IMPLEMENTATION OF BICYCLE SHARING SYSYTEM WITH TRADABLE PERMITS AUCTION AND BEHAVIOR ANALYSIS

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ABSTRACT

This study implements the bicycle sharing system with tradable permits in Yokohama Minato-Mirai Area, Japan. Tradable permits is the rights that users take bicycle sharing system on some time of day and their auction system shifts the load from system administrator by encouraging free sales transaction between users. We first explore relations between the trade behavior and frequency of visiting the area. Then we formulate users' trade behavior of future decision-making under uncertainty by discrete choice model. The estimation results indicate that permits attributes such as weekday or weekend, morning or afternoon have an effect on transaction behavior. Finally, it reveals schedule effect utility function is nonlinear and it has 2 jumping point.

Keywords: Bicycle sharing system, Auction, ITS, Discrete choice model

1. INTRODUCTION

1.1. Background

During the past decade or so, all developed countries have depended on automobile too much. As a result, we emitted much $CO₂$ and destroyed attractive pedestrian spaces. However, because of building low carbon society, our vision will shift multimodal transport

society such as public transport, bicycle and pedestrian. Therefore, we need competitive alternatives to private cars. So, that new mobility service and transport policy need following attributes

- 1. easy switch between transport modes
- 2. good connections between modes
- 3. sharing system and car pooling
- 4. flexible price mechanism and charging scheme
- 5. better traffic information

This paper scopes sharing system and flexible price mechanism.

We have witnessed a worldwide trend towards a bicycle sharing system such as Vélib' in Paris. Sharing system is an efficient system in time and space and shifting from automobile to on-demand mobility service such as bicycle sharing can be meaningful in global warming. Additionally, the developments of ICT and ITS are going to make price system flexible. For example, Akamatsu (2007) proposed "tradable bottleneck permits" (TBP) system. It is revealed that TBP system achieves the most efficient use of network capacity. In theory, we know auction mechanism achieves efficient allocation under some assumption (see Vickrey, 1961, Riley and Samuelson, 1981, Myerson, 1981). And in practice, frequency auction in the USA and internet auction like e-bay are brought to international attention as successful experiences (see Milgrom, 2004, Steiglitz, 2007). As empirical studies, Jofre-Bonet and Pesendorfer (2000, 2003) studied bidding behavior and estimated a bidding model.

This paper shows the elements of trade behavior in bicycle sharing tradable permits mechanism in empirical case study. In this case study, we implemented bicycle sharing system, bicycle sharing tradable permits auction system, and probe person system. Sharing permits trade auction is a double auction mechanism and all tradable permits are allocated to all users at random and users can trade them freely on the internet. Probe person system is a method to get travel diary data and positioning data in detail by GPS mobile phone (see Hato and Kitamura, 2008). We collected travel behavior data and trade behavior data in these systems.

1.2. Mobility Sharing and Mobility auction

Bicycle sharing systems have received increasing attention in recent years. Midgley (2009) says that bicycle sharing systems are currently operating in 78 cities in 16 countries using

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around 70000 bikes. In this literature, they differ from traditional, mostly leisure-oriented bicycle rental services in the following ways.

- 1. They can be rented at one location and either returned there or at another location.
- 2. They provide fast and easy access.
- 3. They have diverse business models.
- 4. They make use of applied technology (IC smart cards and/or mobile phones).
- 5. They are often designed as part of the public transport system.

And DeMaio and Gifford (2004), DeMaio (2009) say the information technology such as smart cards and wireless technologies have allowed bicycle sharing to evolve into the current smart bicycle sharing system. In Japan, Watanabe and Hato (2008) implemented bicycle sharing system with mobile phone application in Kashiwa city as a pilot program and assessed the system.

On the other hand, when we design the mobility sharing services such as bicycle sharing, these problems occur: 1) Although total supply exceeds total demand, you can't use or return the mobility due to spatial uneven distribution of demand or supply; 2) You can't use the mobility when you want because total demand exceeds supply. The former problem has been proposed to resolve the spatial imbalance by encouraging a move to a major port from another small-demand port with price incentives. However, if excess demand occurs, the incentive can't solve it.

Another approach is mobility auction system such as TBP system (Akamatsu, 2007). It is difficult for road authorities to know users' desired arrival and willingness to pay due to the asymmetry of information between road authorities and users. In this system, tradable permits auction makes the optimal price without being presented users' preferences directly. In addition, there is a big advantage not to cause congestion because TBP system does not issue permits more than road capacity. In on-demand mobility, we can solve the capacity problem by incorporating TBP system.

