A KALMAN-FILTER APPROACH FOR DYNAMIC OD ESTIMATION IN CORRIDORS BASED ON BLUETOOTH AND WIFI DATA COLLECTION

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ABSTRACT

From the point of view of the information supplied by an ATIS to the motorists entering a freeway of one of the most relevant is the Forecasted Travel Time, that is the expected travel time that they will experience when traverse a freeway segment. From the point of view of ATMS, the dynamic estimates of time dependencies in OD matrices is a major input to dynamic traffic models used for estimating the current traffic state and forecasting its short term evolution. Travel Time Forecasting and Dynamic OD Estimation are thus two key components of ATIS/ATMS and the quality of the results that they could provide depend not only on the quality of the models but also on the accuracy and reliability of the measurements of traffic variables supplied by the detection technology.

The quality and reliability of the measurements produced by traditional technologies, as inductive loop detectors, is not usually the required by real-time applications, therefore one wonders what could be expected from the new ICT technologies, as for example Automatic Vehicle Location, License Plate Recognition, detection of mobile devices and so on. A simulation experiment is proposed prior to deploy the technology for a pilot project. The simulation emulates the logging and time stamping of a sample of equipped vehicles providing real-time estimates of travel times for the whole population of vehicles and OD pattern of the equipped vehicles are considered real-time estimates of the dynamic OD pattern for the whole population of vehicles. The main objective of this paper is to explore the quality of the data produced by the Bluetooth and Wi-Fi detection of mobile devices

equipping vehicles to estimate time dependent OD matrices. Ad hoc procedures based on Kalman Filtering have been designed and implemented successfully and the numerical results of the computational experiments are presented and discussed.

Keywords: Travel Time, Origin Destination Matrices, Estimation Prediction, ATIS, ATM.

INTRODUCTION

Conceptually the basic architectures of Advanced Traffic Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS) share the main model components; Figure 1 depicts schematically that of an integrated generic ATMS/ATIS:

- A road network equipped with detection stations, suitably located according to a detection layout which timely provides the data supporting the applications
- A Data Collection system collecting from sensors the raw real-time traffic data that must be filtered, checked and completed before being used by the models supporting the management system
- An ad hoc Historic Traffic Database storing the traffic data used by traffic models in combination with the real-time data
- Traffic models aimed at estimating and short term forecasting the traffic state fed with real-time and historic data
- Time dependent Origin-Destination (OD) matrices are inputs to Advanced traffic models . The algorithms to estimate the OD matrices combine real-time and historic data along with other inputs (as the target OD matrices) which are not directly observable
  Estimated and predicted states of the road network can be compared with the expected states, if the comparison is OK (predicted and expected by the management strategies are close enough) then there is no action otherwise, depending on the differences found, a decision is made which includes the most appropriate actions (traffic policies) to achieve the desired objectives.
- Examples of such actions could be: ramp metering, speed control, rerouting, information on current status, levels of service, travel time information and so on.

The objective of this paper is to explore the design and implementation of methods to support the short-term forecasting of expected travel times and to estimate the time dependent OD matrices when, new detection technologies complete the current ones. This is the case of the new sensors detecting vehicles equipped with Bluetooth mobile devices, i.e. hands free phones, Tom-Tom, Parrot and similar devices. From a research stand point this means starting to explore the potential of a new technology in improving traffic models at the same time that, for practitioner, provides sound applications, easy to implement, exploiting technologies, as Bluetooth, whose penetration is becoming pervasive.

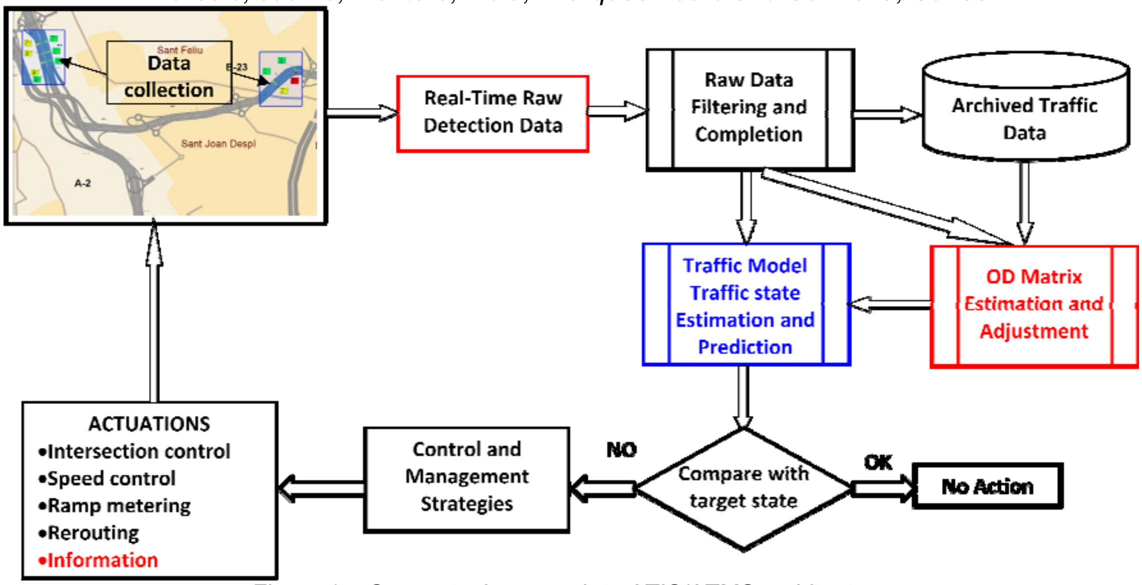


Figure 1 – Conceptual approach to ATIS/ATMS architecture

From the point of view of the information supplied by an ATIS to the motorists entering a freeway there is a wide consensus in considering Forecasted Travel Time one of the most useful from a driver's perspective. Forecasted Travel Time is the expected travel time that they will experience when traversing a freeway segment, instead of the Instantaneous Travel Time, the travel time of a vehicle traversing a freeway segment at time t if all traffic conditions remain constant until the vehicle exits the freeway, which usually under or overestimates travel time depending on traffic conditions; or Reconstructed Travel Time, the travel time realized at time t when a vehicle leaves a freeway segment, which represents a past travel time, see for instance Travis et al (2007).

The dynamic estimates of time dependencies in OD matrices is a major input to dynamic traffic models used in ATMS to estimate the current traffic state as well as to forecast its short term evolution. Travel Time Forecasting and Dynamic OD Estimation are thus two of the key components of ATIS/ATMS and the quality of the results that they can provide depends on the quality of the models as well as on the accuracy and reliability of the traffic measurements of traffic variables supplied by the detection technology. The quality and reliability of the measurements provided by traditional technologies, as inductive loop detectors, usually is not the one required by real-time applications, therefore one wonders what could be expected from the new ICT technologies, i.e. Automatic Vehicle Location, License Plate Recognition, detection of mobile devices and so on. Consequently the main objectives of this paper are: to explore the quality of the data produced by the Bluetooth detection of mobile devices equipping vehicles for Travel Time Forecasting and to estimate time dependent OD matrices.

CAPTURING TRAFFIC DATA WITH BLUETOOTH SENSORS

The sensor integrates a mix of technologies that enable it to audit the Bluetooth and Wi-Fi spectra of devices within its coverage radius. It captures the public parts of the Bluetooth or

Wi-Fi signals. Bluetooth is the global standard protocol (IEEE 802.15.1) for exchanging information wirelessly between mobile devices, using 2.4 GHz short-range radio frequency bandwidth. The captured code consists in the combination of 6 alphanumeric pairs (Hexadecimal). The first 3 pairs are allocated to the manufacturer (Nokia, Panasonic, Sony…) and the type of manufacturer's device (i.e. phone, hands free, Tom-Tom, Parrot….) by the Institute of Electrical and Electronics Engineers (IEEE) and the last 3 define the MAC address, a unique 48-bit address assigned to each wireless device by the service provider company. The uniqueness of the MAC address makes it possible to use a matching algorithm to log the device when it becomes visible to the sensor. The logged device is time stamped and when it is logged again by another sensor at a different location the difference in time stamps can be used to estimate the travel time between both locations. Figure 2 illustrates graphically this process. A vehicle equipped with a Bluetooth device traveling along the freeway is logged and time stamped at time t_1 by the sensor at location 1. After traveling a certain distance it is logged and time stamped again at time t_2 by the sensor at location 2. The difference in time stamps $\tau = t_2 - t_1$ measures the travel time of the vehicle equipped with that mobile device, and obviously the speed assuming the distance between both locations is known. Data captured by each sensor is sent for processing to a central server by GPRS.

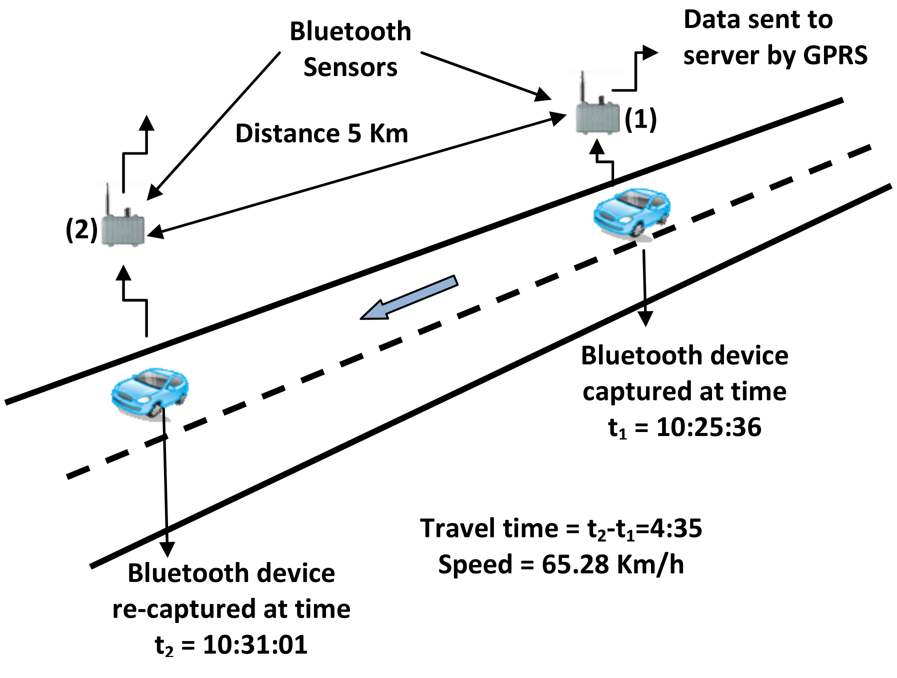


Figure 2: Vehicle monitoring with Bluetooth sensors

Raw measured data cannot be used without a pre-processing aimed at filtering out outliers that could bias the sample, e.g. a vehicle that stops at a gas station between the sensor locations. To remove these data from the sample a filtering process consisting of an adaptive mechanism has been defined, it assumes a lower bound threshold for the free flow speed v_f in that section estimated by previous traffic studies, for example 70Km/h, which defines an upper bound τ_f to the travel time between sensors at 1 and 2 in these conditions. Travel times larger than that threshold are removed as abnormal data. The system monitors every minute the aggregated average speed of the detected vehicles and if it is slowing down and getting closer to the threshold speed, for example average speed – $v_f<\alpha$, for example α=10 Km/h, then the estimate of the speed threshold is decreased to v_f - 2α, and the lower bound threshold for the section is updated accordingly. Smaller values of the average speeds (i.e. 60 Km/h) could be interpreted in terms of a congestion building process and the threshold adaptation continues until a final value of 5 KM/h. If the minute average speeds are increasing the process is reverted accordingly. In some especial conditions like an accident the changes in speed are not fluent and for these situations the rules are changed, if the system is unable to generate any match in more than 2 minutes, the range is open to a maximum time value (5Km/h).

