference.

1 Introduction

The common way to analyse the benefits of transport system improvements is via travel time savings. More recently, the unreliability of travel times has become an important element of research in this domain. A main result of research during the last decade is that travel time reliability plays an essential role in travellers' route choice behaviour (Bates et al., 2001; Lam and Small, 2001; Small et al., 1999; Tseng, 2008). Hence, cost benefit analysis of transport infrastructure improvements that would ignore reliability benefits may miss important aspects. Most researchers of this subject use stated preference data and are focused on car transport. The present study differs from this in two respects: it is based on revealed preference, and it addresses the effect of reliability on railway travel demand. According to the literature in the area of customer satisfaction, travel time reliability is one of the most important quality aspects of railway passengers (Brons and Rietveld, 2008). The more unreliable the travel time is, the higher the so-called 'scheduling costs, and the smaller the likelihood that the train is chosen as a mode of transport. If on a certain route daily commuters are confronted with decreasing reliability, they may tend to choose a different mode of transport, if it is available.

One of the issues is how to measure reliability. As we will see, a wide range of definitions of reliability is available. In the Netherlands, the Dutch Railways (NS) measures reliability by means of arrival punctuality figures on a station level. The Dutch Ministry of Transport defines a train as 'punctual' if it arrives at most three minutes late. Every year the railway company is held accountable for the actual punctuality in relation to the target of 87 per cent of trains that should be punctual (max 3 minutes delay). Lack of punctuality of train services and the delays resulting from that for the total journey play an unmistakably negative role in the evaluation of the service quality by the traveller. One of the points is that the unreliability of train services may induce travellers to take a train earlier to reduce the risk of late arrival. Most of the research about the impact of unreliability is on route choice. Little is known about its impact on travel demand for a certain transport mode (Savelberg et al., 2008)

This paper aims to estimate the elasticity of rail trip demand with respect to travel time reliability. The main issue addressed in this paper is the question: Does a decrease in the reliability of travel time lead to a decline in demand for train travel? And if so: Does improving travel time reliability lead to higher demand for train travel?

A distinguishing feature of this paper is that it makes use of revealed preference data (RP), and thus reflects the actual behaviour that train travellers show when they are confronted with fluctuating travel-time reliability. Bates et al. (2001) have already stated in their article about the valuation of reliability for personal travel that there is a general economic tradition to favour data that is related to observed choices. However, there are often problems with finding real choice situations with sufficient variation to allow statistically reliable estimates to be obtained at the level of detail that is required. Good studies based on revealed preference data are very rare, and often only modest results are obtained.

We will address two main research questions: How large is the effect of a change in travel time variability on the Dutch railways on the number of season-ticket holders? And: What is the most appropriate way of defining travel time reliability? This will be systematically explored by using a range of unreliability indicators focusing on their different performance as predictors of the travel behaviour of season-ticket holders.

This paper is structured in the following manner. In Section 2 we briefly go into the role reliability plays in transport and discuss how reliability can be defined and measured. In Section 3 data issues are examined. Section 4 provides an analysis of the descriptive statistics, and then in Section 5 we present the results of the statistical analysis of the relation between travel time reliability and season-ticket holders. Section 6 concludes.

2 Travel time reliability

In the literature two distinctions are made concerning the appreciation of reliability of travel times: direct appreciation on the basis of travel time variation, and indirect appreciation on the basis of rescheduling costs (Noland and Polak, 2002). The difference here lies in the monetary valuation of the reliability. Travellers will, in general, attach a different value to early and late arrivals and departures, because they have different consequences. Most of the research literature makes a distinction between these two values by using separate terms: schedule delay early (SDE) and schedule delay late (SDL). Travel-time reliability makes the traveller incur costs in the form of uncertainty of travel time and possible trip scheduling costs. In this paper we focus on the relationship between travel-time reliability figures and the number of season-ticket holders¹, and on obtaining estimates that tell us something about the sensitivity of train travel demand to changes in travel time reliability.

We will compare the six travel time reliability indicators. In the Netherlands, travel time reliability is measured by the Dutch Railways company (NS) in terms of punctuality, i.e. the proportion of trains that arrive less than three minutes late. This study is a revealed preference

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¹ Season-ticket holders for a specific route.

(RP) study, because the actual behaviour of the season-ticket holders is taken into account. Will the season-ticket holders continue to keep their season-ticket or not if the travel time becomes more/less reliable? Qualitative and behavioural studies about travel choice behaviour have concluded that the punctuality and reliability of a transport system are valued as critical aspects, which can influence the perception and level of use of different modes of transport. In some situations travellers value a reduction in travel time variation more than a decrease in the average travel time of specific trip (Small et al., 1999; Bates et al., 2001).