But, we point out the following two problems of TBP system. One is the problem whether we can implement the complex trading system technically in daily life. In this regard, we implemented the system in this research and we conducted the pilot program in real urban space. Second problem is that we assume that people in theory do rational decision-making but that real people confront schedule uncertainty. In auction theory, each user has his or her preference in full recognition. Actually, people cannot recognize their preference due to uncertainty of future schedule. On this point, we conducted the above pilot program and

observed microscopic trading behavior. And we try to clarify the users' cognitive structure under uncertainty.

1.3. Layout

We proposed individual choice behavior model as trade behavior model. It is expressed by discrete choice model and we consider the schedule effect of uncertain future decisionmaking and inter-respondent taste heterogeneity. Next session, we describe the model framework, and pilot program for empirical case study is explained in session 3. The following session, we show the estimation results and discuss the result. And session5 concludes.

2. METHODOLOGY

In this section, we describe our modelling framework that allows for the schedule effect of future decision-making and inter-respondent heterogeneity. We begin with a discussion of existing model structures.

2.1. Basic notation and MNL model

Let $U_{i,n,t}$ be the utility of alternative *i* for respondent *n* in choice situation *t*, where this consists of a modelled component $V_{i,n,t}$, commonly referred to as observed utility, and an unobserved component $\varepsilon_{i,n,t}$ such that

$$
U_{i,n,t} = V_{i,n,t} + \varepsilon_{i,n,t} \tag{1}
$$

Where it is common practice to assume a linear relationship between attributes and tastes, such that

$$
U_{i,n,t} = \beta_{n,t} x_{i,n,t} + \varepsilon_{i,n,t} \tag{2}
$$

with $\beta_{i,n,t}$ giving a vector of taste coefficients and $x_{i,n,t}$ giving a vector of attributes describing alternative i as experienced by respondent n in choice situation t .

Under the further assumptions that the unobserved components are identically and independently distributed according to a type I extreme value distribution, and that the parameters β are fixed across the population and across choice situations, the probability that respondent n chooses alternative i in choice situation t is given by the Multinomial Logit (MNL) model (see McFadden, 1974), with

$$
P_{n,t}(i | \beta) = \frac{e^{V_{i,n,t}}}{\sum_{j=1}^{J} e^{V_{j,n,t}}}
$$
 (3)

where *J* gives the number of alternatives faced by respondent n in choice situation t .

2.2. Taste heterogeneity of MMNL

In a model allowing for random taste heterogeneity, such as Mixed Multinomial Logit (Revelt and Train, 1998), the vector of taste coefficients β is assumed to follow a certain random distribution in the sample, such that we have $β ~ f(β | ∩)$ with $Ω$ representing a set of parameters of the (multivariate) normal distribution of β . So the utility is

$$
U_{i,n,t} = \beta'_{n,t} x_{i,n,t} + \varepsilon_{i,n,t}
$$
\n⁽⁴⁾

where β'_{nt} is a vector of coefficients of these variables for respondent *n* representing that person's tastes, and $\varepsilon_{i,n}$, is a random term that is iid extreme value. The coefficients vary over decision makers in the population with density $f(\beta)$. This density is a function of parameters θ that represent, for example, the mean and covariance of the β 's in the population. This specification is the same as for standard logit except that β varies over decision makers rather than being fixed. If we observed β_{nt} , then the choice probability would be standard logit, since the $\varepsilon_{i,n}$'s are iid extreme value. That is, the probability conditional on $\beta_{n,t}$ is

$$
L_{i,n,t}(\beta_{n,t}) = \frac{e^{\beta_{n,t}x_{i,n,t}}}{\sum_{j=1}^{J} e^{\beta_{n,t}x_{j,n,t}}}
$$
(5)

However, we does not know β_{n} and therefore cannot condition on β . The unconditional choice probability is therefore the integral of $L_{i,n}(\beta_{n})$ over all possible variables of β_{n} :

$$
P_{i,n,t} = \int \left(\frac{e^{\beta_{n,t} x_{i,n,t}}}{\sum_{j=1}^J e^{\beta_{n,t} x_{j,n,t}}} \right) f(\beta) d\beta \tag{6}
$$

which is mixed multinomial logit probability.