Since this sensor system can monitor the path of a vehicle, this could raise questions about the privacy of drivers. However, working with the MAC address of Bluetooth device ensures privacy, since the MAC address is not associated with any other personal data; the audited data cannot be related to particular individuals. Besides, so as to reinforce the security of data, an asymmetric encryption algorithm is applied before data leaves the sensor and gets to the database, making it impossible to recover the original data, SanFeliu *et al.* (2009).

ESTIMATION OF TIME DEPENDENT OD MATRICES

Data collection to estimate time dependent OD

The possibility of tracking vehicles equipped with Bluetooth mobile raises naturally the question of whether this information can be used for estimating the dynamic or time dependent OD matrix whose entries $T_{ij}(k)$ represent the number of vehicles accessing the freeway at time interval *k* by the entry ramp *i* with destination the exit ramp *j*.

A simulation experiment has been conducted prior to deploy the technology for a pilot project. The selected site has been a 11.551 km long section of the Ronda de Dalt, a urban freeway in Barcelona, between the Trinitat and the Diagonal Exchange Nodes. The site has 11 entry ramps and 12 exit ramps (including main section flows) in the studied section in direction Llobregat (to the south of the city), Figure 3 depicts a part of the site with the suggested sensor layout. D_i denotes the location of the i-th sensor at the main section; E_j denotes the sensor located at the j-th entry ramp and S_n the sensor located at the n-th exit ramp. Distance between detectors is shown in Table 1.

Vehicles are generated randomly in the simulation model according to a selected probability distribution, i.e. a exponential shifted time headway, whose mean has been adjusted to generate the expected mean $T_{ij}(k)$ of vehicles for each OD-pair (i,j) at each time interval k.

Once a vehicle is generated it is randomly identified as an equipped vehicle depending on the proportion of penetration of the technology, a 30% in our case according to the available information. The simulation emulates the logging and time stamping of this random sample of equipped vehicles. Sensors are modeled located in each entry and exit ramps and in the main stream immediately after each ramp.

Bluetooth and WiFi data are collected every second, and are matched when the same emulated MAC address is detected by sensors at entry ramps, exit ramps and main sections, providing the corresponding counts for each time interval. As a result travel times between detectors can be obtained (Figure 2). Bluetooth and WiFi sensors provide traffic counts and travel times between pairs of sensors for any time interval up to 0.1 seconds for equipped vehicles. The measured travel times at each time interval are

- Travel times from each on-ramp entering the corridor to every off-ramp exiting the corridor.
- Travel times from each on-ramp entering the corridor to every main-section where a sensor has been installed.

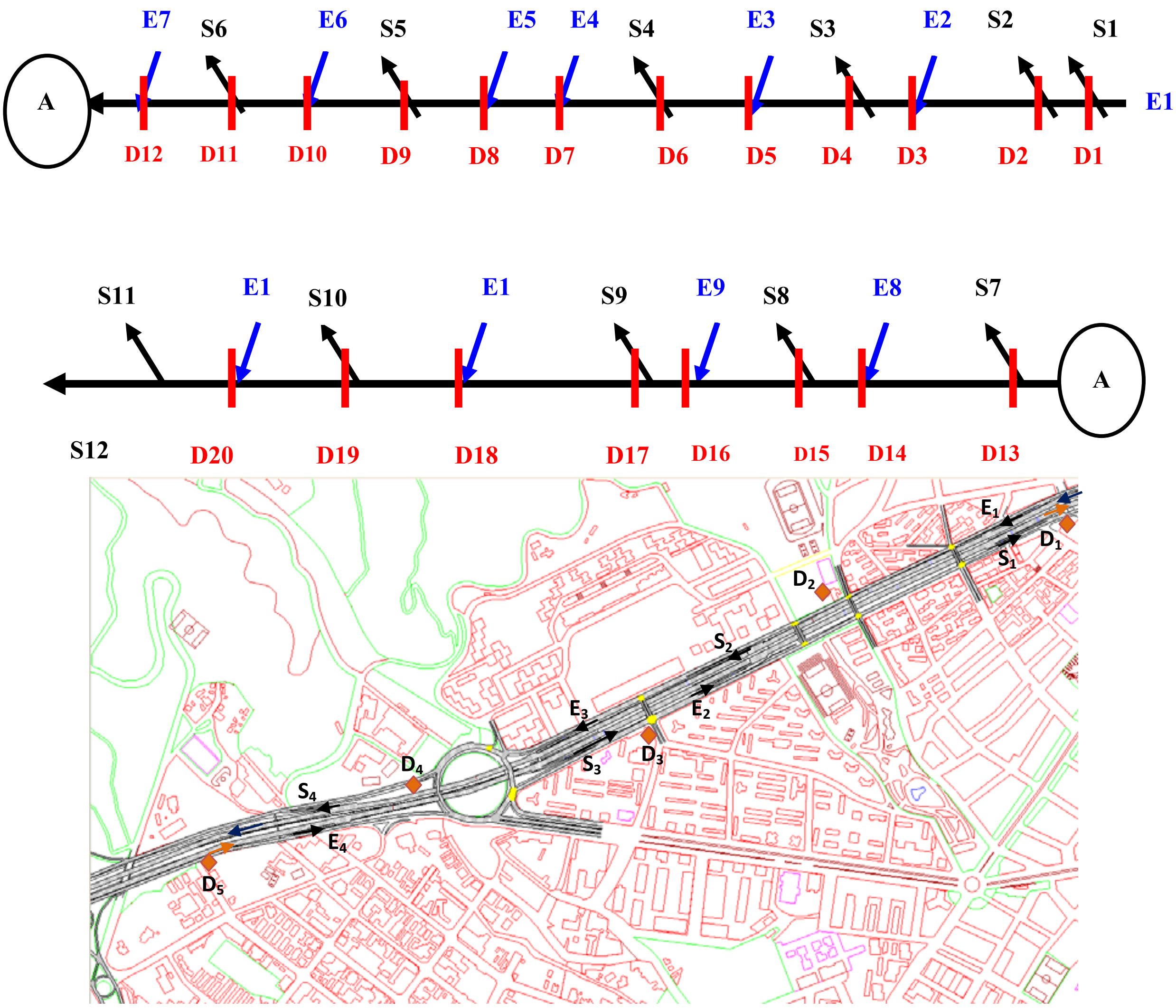


Figure 3. A segment of the site for the OD estimation showing part of the detection layout and diagram with the conceptual structure

Table 1: Distance between main section detectors (in meters)

From	E1	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20
To	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20	S12
Distance	498	841	609	555	470	423	718	507	338	743	950	495	618	435	629	77	210	362	991	303	780

Taking into account that Bluetooth sensors are tagging and time stamping vehicles entering the motorway by entry ramp i at time interval k and tagging them again when leaving the motorway by exit ramp j, then Bluetooth detection is generating a sample $\hat{T}_{ij}(k)$ of the number of vehicles entering the motorway at i during time interval k and later on leaving at j. Therefore it is natural to think of expanding the sample to the whole population to estimate the time-dependent OD matrix $T_{ij}(k)$. This is question that deserves further research. Comparing the number of detected Bluetooth equipped vehicles with the number of vehicles counted by well calibrated inductive loops located at the same position, and taking as reference the inductive loop sample, we found that, although it was a high correlation between both samples and the variability of the Bluetooth sample matched quite well that of the reference sample, the variability of the Bluetooth sample yield unacceptable expansion errors that invalidate any simple expansion procedure. In consequence it is still risky to do a straightforward estimation of OD matrices based only on Bluetooth counting of vehicles but, on the other hand the accuracy in measuring speeds and travel times opens the door to more efficient possibilities of using Kalman Filtering for OD estimates, simplifying the equations and replacing state variables by measurements, as described in the next section.

A Kalman Filter approach for estimating time dependent OD matrices

The estimation of OD matrices from traffic counts has received a lot of attention in the past decades. The extension to dynamic OD estimation in a dynamic system environment from time-series traffic counts has been frequently proposed, see Nihan and Davis (1987), Van Der Zijpp and Hamerslag (1994), Chang and Wu (1994) and more recently Chu *et al.* (2005) and Work *et al.* (2008). A review of the studies until 1991 is available in Bell (1991).

The system equations for OD estimation from static counts are underdetermined because there are far more OD pairs than number of equations, but since dynamic methods employ time-series traffic counts then the number of equations is larger than the number of OD and a unique O-D matrix can thus be obtained. Both in the static and the dynamic methods the relations between OD matrices and traffic counts must be usually defined in terms of an assignment matrix. Static methods, usually specialized for urban networks, establish the relations between OD pairs and link flows through static traffic assignment models embed into entropy or bilevel mathematical programming models, depending on the approach, Spiess (1990), Florian and Chen (1995), Codina and Barcelo (2004). The availability of multiple alternative paths for each OD pair is the crucial difference between linear networks, i.e freeways, and more complex network topologies, i.e. urban networks, thus route choice becomes a key component in this case, and therefore the estimates are formulated in terms of the proportions of OD flows using each of the available paths. Approaches are then usually based on an underlying Dynamic Traffic Assignment and are object of research, see for details Ashok and Ben-Akiva(2000), Ben-Akiva et al. (2001), Mahmassani and Zhou (2005).

We have oriented our research to the dynamic OD estimation in linear congested corridors, without route choice strategies since there is only a unique path connecting each OD pair, taking into account travel times between OD pairs affected by congestion. If no congestion exists but a constant delay for each OD pair is considered the problem can be solved by any of the methods proposed by Bell (1990), Van Der Zijpp and Hamerslag (1994); or Nihan and Davis (1987) if OD travel times are negligible compared to the counting interval.

Nihan and Davis (1987) proposed a recursive method based on Kalman filter (Kalman, 1960) and state-space models, where the state variables are OD proportions, constant or time dependent, between an entry and all possible destination ramps, the observation variables are exit flows on ramps for each interval and the relationship between the state variables and the observations includes a linear transformation, where the numbers of departures from entries during time interval k are explicitly considered. Sensors are assumed in all origins and destinations and provide time-varying traffic counts. Average RMS errors are presented for several algorithmic approaches. There are constraints in the OD proportions: non-negativity, the sum of each row in the matrix is 1 and the total number of vehicles entering the system must be equal to the total number of vehicles exiting the system. Unconstrained estimators are computed first and constraints are enforced later, and several proposals are listed. The proposal is well-suited for intersections where OD travel–times are negligible compared to the counting interval time length.

Bell (1990) formulates a space-state model and applies Kalman filter considering for each OD pair a fixed and non negligible OD travel time distribution where no counts on the main section are considered. Stability on traffic conditions is needed during the estimation process and arising congestion cannot be captured by the formulation.

