# WELFARE EFFECT OF URBAN TRANSPORT IMPROVEMENT THROUGH RETAIL STORE LOCATION CHANGE: AGGLOMERATION AND DISPERSION

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## ABSTRACT

Emergence of the hollowing-out of an urban centre's commerce through market interactions is not socially optimal if such a situation involves market failure. We particularly investigate two factors of market failure: imperfect competition among retail stores and shopping externality caused by multipurpose (one-stop) shopping. We derive the mechanics generating a divergence between market equilibrium and social optimum by constructing a model. Next, based on the model, we analyze the welfare effects of a transportation improvement and the mechanism by which transportation improvement affects hollowing-out.

Keywords: hollowing-out, monopolistic competition, shopping externality

## **1. INTRODUCTION**

During the past several decades, the hollowing-out of urban centres' commerce has repeatedly been brought to public attention in many developed countries. Motorization, improvement of transportation networks, and suburban area expansion are widely recognized as the main reasons for such hollowing-out of urban centres' commerce in recent decades. In the United States, the suburbanization of cities advanced explosively during the 1970s, but with concomitant and subsequent decay of city centres. In Germany and France in the 1970s, the necessity for revitalization of urban centres was a common theme; it remains an important policy issue in the United Kingdom. Following such trends in the US and Europe, since regulations of commercial development on suburban areas were loosened in Japan in the 1990s, the hollowing-out of urban centres' commerce in many small cities has increasingly become an important social issue.

As countermeasures against hollowing-out, these countries have implemented several policies during the last decade to promote revitalization of urban centres<sup>1</sup>. Transportation facility improvement between urban centres and housing areas is a typical policy. Furthermore, some land-use regulations have come into force to promote revitalization of urban centres in European countries such as the UK<sup>2</sup> and Germany, and also in Japan.

Such hollowing-out is the outcome of market interactions. Therefore, hollowing-out is not optimal if such a situation involves technological externalities attributed to market failures. In fact, the relevant literature has described several instances of market failure in commerce: spatial price competition of commercial location, imperfect competition among retail stores, and shopping externality (O'Sullivan (1993)) caused by multipurpose (one-stop) shopping.

Spatial price competition of commercial locations is a phenomenon among firms that supply a homogeneous good. Hotelling's "ice cream vendor" problems (Hotelling (1929)) serve as illustrative examples. Each firm decides its price and location under the competition prevailing with neighbouring firms in the market area. Spatial price competition is examined in the framework of that model in many studies (see e.g., Cappoza and Order (1978), Anderson, de Palma and Thisse (1992), and Beckmann (1999)). However, although this spatial competition framework can represent competition among suburban retail stores such as the same size of shopping malls, gas stations, and convenience stores, it is not useful for treating goods of various kinds supplied in the urban centre and the suburban area. Unlike the setting of "spatial price competition", many goods supplied at the urban centre differ from those at suburban areas. We specifically examine hollowing-out at urban centres in this paper. Therefore, we investigate the two remaining factors of market failure.

The first factor is "shopping externality" attributable to multipurpose shopping: consumers can purchase various commodities during shopping at a single location if retail stores are agglomerated. Consequently, agglomeration of retail stores provides a positive externality for consumers, designated as the "shopping externality" (but not fully demonstrated using a model) by O'Sullivan (1993).

The second factor is "imperfect competition" among retail stores. Within an urban centre, widely various retail stores locate and mutually compete for profits. Such a competitive framework is monopolistic competition: each retail store differentiates its services and assortment of goods to compete with other retail stores. The "monopolistic competition" model can represent such competition among retail stores.

The output of market equilibrium is not socially optimal if these two factors of market failure prevail. Our first purpose is to derive the mechanics generating a divergence between the market equilibrium and social optimum by constructing a model. Next, based on the model, we analyze how transportation improvement affects hollowing-out and the welfare effects of a transportation improvement. More specifically, this paper first presents a comparison between the market equilibrium and the social optimum under the existence of the two technical externalities. In particular, we examine how the level of commercial agglomeration under the market equilibrium diverges from the socially optimal level under the existence of

<sup>&</sup>lt;sup>1</sup> Transitions of hollowing-out and the revitalization of urban centres in several countries are explained in detail in Reynolds and Cuthbertson (2003), Chapter 4.

<sup>&</sup>lt;sup>2</sup> Planning policy guidelines (PPG6) is a policy to revitalize the city centre: the hollowing-out took place because of deregulation in the Thatcher era. Actually, PPG6 encourages retailers to emphasize activities in towns rather than out of towns.

the two factors. Second, we investigate how transport improvement affects changes in the commercial agglomeration and social welfare.