2.3. Schedule effect of uncertain future decision-making

To consider the schedule effect of the uncertain future decision-making, we extend the model above. When today's choice have an effect on future and the future's schedule is uncertainty, we think that people can postpone the active decision-making until schedule is certain. As your decision-making is shown in Fig-1, you have a difficulty to decide the schedule 2 or 3 weeks later and will decide to do nothing passively today. To describe the mechanism, we define $S(t)$ as schedule effect utility. And we rewrite the following utility function.

$$
U_{i,n,t} = \beta_{n,t} x_{i,n,t} + S(t) + \varepsilon_{i,n,t}
$$
 (7)

Figure 1 – Future Schedule decision-making under uncertainty

Figure 2 – Example of utility function of schedule effect

 $S(t)$ is the function of the number of days before expiration date. In a general way, the function has upward-sloping curve because the utility of active decision-making increases for decrease of schedule uncertainty as days pass by. We can assume $S(t)$ is linear function like (a) in Fig-2 or nonlinear function like (b) or (c) in Fig-2. Utility function of line (a) is uncertainty-neutral and it indicates utility of active decision-making increases as the same rate every day. In this situation, there is an uncertain schedule but the degree of the uncertainty decreases by constant rate. But it is not real. Utility function of line (b) is uncertainty-averse and it shows people plan their schedule early. If most people's schedule effect utility looks like line (b), the uncertainty of future decision-making is small and reservation systems are workable. Utility function of line (c) is uncertainty-loving. It means that uncertain schedule is not able to be fixed until scheduled date is close. That is, we don't

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know our schedule until near previous day and can't make a decision about the day. As just described, we can classify the attitude of future schedule fixedness into three groups.

However, we don't know what the schedule effect utility function looks like. Therefore, in this research we focus the shape of schedule effect utility function. We define the schedule effect utility function as discrete function.

$$
S(t) = \sum_{s \in T} \gamma_s \delta_s \tag{8}
$$

where γ_s is parameter and δ_s is Kronecker delta; $\delta_s = 1$ if $s = t$, $\delta_s = 0$ otherwise.

In a method previously described, we don't know the shape of schedule effect utility completely. However, this method tells us the shape and character of it wholly. The knowledge from this method enables us to identify the function as a parametrically-defined function. When the data is time series data, we can estimate these parameters in a similar manner to estimation of dummy variables. For example, at the previous day of expiration date, active decision-making utility function has γ_{-1} as a dummy variable but doesn't have γ_s $\forall s \neq -1$. At 2 days before the expiration, active decision-making utility function has γ_{-2} . However, passive decision-making utility function, that is postponing decision-making, doesn't have γ_s at all times. In this way, we can estimate γ_s as dummy variables.

If these parameters γ_s demonstrate an upward trend as the deadline draw on, it indicates that people can't make a decision and postpone judging until schedule is certain. And whether the shape of utility function is uncertainty-neutral, uncertainty-averse or uncertaintyloving shows the relationship between schedule effect under uncertainty and active/passive decision-making. In addition, this data-oriented process can show the interesting findings. In this way, it is essential to understand people's future decision-making under uncertainty for transportation reservation services and so on.

3. EMPIRICAL ANALYSIS

3.1. Pilot program framework

In empirical analysis, we implemented the following system. Probe Person System, Bicycle sharing system and Tradable permits auction system are mutually connected. Next, Probe Person system, Bicycle Sharing system and Tradable permits system is described. Fig-3 shows the total framework.

Figure 3 – Framework of this pilot program and the relationship of each system

3.1.1. Probe person survey

First, Probe Person system (Hato and Kitamura, 2008) is a method to get travel diary data and positioning data in detail by GPS mobile phone. Users operate the mobile phone when they depart and arrive. Application of the mobile phone records the data of trip OD, travel mode, trip purpose, time of departure, time of mode change, time of arrival, and location data during trip. Using Probe Person survey, we can know more travel behavior data than paperbased survey.

3.1.2. Bicycle sharing tradable permits system

Second, Bicycle Sharing Tradable Permits System is an operation system of a capacitylimited sharing service. The system needs use reservation until the previous day and respondents can reserve bicycle sharing by the application of the mobile phone or the web site. There are two time slot of bicycle sharing and one is morning (9:30 – 13:00) the other is afternoon (13:30 – 19:00). To reserve, respondents need use permit and can participate the following auction to get use permit. Each permit is set the time when you can use bicycle sharing. For example, if you have the permit of $20th$ Nov Morning, you can use bicycle sharing on the time but can't on the different time or date.

3.1.3. Tradable permits Auction system

Next, we will describe the auction system of tradable permits. There are two systems. One is single auction system. In single auction, Seller is the administrator of bicycle sharing system only and buyers are users. Users have a bidding choice when they want. As auction rule is set to second price auction, the highest bidder wins the auction and he or she pay the second price. In this way, permits are distributed according to each user's willingness to pay. Single auction is the mechanism achieving an efficient allocation of resources but it doesn't achieve fairness. So, some researchers suggest double auction mechanism as alternative.