Tseng (2008) surveys the literature on the valuation of trave-time uncertainty, and finds that different definitions of (un)reliability are given. The majority of studies surveyed by Tseng are SP studies that focus on road transport demand, while only a few are based on RP data and focus on rail transport demand (see Appendix A). In the selection of studies, (un)reliability is often measured as the standard deviation of the travel-time distribution (7 studies) or as the difference between the $80th$ (or $90th$) percentile and the median of the travel time distribution (20 studies). In this study we will also make use of these two travel time reliability indicators, four different indicators, two of which are used by the Dutch Railways (NS) (see Section 3). According to the criteria of the NS, a train is on time when this train arrives within three minutes of the scheduled arrival time. Previous research (Brons and Rietveld, 2007) has already criticized the use of only the 3-minute indicator and showed that using multiple punctuality indicators makes a better analysis possible.

3 Data overview

For our analysis we made use of data concerning the number of season-ticket holders on various origin-destination combinations on the Dutch railway network. We included all routes that (i) do not require a transfer and (ii) that have at least 11 season-ticket holders. The decision to drop transfer passengers was based on lack of data for computing unreliability for this group. This results in 288 origin-destination routes spread over the whole Dutch railway network.

The data concerning the season-ticket holders were collected for the period between January 2004 up to and including December 2007. We focus on season-ticket holders who travel on specific routes because these season-ticket holders will indeed travel on the routes for which their season ticket is valid. This is an attractive group for our purposes because they are daily commuters, and they are well informed about travel-time reliability on their route. In addition, we make use of reliability data. These reliability data were collected at the station level and provide the arrival and departure punctuality of all the train series on the Dutch railway network. These data concerning travel time reliability were collected and made available to us

by ProRail, the organisation in charge of the management of the rail network infrastructure. The database covers the period from June 2004 up to and including December 2007, and is aggregated by month (for 43 months). For each station and train series we have the following information with respect to travel time reliability:

- Number of trains with $\{\langle 3; 3-5; 5-7; 7-9; 9 \rangle\}$ minutes of delay on arrival (departure)
- Total number of arriving (departing) trains
- Total number of minutes of delay on arrival (departure)
- Total number of minutes of delay on arrival (departure) for trains that arrive (depart) with more than three minutes of delay

The data are separately available for different parts of the day (morning peak, evening peak and off-peak hours) and different parts of the week (weekdays and weekends). We only look at the weekdays and focus on the morning and evening peaks since commuters mainly travel during these parts of the day and week. To compute unreliability indicators for the selected routes, we first determined which train series goes on which routes. For every route, all the train series delay figures were combined into one figure for every measurement point mentioned above.

The reliability data were collected for the period between June 2004 up and including December 2004, the whole of 2005, 2006 and 2007. On the basis of the available data we will use indicators based on punctuality, indicators based on the size of the delay, and indicators based on travel time variation. We will use six travel time reliability indicators that have already been used in earlier research about travel-time reliability (Brons and Rietveld, 2007, 2008):

- 1: Percentage of delayed trains; more than 3 minutes delay;
- 2: Percentage of very delayed trains; more than 9 minutes of delay;
- 3: Average minutes delay;

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- 4: Average minutes of delay for delayed trains;
- 5: Standard deviation of the arrival/departure time distribution;²
- 6: The $80th$ minus the $50th$ percentile of the arrival/departure time distribution.³

The data concerning the percentage of trains with more than 3 and 9 minutes of delay, and the data concerning the average minutes of delay and the average minutes of delay for delayed trains can directly be inferred from the combined punctuality database. The fifth and sixth

 2^2 See Appendix B for calculations of the standard deviation.

 3 See Appendix B for calculations of the $80th$ minus the $50th$ percentile.

indicators are based on the standard deviation and on the $80th$ minus the $50th$ percentile, but these figures are not directly available, and have to be calculated. The calculations are based on a number of assumptions about the travel time distribution of the available data and are explained more in-depth in Appendix B.