Van Der Zijpp and Hamerslag (1994) proposed a space-state model assuming for each OD pair a fixed and non negligible OD travel time distribution, the state variables are time-varying OD proportions (between an entry and all possible destination ramps), the observation variables are main section counts for each interval, no exit ramp counts are available and the relationship between the state variables and the observations includes a linear transformation that explicitly accounts for the number of departures from each entry during time interval k and a constant indicator matrix detailing OD pairs intercepted by each section detector. Suggestions for dealing with structural constraints on state variables are proposed. The Kalman filter process is interpreted as a Bayesian estimator and initialization and noise properties are widely discussed. Tests with simulated data were conducted comparing several methods and Kalman filter was reported to perform better than the others. Fixed OD travel time delays are not clearly integrated in the space state model, although is somehow considered by the authors.

Chang and Wu (1994) proposed a space-state model considering for each OD pair a non fixed OD travel time estimated from time-varying traffic measures and traffic flow models are implicitly included in the state variables. The state variables are time-varying OD proportions and fractions of OD trips that arrive at each off-ramp m interval after their entrance at interval k. The observation variables are main section and off-ramp counts for each interval and the relationship between the state variables and the observations is complex and nonlinear. An Extended Kalman-filter approach is proposed and two algorithmic variants are implemented, one of them well-suited for on-line applications.

Work *et al* (2008) propose the use of an Ensemble Kalman Filtering approach as a data assimilation algorithm for a new highway velocity model proposal based on traffic data from GPS enabled mobile devices.

Hu et al (2001, 2004) propose an adaptative Kalman Filtering algorithm for the on-line estimation dynamic OD matrices incorporating time-varying model parameters provided by simulation or included as state variables in the model formulation, leading to an Extended KF formulation. The approach applies to freeway corridors where spatial issues of route choice are not present, but temporal issues of traffic dispersion are taken into account. Lin and Chang (2007) proposed an extension of Chang and Wu (1994) to deal with traffic dynamics in a K-F approach assuming travel time information is available.

Bierlaire and Crittin (2004) propose an LSQR algorithm with application to any linear KF approach aimed to solve large-scale problems.

We propose a space-state formulation for dynamic OD matrix estimation in corridors considering congestion that combines elements of Hu et al (2001), Chang and Wu (1994) and Van Der Zijpp and Hamerslag (1994) proposals. A linear Kalman-based filter approach is implemented for recursive state variables estimation. Tracking of the vehicles is assumed by processing Bluetoooth and WiFi signals whose sensors are located as described above. Traffic counts for every sensor and OD travel time from each entry ramp to the other sensors (main section and ramps) are available for any selected time interval length higher than 1 second. Ordinary traffic counts (inductive loops) are assumed for each on ramp entrance. Distribution of travel time delays between OD pairs or between each entry and sensor locations are directly provided by the detection layout and are no longer state variables but measurements, which simplify the approach and make it more reliable.

A basic hypothesis that requires a statistic contrast for test site applications is that equipped and non equipped vehicles are assumed to follow common OD patterns. We assume that it holds in the following. Time interval length is suggested between 1 and 5 minutes to be able to detect arising congestion. Consider a corridor section containing ramps and sensors numbered as in Figure 3.

FORMULATION 1: OD PATTERN AS STATE VARIABLES

The notation is defined in Table 2.

Table 2. Notation for Formulation 1: state variables as OD proportions from each Entry

$q_i(k)$	:	Number of equipped vehicles entering the freeway from on-ramp *i* during interval *k* and $i = 1 \ldots I$
$s_j(k)$	:	Number of equipped vehicles leaving the freeway by off-ramp *j* during interval *k* and $j = 1 \ldots J$
$y_p(k)$	:	Number of equipped vehicles crossing main section sensor *p* and $p = 1 \ldots P$
$G_{ij}(k)$	:	Number of vehicles entering the freeway at on-ramp *i* during interval *k* with destination to off-ramp *j*
$g_{ij}(k)$	:	Number of equipped vehicles entering the freeway from ramp *i* during interval *k* that are headed towards off-ramp *j*
IJ	:	Number of feasible OD pairs depending on entry/exit ramp topology in the corridor. This is the maximum number of IxJ
$t_{ij}(k)$	:	Average measured travel time for equipped vehicles entering from entry *i* and leaving by off-ramp *j* during interval *k*
$t_{ip}(k)$	:	Average measured travel time for equipped vehicles entering from entry *i* and crossing sensor p during interval *k*
$b_{ij}(k)$	:	$= g_{ij}(k)/q_i(k)$ the proportion of equipped vehicles entering the freeway from ramp *i* during interval *k* that are destined to off-ramp *j*.
$U_{ijq}^{h}(k)$	:	= 1 If the average measured time-varying travel time during interval *k* to traverse the freeway section from entry *i* to sensor *q* takes *h* time intervals, where $h = 1 \ldots M$, $q = 1 \ldots Q$ and $Q = J + P$ (the total number of main section and off-ramp sensors), and M is the maximum number of time intervals required by vehicles to traverse the entire freeway section considering a high congestion scenario. = 0 Otherwise
$e(k) = e$	:	A fixed column vector of dimension I containing ones
$z(k)$	:	The observation variables during interval *k*; i.e. a column vector of dimension I+J+P, whose structure is $z(k)^T = (s(k) \quad y(k) \quad e(k))^T$

The state variables are time-varying OD proportions for equipped vehicles entering the freeway from ramp *i* during interval *k* and that are destined to off-ramp *j*. The observation variables are main section and off-ramp counts for each interval *k*. The relationship between the state variables and the observations involves a time-varying **linear transformation** that considers:

- The number of equipped vehicles entering from each entry during time interval *k*, $q_i(k)$.
- M time-varying indicator matrices, $\left[U_{ijq}^{h}(k)\right]$,

The state variables $b_{ij}(k)$ are assumed to be stochastic in nature and evolve in some independent random walk process as shown by the state equation:

$$b_{ij}(k+1) = b_{ij}(k) + w_{ij}(k) \qquad (1)$$

for all feasible OD pairs *(i,j)* where $w_{ij}(k)$'s are independent Gaussian white noise sequences with zero mean and covariance matrix $\mathbf{Q}$. The structural constraints should be satisfied for the state variables,

$$b_{ij}(k) \geq 0 \quad i = 1 \dots I, \quad j = 1 \dots J$$
$$\sum_{j=1}^{J} b_{ij}(k) = 1 \qquad i = 1 \dots I \tag{2}$$

Where $\mathrm{b(k)}$ is the column vector containing all feasible OD pairs ordered by entry ramp. Equality constraints summing one are imposed to ensure the consistence with the definition of state variables in terms of proportions. When solving numerically the measurement equations relating to the state variables and observations the satisfaction of these constraints is checked. This is not usually the case in these implementations of the filter. The measurement equation becomes in this case:

$$z(k) = \begin{pmatrix} \mathbf{H(k)} \\ \mathbf{E} \end{pmatrix} \mathbf{b(k)} + \begin{pmatrix} v'(k) \\ 0 \end{pmatrix} \tag{3}$$

where $v'_{ij}(k)$'s are independent Gaussian white noise sequences with zero mean and covariance matrix $\mathbf{R}'$, leading to a singular covariance matrix for the whole random noise vector $\mathbf{V}[v(k)] = \mathbf{R} = \begin{bmatrix} \mathbf{R}' & 0 \\ 0 & 0 \end{bmatrix}$. The size of matrix $\mathbf{R}$ is (*I*+*J*+*P*).

Since the time varying travel times have to be taken into account to be able to model congestion, then time varying delays from entries to sensor positions have to be considered (they are described in the building process of the observation equations) and thus on ramp entry volumes for M+1 intervals $k, k-1, \dots, k-M$. State variables for intervals $k, k-1, \dots, k-M$ are required to model interactions between time-varying OD patterns, counts on sensors and travel times delays from on-ramps to sensor positions.

Let $\mathbf{b(k)}$ be a column containing state variables for intervals $k, k-1, \dots, k-M$ of dimension (M+1)xIJ.

$$\mathbf{b(k)}^T = (\mathrm{b(k)} \quad \mathrm{b(k-1)} \quad \dots \quad \mathrm{b(k-}M))^T \tag{4}$$

And the state equations have to be written using a matrix operator D for shifting one interval (following Chang and Wu (1994)), which allows eliminating the state variable for the last time interval (i.e., *k*-M) as:

$$\mathbf{b(k+1)} = \mathbf{Db(k)} + \mathbf{w(k)} \quad \text{and} \quad \mathbf{D} = \begin{pmatrix} \mathbf{I}_{IJ} & 0 & \dots & 0 \\ \mathbf{I}_{IJ} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \mathbf{I}_{IJ} & 0 \end{pmatrix} \tag{5}$$

where $\mathbf{w(k)}^T = (w(k) \quad 0 \quad \dots \quad 0)$ is a white noise sequence with zero mean and singular covariance matrix $\mathbf{V}[\mathbf{w(k)}] = \mathbf{W} = \begin{bmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, where **Q** of dimension *IJ* has been previously defined.

In approaches found in the references it is usually a diagonal matrix. We have successfully tested multinomial variance pattern in our computational experiments. Let us detail in Eq. (6), the time-varying linear operator relating OD patterns and current observations for time interval *k* in Eq. (4):

$$\begin{pmatrix} \mathbf{H(k)} \\ \mathbf{E} \end{pmatrix} = \begin{pmatrix} \mathbf{AU(k)^T F(k)} \\ \mathbf{B} \quad 0 \quad \dots \quad 0 \end{pmatrix} \tag{6}$$

Table 3. Formulation 1: Matrix definitions in measurement equations

E	:	Matrix of row dimension I containing 0 for columns related to state variables in time intervals $k-1,\ldots,k-M$ and B for time interval *k* .
B	:	Matrix of dimension I x IJ defining equality constraints (sum to 1 in OD proportions for each entry) for state variable in time interval *k*.
$\mathbf{F(k)}$	:	Matrix of dimensions (1+M)IJ x (1+M)IJ consisting on diagonal matrices $f(k),\ldots,f(k-M)$ containing input on-ramp volumes. This applies to each OD pair and time interval. Each $f(.)$ is a squared diagonal matrix of dimension IJ.
$\mathbf{g(k)}$	:	*Column vector* of OD flows of equipped vehicles for time intervals $k, k-1,\ldots,k-M$
U(*k*)	:	Matrix of dimensions (1+M)IJ x (1+M)(J+P) consisting on diagonal matrices $U(k),\ldots,U(k-M)$ containing 0s or 1s. For $U(k-h)$ is a matrix of dimensions IJx(J+P) containing 1 if travel-time from entry *i* to a given sensor *q* takes *h* intervals for vehicles captured by the *q* sensor at time interval *k* and 0 otherwise. Average measured time – varying travel times are critical and the clue for taking into account congestion effects.
A	:	Matrix of dimensions (J+P) x (1+M)(J+P) that adds up for a given sensor *q* (main section or off-ramp) traffic flows from any previous on-ramps arriving to sensor at interval *k* assuming their travel times are $t_{iq}(k)$

Let be a part of the observation equations, where the linear operator $\mathbf{H(k)}$ relates dynamic OD proportions, dynamic travel time delays and dynamic on-ramp entry flows with dynamic counts on sensors (main section and off-ramp) for equipped vehicles.

$$\mathbf{H(k)\,b(k)} = \mathbf{AU(k)^T F(k)\,b(k)} \approx \begin{pmatrix} s(k) \\ y(k) \end{pmatrix} \qquad (7)$$

The space-state formulation is almost completed,

$$z(k) = \begin{pmatrix} \mathbf{H(k)} \\ \mathbf{E} \end{pmatrix} \mathbf{b(k)} + \begin{pmatrix} v'(k) \\ 0 \end{pmatrix} = \mathbf{R(k)b(k)} + v(k) \qquad (8)$$

A recursive **linear** Kalman-filter approach, well-suited for on-line applications, has been implemented in MatLab, using simulated data for the Test-Site. Matlab has been selected by its ability in working with the large matrices involved in this approach.