The other is double auction system. First, all tradable permits are allocated to all users at random for achieving fairness. Then, if users don't intend to use permits, they can offer them for sale. It is a phase for achieving efficient allocation. This pilot program implemented both single auction and double auction. And this research scopes the double auction mechanism.

3.2. Data

The data used were obtained in 2008 from "Yokohama Mobility Design Survey" by using the system noted above. Table-1 shows the overview of the data. Surveillance period is from $10th$ Nov to $24th$ Dec and it is 44 days. The activity and trip of respondents are recorded by mobile phone with GPS and Web diary. Respondents who did both probe person survey and web diary survey are 118 people and respondents who did only web diary survey are 11 people. Only 19 people of the formers participated in pilot program of bicycle sharing service and we analyzed the small sample data in this paper.

The double auction executed from $14th$ Nov to $2nd$ Dec and single auction did from $3rd$ Dec to 24th Dec. Auction site is linked from each respondent's web diary page. In this auction site, respondents can buy or sell use permit freely by presenting the sales information and purchase intention. In this transaction, they can't pay by real money and can pay by virtual money: ECO point. ECO point is not exchangeable real money but each respondent is paid a fee depending on their collecting ECO point at the finish time of survey. It is inspired by induced value theory.

3.3. Basic Results

3.3.1. Basic analysis

Ten people show the action of the sales intention and the purchase intention, that is, almost half respondents participate in the auction by some form. In double auction, there are 24 trade logs and in single there are 11 logs. And five people (monitor id: ym205, ym209, ym210, ym215, and ym219) occupied 25 among these, so auction user's bias are a little large.

Figure 4 – The relationship between trade and visiting area

In double auction, the sales intentions ware 18 and purchase intentions ware 6, however, the buying and selling approval was only 1. The seller had the selling intention with 100 point or more and there were three people who differentiated clearly weekday permit and holiday permit by price. Contents of transaction to which the buying and selling was approved were the use permit of $24th$ Nov (holiday) in the morning, and the price of approval was 100 point. This is a content that the buyer previously showed the purchase intention, and the seller got on it. In the other five purchase intentions, these use permits were not use by the owners and not got up on the auction. That is, these use permits were not allocated optimally and matching miss set up unfortunately. This is because the number of respondents is few and it becomes thin market. Another reason is that some respondents couldn't transact their use permits for the uncertainty of their schedule. We clarify the elements of trade behavior next session.

3.3.2. Auction trade and activity pattern

Next, the relation between trade behavior and activity pattern is analyzed by using double auction periods data which is the major target of this paper. The demand for the use permit (purchase intention) decreases when it is not possible to visit Minato-Mirai area because of time and the spatial restriction. On the other hand, the supply of use permit must be increasing. The time series variation of MM area visits and sales/purchase intention frequency is shown in Fig-4. In this figure, we understand the correlation between the number of MM visitors outside business purpose and trade intentions. The purchase intentions have been increased on the weekend and holiday in which the number of MM visitor outside business purpose tend to be large but sell intentions have been decreased oppositely. It is understood that the kind of the use permits like holiday or weekday gives bias.

Figure 5 – The assumption of the day to day decision-making under uncertain schedule

4. ESTIMATION RESULTS

4.1. Model assumption

The individual decision-making mechanism in double auction is modelled based on a basic analysis. The number of respondents in this pilot program is 19, they are small samples, so it is difficult to generalize by bias of respondents' socioeconomic characteristic. Let me suppose the following assumption.

- All respondents make a decision about their all permits once a day (Fig-5).
- When they have permit of a certain day, their choice set is "use it", "offer it for sale", "do nothing".
- When they don't have permits of a certain day, their choice set is "buy it", "do nothing".
- Whether a respondent visit the area of bicycle sharing port is discriminated by probe person data.

As a result, because there are 19 respondents for 23 days double auction period, the number of total data is 437 people data. As we assume that all respondents make a decision about their own permit every day, the number of samples with permits is 1398 observations and the number of samples without permits is 3056 observations. These assumptions enable estimation of the model's parameter and we considered time series trade behavior. So, it is thought that this model has constant generality for the purpose of clarifying the element of the trade behavior.

