Ports' Competition Under Services Differentiation and Uncertainty: Governments' Simultaneous Moves

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Abstract

This paper investigates governments' optimal facility investments and ports' optimal pricing under service differentiation and uncertainty settings, considering both additive and multiplicative uncertainty types. It also examines how ports' equilibrium prices are induced by their facilities, marginal costs, service substitution degree, as well as uncertainty conditions. By introducing two-stage game-theoretic models, we show that, in additive uncertainty, the difference of optimal facilities under uncertainty and no uncertainty is completely determined by the mean stochastic demand. However, this difference under multiplicative uncertainty also depends on factors such as ports' marginal costs, service substitution degree, and the variance of stochastic demand. Moreover, we discover that variances of uncertainty cannot affect governments' optimal facility investments under additive uncertainty, but the opposite could be true under multiplicative uncertainty.

Keywords: port competition, service differentiation, sequential game, additive uncertainty, multiplicative uncertainty

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1. Introduction

With increasing globalization, trade amounts across borderlines grow rapidly. Container hub port development has been considered not only as the point of convergence between intermodal transportation and both gateway and transshipment cargoes, but also as an important integrated logistics system in the context of hub-and-spoke system. Therefore, understanding more thoroughly about interactions among governments, ports, and shipping lines, as well as their decision-making behaviors becomes necessary and conspicuous. Game theory as a tool can certainly help to achieve this goal because it is a mathematical framework characterizing interactions among multiple agents seeking their maximum payoffs. Researchers in the field of maritime economics have noticed this and begun applying it. The earlier works include Jankowski (1989), Yang (1999), Song and Panayides (2002), Imai et al. (2006), and so on.

Recently, in De Borger et al.'s (2008) significant paper, they study optimal prices of two ports having downstream congestible transport networks to a common hinterland. This paper allows governments to decide their optimal facility investments, and finds that ports will internalize the hinterland congestion cost and charge their customers accordingly, and ports' capacities are negatively correlated with their pricing. Following De Borger et al., Zhang (2009) analyzes how hinterland access conditions affect ports' competition. However, Zhang focuses on un-congestible ports and allows them to compete in terms of both price and quantity. He discovers that in quantity competition, expanding inland road capacity may not increase port's service and profit. Oppositely, under price competition, enlarging a region's corridor capacity will raise its port's service price and reduce the competitor's. Using Cournot and Stackelberg games, Gkonis and Paraftis (2009) examine optimal transportation capacities of two liquefied natural gas companies. Permitting a landlord port to lease terminals to its competing operators, Saeed and Larson (2009) explore influences of various contracts between the port authority and terminal operators on Bertrand-competition port's pricing in

Pakistan. Their simulation outcomes show that optimal contracts offered by the port authority have high unit fee and low annual rent. Anderson et al. (2008b) employ a simple game to study whether two competing ports will invest in new facilities. They find that the investment decisions would depend on costs of the new facilities. Also, Anderson et al. (2008a) analyze how investment decisions of individual ports affect the market as well as the actions of all other competiting ports. On the other hand, Gkonis et al. (2009) use game theory to tackle the security problem in merchant shipping. Under cooperative-game frameworks, Saeed and Larsen (2010) study the coalition behaviors of three terminals in Karachi port of Pakistan, while Park et al. (2009) investigate the delay-cost problem between charter and ship owners.

In the real world, especially Asia, we observe that more and more emerging markets' governments rush into building port-related infrastructures and facilities to satisfy countries' trading needs or to attract port users. Lee and Flynn (2010) argue that the Asian port developments have been driven by multi-dimensional roles of the central governments, i.e., a port designer, developer, operator, port pricing maker, mediator, and investor. Thus, the function of Asian ports is closely related to governments' economic development plans. Although previous game-theoretic models mentioned above inspect ports' pricing from various perspectives, few investigate governments' optimal facility investments under rapid-changing and uncertain environments of the world economy. Also, little research focuses on port's pricing under uncertainty. On the other hand, as pointed out by Zhang (2009), ports' services should be regarded as differential products due to their dissimilar locations and hinterlands. And few works scrutinize how ports' differential services affect their pricing. Thus, this study tries to analyze governments' optimal facility investments and ports' optimal pricing under uncertainty and service differentiation. We will consider both additive and multiplicative uncertainty types. For each uncertainty type we construct a two-stage game, in which governments choose facility investments first, then ports select their service prices.

Our results show that uncertainty types are crucial in determining governments' and ports' behaviors. Variances of stochastic demands have no impact on optimal facility levels under additive uncertainty, but the opposite could be true under multiplicative uncertainty. Nevertheless, impacts of other parameters, such as the mean stochastic demand, ports' marginal costs, and the service substitution degree, on optimal facility levels are indefinite whichever the uncertainty type is. Moreover, the difference between optimal facility levels under additive uncertainty and no uncertainty depends completely on the mean stochastic demand. In contrast, this difference under multiplicative uncertainty would depend on the variance of stochastic demand, ports' marginal costs, and the service substitution degree. Finally, in additive uncertainty, ports will lower the service prices when facility investments increase, their marginal costs decrease, or the economy deteriorates. However, ports could increase or decrease their prices when the substitution degree of their services increases. Same conclusions can be drawn under multiplicative uncertainty except that the impacts of governments' capital facilities and services differentiation degree on ports' prices become unsure. Our outcomes would remain true when the cost uncertainty or the dependence of the demand and supply of ports' services is considered.

Although Ishii et al.'s (2009) work is the first to explore the impact of stochastic demands on ports' equilibrium prices, their ports' facility levels are fixed, instead of endogenous. Moreover, they consider additive uncertainty only. Moreover, De Borger et al. (2008) do not deal with uncertain shocks in addressing ports' facility investments. Compared to them, we examine uncertainty effects but ignore the part of congestible transport networks to hinterland. Thus, this paper will contribute to maritime economics by shedding lights on how both governments and ports behave under service differentiation and unpredictable economic surroundings, as well as under distinct uncertainty treatments.

The rest of this paper is organized as follows. The additive uncertainty model and its outcomes are presented in Section 2, while those concerning multiplicative uncertainty model are illustrated in Section 3. The two models are extended in Section 4, and our conclusions are drawn in Section 5.

2. Additive Demand Uncertainty

We have ports 1 and 2 in the model. Due to different geographic characteristics and sizes of hinterlands, port users regard services of the two ports as differentiated products. The inverse market demand functions faced by ports 1 and 2 are

$$p_1 = 1 + \theta - q_1 - bq_2 \text{ and} \tag{1}$$

$$p_2 = 1 + \theta - q_2 - bq_1, \text{respectively,} \tag{2}$$

where p_i is the price of unit cargo (e.g., TEU) charged by port i, and q_i is the amount of cargo handled by port i, $i = 1,\ 2$. Parameter $b \in (0,\ 1)$ represents the service substitution degree of the two ports. The larger b is, the higher the service substitution degree is. Let θ be a random variable characterizing demand uncertainty with value range of $(-1,\ 1)$, mean $\bar{\theta}$, variance $\sigma_\theta^2 > 0$, and probability density function (pdf) $f(\theta)$. Actually, uncertainty could also come from the cost side. In Section 4.1, we show that the results obtained here remain true under cost-side uncertainty. By rearranging (1) and (2), the market demand functions for ports 1 and 2 become

$$q_1 = \frac{1+\theta}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \text{ and} \tag{3}$$

$$q_2 = \frac{1+\theta}{1+b} + \frac{bp_1}{1-b^2} - \frac{p_2}{1-b^2}, \text{ respectively.} \tag{4}$$

Ports 1 and 2 can compete in price or quantity. And the outcomes for price competition are presented only, because quantity competition results are the same and available upon request.

Governments provide basic facilities to promote ports' developments. The facilities include, among others, maritime access infrastructure (e.g., breakwater, channels, and navigated aids), and land access infrastructure (e.g., road, railway and inland water ways). When the facilities are enough, ports' service provision costs will decrease with increasing facilities. In contrast, when the facilities are insufficient, ports' service provision costs cannot be reduced. Thus, it is plausible to assume that port i's service provision cost, given cargo amount q_i and facility level K_i, is

$$C_i(q_i,\ K_i) = \begin{cases} c_i q_i & \text{if } K_i < 1, \\ \frac{c_i}{K_i} q_i & \text{if } K_i \geq 1, \end{cases} \tag{5}$$

where $c_i \in (0,\ 1)$ is port i's marginal (provision) cost when zero or less-than-one-unit facility is provided by its government, $i = 1,\ 2$. Note that the threshold value of one-unit facility can be replaced by any other positive numbers without changing the results. And the results stay true qualitatively if $\frac{c_i q_i}{K_i}$ is replaced with $\frac{c_i q_i}{h(K_i)}$, where $h(\cdot)$ is an increasing and strictly concave function of K_i. To have meaningful analyses, we focus on the facilities levels greater than one unit throughout this paper. Accordingly, by (3)-(5), we can get port i's profit function

$$\pi_i(p_i,\ p_j) = (p_i - \frac{c_i}{K_i})[\frac{1+\theta}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}], \tag{6}$$

for $i,\ j \in \{1,\ 2 \mid i \neq j\}$.

Interactions between governments and ports are characterized by the ensuing two-stage game. In the first stage, both governments select facility levels maximizing their expected social welfares independently and simultaneously. Then, a value of random variable θ is realized and observed by the ports. In the second stage, both ports decide service prices maximizing their profits independently and simultaneously. We presume here that the governments and the ports have asymmetric information about demand shocks, because port authorities can access the market more easily and directly. By backward induction, we can derive the subgame perfect equilibrium (hereafter SPE) of the sequential game as follows.

First, given governments' facility levels $(K_1,\ K_2)$ and a realized value of demand uncertainty variable θ, ports choose optimal prices $(p_1^*,\ p_2^*)$ to solve the following problem.

$$\max_{p_i>0}\ \pi_i(p_i,\ p_j) = (p_i - \frac{c_i}{K_i})[\frac{1+\theta}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}]$$

for $i,\ j \in \{1,\ 2 \mid i \neq j\}$. The first-order conditions for interior solutions are

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1+\theta}{1+b} + \frac{c_1}{(1-b^2)K_1} - \frac{2p_1}{(1-b^2)} + \frac{bp_2}{(1-b^2)} = 0 \text{ and} \tag{7}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1+\theta}{1+b} + \frac{c_2}{(1-b^2)K_2} - \frac{2p_2}{(1-b^2)} + \frac{bp_1}{(1-b^2)} = 0. \tag{8}$$

The second-order and stability conditions for $(p_1^*,\ p_2^*)$ hold because $\frac{\partial^2 \pi_i}{\partial p_i^2} = \frac{-2}{(1-b^2)} < 0$, $\frac{\partial^2 \pi_i}{\partial p_j \partial p_i} = \frac{b}{(1-b^2)} > 0$, and $\frac{\partial^2 \pi_i}{\partial p_i^2}\frac{\partial^2 \pi_j}{\partial p_j^2} - (\frac{\partial^2 \pi_i}{\partial p_j \partial p_i})^2 = \frac{4-b^2}{(1-b^2)^2} > 0$ for $i,\ j \in \{1,\ 2 \mid i \neq j\}$. Thus, the first-order conditions are also sufficient for equilibrium prices. By solving (7)-(8), we can obtain

$$p_1^* = \frac{(1-b)(1+\theta)}{(2-b)} + \frac{1}{(4-b^2)}\left[\frac{c_2 b}{K_2} + \frac{2c_1}{K_1}\right] \text{ and} \tag{9}$$

$$p_2^* = \frac{(1-b)(1+\theta)}{(2-b)} + \frac{1}{(4-b^2)}\left[\frac{c_1 b}{K_1} + \frac{2c_2}{K_2}\right]. \tag{10}$$

Equations (9)-(10) imply that these optimal prices are affected by ports' facility levels, marginal costs, and service differentiation degree, as well as the realized value of demand uncertainty. The relations are summarized below.