4 Analysis of the descriptive statistics

Table 1 shows the number of trains and the number of season-ticket holders for the selected routes for the years 2004 up to and including 2007. The month and year season-ticket holders are combined in these figures. On average 8.6 per cent of the total number of train travellers belong to the group of season-ticket holders that we are studying (KTO 2002 - 2005⁴). And about 10.1 per cent of the train travellers have a NS/OV-year season ticket. These NS/OVyear season-ticket holders will probably also travel on one route frequently, but on longer routes the costs of a season ticket will be higher than the costs of an NS/OV-year season ticket and thus less attractive to purchase. Given the lack of information on the route they travel, this group cannot be used for our purpose. There is also a group of special season tickets, which form around 2.0 per cent of the train travellers. Altogether the season-ticket holders group forms around 20.7 per cent of the train travellers. An even larger group (around 24.9 per cent) is formed by the OV student ticket holders; this group will be less affected by changing travel-time reliability since most of them do not own a car.

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⁴ Customer Satisfaction Research of the Dutch Railways 2002-2005 (Klant tevredenheidonderzoek)

Figure 1 clearly shows that in 2005, with respect to 2004, there was a decline in both the number of trains and in the number of season-ticket holders. The years 2006 and 2007 on the other hand, show an increase in both the number of trains and in the number of season-ticket holders. Concerning the travel time reliability indicators it is noticeable that the year 2007 was one with improving figures; all of the indicators improved with respect to the previous year. In that year the Dutch Railways (NS) in collaboration with ProRail introduced a whole new timetable (officially on the 10 December 2006).

Figure 1: Graphical display of the number of train-movements and season-ticket holders, with 2004 as the index year

Table 2 shows for each of the six travel time reliability indicators the average level and the spread of the distribution over the 288 routes. The results in the first column are weighted by the number of trains that run on each of the routes. As mentioned before, the figures for departure and arrival are combined into one figure for each route. The unpunctuality (more than 3 minutes late), as the NS measures it, is according to our calculations 16.7 per cent. This figure corresponds with previous studies (16.3 per cent Brons and Rietveld, 2007). The percentage of trains that are more than 9 minutes late is 3.03 per cent (3.25 per cent Brons and Rietveld, 2007). The average delay of all the trains and all delayed trains is respectively, 1.62 and 6.71 minutes. Concerning the level of season-ticket holders on the selected routes, we can see that there are on average 78 season-ticket holders, with a minimum of 11 and a maximum of 809 season-ticket holders.

Table 2: Descriptive statistics concerning the travel time reliability per route for each of the six travel time reliability indicators

A comparison with the punctuality figures for the years 2004 – 2007 of NS (www.prorail.com) is shown in Table 3. The figures from NS show that the country average of the percentage of trains delayed by more than 3 minutes is below the average percentage

on the 288 selected routes.

These are three possible explanations for the difference between our figures and those of NS. First, our set of routes differs from that of NS. We have a selection of routes on which a minimum of season-ticket holders travels, and we have excluded 'thinner' routes from the analysis. Those 'thinner' routes are probably to a larger degree located in rural/provincial areas, where the punctuality is likely to be higher. Secondly, we only included the punctuality figures from the morning and evening peak hours. The punctuality during peak hours is probably lower than it is during off-peak hours. And thirdly, NS focuses on arrival punctuality, while we combined both arrival and departure into one punctuality figure.

5 Analysis

5.1 Correlations between reliability indicators.

In Table 4 the correlation coefficients between the six travel-time reliability indicators and the relative change in the number of season-ticket holders are shown.

Table 4 can be used to check the consistency between the six travel time reliability indicators. The results are similar to earlier research (Brons and Rietveld, 2007) where both the indicators "Very delayed" and "Average delay of all trains" also showed the largest correlation with the unpunctuality indicator used by NS. The indicator based on travel time variation $(80th – 50th$ percentile) is derogatory with the results of earlier research; this indicator shows a similar correlation with the indicator used by NS, 0.640, as the indicators very delayed (0.676) and average minutes of delay (0.939).

Correlation is significant at the 0.05 level (2-tailed). ** Correlation is significant at the 0.01 level (2-tailed).

All correlation coefficients are based on 288 observations per year for

the years 2005, 2006 and 2007.

When we compare the correlation coefficients of the relative change of the six travel time reliability indicators with the relative change in the number of season-ticket holders on each route in Table 5, it shows that the sixth indicator, the $80th$ minus the $50th$ percentile has the highest correlation in absolute numbers, followed by the percentage of delayed trains (>3mins), the size of the delay in minutes, and the percentage of very delayed trains (>9mins). The second indicator (standard deviation) based on travel time variation shows a lower correlation, and for the indicator related to the size of the delay the correlation is almost equal to zero.