The remainder of the paper is organized as follows: the remainder of this section presents a description of related reports of the literature to show the relation of the current study to the vastly numerous past studies. Section 2 presents a description of construction of the model. Section 3 explains the derivation of the market area of each commercial area in the market equilibrium and in the social optimum using the model. We compare them and analyze technical externalities brought by "multipurpose (one time) shopping" and "monopolistic competition". Section 4 explains how social welfare changes with respect to transport facility improvement under the market equilibrium, with existing shopping externality, and with monopolistic competition. Section 5 concludes the paper.

## **Related literature**

Shopping externality is explained by O'Sullivan (1993) as occurring when a store enters an area. In fact, shopping externality arises if the demand accruing to neighbouring stores, which sell imperfectly substitute goods and complementary goods, becomes great. Consumers have the merit of saving the cost of window-shopping for comparison of goods if a commercial agglomeration exists which sells imperfectly substitute goods. Furthermore, if a commercial agglomeration sells complementary goods, then consumers have the benefit of purchasing goods of several kinds during one-stop shopping and can therefore save transport costs. These merits for consumers increase the demand accruing to neighbouring stores.

The mechanics generating commercial agglomeration by the existence of shopping externality has been explained in some papers. Wolinsky (1983) shows that a commercial agglomeration that sells imperfectly substitute goods is organized if information asymmetries exist between stores and consumers: consumers do not know the price and the quality of goods perfectly until they visit each store. Actually, de Palma et al. (1985) describe that the agglomerated configuration of retail stores at the market centre is a Nash equilibrium if goods are sufficiently differentiated and if transport costs are sufficiently low. Ago (2008) presents the same result in the case of monopolistic competition. These papers present interesting implications for the formation of agglomeration, but the change in utility level by transportation improvement, which the current paper targets, is not addressed.

Monopolistic competition is modelled by Dixit and Stiglitz (1977). Dixit–Stiglitz monopolistic competition is founded on a drastic abstraction of actual competition based on price and location. However, the model is tractable and flexible. The Dixit–Stiglitz model has already been used in many papers for analyses of various problems related to spatial aspects<sup>3</sup>. The formulation of monopolistic competition in our model follows that of Fujita et al. (1999).

Multipurpose (one stop) shopping behaviour of consumers is an activity through which consumers purchase goods of several kinds at one time to save the transport cost for shopping. A consumer can decrease the transport cost per unit of a good if purchasing a large variety of goods. Multipurpose (one-stop) shopping behaviour of consumers is

<sup>&</sup>lt;sup>3</sup> See, e.g. Fujita et al. (1999) and Baldwin et al. (2003).



Figure 1 – Residential area and Commercial areas.

modelled by several reports in the literature<sup>4</sup>. In actual shopping behaviour, consumers visit some stores and marketplaces during a specified period. Several papers model such consumer behaviour using random utility frame works (see e.g., Popkowski et al. (2004) and Sinha (2000)) or a hazard model (see e.g. Popkowski et al. (2000)). In these models, all consumers buy goods at all shopping clusters scattered geographically. The analyses described in the current paper adopt a simpler setting in which each consumer buys multiple goods at one shopping cluster. If products supplied at a smaller marketplace are supplied in a larger marketplace, then consumers visit only one marketplace. This situation holds in hierarchical marketplaces described by Christaller (1933). In this case, modelling one-stop shopping behaviour is sufficient for our manuscript's purpose by appropriately adjusting the time interval<sup>5</sup>. Eaton and Lipsey (1982), modelling one-stop shopping of consumers, show that such behaviour causes commercial agglomeration. However, they do not analyze the endogenous prices of goods and the presence of shopping externality, each of which is analyzed in this paper.

## 2. THE MODEL

For simplicity, we consider a linear residential area, a housing area that is represented as a line segment along which homogeneous consumers are distributed uniformly and continuously. The total population of consumers is fixed as  $\bar{N}$ ; they reside on a plot of land. Each plot's length is normalized to 1. Consequently, the line segment length is equivalent to  $\bar{N}$ .

Each of the two ends of the interposed residential area has a transportation facility to a commercial area. Figure 1 shows that the transportation facility is represented as "TF1" and "TF2". The two commerce areas are, respectively, called "region 1" and "region 2". Stores

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<sup>&</sup>lt;sup>4</sup> See, e.g., Hanson (1980), Mulligan (1985, 1987), and Ingene and Ghosh (1990).

<sup>&</sup>lt;sup>5</sup> For example, Henkel et al. (2000) models one time shopping behaviour to analyze coalition formation among suppliers of retail services. Tabuchi (2009) models self-organization marketplaces under a one-stop shopping situation.

locate in the two commerce regions 1 and 2, but they are not allowed to locate in the housing area. In real cities also, zoning regulations restrict large-size stores' locations in housing areas. Moreover, most cities have a central commercial area and suburban shopping centres. The two commerce regions in the model can be interpreted as those. In both regions, retail stores locate in the existence of monopolistic competition. Considering the transportation cost and the variety of the commodities supplied, each consumer purchases goods at an either–or region. No congestion arises in relation to their shopping trips. For simplification, we do not model the land market of commercial areas. In our model, retail stores can locate anywhere with no land rent. That latter assumption might appear to be strict, but our conclusions are fundamentally identical to those in the case in which the land market is introduced into the model if no market failure exists in the land market.