Lemma 1. For $i,\ j \in \{1,\ 2 \mid i \neq j\}$, *we have*
(i) $\frac{\partial p_i^*}{\partial K_i} = \frac{-2c_i}{(4-b^2)K_i^2} < 0$,
(ii) $\frac{\partial p_i^*}{\partial K_j} = \frac{-c_j b}{(4-b^2)K_j^2} < 0$,
(iii) $\frac{\partial p_i^*}{\partial c_i} = \frac{2}{(4-b^2)K_i} > 0$,
(iv) $\frac{\partial p_i^*}{\partial c_j} = \frac{b}{(4-b^2)K_j} > 0$,
(v) $\frac{\partial p_i^*}{\partial \theta} = \frac{(1-b)}{(2-b)} > 0$, *and*
(vi) $\frac{\partial p_i^*}{\partial b} \geq\ (\leq)\ 0$ *iff* $\theta \leq\ (\geq)\ \theta_i^*$,
where $\theta_i^* = \frac{(4+b^2)c_j}{(2+b)^2 K_j} + \frac{4bc_i}{(2+b)^2 K_i} - 1 \in (-1,\ 0)$.

Proof. The proofs are straightforward, thus omitted.

When government i provides more facilties, port i's marginal costs and charging prices would decrease. Port j behaves the same because ports' service prices are strategic complements under the Bertrand competition. These are what Lemma 1(i)-(ii) say. On the other hand, as port i's marginal costs rise, it will charge its customers higher prices. Again, port j would do the same. These are the contents of Lemma 1(iii)-(iv). As market demands become larger, ports will raise their service prices as indicated by Lemma 1(v). Finally, part (vi) shows that impacts of the service substitution degree on ports' equilibrium prices are uncertain. When market demands are large enough ($\theta \geq \theta_i^*$), port i will lower prices to attract more customers as competition with the other port becomes severer, i.e., b increases. In contrast, when market demands are not large enough ($\theta < \theta_i^*$), port i will raise prices to compensate for its losses from the rising competition.

Substituting equilibrium prices ($p_1^*,\ p_2^*$) into ports' profit functions in (6) yields their equilibrium profits, which are functions of facility levels and realized values of demand uncertainty as shown below.

$$\pi_1^*(K_1,\ K_2,\ \theta) = A_\theta + B_\theta(\frac{c_1}{K_1}) + D_\theta(\frac{c_2}{K_2}) + \frac{Fc_1c_2}{K_1K_2} + G(\frac{c_1}{K_1})^2 + H(\frac{c_2}{K_2})^2 \text{ and} \tag{11}$$

$$\pi_2^*(K_1,\ K_2,\ \theta) = A_\theta + B_\theta(\frac{c_2}{K_2}) + D_\theta(\frac{c_1}{K_1}) + \frac{Fc_1c_2}{K_1K_2} + G(\frac{c_2}{K_2})^2 + H(\frac{c_1}{K_1})^2, \tag{12}$$

with

$$A_\theta = \frac{(1-b)(1+\theta)^2}{(1+b)(2-b)^2} > 0, \tag{13}$$

$$B_\theta = \frac{2(1+\theta)(b^2-2)}{(1+b)(2+b)(2-b)^2} < 0, \tag{14}$$

$$D_\theta = \frac{2b(1+\theta)}{(1+b)(2+b)(2-b)^2} > 0, \tag{15}$$

$$F = \frac{2b(b^2-2)}{(1-b^2)(4-b^2)^2} < 0, \tag{16}$$

$$G = \frac{(b^2-2)^2}{(1-b^2)(4-b^2)^2} > 0, \text{ and} \tag{17}$$

$$H = \frac{b^2}{(1-b^2)(4-b^2)^2} > 0. \tag{18}$$

In particular, by letting $\theta = 0$ in (11)-(12), we acquire ports' equilibrium profits under no uncertainty,

$$\pi_1^*(K_1,\ K_2,\ 0) = A + B(\frac{c_1}{K_1}) + D(\frac{c_2}{K_2}) + \frac{Fc_1c_2}{K_1K_2} + G(\frac{c_1}{K_1})^2 + H(\frac{c_2}{K_2})^2, \text{ and} \quad (19)$$

$$\pi_2^*(K_1,\ K_2,\ 0) = A + D(\frac{c_1}{K_1}) + B(\frac{c_2}{K_2}) + \frac{Fc_1c_2}{K_1K_2} + G(\frac{c_2}{K_2})^2 + H(\frac{c_1}{K_1})^2, \quad (20)$$

with

$$A = \frac{(1-b)}{(1+b)(2-b)^2} > 0, \quad (21)$$

$$B = \frac{2(b^2-2)}{(1+b)(2+b)(2-b)^2} < 0, \text{ and} \quad (22)$$

$$D = \frac{2b}{(1+b)(2+b)(2-b)^2} > 0. \quad (23)$$

Next, given ports' equilibrium price $(p_1^*,\ p_2^*)$, government i will select facility level K_i^* to maximize her expected social welfare,

$$SW_i(K_i,\ K_j) \equiv E\pi_i^*(K_i,\ K_j) - \gamma_i K_i, \quad (24)$$

where $E\pi_i^*(K_i,\ K_j) = \int_{-1}^{1} \pi_i^*(K_i,\ K_j,\ \theta) f(\theta) d\theta$ is government i's expected profit earned by port i under facility level $(K_i,\ K_j)$, and $\gamma_i > 0$ is unit investment cost faced by government $i,\ i = 1,\ 2$. For simplicity, each government's social welfare equals her corresponding port's profit subtracting facility investment cost without considering the surpluses of shipping lines and consumers located in ports' hinterlands. That is because the benefit of shipping lines belongs to foreign firms and the consumers' is ignored due to our neglect of congestible transport networks to hinterlands.

Taking the expectation of (11) and (12) with respect to θ produces

$$E\pi_i^*(K_i,\ K_j) = \pi_i^*(K_i,\ K_j,\ 0) + \frac{c_i\bar{\theta}B}{K_i} + \frac{c_j\bar{\theta}D}{K_j} + A(2\bar{\theta} + \bar{\theta}^2 + \sigma_\theta^2) \quad (25)$$

for $i,\ j \in \{1,\ 2|\ i \neq j\}$. By (24)-(25), we can get

$$SW_i(K_i,\ K_j) \equiv SW_i^{NU}(K_i,\ K_j) + \frac{c_i\bar{\theta}B}{K_i} + \frac{c_j\bar{\theta}D}{K_j} + A(2\bar{\theta} + \bar{\theta}^2 + \sigma_\theta^2), \quad (26)$$

where $SW_i^{NU}(K_i,\ K_j) \equiv \pi_i^*(K_i,\ K_j,\ 0) - \gamma_i K_i$ represents government i's social welfare function under no demand uncertainty. Denote $K^{NU} = (K_1^{NU},\ K_2^{NU})$ governments' optimal facility investments under no uncertainty. That is, K^{NU} must meet the conditions of

$$\frac{\partial SW_i^{NU}(K^{NU})}{\partial K_i} = 0,\ i = 1,\ 2. \tag{27}$$

Differentiating (26) with respect to K_i generates

$$\frac{\partial SW_i(K_i,\ K_j)}{\partial K_i} = \frac{\partial SW_i^{NU}(K_i,\ K_j)}{\partial K_i} - \frac{c_i \bar{\theta} B}{K_i^2},\ i = 1,\ 2. \tag{28}$$

Accordingly, $K^* = (K_1^*,\ K_2^*)$ must satisfy

$$\frac{\partial SW_i(K^*)}{\partial K_i} = \frac{\partial SW_i^{NU}(K^*)}{\partial K_i} - \frac{c_i \bar{\theta} B}{(K_i^*)^2} = 0,\ i = 1,\ 2. \tag{29}$$

To make (29) sufficient conditions for K^* as well, the following assumption is needed.

Assumption 1: The Hessian matrix, H, associated with social welfare functions SW_1 and SW_2 is negative definitive for all K, where

$$H = \begin{bmatrix} \frac{\partial^2 SW_1(K)}{\partial K_1^2} & \frac{\partial^2 SW_1(K)}{\partial K_2 \partial K_1} \\ \frac{\partial^2 SW_2(K)}{\partial K_1 \partial K_2} & \frac{\partial^2 SW_2(K)}{\partial K_2^2} \end{bmatrix}.$$

Analyzing (27)-(29), we can obtain the relation between K_i^* and K_i^{NU} as follows.

Proposition 1. Under Assumption 1, we have
(i) *If* $\bar{\theta} > 0$, *then* $K_i^* > K_i^{NU}$ *for* $i = 1,\ 2$;
(ii) *If* $\bar{\theta} = 0$, *then* $K_i^* = K_i^{NU}$ *for* $i = 1,\ 2$; and
(iii) *If* $\bar{\theta} < 0$, *then* $K_i^* < K_i^{NU}$ *for* $i = 1,\ 2$.
Proof. See the Appendix.

Proposition 1 shows that relative sizes of equilibrium facilities under uncertainty and no uncertainty completely depend on governments' expectations about future market demands. If average market demands are anticipated to rise ($\bar{\theta} > 0$), governments will invest more facilities under uncertainty than under no uncertainty. In contrast, if average market demands are expected to fall ($\bar{\theta} < 0$), governments will invest fewer facilities under unsure situations. Nevertheless, if stable market demands are expected, governments' facility investments under both uncertainty and no uncertainty will be the same.

Moreover, equation (29) implies that optimal facility investments are affected by the mean stochastic demand, ports' marginal costs, and the service differentiation degree, but unaffected by the variance of stochastic demand. These relations are summarized below.