5.2 Regressions

We use the following regression model for the analysis of changes in season-ticket holdership.

$$
\frac{y_{ij(t)} - y_{ij(t-1)}}{y_{ij(t-1)}} = \alpha_t + \beta \frac{x_{ij(t)} - x_{ij(t-1)}}{x_{ij(t-1)}} + \gamma \frac{z_{ij(t)} - z_{ij(t-1)}}{z_{ij(t-1)}} + \theta \frac{y_{(t)}}{1000} + \omega_{2006} D_{2006} + \omega_{2007} D_{2007}
$$

Where j and i are the origin and destination of the selected routes, x is one of the six travel time reliability indicators; and *y* is the number of season-ticket holders. We assume a linear relationship between the relative change in the number of season-ticket holders (*y*) and the relative change in travel time reliability (*x*), with the corresponding parameter β . Further, *z* is the number of trains that run on the selected routes, with the corresponding frequency parameter γ , the parameter θ represents the influence of the number of season-ticket holders on the routes, $D_{\!200}$ and $D_{\!2007}$ are two year dummies for the years 2006 and 2007. Note that this specification implies that we focus on the impact of route-specific variation on routespecific demand by season-ticket holders. The impact of a *general* improvement of reliability on route-specific demand is captured by the year dummies. The year dummies reflect changes in a large number of other factors that do not differ across all routes, such as employment conditions, season ticket tariffs and also the average development of travel-time reliability in

the country as a whole. Thus, our analysis focuses on differences in the development of season-ticket holdership between routes. A much longer time series would be needed to determine the effect of overall improvements in reliability on overall ridership. The conclusion is therefore, that the coefficients we estimate for reliability probably, *underestimate* the total effect of reliability on demand.

Table 6: Regression results – for the relative change in the number of season-ticket holders per route under various definitions of travel-time reliability

Explaining variables	Parameters	% Delayed $(>\!\!3min)$	% Very delayed (>9min)	Average delay of all trains	Average delay delayed trains	Standard deviation	$80^{th} - 50^{th}$ percentile
(Constant)	α	$-0.044**$	$-0.044**$	$-0.043**$	$-0.045**$	$-0.046**$	$-0.040**$
Relative change in reliability Relative change	β	-0.006	-0.001	-0.008	0.073	0.020	$-0.147*$
in the number of trains Number of	γ	$0.018*$	$0.018*$	$0.018*$	$0.019*$	$0.019*$	$0.019*$
season-ticket holders /1000	θ	$0.148*$	$0.148*$	$0.147*$	$0.149*$	$0.148*$	$0.151*$
Dummy 2006	$\omega_{_{2006}}$	$0.092**$	$0.093**$	$0.092**$	$0.092**$	$0.094**$	$0.090**$
Dummy 2007	$\omega_{_{2007}}$	$0.091**$	$0.092**$	$0.090**$	$0.094**$	$0.095**$	$0.083**$
Adjusted R N		0.051 864	0.051 864	0.051 864	0.052 864	0.051 864	0.057 864
* Coefficient is significant at the 0.05 level (2-tailed).							
** Coefficient is significant at the 0.01 level (2-tailed).							

All coefficients are based on 288 observations per year for the years 2005, 2006 and 2007.

Table 6 shows that only the coefficient of the sixth travel time reliability indicator is significant. This indicator concerns the spread of the delay. When the spread increases, the number of season-ticket holders decreases. The results can be interpreted in the following manner: For the sixth indicator (the $80th$ minus the $50th$ percentile), the constant for the year 2005 is -4 per cent, meaning that, with a constant level of the sixth indicator the demand will decrease by 4 per cent. When the sixth indicator increases by 10 per cent, for example, from 1.5 too 1.65, then this further decreases the demand down to -5.47 per cent. For the year 2006 there is a uniform growth of 5.0 per cent (this means: -4 per cent of 2005 plus 9.0 per cent of dummy for 2006) so that the demand will increase to 6.67 per cent with an improvement of 10 per cent of the sixth travel time reliability indicator.

Earlier studies show that travellers have a higher appreciation for a decrease in the spread of the travel time than for a decrease in the average duration of a trip (Bates et al., 2001; Brons and Rietveld, 2008). The results indicate that a decrease in the spread of the travel time has a significant impact on demand, but also that a decrease in average delay (and thus the average travel time) has no significant impact. We also find that a positive change in the number of

trains has a positive influence on the number of season-ticket holders. An increase of 10 per cent in the number of trains results in 0.19 per cent extra growth in the number of seasonticket holders. In European urban transportation usually higher (up to 0.1) frequency elasticities are observed (Transtec, 2006). In addition the number of season-ticket holders is positively related to the growth in the number of season-ticket holders. In other words: the higher the number of season-ticket holders, the larger is the relative change in the following year. On busy routes the relative growth is stronger than on less busy routes. We conclude that the sixth indicator of travel-time reliability appears the best candidate to explain relative changes in the number of season-ticket holders.