Table 4. Formulation 1: KF Algorithm

KF Algorithm	:	Let K be the total number of time intervals for estimation purposes and M maximum number of time intervals for the longest trip
Initialization	:	k=0; Build constant matrices and vectors: e, A, B, D, E, R, W $\mathbf{b}_k^k = \mathbf{b}(0)$ and $\mathbf{P_k^k} = \mathbf{V}[\mathbf{b(0)}]$
Prediction Step	:	$\mathbf{b_{k+1}^k} = \mathbf{Db_k^k}$ $\mathbf{P_{k+1}^k} = \mathbf{DP_k^k D^T} + \mathbf{W}$
Kalman gain computation	:	Get observations of counts and travel times: $q(k+1),\quad s(k+1),\quad y(k+1),\quad t_{ij}(k+1)\quad t_{ip}(k+1)$. Build $z(k+1)$, $\mathbf{F(k+1)}$, $\mathbf{U(k+1)}$. Build $\mathbf{R_{k+1}} = \mathbf{R(k+1)}$. Compute $\mathbf{G_{k+1}} = \mathbf{P_{k+1}^k R_{k+1}^T}\left(\mathbf{R_{k+1}P_{k+1}^k R_{k+1}^T} + \mathbf{R}\right)^{-}$
Filtering	:	Compute $\mathbf{d_{k+1}} = \mathbf{G_{k+1}}\left(z(k+1) - \mathbf{R_{k+1}b_{k+1}^k}\right)$ filter for state variables and errors

		$\boldsymbol{\varepsilon}_{\mathbf{k+1}} = \left(z(k+1) - \mathbf{R}_{\mathbf{k+1}}\mathbf{b}^{\mathbf{k}}_{\mathbf{k+1}}\right)$
		Search maximum step length $0 \le \alpha \le 1$ such that $\mathbf{b}^{\mathbf{k+1}}_{\mathbf{k+1}} = \mathbf{b}^{\mathbf{k}}_{\mathbf{k+1}} + \alpha\, \mathbf{d}_{\mathbf{k+1}} \ge 0$
		$\mathbf{P}^{\mathbf{k+1}}_{\mathbf{k+1}} = \left(\mathbf{I} - \mathbf{G}_{\mathbf{k+1}}\mathbf{R}_{\mathbf{k+1}}\right)\mathbf{P}^{\mathbf{k}}_{\mathbf{k+1}}$
Iteration	:	*k*=*k*+1 if *k*=K EXIT otherwise GOTO Prediction Step
Exit	:	*Print results*

FORMULATION 2: OD FLOWS AS STATE VARIABLES

According to Ben Akiva *et al* (2001), formulations on OD flows present more difficulties on the estimation process, but have a higher performance in the prediction process and thus seem more suitable for Advanced Traffic Management Systems (ATMS).

Table 5. Notation for Formulation 2: state variables as OD flows

$Q_i(k)$	:	Number of vehicles entering the freeway from on-ramp *i* during interval *k* and $i = 1\ldots I$
$q_i(k)$	:	Number of equipped vehicles entering the freeway from on-ramp *i* during interval *k* and $i = 1\ldots I$
$s_j(k)$	:	Number of equipped vehicles leaving the freeway by off-ramp *j* during interval *k* and $j = 1\ldots J$
$y_p(k)$	:	Number of equipped vehicles crossing main section sensor *p* and $p = 1\ldots P$
$G_{ij}(k)$	:	Number of vehicles entering the freeway at on-ramp *i* during interval *k* with destination to off-ramp *j*
$g_{ij}(k)$	:	Number of equipped vehicles entering the freeway from ramp *i* during interval *k* that are headed towards off-ramp *j*
IJ	:	Number of feasible OD pairs depending on entry/exit ramp topology in the corridor. This is the maximum number of IxJ
$\bar{t}_{ij}(k)$	:	Average measured travel time for equipped vehicles entering from entry *i* and leaving by off-ramp *j* during interval *k*
$\bar{t}_{ip}(k)$	:	Average measured travel time for equipped vehicles entering from entry *i* and crossing sensor p during interval *k*
$b_{ij}(k)$	:	$= g_{ij}(k)/q_i(k)$ the proportion of equipped vehicles entering the freeway from ramp *i* during interval *k* that are destined to off-ramp *j*.
$u^h_{iq}(k)$	:	Fraction of vehicles that takes h time intervals to reach sensor q at time interval k that entered the corridor from on ramp i (during time in interval $\left[(k-h-1)\Delta t \;\;,\;\; (k-h)\Delta t\right]$).
$u^h_{ijq}(k)$	:	Fraction of vehicles, according time-varying model parameters (updated from measured travel time on equipped vehicles), that during interval *k* are to be detected since they traverse the freeway section from entry *i* to sensor *q* in *h* time intervals, where $h = 1\ldots M$, $q = 1\ldots Q$ and $Q = J + P$ (the total number of main section and off-ramp sensors), and M is the maximum number of time intervals required by vehicles to traverse the entire freeway section considering a high congestion scenario.
$\mathrm{z}(\mathrm{k})$	:	The observation variables during interval *k*; i.e. a column vector of dimension J+P+I, whose structure is $z(k)^T = \left(s(k) \quad y(k) \quad q(k)\right)^T$

The state variables are time-varying OD flows for equipped vehicles entering the freeway from ramp *i* during interval *k* and that are destined to off-ramp *j*. The observation variables are main section and off-ramp counts for each interval *k*. The relationship between the state variables and the observations involves a time-varying **linear transformation** that considers:

- The number of equipped vehicles entering from each entry during time interval *k*, $q_i(k)$.
- M time-varying model parameters in form of fraction matrices, $\left[U_{ijq}^{h}(k)\right]$.

Even for stable traffic conditions with no congestion, in the view of speed variation among drivers, it is reasonable to assume that at crossing time k of sensor q for vehicles from on-ramp entry i to any feasible exit ramp j are distributed among time intervals k, k-1, …k-M, where M is the maximum number of intervals needed for vehicles to traverse the entire freeway section. This is, there is a travel time distribution from on ramp i to sensor q (either an exit or a main section sensor), that under stability conditions of traffic could be defined as a random variable T_{iq} with a density function $t_{iq}(t)$ for t>0 with expected value μ_{iq} and variance σ_{iq}^2. The fraction of the vehicles that takes h intervals to travel from on ramp i to sensor q at time interval k is u_{iq}^{h} and can be interpreted as the probability of the discretization of the density function to bins whose length is equal to the time interval used Δt. A fixed time interval between 1.2 and 5 minutes is documented in literature.

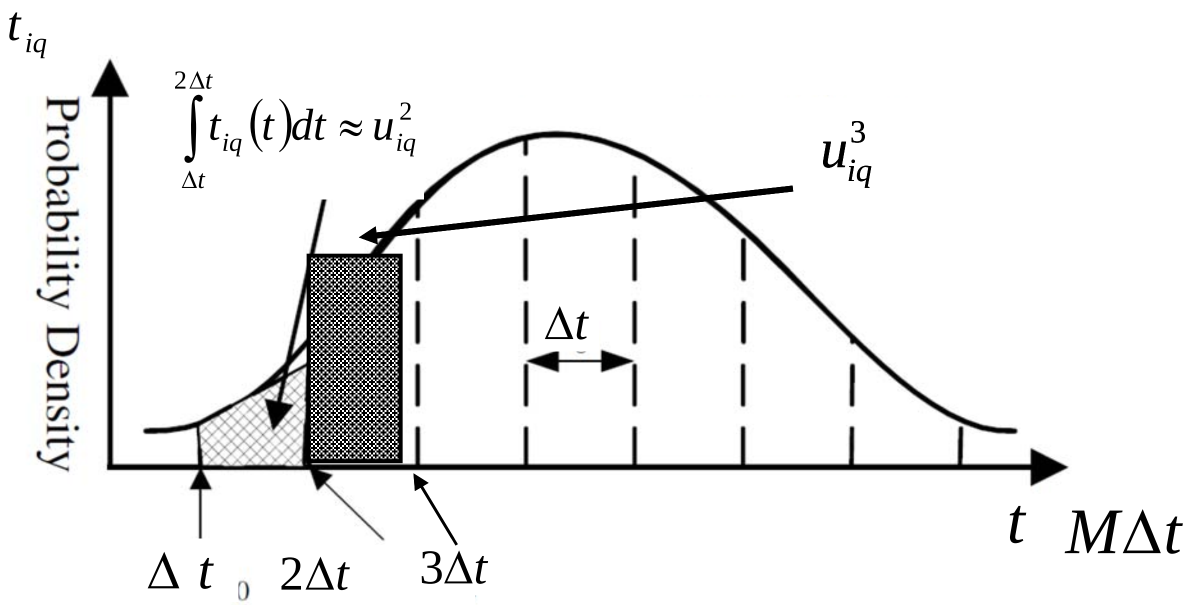


Figure 4.- Travel time distribution approximation in M bins

Model parameters to account with temporal traffic dispersion are fractions u_{iq}^{h}, but in fact to account **for variable traffic conditions**, time-varying model parameters are needed and thus $u_{iq}^{h}(k)$ and structural constraints have to be satisfied, where H<M:

$$\begin{aligned} & u_{iq}^{h}(k) \geq 0 \quad i = 1 \ldots I, \quad q = 1 \ldots Q, h = 1 \ldots H \\ & \sum_{h=1}^{H} u_{iq}^{h}(k) = 1 \quad i = 1 \ldots I, \quad ,q = 1 \ldots Q, \end{aligned} \tag{9}$$

According to traffic conditions evolution, T_{iq} with a density function $t_{iq}(t)$ is not stationary and a more general formulation leading to define $T_{iq}(k)$ (travel time distribution for vehicles reaching sensor *q* at time interval *k* that entered from on-ramp *i*) with density function $t_{iq}^{(k)}(t)$ is proposed, $t_{iq}^{(k)}(t)$ has approximation bins $u_{iq}^{h}(k)$ that have to be updated **from the (assumed random) sample** of on-line travel time data of equipped vehicles. A discretization in H<M time intervals will be initially assumed, but it has to be studied if a (i,q) (entry, sensor) dependent horizon is more suitable (see Figure 4).

Fractions $u_{iq}^{h}(k)$ have to be extended **to all feasible OD pairs and in fact, for all feasible exit ramps *j*** (according to network topology) $u_{ijq}^{h}(k) \leftarrow u_{iq}^{h}(k)$.