Proposition 2. Under Assumption 1, we have
(i) $\frac{\partial K_i^*}{\partial \sigma_\theta^2} = 0$,
(ii) $\frac{\partial K_i^*}{\partial \bar{\theta}} \geq\ (\leq)\ 0$ iff $\frac{c_i B H_{jj}}{(K_i^*)^2} \geq\ (\leq)\ \frac{c_j B H_{ij}}{(K_j^*)^2}$,
(iii) $\frac{\partial K_i^*}{\partial c_i} \geq\ (\leq)\ 0$ iff $H_{jj}[(1+\bar{\theta})B + \frac{c_j F}{K_j^*} + \frac{4c_i G}{K_i^*}] \geq\ (\leq)\ \frac{c_i c_j^2 F^2}{K_i^*(K_j^*)^4}$,
(iv) $\frac{\partial K_i^*}{\partial c_j} \geq\ (\leq)\ 0$ iff $H_{jj} \leq\ (\geq)\ \frac{c_j}{(K_j^*)^3}[(1+\bar{\theta})B + \frac{c_i F}{K_i^*} + \frac{4c_j G}{K_j^*}]$, *and*
(v) $\frac{\partial K_i^*}{\partial b} \geq\ (\leq)\ 0$ iff $\Phi_i H_{jj} \geq\ (\leq)\ \Phi_j H_{ij}$,
where $H_{ii} \equiv \frac{\partial^2 SW_i}{\partial K_i^2} = \frac{2c_i B}{(K_i^*)^3} + \frac{2c_i c_j F}{(K_i^*)^3 K_j^*} + \frac{6c_i^2 G}{(K_i^*)^4} < 0,\ H_{ij} = H_{ji} \equiv \frac{\partial^2 SW_i}{\partial K_j \partial K_i} = \frac{c_i c_j F}{(K_i^* K_j^*)^2} < 0,$
and $\Phi_i = \frac{c_i}{(K_i^*)^2}[(1+\bar{\theta})\frac{\partial B}{\partial b} + \frac{c_j}{K_j^*}\frac{\partial F}{\partial b} + \frac{2c_i}{K_i^*}\frac{\partial G}{\partial b}]$ *for* $i,\ j \in \{1,\ 2 \mid i \neq j\}$.
Proof. See the Appendix.

Although the variance of stochastic demand (σ_θ^2) has no effect on optimal facility levels, it is worthy mentioning that governments' equilibrium expected social welfares are positively correlated with σ_θ^2. Equations (11)-(12) suggest that ports' equilibrium profits are convex functions of realized values of stochastic demand by $0 < b < 1$. This indicates that governments are risk-lovers when facing demand uncertainty. Thus, the larger the variance is, the higher benefits governments would obtain as stated in (26) by $A > 0$. In contrast, impact of the mean stochastic demand on optimal facility levels is unsure. When government i expects average market demands to rise (i.e., a higher $\bar{\theta}$), her marginal expected social welfare of facilities ($\frac{\partial SW_i}{\partial K_i}$) will increase as stated in (29) by $B < 0$ and she will invest more. This is the direct effect of $\bar{\theta}$ on government i's optimal facility level (K_i^*), which is represented by the positive term of $\frac{c_i B H_{jj}}{(K_i^*)^2}$. Nevertheless, government j will also invest more as $\bar{\theta}$ increases. And larger K_j^* will lower government i's marginal expected social welfare of facilities because $H_{ji} = \frac{\partial^2 SW_i}{\partial K_j \partial K_i} < 0$, hence government i will invest fewer. This is the indirect effect of $\bar{\theta}$ on K_i^*, which is reflected by the negative term of $\frac{-c_j B H_{ij}}{(K_j^*)^2}$. If the direct effect dominates, government i will invest more facilities as $\bar{\theta}$ increases. In contrast, government i will provide fewer facilities when the indirect effect dominates. And $\bar{\theta}$ will have no impact on K_i^* when both effects cancel each other out. This is the content of Proposition 2(ii).

Next, Proposition 2(iii) claims that the increase of port i's marginal cost (c_i) may raise or lower government i's optimal facility level (K_i^*). The impact of c_i on K_i^* consists of a direct and an indirect effects. When c_i rises, government i's marginal expected social welfare of facilities could increase or decrease. Although ports will raise their service prices due to higher c_i by Lemma 1(iii)-(iv); the equilibrium cargo amount,

$$q_i^* = \frac{1+\theta}{(1+b)(2-b)} + \frac{b}{(1-b^2)(4-b^2)}(\frac{c_j}{K_j}) + \frac{(b^2-2)}{(1-b^2)(4-b^2)}(\frac{c_i}{K_i}),$$

derived by plugging (9)-(10) into (3)-(4), will fall because $\frac{\partial q_i^*}{\partial c_i} = \frac{(b^2-2)}{(1-b^2)(4-b^2)K_i} < 0$. Thus, rising c_i may cause port i's equilibrium profit as well as government i's expected social welfare to increase or decrease. This is the direct effect of c_i on K_i^*, reflected by the term of $H_{jj}[(1+\bar{\theta})B + \frac{c_j F}{K_j^*} + \frac{4c_i G}{K_i^*}]$. On the other hand, government j's marginal expected social welfare of facilities will increase with rising c_i because $\frac{\partial \pi_j^*}{\partial c_i \partial K_j} = -\frac{F c_j}{K_i^*(K_j^*)^2} > 0$ by $F < 0$. Thus, government j's optimal facility level (K_j^*) will rise. And higher K_j^* will in turn decrease government i's marginal expected social welfare of facilities due to $H_{ji} < 0$, hence government i will provide fewer facilities. This the indirect effect of c_i on K_i^*, which is represented by the positive term of $\frac{c_i c_j F^2}{K_i^*(K_j^*)^4}$. A large positive direct effect could dominate the indirect effect, then government i will invest more facilities as port i's marginal costs increase. In contrast, if the direct effect is negative or a small positive one, then government i's optimal facility investments will decrease with port

i's increasing marginal costs. By similar arguments, we can explain Proposition 2(iv).

Finally, Proposition 2(v) shows that impact of the service substitution degree (b) on optimal facilities is unsure as well. Changing b has a direct and an indirect effects on K_i^*. Lemma 1(vi) displays that ports' equilibrium prices and cargo amounts could rise or fall as b increases. Accordingly, impacts of b on ports' equilibrium profits are uncertain, so are impacts of b on governments' marginal expected social welfares of facilities. This means that the direct and indirect influences of b on K_i^* have no clear directions, which are reflected by the terms of $\Phi_i H_{jj}$ and $\Phi_j H_{ij}$, respectively. Again, the sign of $\frac{\partial K_i^*}{\partial b}$ would depend on relative sizes of the direct and indirect effects.

3. Multiplicative Demand Uncertainty

Settings of this section are the same as those in Section 2 except that demand uncertainty now owns a multiplicative form. Suppose that the inverse market demand functions faced by ports 1 and 2 are

$$p_1 = 1 + \epsilon \cdot g(K_1) - q_1 - bq_2 \text{ and} \tag{30}$$

$$p_2 = 1 + \epsilon \cdot g(K_2) - q_2 - bq_1, \text{respectively,} \tag{31}$$

where ϵ is a random variable representing demand uncertainty with value range of $(-1,\ 1)$, mean $\bar{\epsilon}$, and variance σ_ϵ^2. Unlike the additive shock, the impact strength of demand uncertainty on service prices would depend on ports' facility levels. The strength factor is characterized by function $g : [1,\ \infty) \rightarrow (0,\ \bar{g})$ for some positive upper bound $\bar{g}$, which is assumed concave with a positive or negative first derivative. If $g'(K) > 0$, the port with larger facilities will meet more cargo growths than that with smaller facilities when the economy booms (i.e., a higher ϵ). In contrast, the port with larger facilities will suffer more from cargo drops than that with smaller facilities when the economy declines (i.e., a lower ϵ). However, if $g'(K) < 0$, opposite situations occur. Both $g'(\cdot) > 0$ and $g'(\cdot) < 0$ are possible in the real world. We will show later that the sign of $g'(\cdot)$ plays an important role in determining ports' equilibrium prices and governments' optimal facility investments.

By rearranging (30) and (31), the market demand functions of ports 1 and 2 become

$$\begin{aligned} q_1 &= \frac{1+\epsilon g(K_1)}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \text{ and} \\ q_2 &= \frac{1+\epsilon g(K_2)}{1+b} + \frac{bp_1}{1-b^2} - \frac{p_2}{1-b^2}, \text{ respectively.} \end{aligned}$$

Port i's cost function is still presumed to be (5). Accordingly, port i's profit function is

$$\pi_i(p_i,\ p_j) = (p_i - \frac{c_i}{K_i})\left\{\frac{[1+\epsilon g(K_i)]}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}\right\},\ i,\ j = 1,\ 2. \tag{32}$$

Timing of the sequential game here is the same as that in Section 2. Thus, the SPEs are solved by backward induction as follows.

First, given $(K_1,\ K_2)$ and ϵ, ports 1 and 2 choose optimal price $(\hat{p}_1,\ \hat{p}_2)$, which is the solution of the following problem.

$$\max_{p_i>0}\ \pi_i(p_i,\ p_j) = (p_i - \frac{c_i}{K_i})\left\{\frac{[1+\epsilon g(K_i)]}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}\right\},\ i,\ j = 1,\ 2.$$

The first-order conditions for interior $(\hat{p}_1,\ \hat{p}_2)$ are

$$\frac{\partial \pi_1}{\partial p_1} = \frac{[1+\epsilon g(K_1)]}{1+b} + \frac{c_1}{(1-b^2)K_1} - \frac{2\hat{p}_1}{(1-b^2)} + \frac{b\hat{p}_2}{(1-b^2)} = 0 \text{ and} \tag{33}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{[1+\epsilon g(K_2)]}{1+b} + \frac{c_2}{(1-b^2)K_2} - \frac{2\hat{p}_2}{(1-b^2)} + \frac{b\hat{p}_1}{(1-b^2)} = 0. \tag{34}$$

As in Section 2, the second-order and stability conditions hold at $(\hat{p}_1,\ \hat{p}_2)$. Thus, conditions (33)-(34) are sufficient for equilibrium prices as well. By solving (33)-(34), we can get

$$\hat{p}_1 = \frac{(1-b)}{(2-b)} + \frac{(1-b)\epsilon}{(4-b^2)}[2g(K_1) + bg(K_2)] + \frac{1}{(4-b^2)}\left[\frac{c_2 b}{K_2} + \frac{2c_1}{K_1}\right] \text{ and} \tag{35}$$

$$\hat{p}_2 = \frac{(1-b)}{(2-b)} + \frac{(1-b)\epsilon}{(4-b^2)}[bg(K_1) + 2g(K_2)] + \frac{1}{(4-b^2)}\left[\frac{c_1 b}{K_1} + \frac{2c_2}{K_2}\right]. \tag{36}$$

Equations (35)-(36) imply that ports' optimal prices are affected by facility levels, ports' marginal costs, the service differentiation degree, and realized values of demand uncertainty. The relations are summarized below.