For convenience of our consideration, we merely assume that region 1 is an urban centre, whereas region 2 is on the outskirts of a city. However, no technical difference exists between retail stores in regions 1 and 2 in the model; the exchange of 1 and 2 does not influence the outcome of the following analysis.

## **Transport cost**

Transportation in the housing area and transportation facilities requires some monetary expense, but it is costless in commercial areas. This assumption corresponds to the situation in which transport costs associated with travel between a shopper's home and the commercial area are much greater than the costs among retail stores in the commercial area. Therefore, rational consumers purchase goods of many kinds during one shopping trip.

It is assumed that the transport cost per unit distance in the housing area is one, and that the travel cost of TF1 is  $t_1$ , and that of TF2 is  $t_2$ . Therefore, for the *nth* consumer from region 1, the respective transport costs  $L_1(n)$  for region 1 and  $L_2(n)$  for region 2 are given as  $L_1(n) = n + t_1$  and  $L_2(n) = (\overline{N} - n) + t_2$ .

## **Consumer behaviour**

In our model, we specifically examine consumers' shopping behaviour during a certain fixed time interval, as assumed by Eaton and Lipsey (1982). Each consumer shares a log linear utility function:

$$V_i = \mu \ln M_i + (1 - \mu) \ln A_i.$$
 (1)

Therein, subscript *i* signifies that the consumer shops at region *i*,  $M_i$  represents a composite index of the consumption of commercial goods,  $A_i$  denotes the consumption of a numeraire good of which the price is one, and  $\mu$  is a constant representing the expenditure share of commercial goods. The quantity index,  $M_i$ , represents a sub-utility function defined over a continuum of varieties of commercial goods. In addition,  $m_i(x)$  denotes the consumption of each available variety;  $f_i$  stands for the number of goods sold at region i — which is equivalent to the number of retail stores, and  $\sigma$  is the elasticity of substitution

between any two varieties. We assume that  $M_i$  is defined using a constant elasticity of substitution (CES) function:

$$M_{i} = \left[\int_{0}^{f_{i}} m_{i}(x)^{\sigma-1/\sigma} dx\right]^{\sigma/\sigma-1}.$$

Given income *Y* and price  $p_i(x)$  for each commercial good, and transport costs for region *i*, the consumers' budget constraint is

$$A_i + \int_0^{f_i} p_i(x)m_i(x) dx + L_i(n) = Y$$
.

Consumers' utility maximization is represented as shown below.

$$\max_{M_{i},A_{i}} V_{i} \qquad s.t. \ A_{i} + G_{i}M_{i} + L_{i}(n) = Y$$
(2)

Therein,  $G_i$  is the price index of commercial goods supplied in region *i*. Because it is assumed that no technical difference exists among retail stores, all commercial goods are sold at the same price  $p^*$ . Therefore,  $G_i$  is represented as

$$G_{i} = \left[\int_{0}^{f_{i}} p(x)^{1-\sigma} dx\right]^{1/1-\sigma} = p^{*} f_{i}^{\frac{1}{1-\sigma}}.$$
 (3)

The maximized utility by consumers' utility maximization (2), is expressed as a function of income, price of retail stores and number of goods, giving the following indirect utility function

$$V_{i} = \ln\left(\frac{\mu}{p^{*}}\right)^{\mu} \left(1-\mu\right)^{1-\mu} + \ln\left(Y-L_{i}\right) + \frac{\mu}{\sigma-1} \ln f_{i}.$$
 (4)

Equation (4) is derived from maximizing the utility of an *nth* consumer who goes shopping in region i. The first term in eq. (4) is a function of the price of a good; the second term is that of income. The third term is that of the number of kinds of goods.

## **Retail store behaviour**

Each retail store supplies a horizontally differentiated good under conditions of free entry and exit. Under monopolistic competition, none supplies the same kind of good as the others. Therefore, their number in a region is equivalent to the number of kinds of goods.

Their technology is the same in both regions: it involves a fixed input cost F and marginal input cost requirement c. Consequently, the production of a quantity q of any good at any location requires the cost given as F + cq.