Let $\mathrm{g(k)}$ be a column vector containing state variables $g_{ij}(k)$ for interval k for all feasible OD pairs *(i,j)* ordered by entry ramp; i.e, in the test site there are IJ=74 OD pairs. The state variables $g_{ij}(k)$ are assumed to be stochastic in nature and OD flows at current time *k* are related to the OD flows of previous time intervals by an autoregressive model of order *r* <<*M*:

$$\mathrm{g(k+1)} = \sum_{w=1}^{r} \mathrm{D(w)g(k-w+1)} + C_k \ \mathrm{w(k)} \qquad (10)$$

where $w_{ij}(k)$'s are independent Gaussian white noise sequence with zero mean and covariance matrix $\mathbf{Q_k}$, C_k *IJxIJ* are known matrices ($C_k = I$ in Formulation 1) and $\mathrm{D}(w)$ are *IJxIJ* transition matrices which describes the effects of previous OD flows $g_{ij}(k-w+1)$ on current flows $g_{ij}(k+1)$ for *w=1..r* and all feasible OD pairs *(i,j). AR(1) is assumed in preliminary results.* The structural constraints should be satisfied for the state variables,

$$\begin{aligned} & g_{ij}(k) \geq 0 \qquad i = 1 \ldots I, \quad j = 1 \ldots J \\ & \sum_{j=1}^{J} g_{ij}(k) = q_i(k) \qquad i = 1 \ldots I \end{aligned} \qquad (11)$$

Equality constraints have been explicitly considered in the observation equations through the definition of dummy sums sensor counts where no measurement error is allowed adapting a suggested but not tested proposal found in Van Der Zijpp and Hamerslag (1994).

Structural constraints should also be satisfied for the time-varying model parameters $u_{ijq}^{h}(k)$ reflecting temporal traffic dispersion,

$$\begin{aligned} & u_{ijq}^{h}(k) \geq 0 \qquad i = 1 \ldots I, \quad j = 1 \ldots J, q = 1 \ldots Q, h = 1 \ldots H \\ & \sum_{h=1}^{H} u_{ijq}^{h}(k) = 1 \qquad i = 1 \ldots I, \quad j = 1 \ldots J, q = 1 \ldots Q, \end{aligned} \qquad (12)$$

Structural constraints for $u_{ijq}^{h}(k)$ have not been explicitly considered in the observation equations since it can be guaranteed by the updating process of time-varying model parameters from travel-time data on equipped vehicles at current time.

Since the time varying travel times have to be taken into account to be able to model congestion, then time varying delays from entries to sensor positions have to be considered (they are described in the building process of the observation equations) and thus on ramp entry volumes for M+1 intervals $k, k-1, \ldots, k-M$. State variables for intervals $k, k-1, \ldots, k-M$ are required to model interactions between time-varying OD patterns, counts on sensors and distribution of travel times delays (traffic dispersion) from on-ramps to sensor positions.

Let $\mathbf{g(k)}$ be a column containing state variables for intervals $k, k-1, \ldots, k-M$ of dimension (M+1)xIJ.

$$\mathbf{g(k)}^T = \left(\mathrm{g(k)} \quad \mathrm{g(k-1)} \quad \ldots \quad \mathrm{g(k-}M)\right)^T \tag{13}$$

The relationship between the state variables and the observations takes into account equality constraints for the current state variables $g_{ij}(k)$ as non-error conservation flows for on-ramp observations and a Gaussian error measurement for observed sensor counts during time interval *k*. It will be justified that the following equation holds:

$$\mathrm{z(k)} = \begin{pmatrix} \mathbf{H(k)} \\ \mathbf{E} \end{pmatrix} \mathbf{g(k)} + \begin{pmatrix} \mathrm{v'(k)} \\ 0 \end{pmatrix} \tag{14}$$

where $v'_{ij}(k)$'s are independent Gaussian white noise sequence with zero mean and covariance matrix $\mathbf{R}_\mathbf{k}^1$ (related to traffic counts on exits and main sections) and $\mathbf{R}_\mathbf{k}^2$ related to traffic counts on entry ramps and modeling the structural equation (11). In Formulation 1, no error was accepted for structural constraints and thus $\mathbf{R}_\mathbf{k}^2 = 0$ leading to a singular covariance matrix for the whole random noise vector $\mathbf{V}[\mathrm{v(k)}] = \mathbf{R}_\mathbf{k} = \begin{bmatrix} \mathbf{R}_\mathbf{k}^1 & 0 \\ 0 & \mathbf{R}_\mathbf{k}^2 \end{bmatrix}$. The size of the squared matrix $\mathbf{R}_\mathbf{k}$ is (*I*+*J*+*P*).

The state equations have to be written using a matrix operator **D** for shifting and modelling autoregressive intervals (following Chang and Wu (1994) and Hu (2001)), that allows eliminating the state variable for the last time interval (i.e., *k*-M) as:

$$\mathbf{g}(\mathbf{k}+\mathbf{1}) = \mathbf{D}\mathbf{g}(\mathbf{k}) + \mathbf{C}_\mathbf{k}\mathbf{w}(\mathbf{k}) \tag{15}$$

where $\mathbf{w}(\mathbf{k})^T = (\mathrm{w}(\mathrm{k}) \quad 0 \quad \ldots \quad 0)$ is a white noise sequence with zero mean and singular covariance matrix $\mathbf{V}[\mathbf{w}(\mathbf{k})] = \mathbf{W}_\mathbf{k} = \begin{bmatrix} \mathbf{Q}_\mathbf{k} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, where $\mathbf{Q}_\mathbf{k} = \mathbf{Q}$ of dimension *IJ* has been previously defined for Formulation 1 and (it is a diagonal matrix in reported applications) and

$$\mathbf{C}_\mathbf{k} = \begin{pmatrix} \mathrm{C_k} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{D} = \begin{pmatrix} \mathrm{D(1)} & \mathrm{D(2)} & \ldots & 0 \\ \mathbf{I}_{IJ} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \mathbf{I}_{IJ} & 0 \end{pmatrix} \tag{16}$$

For the particular state-space model AR(1) considered in Formulation 1, C, D matrices become ,

$$\mathbf{C_k} = \mathbf{C} = \begin{pmatrix} \mathbf{I}_{IJ} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{D} = \begin{pmatrix} \mathbf{I}_{IJ} & 0 & \dots & 0 \\ \mathbf{I}_{IJ} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \mathbf{I}_{IJ} & 0 \end{pmatrix} \tag{17}$$

Let us detail in Eq. (18), the time-varying linear operator relating OD flows and current observations for time interval *k* in Eq. (14).

$$\begin{pmatrix} \mathbf{H(k)} \\ \mathbf{E} \end{pmatrix} = \begin{pmatrix} \mathbf{AU(k)^T} & & & \\ \mathbf{B} & 0 & \dots & 0 \end{pmatrix} \tag{18}$$

$U(k-h)$ for h=0,…,M reflects corridor structure and travel times delays in terms of fractions on travel time bins (time-varying model parameters). Implicit structural restrictions in Eq 9 are satisfied and $\mathbf{U(k)}$ dimensions are (1+M)IJ x (1+M)(J+P).

$$U(k-h) = \left(\underbrace{\begin{matrix} u_{ijj}^{h} & \cdots \\ \vdots & \ddots \end{matrix}}_{J} \;\; \underbrace{\begin{matrix} u_{ijp}^{h} & \cdots \\ \vdots & \ddots \end{matrix}}_{P} \right) \text{ and } \mathbf{U(k)} = \begin{pmatrix} U(k) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & U(k-M) \end{pmatrix} \tag{19}$$

And thus, $\mathbf{H(k)\,g(k)} = \mathbf{AU(k)^T g(k)} \approx \begin{pmatrix} s(k) \\ y(k) \end{pmatrix}$, the linear operator $\mathbf{H(k)}$ relate (20) OD flows, dynamic travel time delays and dynamic on-ramps entry flows with dynamic counts on sensors (main section and off-ramp) for equipped vehicles. The space-state formulation is almost completed,

$$z(k) = \begin{pmatrix} \mathbf{H(k)} \\ \mathbf{E} \end{pmatrix} \mathbf{g(k)} + \begin{pmatrix} \mathrm{v'(k)} \\ 0 \end{pmatrix} = \mathbf{F(k)g(k)} + \mathrm{v(k)}$$

A recursive linear Kalman-filter approach is detailed in Table 6.

Table 6-KF Algorithm for Formulation 2

KF Algorithm	:	Let K be the total number of time intervals for estimation purposes and M maximum number of time intervals for larger trip
Initialization	:	k=0; Build constant matrices and vectors: A, B, C, D, E. Inicialize Q $_\mathbf{k}$ (and W $_\mathbf{k}$), R $_\mathbf{k}$ $\mathbf{g_k^k} = \mathbf{g(0)}$ in Eq. (13) $\mathbf{P_k^k} = \mathbf{V}[\mathbf{w}(\mathbf{0})]$
Prediction Step	:	$\mathbf{g_{k+1}^k} = \mathbf{Dg_k^k}$ and $\mathbf{P_{k+1}^k} = \mathbf{DP_k^k D^T} + \mathbf{W_k}$
Kalman gain computation	:	Get observations of counts and update fractions in travel times bins: $q(k+1),\quad s(k+1),\quad y(k+1),\quad u_{ij}^{h}(k+1)\quad u_{ip}^{h}(k+1)$. Build $\mathrm{z}(\mathrm{k}+1)$ and $\mathbf{U}(\mathbf{k}+\mathbf{1})$ in Eq (19). Build $\mathbf{F_{k+1}} = \mathbf{F}(\mathbf{k}+\mathbf{1})$ in Eq (20). Compute $\mathbf{K}_{\mathbf{k+1}} = \mathbf{P_{k+1}^k F_{k+1}^T} \left(\mathbf{F_{k+1} P_{k+1}^k F_{k+1}^T} + \mathbf{R_k} \right)^{-}$
Filtering	:	Compute $\mathbf{d_{k+1}} = \mathbf{K_{k+1}}\left(\mathrm{z}(\mathrm{k}+1) - \mathbf{F_{k+1} g_{k+1}^k}\right)$ filter for state variables and errors $\boldsymbol{\varepsilon}_{\mathbf{k+1}} = \left(z(k+1) - \mathbf{F_{k+1} g_{k+1}^k}\right)$

		Search maximum step length $0 \le \alpha$ such that $g_{\mathbf{k+1}}^{\mathbf{k+1}} = g_{\mathbf{k+1}}^{\mathbf{k}} + \alpha\, \mathbf{d}_{\mathbf{k+1}} \ge 0$ $\mathbf{P}_{\mathbf{k+1}}^{\mathbf{k+1}} = \left(\mathbf{I} - \mathbf{K}_{\mathbf{k+1}} \mathbf{F}_{\mathbf{k+1}}\right) \mathbf{P}_{\mathbf{k+1}}^{\mathbf{k}}$
Iteration	:	*k*=*k*+1 if *k*=K EXIT otherwise GOTO Prediction Step
Exit	:	*Print results*

FORMULATION 3: DEVIATES FROM HISTORIC OD FLOWS AS STATE VARIABLES

According to Ben Akiva *et al* (2001) and Antoniou *et al* (2007) formulations where state variable are defined as deviations of OD flows with respect to best historical values present several benefits from considering directly OD flows as state variables:

- OD flows have skewed distributions, but Kalman filtering theory is developed for normal variables and thus symmetric distributions. Deviations from historical values would have more symmetric distributions and thus a proper approximation to normal theory.
- OD flow deviates from the best historical values allow to incorporate more historical data in the model formulation *as a priori* structural information.

According to our experience on Formulations 1 and 2 for the dynamic OD flow estimation on corridors, KF iterations on the filtering stage, the step size α becomes 0 very often to prevent creating unfeasibility on state variables; non-negativity constraints on state variables become critical in the evolution of the KF estimates. But KF scheme was developed for normal state variables. Reformulation on state variables being deviates from historical values adapts to a *more normal* scheme.And last, but not least, a higher performance in the prediction process seems a good value for Advanced Traffic Management Systems (ATMS). Historical data from the previous type of day will be easily available. The notation is defined as in the previous section, but increased by definitions given in Table 7.