Lemma 2. For $i,\ j \in \{1,\ 2 \mid i \neq j\}$, *we have*

(i) $\frac{\partial \hat{p}_i}{\partial K_i} \geq\ (\leq)\ 0$ *iff* $\frac{c_i}{K_i^2} \leq\ (\geq)\ \epsilon(1-b)g'(K_i)$;

(ii) $\frac{\partial \hat{p}_i}{\partial K_j} \geq\ (\leq)\ 0$ *iff* $\frac{c_j}{K_j^2} \leq\ (\geq)\ \epsilon(1-b)g'(K_j)$;

(iii) $\frac{\partial \hat{p}_i}{\partial c_i} = \frac{2}{(4-b^2)K_i} > 0$;

(iv) $\frac{\partial \hat{p}_i}{\partial c_j} = \frac{b}{(4-b^2)K_j} > 0$;

(v) $\frac{\partial \hat{p}_i}{\partial \epsilon} = \frac{(1-b)[2g(K_i)+bg(K_j)]}{(4-b^2)} > 0$; *and*

(vi) *If* $b > 0.536$ *or* $b \leq 0.536$ *with* $\Psi_i \leq 0$, *we have*

$$\frac{\partial \hat{p}_i}{\partial b} \geq\ (\leq)\ 0 \textit{ iff } \epsilon \leq\ (\geq)\ \epsilon_i^*, \textit{ and}$$
$$\frac{\partial \hat{p}_i}{\partial b} \geq\ (\leq)\ 0 \textit{ iff } \epsilon \geq\ (\leq)\ \epsilon_i^*$$

if $b \leq 0.536$ *with* $\Psi_i > 0$, *where* $\Psi_i \equiv 2g(K_i)(-b^2+2b-4)+g(K_j)(b^2-8b+4)$ *and* $\epsilon_i^* \equiv \frac{1}{\Psi_i}[(2+b)^2 - \frac{4bc_i}{K_i} - \frac{(4+b^2)c_j}{K_j}]$.

Proof. See the Appendix.

When the signs of ϵ and $g'(K_i)$ are opposite or $g'(K_i)$ is positive but small, Lemma 2(i)-(ii) reach the same conclusions as those in Lemma 1(i)-(ii). However, if ϵ and $g'(K_i)$ have the same signs and $g'(K_i)$ is positive and large, Lemma 2(i)-(ii) would obtain conclusions different from those in Lemma 1(i)-(ii). We will demonstrate this using positive ϵ and $g'(K_i)$. As port i's facilities (K_i) increase, its marginal costs decrease at a speed of $\frac{c_i}{K_i^2}$, hence port i should lower its service prices. However, under multiplicative uncertainty, port i's service demand will increase with expanding facilities at a speed of $g'(K_i)$, which will induce port i to raise its service prices. Large enough $g'(K_i)$ ($\geq \frac{c_i}{\epsilon(1-b_i)K_i^2} > \frac{c_i}{K_i^2}$) implies that the rising speed of port users outpaces the falling speed of marginal costs. Thus, it is optimal for port i to raise its service prices. Port j should also raise prices due to strategic complements of ports' service prices. This outcome differs from De Borger et al.'s (2008) entirely. Similar intuitions apply when ϵ and $g'(K_i)$ are negative and $|g'(K_i)|$ is large.

Intuitions of Lemma 2(iii)-(v) are similar to those of Lemma 1(iii)-(v). Finally, Lemma 2(vi) shows that impact of the service differentiation degree on equilibrium prices under multiplicative uncertainty is the same as that under additive uncertainty except when $b \leq 0.536$ and $\Psi_i > 0$. The former condition suggests that the service substitution degree is low enough, and the latter condition implies $\epsilon_i^* > 0$. Thus, when the economy is good enough ($\epsilon \geq \epsilon_i^*$), port i will face extra demands from multiplicative uncertainty. On the other hand, a low enough substitution degree would prevent ports from losing customer shares if they attempt to raise service prices. Therefore, it is optimal for port i to increase prices as the economy is good even competition becomes severer. In contrast, port i will suffer from extra demand drops when the economy declines. Thus, it will lower prices to keep its customers under strong competition.

Substituting equilibrium price $(\hat{p}_1,\ \hat{p}_2)$ into profit functions in (32) yields ports' equilibrium profits, which would depend on facility levels and realized values of demand shock as follows.

$$\begin{aligned}\hat{\pi}_1(K_1,\ K_2,\ \epsilon) \quad = \quad & A + B\frac{c_1}{K_1} + D\frac{c_2}{K_2} + G(\frac{c_1}{K_1})^2 + H(\frac{c_2}{K_2})^2 + I_\epsilon g(K_1) + J_\epsilon g(K_2) \\ & + L_\epsilon g(K_1)^2 + M_\epsilon g(K_2)^2 + N_\epsilon g(K_1)\frac{c_1}{K_1} + R_\epsilon g(K_2)\frac{c_1}{K_1} \\ & + S_\epsilon g(K_1)\frac{c_2}{K_2} + T_\epsilon g(K_2)\frac{c_2}{K_2} + U_\epsilon g(K_1)g(K_2) + F\frac{c_1 c_2}{K_1 K_2} \qquad (37)\end{aligned}$$

and

$$\begin{aligned}\hat{\pi}_2(K_1,\ K_2,\ \epsilon) \quad = \quad & A + B\frac{c_2}{K_2} + D\frac{c_1}{K_1} + G(\frac{c_2}{K_2})^2 + H(\frac{c_1}{K_1})^2 + I_\epsilon g(K_2) + J_\epsilon g(K_1) \\ & + L_\epsilon g(K_2)^2 + M_\epsilon g(K_1)^2 + N_\epsilon g(K_2)\frac{c_2}{K_2} + R_\epsilon g(K_1)\frac{c_2}{K_2} \\ & + S_\epsilon g(K_2)\frac{c_1}{K_1} + T_\epsilon g(K_1)\frac{c_1}{K_1} + U_\epsilon g(K_1)g(K_2) + F\frac{c_1 c_2}{K_1 K_2}, \qquad (38)\end{aligned}$$

where

$$
\begin{aligned}
I_\epsilon &= \frac{4\epsilon(1-b)}{(1+b)(2-b)(4-b^2)},\ J_\epsilon = \frac{2\epsilon b(1-b)}{(1+b)(2-b)(4-b^2)}, && (39)\\
L_\epsilon &= \frac{4\epsilon^2(1-b)}{(1+b)(4-b^2)^2} > 0,\ M_\epsilon = \frac{\epsilon^2 b^2(1-b)}{(1+b)(4-b^2)^2} > 0, && (40)\\
N_\epsilon &= \frac{4\epsilon(b^2-2)}{(1+b)(4-b^2)^2},\ R_\epsilon = \frac{2\epsilon b(b^2-2)}{(1+b)(4-b^2)},\ \text{and} && (41)\\
S_\epsilon &= \frac{4\epsilon b}{(1+b)(4-b^2)^2},\ T_\epsilon = \frac{2b^2\epsilon}{(1+b)(4-b^2)^2},\ U_\epsilon = \frac{4\epsilon^2 b(1-b)}{(1+b)(4-b^2)^2} > 0. && (42)
\end{aligned}
$$

In particular, by letting $\epsilon = 0$ in (37)-(38), we obtain ports' equilibrium profits under no uncertainty, $\hat{\pi}_1(K_1,\ K_2, 0)$ and $\hat{\pi}_2(K_1,\ K_2, 0)$. It is no surprise to find ports' equilibrium profits under no multiplicative and no additive uncertainty are the same. That is,

$$\hat{\pi}_i(K_i,\ K_j, 0) = \pi_i^*(K_i,\ K_j, 0),$$

where $\pi_i^*(K_i,\ K_j, 0),\ i = 1,\ 2$, are defined in (19)-(20).

Next, given $(\hat{p}_1,\ \hat{p}_2)$, government i will select optimal facility level $\hat{K}_i$ to maximize her expected social welfare,

$$S\hat{W}_i(K_i,\ K_j) \equiv E\hat{\pi}_i(K_i,\ K_j) - \gamma_i K_i,$$

where $E\hat{\pi}_i(K_i,\ K_j) = \int \hat{\pi}_i(K_i,\ K_j,\ \epsilon)d\epsilon$ is government i's expected profit obtained by port i given facilities $(K_i,\ K_j)$ for $i = 1,\ 2$. By simple calculations, we can acquire

$$E\hat{\pi}_i(K_i,\ K_j) \quad = \quad \pi_i^*(K_i,\ K_j,\ 0) + \Delta_i(K_i,\ K_j) \text{ for } i,\ j = 1,\ 2,$$

with

$$
\begin{aligned}
\Delta_i(K_i,\ K_j) \quad = \quad & \bar{\epsilon}\cdot\Big[I_1 g(K_i) + J_1 g(K_j) + \frac{c_i N_1 g(K_i)}{K_i} + \frac{c_i R_1 g(K_j)}{K_i} + \frac{c_j S_1 g(K_i)}{K_j}\\
& + \frac{c_j T_1 g(K_j)}{K_j}\Big] + (\bar{\epsilon}^2 + \sigma_\epsilon^2)[L_1 g(K_i)^2 + M_1 g(K_j)^2 + U_1 g(K_i) g(K_j)],
\end{aligned}
$$

where $I_1,\ J_1,\ L_1, M_1,\ N_1,\ R_1,\ S_1,\ T_1$ and U_1 correspond to the $I_\epsilon,\ J_\epsilon,\ L_\epsilon,\ M_\epsilon,\ N_\epsilon,\ R_\epsilon,$ $S_\epsilon,\ T_\epsilon$ and U_ϵ in (39)-(42) evaluated at $\epsilon = 1$. Accordingly, we have

$$S\hat{W}_i(K_i,\ K_j) = SW_i^{NU}(K_i,\ K_j) + \Delta_i(K_i,\ K_j), \tag{43}$$

where $SW_i^{NU}(K_i,\ K_j) = \hat{\pi}_i(K_i,\ K_j,\ 0) - \gamma_i K_i,\ i = 1,\ 2$, is government i's social welfare under no uncertainty. Recall that $K^{NU} = (K_1^{NU},\ K_2^{NU})$ represents both governments' optimal facility levels under no uncertainty.

Differentiating (43) with respect to K_i produces

$$\frac{\partial S\hat{W}_i(K_i,\ K_j)}{\partial K_i} = \frac{\partial SW_i^{NU}(K_i,\ K_j)}{\partial K_i} + \frac{\partial \Delta_i(K_i,\ K_j)}{\partial K_i},\ i,\ j = 1,\ 2. \tag{44}$$

Consequently, $\hat{K} = (\hat{K}_1,\ \hat{K}_2)$ must meet the condition of

$$\frac{\partial S\hat{W}_i(\hat{K})}{\partial K_i} = \frac{\partial SW_i^{NU}(\hat{K})}{\partial K_i} + \frac{\partial \Delta_i(\hat{K})}{\partial K_i} = 0,\ i = 1,\ 2. \tag{45}$$

To make conditions (45) sufficient as well, the following assumption is needed.