Considering a particular retail store supplying a specific good, its profit,  $\pi$ , is given as

$$\pi = pq - F - cq ,$$

where p is the mill price. Each retail store is assumed to have a price index  $G_i$  as given. The perceived elasticity of demand is therefore  $\sigma$ . Therefore, the first order condition of profit maximization implies that equilibrium price  $p^*$  is

$$p^* = \frac{\sigma}{\sigma - 1}c\tag{5}$$

for all retail stores. Given the pricing rule, the profit is

$$\pi = \frac{1}{\sigma - 1} cq - F \, .$$

Therefore, the zero-profit condition implies that equilibrium output  $q^*$  is

$$q^* = \frac{F(\sigma - 1)}{c}.$$
 (6)

It is constant for every active retail store in the economy.

## 3. MARKET EQUILIBRIUM AND SOCIAL OPTIMUM

### Market area with market equilibrium

Let  $\hat{n}(0 < \hat{n} < \overline{N})$  be the interior market boundary: the location of the marginal consumer who is indifferent to visiting either regions. If the utility visiting one region is higher than that of another region for all consumers, then the interior market boundary does not exist and all consumers go to one region for shopping: either  $\hat{n} = 0$  or  $\hat{n} = \overline{N}$ . This condition is expressed as

$$\hat{n} = \begin{cases} 0 & if \quad V_1(n) < V_2(n) \\ \begin{bmatrix} 0, \overline{N} \end{bmatrix} & if \quad V_1(\hat{n}) = V_2(\hat{n}) \\ \overline{N} & if \quad V_1(n) > V_2(n). \end{cases}$$

A stability condition is necessary for the interior solution ( $\hat{n} = [0, \overline{N}]$ ). It is shown as

$$\partial \left( V_1(\hat{n}) - V_2(\hat{n}) \right) / \partial n < 0$$
,

which expresses that any consumers' change in a region for shopping decreases their own utility level. In this situation, no one has an incentive to change their shopping destination.

Corner solutions (  $\hat{n} = 0, \overline{N}$  ) mean that all consumers go to one region for shopping; accordingly, no retail store locates in the other region. We analyze the change in the market area of each region occurring because of transportation facility improvement. Therefore, we assume the existence of an interior solution and we specifically examine the case of an interior solution from this point on.

We derive the numbers of kinds of goods in each region  $f_i$  with the interior solution. They depend on the demand. Put differently,  $f_i$  is a function of the market area size, equivalent to the number of consumers who visit the region. In fact, p(j)m(j), the expenditure of a good of a consumer, is derived through formulation of consumer behaviour as

$$p(j)m(j) = p(j) \left(\frac{p(j)}{G_i}\right)^{-\sigma} M_i, \qquad (7)$$

where  $M_i = \mu(Y - L_i)/G_i$  derives the solution of consumers' utility maximization (2). Substituting  $M_i = \mu(Y - L_i)/G_i$  and eq. (3) into (5) yields  $p(j)m(j) = \mu(Y - n - t_1)/f_1$  for a good supplied in region 1 and  $p(j)m(j) = \mu(Y - (\overline{N} - n) - t_2)/f_2$  for a good supplied in region 2. The sum of the expenditure with respect to all consumers visiting each region is equal to the sales turnover amount of each retail store  $p^*q^*$ :

$$p^*q^* = \int_0^{\hat{n}} \frac{\mu}{f_1} (Y - n - t_1) dn$$
 in region 1 and (8a)

$$p^{*}q^{*} = \int_{\bar{N}-\hat{n}}^{\bar{N}} \frac{\mu}{f_{2}} \Big( Y - \big(\bar{N} - n\big) - t_{2} \Big) dn \text{ in region 2.}$$
(8b)

Substituting eq. (5) and (6) into eq. (8a) and (8b) yields

$$f_1 = \frac{\mu}{F\sigma} \left( (Y - t_1)\hat{n} - \frac{\hat{n}^2}{2} \right) \text{ and }$$
(9a)

$$f_2 = \frac{\mu}{F\sigma} \left( (Y - t_2) \left( \overline{N} - \hat{n} \right) - \frac{\left( \overline{N} - \hat{n} \right)^2}{2} \right).$$
(9b)

Differentiation of eq. (9a) and eq. (9b) with respect to the number of consumers in one's own market area yields

$$\frac{\partial f_1}{\partial \hat{n}} = \frac{\mu}{F\sigma} (Y - \hat{n} - t_1) > 0 \text{ and}$$
$$\frac{\partial f_2}{\partial (\bar{N} - \hat{n})} = \frac{\mu}{F\sigma} (Y - (\bar{N} - \hat{n}) - t_2) > 0.$$

As they show, the market area expansion increases the variety of goods in the region.

### Market area with social optimum

The market area with social optimum is defined as the market area which maximizes the sum of individuals' utility level. It is represented as SW, social welfare, which is

$$\max_{\hat{n}} SW = \int_{0}^{\hat{n}} V_{1}(n,\hat{n}) dn + \int_{\bar{N}-\hat{n}}^{\bar{N}} V_{2}(n,\hat{n}) dn .$$
 (10)

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The first term in eq. (10) is the sum of consumers' utility who visit region 1. The second term is the sum of the utility of consumers who visit region 2.