Table 7. New notation for Formulation 3

$\tilde{Q}_i(k) = \tilde{Q}_i$	:	Historic number of vehicles entering the freeway from on-ramp *i* for a time interval *length Δt* and $i = 1 \ldots I$
$\tilde{q}_i(k) = \tilde{q}_i$	:	Historic number of equipped vehicles entering the freeway from on-ramp *i* for a time interval *length Δt* and $i = 1 \ldots I$
$\tilde{s}_j(k) = \tilde{s}_j$	:	Historic number of equipped vehicles leaving the freeway by off-ramp *j* during a time interval *length Δt* and $j = 1 \ldots J$
$\tilde{y}_p(k) = \tilde{y}_p$	:	Historic number of equipped vehicles crossing main section sensor *p* and $p = 1 \ldots P$ for a time interval *length Δt*
$G_{ij}(k)$	:	Number of vehicles entering the freeway at on-ramp *i* during interval *k* with destination to off-ramp *j*
$\tilde{g}_{ij}(k) = \tilde{g}_{ij}$	:	Historic flow of equipped vehicles entering the freeway from ramp *i* for a time interval *length Δt* that are headed towards off-ramp *j*
$\Delta g_{ij}(k)$	:	Deviate of equipped vehicles entering the freeway from ramp *i* during interval *k* that are headed towards off-ramp *j* with respect to historic flow $\Delta g_{ij}(k) = g_{ij}(k) - \tilde{g}_{ij}(k)$.
$\tilde{\mathrm{z}}(\mathrm{k})$	:	The historic observation variables during interval k when historic OD flows for a time interval *length Δt*; are considered given current model parameters (observed travel time delays), i.e. a column vector of dimension J+P+I, whose structure is $\tilde{z}(k)^T = \left(\tilde{s}(k) \quad \tilde{y}(k) \quad \tilde{q}(k)\right)^T$

The state variables are time-varying OD flow deviates from historic OD flows for interval length *Δt* for equipped vehicles entering the freeway from ramp *i* during interval *k* and that are destined to off-ramp *j*. Let $\mathbf{\Delta g(k)}$ be a column containing state variables (in the deviation form) for intervals $k, k-1, \ldots, k-M$ of dimension (M+1)xIJ.

$$\mathbf{\Delta g(k)}^T = \left(\Delta \mathrm{g(k)} \quad \Delta \mathrm{g(k-1)} \quad \ldots \quad \Delta \mathrm{g}(\mathrm{k}-M)\right)^T \tag{21}$$

The state equations have to be written using a matrix operator **D** for shifting and modeling autoregressive intervals (following Chang and Wu (1994) and Hu (2001) , that allows eliminating the state variable for the last time interval (i.e., *k*-M) as:

$$\mathbf{\Delta g(k+1)} = \mathbf{D\Delta}\,\mathrm{g}(\mathbf{k}) + \mathbf{w}(\mathbf{k}) \tag{22}$$

where $\mathbf{w}(\mathbf{k})^T = \left(\mathrm{w(k)} \quad 0 \quad \ldots \quad 0\right)$ is a white noise sequence with zero mean and singular covariance matrix $\mathbf{V}[\mathbf{w}(\mathbf{k})] = \mathbf{W_k}$, has been previously defined in the former section Formulation 2

The relationship between the state variables (in the deviation form) and the observations takes into account equality constraints for the current state variables $\Delta g_{ij}(k)$ as non-error conservation flows for on-ramp observations and a Gaussian error measurement for observed sensor counts during time interval *k*. It will be justified that the following equation holds:

$$\mathbf{z(k)} = \mathbf{F(k)}\left(\mathbf{g(k)} - \tilde{\mathbf{g}}(\mathbf{k}) + \tilde{\mathbf{g}}(\mathbf{k})\right) + \mathrm{v(k)} \rightarrow \mathbf{\Delta z(k)} = \mathbf{z(k)} - \tilde{\mathbf{z}}(\mathbf{k}) = \mathbf{F(k)}\left(\underbrace{\mathbf{g(k)} - \tilde{\mathbf{g}}(\mathbf{k})}_{\mathbf{\Delta g(k)}}\right) + \mathrm{v(k)} \tag{23}$$

where $\tilde{\mathbf{z}}(\mathbf{k}) = \mathbf{F(k)}\tilde{\mathbf{g}}\mathbf{(k)}$ assign historic OD flows according to current model parameters (travel time delays) and $v_{ij}(k)$'s are independent Gaussian white noise sequence with zero mean and covariance matrix $\mathbf{R_k^1}$ (related to traffic counts on exits and main sections) and $\mathbf{R_k^2}$ related to traffic counts on entry ramps and flow conservation giving a covariance matrix for the whole random noise vector $\mathbf{V}[\mathrm{v(k)}] = \mathbf{R_k} = \begin{bmatrix} \mathbf{R_k^1} & 0 \\ 0 & \mathbf{R_k^2} \end{bmatrix}$ of size $(I+J+P)$.

The time-varying $\mathbf{F(k)}$ linear operator relating the difference on the current OD flows minus the historic OD flows to the current deviate observations for time interval *k is defined as in Formulation 2.* $\mathbf{F(k)}$ is meaningful when applied to non-negative OD flows and thus has to be computed for current $\mathbf{g(k)} = \mathbf{\Delta g(k)} + \tilde{\mathbf{g}}(\mathbf{k})$ OD flows and for historic OD flows $\tilde{\mathbf{g}}(\mathbf{k})$.

And thus, $\mathbf{F(k)}\,\mathbf{g(k)} = \begin{pmatrix} \mathbf{AU(k)^T} \\ \mathbf{E} \end{pmatrix}\mathbf{g(k)} \approx \begin{pmatrix} s(k) \\ y(k) \\ q(k) \end{pmatrix}$ and $\mathbf{F(k)}\,\tilde{\mathbf{g}}\mathbf{(k)} = \begin{pmatrix} \mathbf{AU(k)^T} \\ \mathbf{E} \end{pmatrix}\tilde{\mathbf{g}}\mathbf{(k)} \approx \begin{pmatrix} \tilde{s}(k) \\ \tilde{y}(k) \\ \tilde{q}(k) \end{pmatrix}$ the

linear operator $\mathbf{F(k)}$ relates dynamic OD flows, dynamic travel time delays and dynamic on-ramps entry flows with dynamic counts on sensors for equipped vehicles. The space-state formulation is completed and the adapted KF Algorithm is described in Table 8.

Table 8 - KF Algorithm for Formulation 3

KF Algorithm	:	Let K be the total number of time intervals for estimation purposes and M maximum number of time intervals for larger trip
Initialization	:	k=0; Build constant matrices and vectors: A, B, C, D, E. Inicialize Q $_{\mathbf{k}}$ (and W $_{\mathbf{k}}$), R $_{\mathbf{k}}$ $\mathbf{\Delta g_k^k} = \mathbf{0} \quad \mathbf{P_k^k} = \mathbf{V}[\mathbf{w}(\mathbf{0})]$
Prediction Step	:	$\mathbf{\Delta g_{k+1}^k} = \mathbf{D}\Delta \mathrm{g}_{\mathbf{k}}^{\mathbf{k}}$ and $\mathbf{P_{k+1}^k} = \mathbf{DP_k^k D^T} + \mathbf{W_k}$
Kalman gain computation	:	Get observations of counts and update fractions in travel times bins: $q(k+1), \quad s(k+1), \quad y(k+1), \quad u_{ij}^h(k+1) \quad u_{ip}^h(k+1)$. Build $\mathbf{\Delta}\ \mathrm{z}(\mathrm{k}+1)$ and $\mathbf{U}(\mathbf{k}+\mathbf{1})$. Build $\mathbf{F_{k+1}} = \mathbf{F}(\mathbf{k}+\mathbf{1})$. Compute $\mathbf{K}_{\mathbf{k}+\mathbf{1}} = \mathbf{P_{k+1}^k F_{k+1}^T} \left(\mathbf{F_{k+1} P_{k+1}^k F_{k+1}^T} + \mathbf{R_k}\right)^{-}$
Filtering	:	Compute $\mathbf{d_{k+1}} = \mathbf{K_{k+1}}\left(\Delta \mathrm{z}(\mathrm{k}+1) - \mathbf{F_{k+1}\Delta g_{k+1}^k}\right)$ filter for state variables and errors $\boldsymbol{\varepsilon}_{\mathbf{k+1}} = \left(\Delta \mathrm{z}(k+1) - \mathbf{F_{k+1}\Delta g_{k+1}^k}\right)$. Search maximum step length α such that $\Delta g_{\mathbf{k+1}}^{\mathbf{k+1}} = \Delta g_{\mathbf{k+1}}^{\mathbf{k}} + \alpha\ \mathbf{d_{k+1}} \geq -\tilde{\mathbf{g}}(\mathbf{k})$ and $\mathbf{P_{k+1}^{k+1}} = \left(\mathbf{I} - \mathbf{K_{k+1}F_{k+1}}\right)\mathbf{P_{k+1}^k}$
Iteration	:	*k*=*k*+1 if *k*=K EXIT otherwise GOTO Prediction Step
Exit	:	*Print results*

COMPUTATIONAL RESULTS

Two sets of computational experiments have been conducted with simulated data, assuming for debugging purposes in all cases a 100% rate of equipped vehicles. *OD pattern is fixed for all the experiments* and OD flows are defined variable according to 1 slice (congested/uncongested situations) in the First Set and 4 slices in the Second Set. OD pattern and OD flows for the 1 slice uncongested situation are shown in Table 9.

Table 9- First Test Set – Uncongested OD flows (veh/h) and OD Pattern

OD Flows

Entry/Exit	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	93	245	96	169	146	106	136	153	256	92	329	2513	**4334**
2			10	0	8	9	19	21	24	9	25	113	**238**
3				0	0	0	20	23	21	5	30	119	**218**
4					0	0	11	14	21	0	26	99	**171**
5					0	0	0	0	15	0	32	98	**145**
6						0	0	0	0	0	34	77	**111**
7							1	0	0	0	26	60	**87**
8								0	2	0	25	73	**100**
9									0	0	75	101	**176**

10									13	25	53	**91**
11										26	50	**76**
Total	93	245	106	169	154	115	187	211	339	119	653	3356	**5747**

OD Pattern

Entry/Exit	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	2.1%	5.7%	2.2%	3.9%	3.4%	2.4%	3.1%	3.5%	5.9%	2.1%	7.6%	58.0%	**100%**
2			4.2%	0.0%	3.4%	3.8%	8.0%	8.8%	10.1%	3.8%	10.5%	47.5%	**100%**
3				0.0%	0.0%	0.0%	9.2%	10.6%	9.6%	2.3%	13.8%	54.6%	**100%**
4					0.0%	0.0%	6.4%	8.2%	12.3%	0.0%	15.2%	57.9%	**100%**
5					0.0%	0.0%	0.0%	0.0%	10.3%	0.0%	22.1%	67.6%	**100%**
6						0.0%	0.0%	0.0%	0.0%	0.0%	30.6%	69.4%	**100%**
7							1.1%	0.0%	0.0%	0.0%	29.9%	69.0%	**100%**
8								0.0%	2.0%	0.0%	25.0%	73.0%	**100%**
9									0.0%	0.0%	42.6%	57.4%	**100%**
10										14.3%	27.5%	58.2%	**100%**
11											34.2%	65.8%	**100%**
Total	***1.6%***	***4.3%***	***1.8%***	***2.9%***	***2.7%***	***2.0%***	***3.3%***	***3.7%***	***5.9%***	***2.1%***	***11.4%***	***58.4%***	**100%**

In the First Set of computational experiments, two fixed OD patterns with static OD flows have been used for testing purposes, for a time horizon of 1 hour. An OD flows for uncongested conditions and another one for congested.