Assumption 2: The Hessian matrix, $\hat{H}$, associated with social welfare functions $S\hat{W}_1$ and $S\hat{W}_2$ is negative definite for all K, where

$$\hat{H} = \begin{bmatrix} \frac{\partial^2 S\hat{W}_1(K)}{\partial K_1^2} & \frac{\partial^2 S\hat{W}_1(K)}{\partial K_2 \partial K_1} \\ \frac{\partial^2 S\hat{W}_2(K)}{\partial K_1 \partial K_2} & \frac{\partial^2 S\hat{W}_2(K)}{\partial K_2^2} \end{bmatrix}.$$

Under Assumption 2, $\hat{K}$ always exists by the implicit function theorem and owns the following properties.

Proposition 3. Under Assumption 2, we have
(i) *If* $\frac{\partial \Delta_1(K^{NU})}{\partial K_1} > 0$ *and* $\frac{\partial \Delta_2(K^{NU})}{\partial K_2} > 0$, *then* $\hat{K}_i > K_i^{NU}$ *for* $i = 1,\ 2$;
(ii) *If* $\frac{\partial \Delta_1(K^{NU})}{\partial K_1} > 0$ *and* $\frac{\partial \Delta_2(K^{NU})}{\partial K_2} < 0$, *then* $\hat{K}_1 > K_2^{NU}$ *and* $\hat{K}_2 < K_2^{NU}$;
(iii) *If* $\frac{\partial \Delta_1(K^{NU})}{\partial K_1} < 0$ *and* $\frac{\partial \Delta_2(K^{NU})}{\partial K_2} > 0$, *then* $\hat{K}_1 < K_2^{NU}$ *and* $\hat{K}_2 > K_2^{NU}$; *and*
(iv) *If* $\frac{\partial \Delta_1(K^{NU})}{\partial K_1} < 0$ *and* $\frac{\partial \Delta_2(K^{NU})}{\partial K_2} < 0$, *then* $\hat{K}_i < K_i^{NU}$ *for* $i = 1,\ 2$.
Here $\frac{\partial \Delta_i(K^{NU})}{\partial K_i} = \bar{\epsilon} \cdot \{I_1 g'(K_i^{NU}) + \frac{c_i N_1 g'(K_i^{NU})}{K_i^{NU}} - \frac{c_i[N_1 g(K_i^{NU}) + R_1 g(K_j^{NU})]}{(K_i^{NU})^2} + \frac{c_j S_1 g'(K_i^{NU})}{K_j^{NU}}\} +$ $(\bar{\epsilon}^2 + \sigma_\epsilon^2)[2L_1 g'(K_i^{NU}) + U_1 g'(K_i^{NU}) g(K_j^{NU})]$ *for* $i,\ j \in \{1,\ 2 \mid i \neq j\}$.
Proof. See the Appendix.

Unlike in additive uncertainty, here relative sizes of facility levels under uncertainty and no uncertainty depend not only on governments' expected stochastic demands, but on factors such as the service substitution degree, ports' marginal costs, and variances of the stochastic demands. Under additive uncertainty, when two governments expect rising (falling) average market demands, $\bar{\epsilon}$, at the same time, their optimal facility investments will be higher (lower) than those under no uncertainty. However, under multiplicative uncertainty, we could have one government's optimal facilities larger and the other government's facilities lower than those under no uncertainty even they have the same expectation about the future demands, as shown by Proposition 3(ii)-(iii).

Next, we will demonstrate how variances of uncertainty (σ_ϵ^2) affect relative sizes of governments' optimal facilities under both multiplicative uncertainty ($\hat{K}_i$) and no uncertainty (K_i^{NU}). Equations (44)-(45) suggest that the difference between $\hat{K}_i$ and K_i^{NU} is determined by the term of $\frac{\partial \Delta_i}{\partial K_i}$ with

$$\frac{\partial^2 \Delta_i}{\partial \sigma_\epsilon^2 \partial K_i} = 2L_1 g'(K_i^{NU}) + U_1 g'(K_i^{NU}) g(K_j^{NU}) \text{ for } i,\ j = 1,\ 2.$$

Since L_1, U_1, and $g(\cdot)$ are positive, the sign of $\frac{\partial^2 \Delta_i}{\partial \sigma_\epsilon^2 \partial K_i}$ would depend on the sign of marginal impact strength of multiplicative uncertainty on price, $g'(\cdot)$. If $g'(\cdot) > 0$, we have $\frac{\partial^2 \Delta_i}{\partial \sigma_\epsilon^2 \partial K_i} > 0,\ i = 1,\ 2$. It implies that differences between $\hat{K}_i$ and K_i^{NU} increase with rising variances of multiplicative uncertainty. In contrast, if $g'(\cdot) < 0$, the differences decrease with rising variances of multiplicative uncertainty.

And by analyzing (45), we could obtain the followings.

Proposition 4. Under Assumption 2, we have
(i) $\frac{\partial \hat{K}_i}{\partial \sigma_\epsilon^2} \geq (\leq)\ 0$ *iff* $\hat{H}_{ij} \frac{\partial^2 \Delta_j}{\partial \sigma_\epsilon^2 \partial K_j} \geq (\leq)\ \hat{H}_{jj} \frac{\partial^2 \Delta_i}{\partial \sigma_\epsilon^2 \partial K_i}$,
(ii) $\frac{\partial \hat{K}_i}{\partial \bar{\epsilon}} \geq (\leq)\ 0$ *iff* $\hat{H}_{ij} \frac{\partial^2 \Delta_j}{\partial \bar{\epsilon} \partial K_j} \geq (\leq)\ \hat{H}_{jj} \frac{\partial^2 \Delta_i}{\partial \bar{\epsilon} \partial K_i}$,
(iii) $\frac{\partial \hat{K}_i}{\partial c_i} \geq (\leq)\ 0$ *iff* $\hat{H}_{jj}[\frac{\partial^2 \pi_i^*(K_i^*, K_j^*, 0)}{\partial c_i \partial K_i} + \frac{\partial^2 \Delta_i}{\partial c_i \partial K_i}] \geq (\leq)\ \hat{H}_{ij}[\frac{\partial^2 \pi_j^*(K_i^*, K_j^*, 0)}{\partial c_i \partial K_j} + \frac{\partial^2 \Delta_j}{\partial c_i \partial K_j}]$,
(iv) $\frac{\partial \hat{K}_i}{\partial c_j} \geq (\leq)\ 0$ *iff* $\hat{H}_{jj}[\frac{\partial^2 \pi_i^*(K_i^*, K_j^*, 0)}{\partial c_j \partial K_i} + \frac{\partial^2 \Delta_i}{\partial c_j \partial K_i}] \geq (\leq)\ \hat{H}_{ij}[\frac{\partial^2 \pi_j^*(K_i^*, K_j^*, 0)}{\partial c_j \partial K_j} + \frac{\partial^2 \Delta_j}{\partial c_j \partial K_j}]$, *and*
(v) $\frac{\partial \hat{K}_i}{\partial b} \geq (\leq)\ 0$ *iff* $\hat{H}_{jj}[\frac{\partial^2 \pi_i^*(K_i^*, K_j^*, 0)}{\partial b \partial K_i} + \frac{\partial^2 \Delta_i}{\partial b \partial K_i}] \geq (\leq)\ \hat{H}_{ij}[\frac{\partial^2 \pi_j^*(K_i^*, K_j^*, 0)}{\partial b \partial K_j} + \frac{\partial^2 \Delta_j}{\partial b \partial K_j}]$,
where $\hat{H}_{ii} = \frac{\partial^2 \pi_i^*(K_i^*, K_j^*, 0)}{\partial K_i^2} + \frac{\partial^2 \Delta_i(K^*)}{\partial K_i^2} < 0$, $\hat{H}_{ij} = \hat{H}_{ji} = \frac{c_i c_j F}{(K_i^*)^2 (K_j^*)^2} + \frac{\partial^2 \Delta_i}{\partial K_j \partial K_i}$, *and* $i,\ j \in \{1,\ 2 | i \neq j\}$.

Proof. See the Appendix.

Unlike in additive uncertainty, variances of multiplicative uncertainty (σ_ϵ^2) could affect governments' optimal facility investments as displayed in Proposition 4(i). However, the impact of σ_ϵ^2 on optimal facility levels is unsure. The reason is given below. Rising σ_ϵ^2 may lead to larger or smaller marginal expected social welfares of governments because the sign of $\frac{\partial^2 \Delta_i}{\partial \sigma_\epsilon^2 \partial K_i} = 2L_1 g'(K_i) + U_1 g'(K_i) g(K_j)$ is positive if $g'(K_i) > 0$ and negative if $g'(K_i) < 0$. Suppose $g'(K_i) > 0$ for $i = 1,\ 2$. Then governments' marginal expected social welfares of facilities increase with rising σ_ϵ^2. So, governments will provide more facilities. This is the direct effect of σ_ϵ^2 on K_i^*, which is represented by the positive term of $-\hat{H}_{jj} \frac{\partial^2 \Delta_i}{\partial \sigma_\epsilon^2 \partial K_i}$ for $i,\ j = 1,\ 2$. However, unlike in additive uncertainty, higher government j's facility levels may increase or decrease government i's marginal expected social welfare of facilities because the sign of $\hat{H}_{ij}$ $(= \frac{\partial^2 S\hat{W}_i}{\partial K_j \partial K_i})$ is indefinite. If $\frac{\partial^2 \Delta_i}{\partial K_j \partial K_i} < 0$ or $0 < \frac{\partial^2 \Delta_i}{\partial K_j \partial K_i} < \frac{-c_i c_j F}{(K_i^*)^2 (K_j^*)^2}$, we have negative H_{ij} as the case in additive uncertainty, meaning that government i's marginal expected social welfare of facilities decreases with rising facilities of government j. Therefore, we have a negative indirect effect reflected by the term of $\hat{H}_{ij} \frac{\partial^2 \Delta_j}{\partial \sigma_\epsilon^2 \partial K_j}$. Then, government i's optimal facilities will increase when the direct effect dominates, decrease when the indirect effect dominates, and stay unchanged if the two effects cancel each other out. However, if $\frac{\partial^2 \Delta_i}{\partial K_j \partial K_i} > \frac{-c_i c_j F}{(K_i^*)^2 (K_j^*)^2}$, then $H_{ij} > 0$, suggesting that government i's marginal expected social welfare of facilities will increase with government j's facility levels. Thus, the indirect effect is positive. Combined with a positive direct effect, government i will invest more facilities as σ_ϵ^2 rises. Similar arguments can be applied to the case of $g'(K_i) < 0$ for $i = 1,\ 2$.

Finally, Proposition 4(ii)-(v) state that impacts of other factors, such as the mean stochastic demand, ports' marginal costs, and the service differentiation degree, on optimal facilities are unsure as well. Since the intuitions are similar to those of Proposition 4(i), explanations are omitted.

4. Extensions

In this section, we show that the outcomes acquired in Sections 2-3 stay true qualitatively when uncertainty comes from the cost side or when services demanded by port users depend on facility levels.