To derive the condition for a socially optimal market area, the first-order condition of social welfare maximization with respect to market area,  $dSW/d\hat{n} = 0$ , is derived as

$$V_1(\hat{n}) + EX_1 = V_2(\hat{n}) + EX_2,$$
 (11)

where

$$EX_{1} = \int_{0}^{\hat{n}} \frac{\partial V_{1}(n,\hat{n})}{\partial \hat{n}} dn = \frac{\mu}{\sigma - 1} \frac{\hat{n}}{f_{1}} \frac{\partial f_{1}}{\partial \hat{n}} \text{ and}$$
(12a)

$$EX_{2} = \int_{\hat{n}}^{\bar{N}} \frac{\partial V_{2}(n,\hat{n})}{\partial (\bar{N}-\hat{n})} dn = \frac{\mu}{\sigma-1} \frac{\bar{N}-\hat{n}}{f_{2}} \frac{\partial f_{2}}{\partial (\bar{N}-\hat{n})}.$$
 (12b)

Comparison of eq. (11) to  $V_1(\hat{n}) = V_2(\hat{n})$ , the condition of the interior solution with market equilibrium, show that the difference between the social optimum and market equilibrium is the second term in eq. (11), represented as  $EX_1$  and  $EX_2$ . They express that technical externality arises from shopping externality and monopolistic competition. Equations (12a) and (12b) show that the change in utility caused by the infinitesimal change in market area  $\partial V_i(n,\hat{n})/\partial \hat{n}$  applies to all consumers who visit region *i*. Put differently, if consumers switch their own personal destination from a marketplace to the other marketplace, then it changes not only their own utility but also that of all other consumers.

**Proposition 1.** Technical externalities arise in the existence of "multiple purpose (one time) shopping" and "monopolistic competition". "Multiple purpose shopping" is that by which one can purchase some good in one-stop shopping by visiting a commercial agglomeration. "Monopolistic competition" is that by which various goods supplied in a region increase concomitantly with increased regional demand.

We assume that transportation within commercial areas is costless. It invariably compels consumers to purchase widely various goods in one shopping trip to save transport costs between their residence and the commercial area. The greater the amount of goods obtained in one shopping trip, the lower the transport cost per good. Put differently, scale economies apply to the consumers' shopping trips.

The convexity of *SW* is a necessary condition making eq. (11) the condition of social welfare maximization. We assume that  $d^2SW/d\hat{n}^2 < 0$ , where *SW* is convex in the model.

The social optimum market area which maximizes social welfare is the corner solution  $\hat{n} = \overline{N} (\hat{n} = 0)$  if  $dSW/d\hat{n} > 0 (<0)$  at  $\hat{n} \in [0, \overline{N}]$ . It shows such a situation that all consumers visit either region and all retail stores also locate to either region.

### Market equilibrium and social optimum

We compare the market area with the market equilibrium derived by the condition of  $V_1(\hat{n}) = V_2(\hat{n})$  and that with social optimum derived by condition (11).



Figure 2 – Utility, technical externality, and market area.

To capture the configuration of indirect utility function  $V_i(\hat{n})$ , differentiating  $V_1(\hat{n})$  and  $V_2(\hat{n})$  with respect to its market area  $\hat{n}$  and  $\overline{N} - \hat{n}$  yields

$$\frac{\partial V_1(\hat{n})}{\partial \hat{n}} = -\frac{1}{Y - \hat{n} - t_1} + \frac{\mu}{\sigma - 1} \frac{\partial f_1}{\partial \hat{n}} \frac{1}{f_1} \text{ and}$$
(13a)

$$\frac{\partial V_2(\hat{n})}{\partial \left(\bar{N}-\hat{n}\right)} = -\frac{1}{Y - \left(\bar{N}-\hat{n}\right) - t_2} + \frac{\mu}{\sigma - 1} \frac{\partial f_2}{\partial \left(\bar{N}-\hat{n}\right)} \frac{1}{f_2}.$$
 (13b)