The Second Set of computational experiments has been conducted with time sliced OD flows sharing the same OD Pattern as in the First Test Set, but the time horizon (1h 40min) splits in four time intervals of 25 minutes and demand is distributed to account for the 15%, 25%, 35% and 25% of the total demand in each slice. We remark that the OD pattern is still fixed, but not the OD flows which are slice dependent.

RESULTS FOR FORMULATION 1

Tests in the First Set show that the proposed Kalman Filtering approach for Formulation 1converges successfully to the true results in few iterations for the uncongested and congested situations. Figure 5 graphically depicts the convergence progress for OD proportions from entry 1 to 11 of the feasible off-ramps for the uncongested matrix of OD flows. KF filter initialization is always non-informative (every off-ramp of one on-ramp has the same probability), for the 74 OD pair in the site's model, which is considered a very difficult initialization.

Table 10 summarizes the values of the RMSE for each OD proportion at the end of the process. And compares the RMSE error values for congested and uncongested tests for some OD pairs (Root Mean Squared Error on OD proportions). The results show no significant differences in the accuracy of the estimates of target OD proportions. Initialization of covariance has a key effect on convergence in accordance with the experience reported by other researchers.

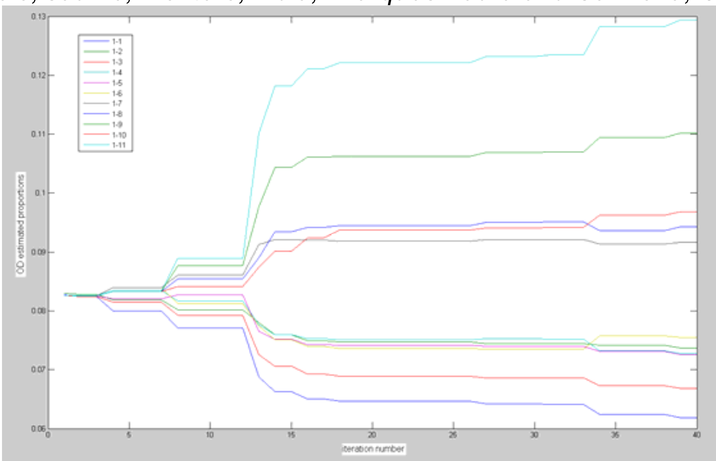


Figure 5- OD Pairs 1-1 to 1-11. Convergence to truly OD Proportion for constant OD pattern without congestion (time horizon 1h)

Table 10- First Test Set - RMSE values (multiplied by 10^{-2}) for a sample of OD pairs

RMSE $x10^{-2}$			Some OD pairs – Fix OD Pattern – Static OD flows											
			1-1	**1-2**	**1-3**	**1-4**	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**	**1-12**
Interval length (seconds)	**90**	**Uncongested**	4.63	2.49	4.84	3.98	4.20	5.15	6.02	4.82	4.98	7.30	4.55	5.40
	150	**Congested**	5.14	2.54	5.22	4.26	4.75	6.92	6.87	5.56	6.98	0.83	5.52	4.10

The results for the Second Set of computational experiments can be summarized as follows: for time intervals where traffic flow varies from free flow to dense but not yet saturation conditions the filtering approach works as expected and its performance seems not to be affected for congested flows. RMSE values are of a similar order of magnitude. The equivalent results to those in Figure 5 for the same set of OD pairs are depicted in Figure 6. Table 11 summarizes the values of the RMSE for some OD proportions for the 4th time slice.

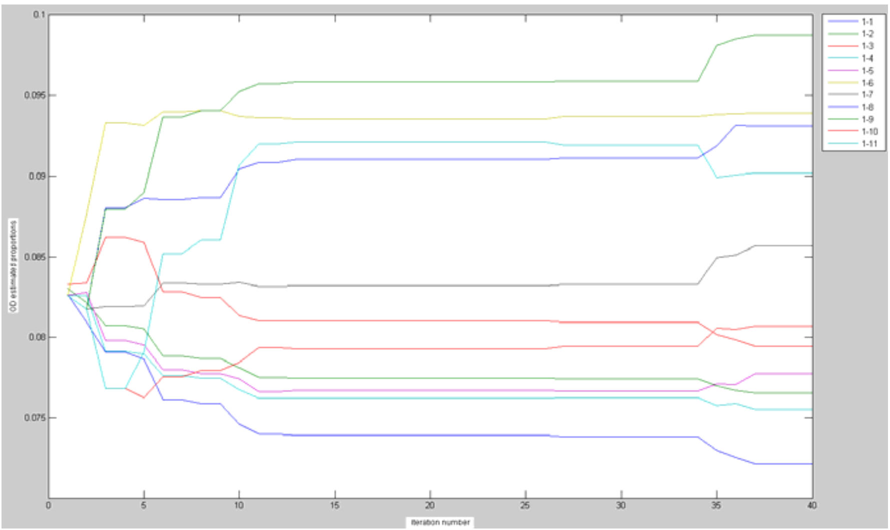


Figure 6- OD Pairs 1-1 to 1-11. Convergence to truly OD Proportions for time-sliced OD (time horizon 1h40min)

Table 11 - Second Test Set - RMSE values (multiplied by 10^{-2}) for a sample of OD proportions (from Entry 1).

4 th time slice: RMSEx10^{-2}	Some OD pairs – Fix OD Pattern – Time Sliced OD flows											
	1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1- 11	1- 12
Interval length150 seg	5.61	2.77	6.07	4.14	4.52	7.09	5.22	5.70	3.88	5.84	1.23	5.21

RESULTS FOR FORMULATION 2

Figure 7 graphically depicts the convergence progress for OD flows from entry 1 to each of the 12 off-ramps for the static and uncongested OD matrix. The KF approach for Formulation 2 is shown to be stable under variance-covariance pattern initialization and no sensitive to diagonal or multinomial blocked variance of state variables in the uncongested situation. Convergence and results seem more seriously affected by the initialization of OD flows than Formulation 1.

Table 12 summarizes the values of the RMSE for OD flows of entry 1 at the end of the process and compares the RMSE error values for the uncongested in terms of OD proportions (as in Formulation 1). The results show no important differences in the accuracy of the estimates of target OD flows or target OD Proportions, in relative terms. A discretization in 3 bins for travel time distributions has been used. Variance-covariance matrices are diagonal proportional to historic variance for OD flows, entry, exit and section counts.

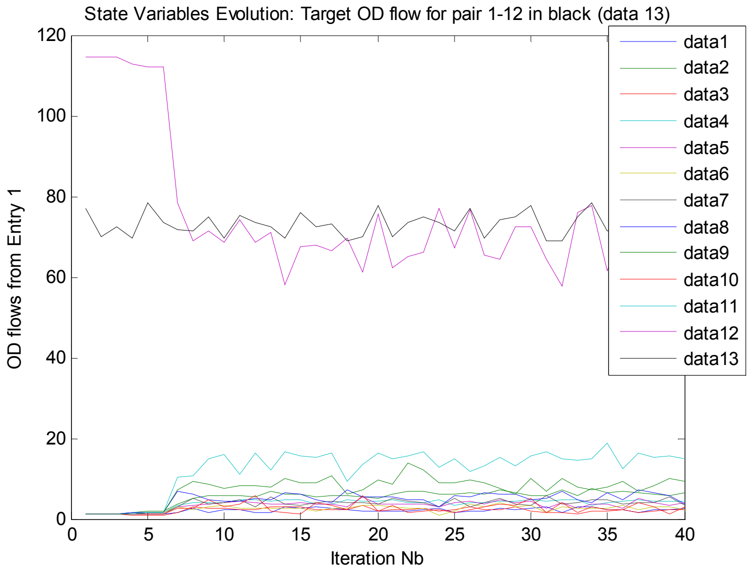


Figure 7. OD Pairs 1-1 to 1-12. Convergence to OD flows for First Set experiments without congestion (time horizon 1h)

Table 12- First Set Uncongested Situation: RMSE values for a sample of OD pairs (interval length 90 seg)

RMSE	Some OD pairs – Fix OD Pattern – Static OD flows (Uncongested Situation)											
	1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1-11	1-12
ODPattern x10^{-2}	0,51	0,77	0,40	0,66	0,52	0,38	0,67	0,80	1,12	1,08	1,32	2,47
ODflows	0,66	1,02	0,50	0,86	0,63	0,48	0,87	0,98	1,42	1,34	1,65	3,90
Target flows	2,72	7,15	2,81	4,93	4,26	3,10	3,97	4,47	7,48	2,68	9,60	73,34

Considering a very poor informative starting point, equivalent to the maximum entropy for each entry (no previous information on destinations proportions is assumed and equal probability is set) and diagonal variance matrices based on historical variance KF convergence is found without difficulties for uncongested situations, but errors are greater (see Figure 8 and Table 13).

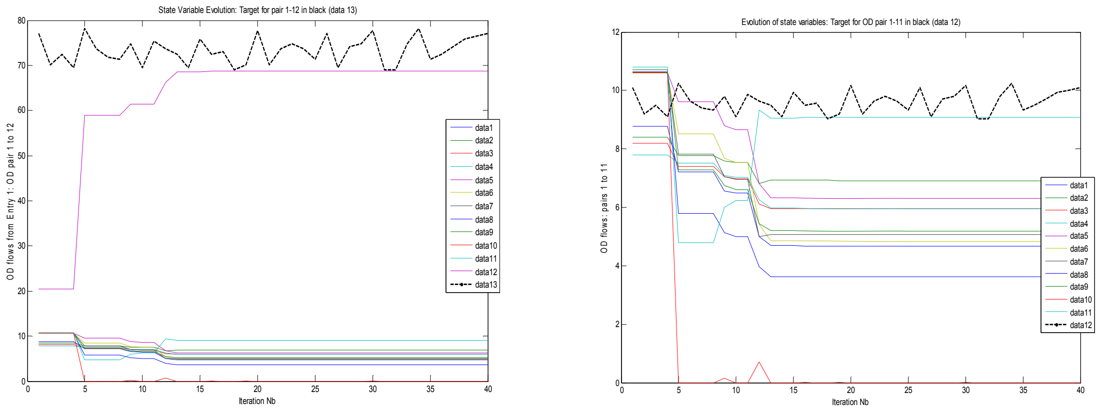


Figure 8. OD Pairs 1-1 to 1-12. Convergence to OD flows for First Set experiments without congestion (time horizon 1h): no information as starting point

Table 13- First Set Uncongested Situation: RMSE values for a sample of OD pairs (interval length 90 seg) : no information as starting point

		Some OD pairs – Fix OD Pattern – Static OD flows											
		1-1	**1-2**	**1-3**	**1-4**	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**	**1-12**
Interval length 90 seg	**RMSE x10^{-2} ODPattern**	2,38	0,33	2,85	1,20	2,58	2,69	2,08	1,63	1,73	2,79	1,55	14,62
	RMSE x10^{-2} ODPattern(no transient)	1,67	0,17	2,54	0,88	1,79	1,60	1,01	0,64	1,75	2,10	0,81	4,51
	RMSE ODflows	3,10	0,59	3,68	1,63	3,39	3,52	2,74	2,15	2,24	3,57	1,94	18,07
	RMSE ODflows (no transient)	2,14	0,38	3,23	1,15	2,29	2,05	1,31	0,85	2,24	2,66	1,05	6,16
	Target flows	**2,72**	**7,15**	**2,81**	**4,93**	**4,26**	**3,10**	**3,97**	**4,47**	**7,48**	**2,68**	**9,60**	**73,34**
	Average of filtered values	5,49	7,20	6,40	6,39	7,26	6,01	6,04	4,65	6,07	1,08	8,60	62,31
	Average of filtered values (no transient)	4,81	6,96	6,02	6,04	6,47	5,04	5,19	3,72	5,29	0,02	8,89	68,17

Figure 9 depicts the convergence progress for OD flows from entry 1 to each of the 12 off-ramps for the First Set of tests in a congested situation. The KF approach for Formulation 2 is shown to be stable under variance-covariance pattern initialization and no sensitive to diagonal or multinomial blocked variance of state variables in the uncongested situation. Convergence and results seem to be affected by the initialization of OD flows.