4.1. Cost Uncertainty

In the real world, uncertainty could come from the cost side, such as oil price shocks, random cargo loading and unloading situations, and wage disputes between port authorities and labor unions. To save space, we present the outcomes under additive cost uncertainty merely. Multiplicative uncertainty results are available upon request.

The inverse market demand faced by ports 1 and 2 are

$$q_1 = \frac{1}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \text{ and} \tag{46}$$

$$q_2 = \frac{1}{1+b} + \frac{bp_1}{1-b^2} - \frac{p_2}{1-b^2}, \text{ respectively.} \tag{47}$$

And port i's service provision cost is

$$C_i(q_i,\ K_i,\ \delta) = \begin{cases} (c_i - \delta)q_i & \text{if } K_i < 1, \\ (\frac{c_i}{K_i} - \delta)q_i & \text{if } K_i \geq 1, \end{cases} \tag{48}$$

where δ is the random variable representing uncertainty from the cost side with value range of $(-\max\{c_1,\ c_2\},\ \min\{c_1,\ c_2\})$, mean $\bar{\delta}$, variance σ_δ^2, and pdf $\tilde{f}(\delta)$. The larger δ is, the lower marginal costs port i has. As in Sections 2-3, we focus on the facility level bigger than one-unit. By (46)-(48), we can get port i's profit function,

$$\pi_i(p_i,\ p_j) = [p_i - \frac{c_i}{K_i} + \delta][\frac{1}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}] \tag{49}$$

for $i,\ j \in \{1,\ 2 \mid i \neq j\}$. Our sequential game is the same as that in Section 2 except that random variable θ is replaced with δ. Accordingly, we can obtain ports' equilibrium prices

$$\begin{aligned}\tilde{p}_1 &= \frac{1-b}{2-b} + \frac{1}{(4-b^2)}\left[2(\frac{c_1}{K_1}-\delta)+b(\frac{c_2}{K_2}-\delta)\right] \text{ and}\\ \tilde{p}_2 &= \frac{1-b}{2-b} + \frac{1}{(4-b^2)}\left[b(\frac{c_1}{K_1}-\delta)+2(\frac{c_2}{K_2}-\delta)\right].\end{aligned}$$

It is easy to check that these equilibrium prices own the same properties as those in Lemma 1.

Substituting $(\tilde{p}_1,\ \tilde{p}_2)$ into ports' profit functions in (49) yields ports' equilibrium profits,

$$\begin{aligned}\tilde{\pi}_1(K_1,\ K_2,\ \delta) &= A + B[\frac{c_1}{K_1}-\delta] + D[\frac{c_2}{K_2}-\delta] + F[\frac{c_1}{K_1}-\delta][\frac{c_2}{K_2}-\delta]\\ &\quad +G[\frac{c_1}{K_1}-\delta]^2 + H[\frac{c_2}{K_2}-\delta]^2 \text{ and} \qquad (50)\\ \tilde{\pi}_2(K_1,\ K_2,\ \delta) &= A + D[\frac{c_1}{K_1}-\delta] + B[\frac{c_2}{K_2}-\delta] + F[\frac{c_1}{K_1}-\delta][\frac{c_2}{K_2}-\delta]\\ &\quad +G[\frac{c_2}{K_2}-\delta]^2 + H[\frac{c_1}{K_1}-\delta]^2, \qquad (51)\end{aligned}$$

where $A,\ B,\ D,\ F,\ G$ and H are defined in (16)-(18) and (21)-(23). By letting $\delta = 0$ in (50)-(51), we can get ports' equilibrium profits under no uncertainty, which equal $\pi_1^*(K_1,\ K_2,\ 0)$ and $\pi_2^*(K_1,\ K_2,\ 0)$ defined in (19)-(20), respectively.

Given ports' optimal price $(\tilde{p}_1,\ \tilde{p}_2)$, government i will choose optimal facility level $\tilde{K}_i$ to maximize her expected social welfare

$$\tilde{SW}_i \equiv E\tilde{\pi}_i(K_i,\ K_j) - \gamma_i K_i,\ \ i,\ j = 1,\ 2,$$

where $E\tilde{\pi}_i(K_i,\ K_j) = \int \tilde{\pi}_i(K_i,\ K_j,\ \delta)\tilde{f}(\delta)d\delta$ is government i's expected profit obtained by port i given facilities $(K_i,\ K_j)$. By simple calculations, we have

$$\begin{aligned}E\tilde{\pi}_i(K_i,\ K_j) &= \pi_i^*(K_i,\ K_j,\ 0) - \bar{\delta}[B + D + F(\frac{c_i}{K_i}+\frac{c_j}{K_j}) + \frac{2c_iG}{K_i} + \frac{2c_jH}{K_j}]\\ &\quad +(\bar{\delta}^2+\sigma_\delta^2)[F+G+H]\end{aligned}$$

for $i,\ j \in \{1,\ 2 \mid i \neq j\}$. Consequently,

$$\begin{aligned}\tilde{SW}_i(K_i,\ K_j) &= SW_i^{NU}(K_i,\ K_j) - \bar{\delta}[B + D + F(\frac{c_i}{K_i} + \frac{c_j}{K_j}) + \frac{2c_iG}{K_i} + \frac{2c_jH}{K_j}] \\ &\quad +(\bar{\delta}^2 + \sigma_\delta^2)[F + G + H],\ i,\ j = 1,\ 2. \qquad (52)\end{aligned}$$

Differentiating (52) with respect to K_i and evaluating at $K^{NU} = (K_1^{NU},\ K_2^{NU})$ produces

$$\frac{\tilde{SW}_i(K^{NU})}{\partial K_i} = \frac{SW_i^{NU}(K^{NU})}{\partial K_i} + \frac{c_i\bar{\delta}(F + 2G)}{(K_i^{NU})^2} = \frac{c_i\bar{\delta}(F + 2G)}{(K_i^{NU})^2},$$

because $\frac{\partial SW_i^{NU}(K^{NU})}{\partial K_i} = 0$ for $i = 1,\ 2$. Since $F + 2G = \frac{2(b^2-2)}{(4-b^2)^2(1-b^2)}[b^2 + b - 2] > 0$ by $0 < b < 1$, relative sizes of $\tilde{K}_i$ and K_i^{NU} depend on the sign of the mean stochastic cost $(\bar{\delta})$. This suggests that Proposition 1 holds under additive cost uncertainty. Moreover, it is not difficult to see that Proposition 2 still hold qualitatively.

4.2. Dependence of Demand and Supply of Ports' Services

In Sections 2, the demand and supply of ports' services are implicitly assumed independent. However, they may not be. For instance, the port with larger facility levels could attract more users than its competitor. We will address this issue here. And we discover that considering the dependence of demand and supply of ports' services will not alter the results.

The inverse market demand functions faced by ports 1 and 2 are assumed

$$p_1 = 1 + \theta + K_1 - q_1 - bq_2 \text{ and} \qquad (53)$$

$$p_2 = 1 + \theta + K_2 - q_2 - bq_1, \text{respectively.} \qquad (54)$$

The larger facilities a port has, the more users it can attract. Rearranging (53)-(54) generates market demands for ports 1 and 2,

$$\begin{aligned}q_1 &= \frac{1+\theta}{1+b} + \frac{K_1 - bK_2}{1-b^2} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \text{ and} \\ q_2 &= \frac{1+\theta}{1+b} + \frac{K_2 - bK_1}{1-b^2} + \frac{bp_1}{1-b^2} - \frac{p_2}{1-b^2},\ \text{respectively.}\end{aligned}$$

The cost functions of ports are the same as those in (5). Then port i's profit function is

$$\pi_i(p_i,\ p_j) = (p_i - \frac{c_i}{K_i})[\frac{1+\theta}{1+b} + \frac{K_i - bK_j}{1-b^2} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}],\ i,\ j = 1,\ 2. \tag{55}$$

Consequently, we can obtain ports' equilibrium prices,

$$\begin{aligned}\bar{p}_1 &= \frac{(1-b)(1+\theta)}{(2-b)} + \frac{1}{(4-b^2)}\left[\frac{c_2 b}{K_2} + \frac{2c_1}{K_1}\right] + \frac{(2-b^2)K_1 - bK_2}{4-b^2} \text{ and} \\ \bar{p}_2 &= \frac{(1-b)(1+\theta)}{(2-b)} + \frac{1}{(4-b^2)}\left[\frac{c_1 b}{K_1} + \frac{2c_2}{K_2}\right] + \frac{(2-b^2)K_2 - bK_1}{4-b^2}.\end{aligned}$$

These equilibrium prices own the same properties as those in Lemma 1 except that the impact of ports' own facility levels on their equilibrium prices is unsure.[1]

Substituting equilibrium price $(\bar{p}_1,\ \bar{p}_2)$ into profit functions in (55) yields ports' equilibrium profit functions,

$$\begin{aligned}\bar{\pi}_1(K_1,\ K_2,\ \theta) &= A_\theta + B_\theta(\tfrac{c_1}{K_1}) + D_\theta(\tfrac{c_2}{K_2}) + \tfrac{Fc_1c_2}{K_1K_2} + G(\tfrac{c_1}{K_1})^2 + H(\tfrac{c_2}{K_2})^2 \\ &\quad + \tfrac{[(2-b^2)K_1 - bK_2]^2}{(4-b^2)^2(1-b^2)} + \tfrac{2(1+\theta)[(2-b^2)K_1 - bK_2]}{(4-b^2)(1+b)(2-b)} \\ &\quad + \tfrac{c_1}{K_1}[\tfrac{2(b^2-2)[(2-b^2)K_1 - bK_2]}{(4-b^2)^2(1-b^2)}] + \tfrac{c_2}{K_2}[\tfrac{2b[(2-b^2)K_1 - bK_2]}{(4-b^2)^2(1-b^2)}]\end{aligned} \tag{56}$$

and

$$\begin{aligned}\bar{\pi}_2(K_1,\ K_2,\ \theta) &= A_\theta + B_\theta(\tfrac{c_2}{K_2}) + D_\theta(\tfrac{c_1}{K_1}) + \tfrac{Fc_1c_2}{K_1K_2} + G(\tfrac{c_2}{K_2})^2 + H(\tfrac{c_1}{K_1})^2 \\ &\quad + \tfrac{[(2-b^2)K_2 - bK_1]^2}{(4-b^2)^2(1-b^2)} + \tfrac{2(1+\theta)[(2-b^2)K_2 - bK_1]}{(4-b^2)(1+b)(2-b)} \\ &\quad + \tfrac{c_2}{K_2}[\tfrac{2(b^2-2)[(2-b^2)K_2 - bK_1]}{(4-b^2)^2(1-b^2)}] + \tfrac{c_1}{K_1}[\tfrac{2b[(2-b^2)K_2 - bK_1]}{(4-b^2)^2(1-b^2)}],\end{aligned} \tag{57}$$

where the definitions of $A_\theta,\ B_\theta,\ D_\theta,\ F, G$, and H are in (13)-(18). By letting $\theta = 0$ in (56)-(57), we can get ports' equilibrium profits under no uncertainty, $\bar{\pi}_1(K_1,\ K_2,\ 0)$ and $\bar{\pi}_2(K_1,\ K_2,\ 0)$.