Figure 2: Frequency distribution of the 80th – 50th percentile indicator.

Figure 2 illustrates the frequency distribution of the sixth travel-time reliability indicator (the $80th$ minus the 50th percentile). The distribution concerns the 288 selected routes for the total time period between June 2004 and December 2007. Improving the sixth indicator implies that the distance between the $80th$ and the $50th$ percentile decreases. Consider, for example, the route Tilburg – Oss, where the sixth indicator has a value of 2.73 minutes in 2007. If this value improves to the mean value of the sixth indicator (1.83 minutes), which is an improvement of 33 per cent, the number of season-ticket holders will increase by 4.85 per cent.

6 Conclusions and discussion

This study has focused on the impact of travel time reliability on the number of season-ticket holders on origin-destination routes on the Dutch rail network. The main concern of this study was the question: Does unreliability lead to an actual decrease in demand? With respect to the six travel-time reliability indicators introduced in Section 3, the results indicate that the sixth indicator (the $80th$ minus the $50th$ percentile) best reflects the fluctuations in the number of season-ticket holders. An improvement of travel time reliability indicator has a positive influence on the growth of the number season-ticket holders. A 10 per cent improvement of the $80th$ minus the $50th$ percentile indicator results in a 1.47 per cent higher growth in the number of season-ticket holders. In an earlier study (Brons and Rietveld, 2008) it was proven that both the fifth (standard deviation) and the sixth (the $80th$ minus the $50th$ percentile) indicators reflect the reliability perception of the traveller significantly. In the present study the relation with the fifth indicator was not significant. A possible explanation of the difference between the two studies is that the former has been carried out for all travellers, whereas the present one is only for daily travellers.

An important implication of this study is that, since unreliability affects travel behaviour, it apparently is a component of generalized costs, and hence should enter accessibility measures as surveyed among others by Banister and Berechman, 2000. If unreliability were the same in all parts of the network, incorporating it in accessibility measures will not lead to new insights. However, as Figure 2 shows, there is a rather broad variation in reliability across various routes, and this would imply that incorporating reliability might have distinct consequences for the resulting accessibility patterns.

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Appendix A: Overview of empirical studies

Appendix B: Calculation of the standard deviation and the 80th minus the 50th percentile

As discussed in Section 3 we have the following data available with respect to travel time reliability:

- Number of trains with $\{<\;3; 3-5; 5-7; 7-9; >\;9\}$ minutes of delay on arrival (departure)
- Total number of arriving (departing) trains;
- Total number of minutes of delay on arrival (departure);
- Total number of minutes of delay on arrival (departure) for trains that arrive (depart) with more than 3 minutes of delay.

First, we assume that the distribution of arrival and departure times are uniform within each of the five categories of delay. Standard deviation and percentiles can then be calculated based on the middle point of the intervals. With respect to the three intermediate intervals the middle points are known. However, since the first interval's lower limit and the last interval's upper limit are not known, the middle points cannot be readily calculated. The middle point of the last interval can be estimated according to the following equation:

$$
X^{L} = \sum_{i=2}^{5} M_{i} N_{i} \Longleftrightarrow M_{5} = \frac{\overline{X}^{L} - \sum_{i=2}^{4} M_{i} N_{i}}{N_{5}}
$$

Where X^L represents the total delay in minutes for delayed trains, M_i represents the middle point of interval *i*, N_i represents the number of trains in category *i* and *X* represents the total delay in minutes. Next, the middle point of the first interval can be calculated as follows:

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$$
X = \sum_{i=1}^{5} M_i N_i \Longleftrightarrow M_1 = \frac{\overline{X} - \sum_{i=2}^{5} M_i N_i}{N_1}.
$$

Based on the estimated and calculated middle points, the standard deviation can be computed as follows:

$$
SD = \sqrt{\frac{\sum_{i=1}^{5} (M_i - \overline{X})^2 N_i}{\left(\sum_{i=1}^{5} N_i\right) 1}}
$$

The 50th and the 80th percentile can be both calculated as follows:

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$$
P^{per} = \left(M_1 - \frac{1}{2}R_1\right) + \sum_{i=1}^{k-1} R_i + \frac{\left(\text{perc} - \sum_{i=1}^{k-1} S_i\right)}{S_k} R_k
$$

where *perc* denotes the percentile to be calculated; R_i represents the bandwidth of interval *i*; *k* represents the interval within which the percentile is located; and S_i represents the percentage of trains in interval *i*.