Table 14 summarizes the values of the RMSE results for OD flows of entry 1 relative to target OD flows and shows the RMSE error values for the congested First Set situation in OD proportions (as in Formulation 1) and OD flows, for the whole time horizon with and without transient time intervals. The results show no important differences in the accuracy of the estimates of target OD flows or target OD Proportions compared to the uncongested situation.

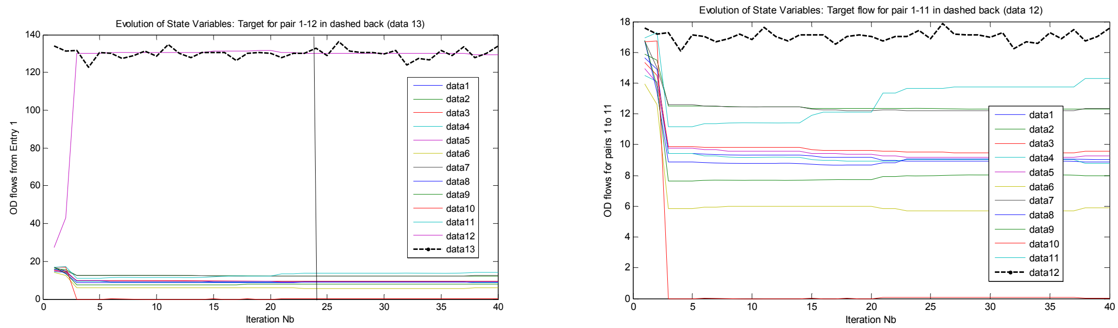
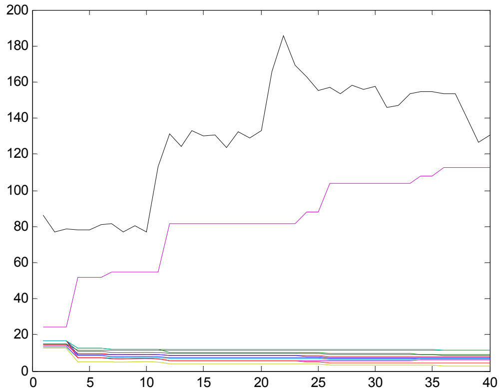


Figure 9. OD Pairs 1-1 to 1-12. Convergence to OD flows for First Set experiments with congestion (time horizon 1h40min)

The Second Set of Experiments with Variable OD flows in 4 slices (but estable OD Pattern) resulting a combination of very uncongested, uncongested, congested and congested slices are a challenge for the KF algorithm in Formulation 2. The number of bins in the empirical discrete travel time distributions has a serious effect in the performance as depicted in Figure 10. The starting point is set to equiprobability in OD Pattern for each Entry. RMSE errors for H=3 bins are shown in Table 15.

Table 14- First Set Congested Situation: RMSE values for a sample of OD pairs (interval length 150 seg)

Interval length 150 seg	Some OD pairs – Fix OD Pattern – Static (1-Slice) OD flows with Congestion											
	1-1	**1-2**	**1-3**	**1-4**	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**	**1-12**
RMSE $x10^{-2}$ ODPattern	2,19	0,62	1,96	0,67	0,76	0,94	2,09	0,94	2,55	2,47	1,82	8,82
RMSE $x10^{-2}$ ODPattern (no transient)	1,84	0,49	1,66	0,15	0,21	0,16	1,88	0,34	2,57	2,12	1,70	0,96
ODflows- RMSE	4,69	1,33	4,17	1,13	1,28	1,73	4,45	1,65	5,94	5,43	4,32	22,05
ODflows (no transient) - RMSE	4,13	1,35	3,70	0,52	0,39	0,31	4,16	0,68	6,07	4,89	4,12	2,65
Target flows (equipped)	4	11	5	8	7	4	6	7	11	4	15	118
Target flows	5	14	6	9	9	6	8	8	14	5	17	130
Average of filtered values	9,4	12,6	9,9	9,3	9,6	6,2	12,5	9,2	8,2	0,9	13,0	125,6
Av. filtered values (no transient)	9,0	12,4	9,6	9,0	9,3	5,8	12,3	8,9	7,9	0,0	13,1	130,3



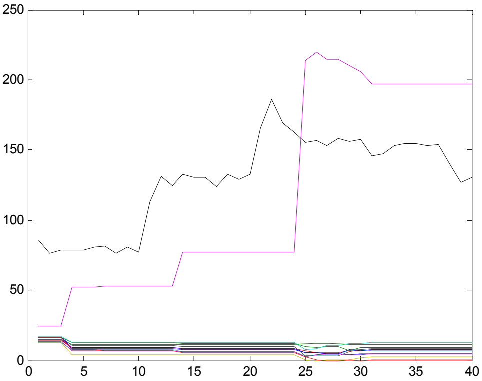


Figure 10. OD Pairs 1-1 to 1-12. Convergence to OD flows for the Second Set of experiments with 4 sliced OD flows (time horizon 1h40min). Discrete travel time distributions in H=3 bins (left) and H=1 bin (right). Target in black for OD pair 1-12.

Table 15- Second Set of Tests: RMSE values for a sample of OD pairs (interval length 150 seg)

Interval length 150 seg		Some OD pairs – Fix OD Pattern – 4 Sliced OD flows											
		1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1-11	1-12
RMSE	$x10^{-2}$ ODPattern	2,59	1,01	2,72	1,08	1,17	1,28	2,54	1,94	1,52	2,28	1,17	16,58
	ODflows	4,1	4,4	4,1	4,4	4,5	3,5	4,1	4,0	6,0	4,4	6,9	51,4
OD flows Target	Slice 1: 15 min	2	6	3	4	4	2	3	4	7	2	7	52
	Slice 2: 15 min	4	10	4	7	7	5	6	7	13	4	14	103
	Slice 3: 15 min	5	13	6	9	8	6	7	7	14	4	15	117
	Slice 4: 15 min	5	13	5	9	8	4	7	7	11	4	15	118

PRELIMINARY RESULTS FOR FORMULATION 3

The deviate formulation relative to historic values does not seem to take benefit over Formulation 2 for the **uncongested situation**. Diagonal var-cov matrices with values proportional to historical flows are used. State variables defined as deviates from historical OD flows of equipped vehicles in Formulation 3 does not improve accuracy with respect to Formulation 2. Figure 11 depicts the convergence progress for OD flows from entry 1 to each of the 12 off-ramps for the First Set of tests in the uncongested situation and Table 16 summarizes the values of the RMSE for OD flows of entry 1 and shows RMSE error values relative to target values for OD flows for the whole time horizon with and without transient time intervals.

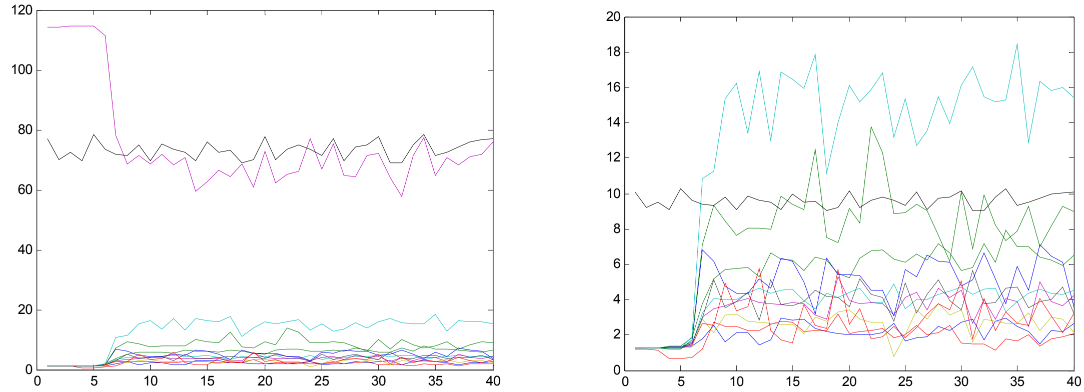


Figure 11. OD Pairs 1-1 to 1-12. Convergence to OD flows for First Set experiments without congestion (time horizon 1h)

Table 16- First Set Uncongested Situation: RMSE values for a sample of OD pairs (interval length 90 seg) : no information as starting point

		Some OD pairs – Fix OD Pattern – Static OD flows – No Congestion											
		1-1	**1-2**	**1-3**	**1-4**	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**	**1-11**	**1-12**
Interval length 90 seg	**ODflows- RMSE**	0,82	2,50	1,08	1,61	1,34	0,93	1,26	1,76	3,05	1,19	6,19	16,71
	ODflows (no transient) - RMSE	0,63	1,05	0,85	0,81	0,73	0,67	0,75	1,33	2,11	1,13	5,92	6,34
	Target flows	**2,72**	**7,15**	**2,81**	**4,93**	**4,26**	**3,10**	**3,97**	**4,47**	**7,48**	**2,68**	**9,60**	**73,34**
	Av. filtered values	2,09	5,47	1,93	3,78	3,42	2,52	3,67	4,71	7,67	2,79	13,00	75,37
	Av. filtered values (no transient)	2,26	6,31	2,10	4,25	3,83	2,74	4,10	5,25	8,83	3,02	15,29	68,17

CONCLUSIONS AND FUTURE RESEARCH

Bluetooth sensors to detect mobile devices have proved to be a mature technology that provides sound measurements of average speeds and travel times between sensor locations. These sensors are already in operation at the AP-7 Motorway in Spain between Barcelona and the French border. This paper has explored how data available from this technology can be used to estimate dynamic origin to destination matrices in motorways by proposing several *ad hoc* linear Kalman Filter approaches. The results on the conducted experiments using simulation data prove that the approach works fine for uncongested and congested conditions but properly tuning on the initialization points and matrices is critical in some situations. Further research is necessary to develop a robust algorithm in congested situations that can be used to estimate time-dependent OD matrices from the direct vehicle logging by Bluetooth, since precision on the estimated OD pattern is also affected by interval length: an adaptive time varying scheme for time interval length according to congestion should be included in the near future.

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