[1] Contrary to Lemma 1(i), we have $\frac{\partial \bar{p}_i}{\partial K_i} = \frac{-2c_i + (2-b^2)K_i^2}{(4-b^2)K_i^2}$, which implies $\frac{\partial \bar{p}_i}{\partial K_i} \geq\ (\leq)\ 0$ iff $2c_i \leq (\geq)\ (2-b^2)K_i^2$.

Given ports' optimal price $(\bar{p}_1,\ \bar{p}_2)$, government i will select optimal facility level $\bar{K}_i$ to maximize her expected social welfare,

$$S\bar{W}_i(K_i,\ K_j) \equiv E\bar{\pi}_i(K_i,\ K_j) - \gamma_i K_i, \ \ i = 1,\ 2,$$

where

$$E\bar{\pi}_i(K_i,\ K_j) = \bar{\pi}_i(K_i,\ K_j,\ 0) + \bar{\theta}\{2A + B + D + \frac{2[(2-b^2)K_i - bK_j]}{(4-b^2)(1+b)(2-b)}\} + A(\bar{\theta}^2 + \sigma_\theta^2)$$

is government i's expected profit earned by port i, and the definitions of $A,\ B$, and D are in (21)-(23). Then, we have

$$\begin{aligned} S\bar{W}_i(K_i,\ K_j) &= SW_i^{NU}(K_i,\ K_j) + \bar{\theta}\{2A + B + D + \frac{2[(2-b^2)K_i - bK_j]}{(4-b^2)(1+b)(2-b)}\} \\ &\quad + A(\bar{\theta}^2 + \sigma_\theta^2), \end{aligned} \tag{58}$$

where $SW_i^{NU}(K_i,\ K_j) = \bar{\pi}_i(K_i,\ K_j,\ 0) - \gamma_i K_i$ for $i = 1,\ 2$. Differentiating (58) with respect to K_i and evaluating at K^{NU} produces

$$\frac{\partial S\bar{W}_i(K^{NU})}{\partial K_i} = \frac{\partial SW_i^{NU}(K^{NU})}{\partial K_i} + \frac{2(2-b^2)\bar{\theta}}{(4-b^2)(1+b)(2-b)} = \frac{2(2-b^2)\bar{\theta}}{(4-b^2)(1+b)(2-b)}$$

by $\frac{\partial SW_i^{NU}(K^{NU})}{\partial K_i} = 0$ for $i = 1,\ 2$. Thus, as in Section 2, relative sizes of $\bar{K}_i$ and K_i^{NU} completely depend on the sign of $\bar{\theta}$ because $0 < b < 1$. This means that the outcomes of Proposition 1 remain true qualitatively. We can show that Proposition 2 stays true qualitatively, and their proofs are available upon request.

5. Conclusions and Policy Implications

This paper studies ports' optimal pricing and governments' optimal facility investments under service differentiation and uncertainty settings. Both additive and multiplicative uncertainty types are considered. Under each type, a two-stage game is constructed to characterize interactions between ports and governments.

Our first finding is that in additive uncertainty the difference of optimal facilities under uncertainty and no uncertainty is completely determined by the mean stochastic demand. However, in multiplicative uncertainty this difference also depends on factors such as ports' marginal costs, service substitution degree, and variances of stochastic demand. The second finding is that variances of stochastic demand cannot affect governments' optimal facility investments under additive uncertainty, while the opposite could be true under multiplicative uncertainty. Impacts of model's other parameters, such as the mean of uncertainty, ports' marginal costs and service substitution degree, on optimal facility levels have no definite directions in both uncertainty types. The third finding is that the impacts of economic conditions and ports' marginal costs on their optimal pricing are similar under the two uncertainty types, while influences of facility investments and the service substitution degree on ports' optimal pricing could be different.

We also show that all the above outcomes remain true qualitatively when the models are extended to consider the cost-side uncertainty and the dependence of demand and supply of ports' services. Since in this paper we do not take congestible transport networks to ports' hinterlands into account, how both governments and ports react when facing uncertainty and congestible transport networks at the same time is a good topic for future research. Moreover, in real world, we often observe that different countries develop their ports at distinct times, instead of simultaneously. Thus, it is worth exploring optimal behaviors of governments and ports when the governments move sequentially and face uncertainty.

Finally, two policy implications could be drawn based on our results. First, it is important for governments to identify the uncertainty types before making decisions because it can cause distinct outcomes. For instance, governments may ignore the variances of stochastic demand under additive uncertainty, but they should consider them under multiplicative uncertainty in deciding optimal facility investments. Second, ports' pricing behaviors under different uncertainty settings could be dissimilar. Thus, researchers ought to be careful as conducting empirical tests.

Appendix

Proof of Proposition 1: Evaluating (28) at $K^{NU} = (K_1^{NU},\ K_2^{NU})$ yields

$$\frac{\partial SW_i(K^{NU})}{\partial K_i} = \frac{\partial SW_i^{NU}(K^{NU})}{\partial K_i} - \frac{c_i\bar{\theta}B}{(K_i^{NU})^2} = -\frac{c_i\bar{\theta}B}{(K_i^{NU})^2}$$

by (27) for $i = 1,\ 2$. Since $B < 0$ by (22), we have

$$\frac{\partial SW_i(K^{NU})}{\partial K_i} \geq\ (\leq)\ 0 \text{ iff } \bar{\theta} \geq\ (\leq)\ 0.$$

Under Assumption 1, strictly concavity of SW_i and (29) imply that

$$K_i^* \geq\ (\leq)\ K_i^{NU} \text{ iff } \bar{\theta} \geq\ (\leq)\ 0 \text{ for } i = 1,\ 2.$$

□

Proof of Proposition 2: (i) It is trivial.

(ii) Differentiating (29) with respect to $\bar{\theta}$ and evaluating at $K^* = (K_1^*,\ K_2^*)$ produces

$$H \cdot \begin{bmatrix} \frac{\partial K_1^*}{\partial \bar{\theta}} \\ \frac{\partial K_2^*}{\partial \bar{\theta}} \end{bmatrix} = \begin{bmatrix} \frac{c_1 B}{(K_1^*)^2} \\ \frac{c_2 B}{(K_2^*)^2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\frac{\partial K_1^*}{\partial \bar{\theta}} = \frac{1}{|H|}[\frac{c_1 BH_{22}}{(K_1^*)^2} - \frac{c_2 BH_{12}}{(K_2^*)^2}] \text{ and } \frac{\partial K_2^*}{\partial \bar{\theta}} = \frac{1}{|H|}[\frac{c_2 BH_{11}}{(K_2^*)^2} - \frac{c_1 BH_{21}}{(K_1^*)^2}].$$

Since $B < 0, H_{ii} < 0$, and $H_{ij} < 0$, we have $\frac{\partial K_i^*}{\partial \bar{\theta}} \geq\ (\leq)\ 0$ iff $\frac{c_i BH_{jj}}{(K_i^*)^2} \geq\ (\leq)\ \frac{c_j BH_{ij}}{(K_j^*)^2}$.

(iii)-(iv) Differentiating (29) with respect to c_1 and evaluating at $K^* = (K_1^*,\ K_2^*)$ generates

$$H \cdot \begin{bmatrix} \frac{\partial K_1^*}{\partial c_1} \\ \frac{\partial K_2^*}{\partial c_1} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_1^*}{\partial c_1 \partial K_1} + \frac{\bar{\theta}B}{(K_1^*)^2} \\ -\frac{\partial^2 \pi_2^*}{\partial c_1 \partial K_2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\begin{aligned} \frac{\partial K_1^*}{\partial c_1} &= \frac{1}{|H|(K_1^*)^2}\{H_{22}[(1+\bar{\theta})B + \frac{c_2 F}{K_2^*} + \frac{4c_1 G}{K_1^*}] - \frac{c_1 c_2^2 F^2}{K_1^*(K_2^*)^4}\} \\ &\geq\ (\leq)\ 0 \text{ iff } H_{22}[(1+\bar{\theta})B + \frac{c_2 F}{K_2^*} + \frac{4c_1 G}{K_1^*}] \geq\ (\leq)\ \frac{c_1 c_2^2 F^2}{K_1^*(K_2^*)^4} \end{aligned}$$

and

$$\begin{aligned}\frac{\partial K_2^*}{\partial c_1} &= \frac{c_2 F}{|H| K_1^* (K_2^*)^2}\{H_{11} - \frac{c_1}{(K_1^*)^3}[(1+\bar{\theta})B + \frac{c_2 F}{K_2^*} + \frac{4c_1 G}{K_1^*}]\} \\ &\geq \ (\leq)\ 0 \text{ iff } H_{11} \leq \ (\geq) \frac{c_1}{(K_1^*)^3}[(1+\bar{\theta})B + \frac{c_2 F}{K_2^*} + \frac{4c_1 G}{K_1^*}].\end{aligned}$$

Similarly, differentiating (29) with respect to c_2 and evaluating at $K^* = (K_1^*,\ K_2^*)$ yields

$$H \cdot \begin{bmatrix} \frac{\partial K_1^*}{\partial c_2} \\ \frac{\partial K_2^*}{\partial c_2} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi_1^*}{\partial c_2 \partial K_1} \\ -\frac{\partial^2 \pi_2^*}{\partial c_2 \partial K_2} + \frac{\bar{\theta} B}{(K_2^*)^2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\begin{aligned}\frac{\partial K_2^*}{\partial c_2} &= \frac{1}{|H|(K_2^*)^2}\{H_{11}[(1+\bar{\theta})B + \frac{c_1 F}{K_1^*} + \frac{4c_2 G}{K_2^*}] - \frac{c_2 c_1^2 F^2}{K_2^* (K_1^*)^4}\} \\ &\geq \ (\leq)\ 0 \text{ iff } H_{11}[(1+\bar{\theta})B + \frac{c_1 F}{K_1^*} + \frac{4c_2 G}{K_2^*}] \geq \ (\leq)\ \frac{c_2 c_1^2 F^2}{K_2^* (K_1^*)^4}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial K_1^*}{\partial c_2} &= \frac{c_1 F}{|H| K_2^* (K_1^*)^2}\{H_{22} - \frac{c_2}{(K_2^*)^3}[(1+\bar{\theta})B + \frac{c_1 F}{K_1^*} + \frac{4c_2 G}{K_2^*}]\} \\ &\geq \ (\leq)\ 0 \text{ iff } H_{22} \leq \ (\geq) \frac{c_2}{(K_2^*)^3}[(1+\bar{\theta})B + \frac{c_1 F}{K_1^*} + \frac{4c_2 G}{K_2^*}].\end{aligned}$$