4.2. Estimation results and discussion

In the parameter estimation, three variables were newly set. "Distance" is defined as the distance in a straight line between Minato-Mirai station and respondent's home. "Area visiting dummy" is the dummy variable that becomes 1 when she visits MM area at the date of her permit. "A.M. dummy" is the dummy variable that is 1 when the time of her permit is set in the morning.

The utility of "use" is the linear sum of holiday dummy, log (distance), area visiting dummy and male dummy. The utility of "sell" is the linear sum of weekday dummy, A.M. dummy, log (distance), area visiting dummy and male dummy. The utility of "do nothing" is the linear sum of alternative specific constant and middle age dummy.

We can take the utility of this model as three parts. One part is the value of tradable permit. Holiday dummy, weekday dummy and A.M. dummy are the element of tradable permit itself. Second part is the value by individual. Distance, visiting dummy, male dummy and middle age dummy differ from one person to another. In fact, the tradable permits' value and individual attributes have impact on the trade behavior. And third part is schedule effect utility. As Eq. (8), we set the utility function of "use" and "sell". These choices are active choices and "do nothing" is passive choice. The parameters show the difficulty of active choice in uncertainty. These parameters indicate how respondents decide their schedule and whether they use or sell permits.

Three models are estimated in this paper. Model 1 is Multinomial Logit model without schedule effect. In model 2, we considered the schedule effect. And model 3 is Mixed Logit model considering schedule effect. The results are shown in Table-2. The implication is that people tends to use permit and bicycle sharing when the distance is short, they visit the area and the date of permit is holiday. On the other hand, when the time of permit is weekday or afternoon, respondents want to offer their permit for sale. And middle ages don't want to participate in mobility auction.

In model 2 and model 3 results, schedule effect is considered. Each parameter is estimated as dummy variable and Table-2 shows parameters. To compare two models, Fig-6 shows that schedule effect's estimates are scaled from 0 to 1. In both models, min is $t = -15$ (two weeks before) and max is $t = -1$ (the day before), and the graph is mainly upwardsloping curve. Its shape looks like uncertainty-loving and it indicates that people can't make a decision about their own permits until one day or two days before. In addition, interesting finding is the gradient from 9 days before to 8 days before is very large and it is shown that some people think of their schedule just 1 week before and they decide how to use their permits. That is, in our scheduling mechanism, it is interpreted that the time of about 1 week before is first big jumping point and the time of 1 or 2 days before is next big jumping point. In spite of the difference of the estimate parameters, interestingly, the shape of the schedule effect's curve is about same. It is the robustness of the schedule effect of this data.

Figure 6 – Schedule effect utility function under uncertain future decision-making

4.3. Parametric-defined schedule effect utility

The knowledge from Fig-6 implies the shape of schedule effect utility function seems uncertainty-loving. As a result, we can assume schedule effect utility function $S(t)$ as a parametrically -defined function. It is logistic function as below.

$$
S(t) = \frac{a_1}{1 + \exp(a_2 \cdot t)}
$$
\n(9)

where a_1, a_2 are unknown parameter and *t* is time such as $t = -1, -2, \dots$

The estimation result of schedule effect utility function Eq. (9) is Table-3. Though the number of parameters in this model is less than models with discrete schedule effect utility function, improvements of model fit is achieved. Two parameters a_1 , a_2 replace twenty one parameters γ_s . Fig-7 shows that schedule effect's estimates are scaled from 0 to 1 comparing Model2 and Model3. This result implies that respondents in this pilot program tend to be uncertainty-loving and the fact is a difficult problem in all transport service in which the future decision-making relates.

Table 3 - Estimation results of parametric schedule effect utility function

Figure 7 – Schedule effect utility function under uncertain future decision-making

5. CONCLUSIONS

We implemented bicycle sharing system and the use permit auction system and proposed a dynamic framework for estimating schedule effect of uncertain future decision-making. First, we estimate discrete schedule effect function because of no information. Second, we estimate parametric schedule effect function by the former result. And, we applied the framework to the trade behavior data and clarified the elements of trade behavior. From the result, participating in use permit auction is effected by the distance between users' home and the bicycle port area, whether they visit the area on the day, elements of the use permit and schedule effect. In special, we understand that people tend to remind use permit about one week before the day and remind it strongly about 1 or 2 days before. These two jumping points are interesting findings. However, as thin market and this schedule effect can make the auction inefficient, it implicates that the reservation system of transport service is difficult without consideration future schedule.

As future tasks, the transaction cost of auction, effect of a thin market and the users' cognitive difference between double auction and single auction will be studied. In particular, ognitive transaction cost would be an interesting topic for further study. c

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