The first terms in eqs. (13a) and (13b) are negative, whereas the second terms are positive. We can derive the following properties:  $\partial V_1(\hat{n})/\partial \hat{n} \to +\infty$  at  $\hat{n} \to 0$ ,  $\partial V_2(\hat{n})/\partial (\bar{N}-\hat{n}) \to +\infty$  at  $\bar{N}-\hat{n}\to 0$ ,  $\partial^2 V_1(\hat{n})/\partial \hat{n}^2 < 0$ , and  $\partial^2 V_2(\hat{n})/\partial (\bar{N}-\hat{n})^2 < 0$ . Therefore,  $V_i(\hat{n})$  is convex upward with respect to its market area. Furthermore, we can derive the following properties from eq. (12a) and (12b):  $EX_1 \to 0$  at  $\hat{n} \to 0$ ,  $EX_2 \to 0$  at  $\bar{N}-\hat{n}\to 0$ ,  $\partial EX_1/\partial \hat{n} > 0$ , and  $\partial EX_2/\partial (\bar{N}-\hat{n}) > 0$ . From these properties, we can draw the shape of  $V_i(\hat{n})$  and  $V_i(\hat{n}) + EX_i$  as Fig. 2, in which the horizontal axis expresses the distribution of consumers:  $n_1$ —starting from the left—is the number of consumers who visit region 1 and  $n_2$ —starting from the right—represents the number of consumers who visit region 2. In Fig. 2,  $V_i(\hat{n})$  is drawn as a solid line, whereas  $V_i(\hat{n}) + EX_i$  is drawn as a broken line. The difference in shape between  $V_1(\hat{n})$  and  $V_2(\hat{n})$  arises from the difference of  $t_i$ , the transport cost for transportation facility, which is included in the second and third terms in eq. (4). The intersection of  $V_1(\hat{n})$  and  $V_2(\hat{n})$  corresponds to  $\hat{n}^m$ , which is the market area with market equilibrium; the intersection of  $V_1(\hat{n}) + EX_1$  and  $V_2(\hat{n}) + EX_2$  corresponds to  $\hat{n}^s$ , which is the market area with the social optimum.

Figure 2 shows that the market equilibrium is not generally equivalent to the social optimum. The left side of Fig. 2 shows the case in which the socially optimum market area of region 1 is larger than that with the market equilibrium; the right side in Fig. 2 shows the reverse situation. Either case can exist depending on the shape and magnitude relation among  $V_i(\hat{n})$  and  $EX_i$ .

Whether  $\hat{n}^m$  is larger than  $\hat{n}^s$  or not depends on whether or not  $EX_1$  is greater than  $EX_2$  at  $\hat{n}^m$ , the market area with market equilibrium. In fact,  $\Delta EX^m$ —the difference between  $EX_1$  and  $EX_2$  at  $\hat{n}^m$ —is expressed as

$$\Delta EX^{m}(=EX_{1}-EX_{2})=\int_{0}^{\hat{n}^{m}}\frac{\partial V_{1}(n,\hat{n}^{m})}{\partial \hat{n}}dn-\int_{\hat{n}^{m}}^{\bar{N}}\frac{\partial V_{2}(n,\hat{n}^{m})}{\partial (\bar{N}-\hat{n})}dn.$$

If  $\Delta EX^m$  is positive/negative, then  $\hat{n}^m$  is less/more than  $\hat{n}^s$ . Its deformation yields

$$\Delta EX^{m} = \int_{0}^{\hat{n}^{m}} \frac{\partial V_{1}\left(n,\hat{n}^{m}\right)}{\partial f_{1}} dn \frac{\partial f_{1}}{\partial \hat{n}} - \int_{\hat{n}^{m}}^{\bar{N}} \frac{\partial V_{2}\left(n,\hat{n}^{m}\right)}{\partial f_{2}} dn \frac{\partial f_{2}}{\partial \left(\bar{N}-\hat{n}\right)}.$$
 (14)

Equation (14) shows that  $\Delta EX^m$  is the difference between the change in utility multiplied by the change in the number of varieties, which results from the change in the market area.

**Proposition 2.** Under the monopolistic competition of retail stores and shopping externalities of consumers, the market area with market equilibrium is not equivalent to that with a social optimum. The magnitude relation between  $EX_1$  and  $EX_2$  at  $\hat{n}^m$  determines whether  $\hat{n}^m$  is larger than  $\hat{n}^s$ .

Shopping externality, which arises from one-stop shopping, is invariably a technical externality in the market. Therefore, determining if the market area with market equilibrium is larger or smaller than that with social optimum is of importance for a decision for making policies to control the hollowing-out of city centres.

To clarify the magnitude of the relation between  $EX_1$  and  $EX_2$  at  $\hat{n}^m$  in the case of our model, we derive  $\Delta EX^m$ . Substituting eqs. (9a), (9b), (12a), and (12b) to eq. (14) yields

$$\Delta EX^{m} = \frac{\left[\frac{Y - t_{1}}{(Y - t_{1}) + (Y - t_{2})} - \frac{\hat{n}^{m}}{\bar{N}}\right]}{\frac{\bar{N}}{2}\left[(Y - t_{1}) + (Y - t_{2})\right]\left(Y - t_{1} - \frac{\hat{n}^{m}}{2}\right)\left(Y - t_{2} - \frac{\bar{N} - \hat{n}^{m}}{2}\right)}.$$
(15)

The denominator of eq. (15) is positive. Therefore, the sign of the numerator dominates the sign of  $\Delta EX^m$ . The first term in the numerator is the ratio of the income minus transport cost for the first consumer closest to region 1 and  $\overline{N}th$  consumer closest to region 2. The second term in the numerator is the ratio of  $\hat{n}^m$ , the market area of region 1 with market equilibrium, to  $\overline{N}$ , the total housing area. Figure 3 presents an intuitive interpretation of the property of eq. (15).

If the ratio of  $\hat{n}^m$  to  $\bar{N}$  is smaller/larger than the ratio of income consumed in goods for the first consumer closest to region 1, to the sum of that with the first consumer and *nth* consumer closest to region 2, then  $\hat{n}^m$  is excessively smaller/larger than  $\hat{n}^s$ , the market area with the social optimum.

The property of eq. (15) holds under any form of the cost function of retail stores, although it is dependent on the consumers' utility function and form of competition among retail stores.



Figure 3 – Excessively large or small market area with market equilibrium.

Equation (15) is derived under a log-linear utility function of consumers and monopolistic competition among retail stores. However, the property of the log-linear utility function—the ratio of expenditure for which each good is constant—is a general consumption activity and the property of monopolistic competition, as described previously, approximates the real form of competition among retail stores.

## 4. TRANSPORT IMPROVEMENT AND WELFARE CHANGE

### Change in social welfare caused by transport improvement

Under the market equilibrium, we derive  $dSW_1$ , the change in social welfare, with respect to TF1, which is represented as the change in  $t_1$ .  $dSW_1$  derived by eq. (10) as

$$dSW_{1} = \int_{0}^{\hat{n}} \frac{\partial V_{1}}{\partial \hat{n}} dn \frac{\partial \hat{n}}{\partial t_{1}} dt_{1} + \int_{0}^{\hat{n}} \frac{\partial V_{1}}{\partial t_{1}} dn dt_{1} + \int_{\hat{n}}^{\bar{N}} \frac{\partial V_{2}}{\partial (\bar{N} - \hat{n})} dn \frac{\partial (\bar{N} - \hat{n})}{\partial t_{1}} dt_{1}.$$

The benefit of TF1 improvement in monetary terms,  $dB_1$ , is derived by dividing it by the marginal utility of income  $\partial V_i / \partial Y$ , as presented below.

$$dB_{1} = -\hat{n}dt_{1} + \int_{0}^{\hat{n}} \frac{\partial V_{1}/\partial f}{\partial V_{1}/\partial Y} dn \frac{\partial f_{1}}{\partial t_{1}} dt_{1} + \int_{0}^{\hat{n}} \frac{\partial V_{1}/\partial f}{\partial V_{1}/\partial Y} dn \frac{\partial f_{1}}{\partial \hat{n}} \frac{\partial \hat{n}}{\partial t_{1}} dt_{1} + \int_{\hat{n}}^{\bar{N}} \frac{\partial V_{2}/\partial f}{\partial V_{2}/\partial Y} dn \frac{\partial f_{2}}{\partial (\bar{N} - \hat{n})} \frac{\partial (\bar{N} - \hat{n})}{\partial t_{1}} dt_{1}$$

$$\Rightarrow dB_{1} = -\hat{n}dt_{1} + p^{*}q^{*} \frac{\mu}{\sigma - 1} \frac{\partial f_{1}}{\partial t_{1}} dt_{1} + p^{*}q^{*} \frac{\mu}{\sigma - 1} \frac{\partial f_{1}}{\partial \hat{n}} \frac{\partial \hat{n}}{\partial t_{1}} dt_{1} + p^{*}q^{*} \frac{\mu}{\sigma - 1} \frac{\partial f_{1}}{\partial \hat{n}} \frac{\partial \hat{n}}{\partial t_{1}} dt_{1} + p^{*}q^{*} \frac{\mu}{\sigma - 1} \frac{\partial f_{2}}{\partial (\bar{N} - \hat{n})} \frac{\partial (\bar{N} - \hat{n})}{\partial t_{1}} dt_{1} \quad (16)$$

Equation (16) shows that transportation improvement brings about both a direct effect and externalities. The first term in eq. (16) is the direct effect: the benefit of decrease in transport

cost for consumers who visit region 1. The second, third, and fourth terms are the externalities.

The second term in eq. (16) is the benefit of the change in consumer surplus in region 1 with respect to the change in varieties in region 1, which is directly attributable to transport improvement. The third term is the benefit of the change in consumer surplus in region 1 with respect to the change in varieties in region 1, caused indirectly by transport improvement: the change in varieties occurs via the change in market area caused by transport improvement. The fourth term is the benefit of the change in consumer surplus in region 2 with respect to the change in varieties in region 2, which is caused indirectly by transport improvement such as that in the case in the third term.

Comparing the social welfare change caused by TF1 improvement to that caused by TF2 improvement, we can evaluate which improvement is effective from the perspective of social welfare improvement. We assume that the decrease in transport cost per unit construction cost is the same in TF1 and TF2 ( $dt_1 = dt_2 = dt$ ). In fact,  $dB_1 - dB_2$ , the difference between the benefit of TF1 improvement and that of TF2 improvement, is derived as shown below.

$$dB_1 - dB_2 = -\left[\hat{n}(t_1, t_2) - \left(\bar{N} - \hat{n}(t_1, t_2)\right)\right]dt + \left[p^*q^*\frac{\mu}{\sigma - 1}\frac{\partial f_1}{\partial t} - p^*q^*\frac{\mu}{\sigma - 1}\frac{\partial f_2}{\partial t}\right]dt$$
(17)

The TF1 improvement derives greater/less benefit per unit transport cost than TF2 improvement if eq. (17) is positive/negative. The first term in eq. (17) is the difference between the direct effect of TF1 improvement and that of TF2 improvement. The second term is the difference between the externalities in region 1 and region 2. Estimating these terms enables evaluation of which improvement is effective from the social welfare perspective.

**Proposition 3.** The benefit of TF1 is expressed as eq. (16). More effective transportation facility improvement in the perspective of social welfare is determined using eq. (17).

## Change in market area and social welfare

The decrease in  $t_1$  implies an increase in the income of consumers for the consumption of goods. Therefore, the decrease in  $t_1$  raises the utility of consumers. Figure 4 portrays the change in market area and the change in the social welfare with respect to TF1 improvement. The decrease in  $t_1$  raises the utility of consumers. Therefore,  $V_1(\hat{n}) + EX_1$  moves superiorly by TF1 improvement. The value of  $V_1(\hat{n}) + EX_1$  before improvement is shown as a solid line; that of  $V_1(\hat{n}) + EX_1$  after improvement is shown as a broken line.

The right side and left side in Fig. 4 both show that TF1 improvement expands the market area of region 1. However, social welfare on the right side in Fig. 4 decreases, whereas that on the left side in Fig. 4 increases.

Regarding the right side in Fig. 4, the utility of consumers who visit region 1 diminishes as its market area increases, whereas that of consumers who visit region 2 increases because its market area expands. The factors determining these gradients are the relative dimensions of the first term and the second terms in eqs. (13a) and (13b): the first terms are shortcomings of transport cost increase attributable to the increase in the market area,



Figure 4 – Change in market area and social welfare.

whereas the second terms are merits of the variety increase attributable to the increase in the market area.

Regarding the right side in Fig. 4, the shortcoming of transport cost increase is greater than the merit of the variety increase in region 1, although the shortcoming of transport cost increase is greater than the merit of variety increase in region 2. In this situation, the transportation facility improvement in market equilibrium decreases social welfare.

**Proposition 4.** *TF1* improvement expands the market area of region 1, although it is not always true that transport facility improvement increases social welfare under any form or any magnitude of  $V_1(\hat{n}) + EX_1$ .

This paradoxical consequence arises from a combination of the multiple externalities: multiple-purpose shopping at the two regions and the monopolistic competition at the two regions. Typical paradoxical examples arising from a combination of multiple externalities are the Pigou–Knight paradox (Pigou (1920) and Knight (1924)), Braess' paradox (Braess (1968)), and Downs–Thomson paradox (Downs (1962) and Thomson (1977)).

## **5. CONCLUSION**

Conclusions of the paper are the following four.

First, results show that the generation mechanism of technical externalities is derived from the property of one-stop shopping. Given monopolistic competition, consumers can purchase a variety of goods through one-stop shopping if differentiated retail stores are agglomerated in the same region. Variety in a region increases concomitantly with the increased demand.

Second, the market area with market equilibrium and that with social optimum is not the same because of market failure: it derives from a shopping externality and monopolistic competition in our model. Whether the market area with market equilibrium is larger or

smaller than that with social optimum is dependent on the shape and magnitude relation between  $V_i(\hat{n})$  and  $EX_i$ .

Third, we derive the transport improvement benefit evaluation formula of eq. (16). Furthermore, we can determine more effective transportation facility improvement from a social welfare perspective using eq. (17).

Fourth, transportation improvement does not necessarily increase social welfare. The arguments presented herein clarify the conditions under which social welfare decreases.

Based on the public awareness that hollowing-out of urban centres is inefficient from a social welfare perspective, some policies have been implemented to promote urban centre revitalization: improving transportation accessibility of an urban centre is one such policy. However, our conclusions show that such a policy can decrease social welfare according to various circumstances.

Our model incorporates shopping externality and monopolistic competition among retail stores as factors exacerbating market failure: some other factors of market failure, e.g. agglomeration economies and congestion externality, are not addressed in the model. Public awareness that the hollowing-out of urban centres is inefficient might encourage consideration of market failure of all sorts. For that reason, future research should address such externalities.

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