(v) Differentiating (29) with respect to b and evaluating at $K^* = (K_1^*,\ K_2^*)$ produces

$$H \cdot \begin{bmatrix} \frac{\partial K_1^*}{\partial b} \\ \frac{\partial K_2^*}{\partial b} \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix},$$

where $\Phi_i = -\frac{\partial^2 \pi_i^*}{\partial b \partial K_i} + \frac{c_i \bar{\theta}}{(K_i^*)^2}\frac{\partial B}{\partial b} = \frac{c_i}{(K_i^*)^2}[(1+\bar{\theta})\frac{\partial B}{\partial b} + \frac{c_j}{K_j^*}\frac{\partial F}{\partial b} + \frac{2c_i}{K_i^*}\frac{\partial G}{\partial b}],\ i = 1,\ 2.$ By Cramer's rule, we can get $\frac{\partial K_i^*}{\partial b} \geq \ (\leq)\ 0$ iff $\Phi_i H_{jj} \geq \ (\leq)\ \Phi_j H_{ij}$. □

<u>*Proof of Lemma 2*</u>: Since Lemma 2(iii)-(v) are easily checked from (35)-(36), it remains to show parts (i), (ii), and (vi). Differentiating (35) and (36) with respect to K_i generates

$$\frac{\partial \hat{p}_i}{\partial K_i} = \frac{2}{(4-b^2)}[(1-b)\epsilon g'(K_i) - \frac{c_i}{K_i^2}],\ i = 1,\ 2.$$

Thus, we have $\frac{\partial \hat{p}_i}{\partial K_i} \geq \ (\leq)\ 0$ iff $\frac{c_i}{K_i^2} \leq \ (\geq)\ \epsilon(1-b)g'(K_i)$. Similarly, differentiating (35) and (36) with respect to K_j yields

$$\frac{\partial \hat{p}_i}{\partial K_j} = \frac{b}{(4-b^2)}[(1-b)\epsilon g'(K_j) - \frac{c_j}{K_j^2}],\ i = 1,\ 2.$$

Thus, we have $\frac{\partial \hat{p}_i}{\partial K_j} \geq \ (\leq)\ 0$ iff $\frac{c_j}{K_j^2} \leq \ (\geq)\epsilon(1-b)g'(K_j)$.

Finally, differentiating (35) with respect to b produces

$$\begin{aligned}\frac{\partial \hat{p}_1}{\partial b} \quad &= \quad \frac{-1}{(2-b)^2} + \frac{(4+b^2)c_2}{(4-b^2)^2 K_2} + \frac{4bc_1}{(4-b^2)^2 K_1} \\ &\quad + \frac{\epsilon}{(4-b^2)^2}[2g(K_1)(-b^2+2b-4) + g(K_2)(b^2-8b+4)].\end{aligned}$$

Since $0 < b < 1$, we have $-b^2+2b-4 < 0$ and $b^2-8b+4 \geq \ (\leq)\ 0$ iff $b \leq \ (\geq)\ 0.536$. Thus, there are three cases.

<u>Case 1</u>: Suppose $b > 0.536$. Then $\Psi_1 \equiv 2g(K_1)(-b^2+2b-4) + g(K_2)(b^2-8b+4) < 0$, hence

$$\frac{\partial \hat{p}_1}{\partial b} \geq \ (\leq)\ 0 \text{ iff } \epsilon \leq \ (\geq)\ \epsilon_1^* \tag{59}$$

with $\epsilon_1^* \equiv \frac{1}{\Psi_1}[(2+b)^2 - 4bc_1/K_1 - (4+b^2)c_2/K_2]$.

<u>Case 2</u>: Suppose $b \leq 0.536$ and $\Psi_1 < 0$. Then (59) still holds.

<u>Case 3</u>: Suppose $b \leq 0.536$ and $\Psi_1 > 0$. Then we get

$$\frac{\partial \hat{p}_1}{\partial b} \geq \ (\leq)\ 0 \text{ iff } \epsilon \geq \ (\leq)\ \epsilon_1^*.$$

By similar arguments, we can obtain $\frac{\partial \hat{p}_2}{\partial b}$. $\square$

<u>*Proof of Proposition 3*</u>: Evaluating (44) at K^{NU} generates

$$\frac{\partial S\hat{W}_i(K^{NU})}{\partial K_i} = \frac{\partial SW_i^{NU}(K^{NU})}{\partial K_i} + \frac{\partial \Delta_i(K^{NU})}{\partial K_i} = \frac{\partial \Delta_i(K^{NU})}{\partial K_i}$$

by (27) for $i = 1,\ 2$. Thus, relative sizes of $\hat{K}_i$ and K_i^{NU} depend on the signs of $\frac{\partial \Delta_1(K^{NU})}{\partial K_1}$ and $\frac{\partial \Delta_2(K^{NU})}{\partial K_2}$. Recall that $\frac{\partial S\hat{W}_i(\hat{K})}{\partial K_i} = 0,\ i = 1,\ 2$. If $\frac{\partial \Delta_i(K^{NU})}{\partial K_i} > 0$ for $i = 1,\ 2$, then $\frac{\partial \hat{SW}_i(K^{NU})}{\partial K_i} > 0$; hence strict concavity of $\hat{SW}_i$ implies $\hat{K}_i > K_i^{NU},\ i = 1,\ 2$. If $\frac{\partial \Delta_1(K^{NU})}{\partial K_1} > 0$ and $\frac{\partial \Delta_2(K^{NU})}{\partial K_2} < 0$, then $\frac{\partial \hat{SW}_1(K^{NU})}{\partial K_1} > 0$ and $\frac{\partial \hat{SW}_2(K^{NU})}{\partial K_2} < 0$; hence $\hat{K}_1 > K_1^{NU}$ and $\hat{K}_2 < K_2^{NU}$. In contrast, if $\frac{\partial \Delta_1(K^{NU})}{\partial K_1} < 0$ and $\frac{\partial \Delta_2(K^{NU})}{\partial K_2} > 0$, then $\frac{\partial \hat{SW}_1(K^{NU})}{\partial K_1} < 0$ and $\frac{\partial \hat{SW}_2(K^{NU})}{\partial K_2} > 0$; hence $\hat{K}_1 < K_1^{NU}$ and $\hat{K}_2 > K_2^{NU}$. Finally, we have $\hat{K}_i < K_i^{UN}$ if $\frac{\partial \Delta_i(K^{NU})}{\partial K_i} < 0$ for $i = 1,\ 2$. □

Proof of Proposition 4: Differentiating (45) with respect to σ_ϵ^2 and evaluating at $K^* = (K_1^*,\ K_2^*)$ yields

$$\hat{H} \cdot \begin{bmatrix} \frac{\partial \hat{K}_1}{\partial \sigma_\epsilon^2} \\ \frac{\partial \hat{K}_2}{\partial \sigma_\epsilon^2} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \Delta_1}{\partial \sigma_\epsilon^2 \partial K_1} \\ -\frac{\partial^2 \Delta_2}{\partial \sigma_\epsilon^2 \partial K_2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\frac{\partial \hat{K}_1}{\partial \sigma_\epsilon^2} = \frac{1}{|\hat{H}|}[-\hat{H}_{22}\frac{\partial^2 \Delta_1}{\partial \sigma_\epsilon^2 \partial K_1} + \hat{H}_{12}\frac{\partial^2 \Delta_2}{\partial \sigma_\epsilon^2 \partial K_2}] \geq\ (\leq)\ 0 \text{ iff } \hat{H}_{12}\frac{\partial^2 \Delta_2}{\partial \sigma_\epsilon^2 \partial K_2} \geq\ (\leq)\ \hat{H}_{22}\frac{\partial^2 \Delta_1}{\partial \sigma_\epsilon^2 \partial K_1}$$

and

$$\frac{\partial \hat{K}_2}{\partial \sigma_\epsilon^2} = \frac{1}{|\hat{H}|}[-\hat{H}_{11}\frac{\partial^2 \Delta_2}{\partial \sigma_\epsilon^2 \partial K_2} + \hat{H}_{21}\frac{\partial^2 \Delta_1}{\partial \sigma_\epsilon^2 \partial K_1}] \geq\ (\leq)\ 0 \text{ iff } \hat{H}_{21}\frac{\partial^2 \Delta_1}{\partial \sigma_\epsilon^2 \partial K_1} \geq\ (\leq)\ \hat{H}_{11}\frac{\partial^2 \Delta_2}{\partial \sigma_\epsilon^2 \partial K_2}.$$

(ii) Differentiating (45) with respect to $\bar{\epsilon}$ and evaluating at $K^* = (K_1^*,\ K_2^*)$ produces

$$\hat{H} \cdot \begin{bmatrix} \frac{\partial \hat{K}_1}{\partial \bar{\epsilon}} \\ \frac{\partial \hat{K}_2}{\partial \bar{\epsilon}} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \Delta_1}{\partial \bar{\epsilon} \partial K_1} \\ -\frac{\partial^2 \Delta_2}{\partial \bar{\epsilon} \partial K_2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\frac{\partial \hat{K}_1}{\partial \sigma_\epsilon^2} = \frac{1}{|\hat{H}|}[-\hat{H}_{22}\frac{\partial^2 \Delta_1}{\partial \bar{\epsilon} \partial K_1} + \hat{H}_{12}\frac{\partial^2 \Delta_2}{\partial \bar{\epsilon} \partial K_2}] \geq\ (\leq)\ 0 \text{ iff } \hat{H}_{12}\frac{\partial^2 \Delta_2}{\partial \bar{\epsilon} \partial K_2} \geq\ (\leq)\ \hat{H}_{22}\frac{\partial^2 \Delta_1}{\partial \bar{\epsilon} \partial K_1}$$

and

$$\frac{\partial \hat{K}_2}{\partial \bar{\epsilon}} = \frac{1}{|\hat{H}|}[-\hat{H}_{11}\frac{\partial^2 \Delta_2}{\partial \bar{\epsilon} \partial K_2} + \hat{H}_{21}\frac{\partial^2 \Delta_1}{\partial \bar{\epsilon} \partial K_1}] \geq\ (\leq)\ 0 \text{ iff } \hat{H}_{21}\frac{\partial^2 \Delta_1}{\partial \bar{\epsilon} \partial K_1} \geq\ (\leq)\ \hat{H}_{11}\frac{\partial^2 \Delta_2}{\partial \bar{\epsilon} \partial K_2}.$$

(iii)-(iv) Differentiating (45) with respect to c_1 and evaluating at $K^* = (K_1^*,\ K_2^*)$ generates

$$\hat{H} \cdot \begin{bmatrix} \frac{\partial \hat{K}_1}{\partial c_1} \\ \frac{\partial \hat{K}_2}{\partial c_1} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_1 \partial K_1} \\ \frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_1 \partial K_2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\begin{aligned}\frac{\partial \hat{K}_1}{\partial c_1} &= \frac{1}{|\hat{H}|}\{\hat{H}_{22}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_1 \partial K_1}] - \hat{H}_{12}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_1 \partial K_2}]\} \\ &\geq (\leq)\ 0 \text{ iff } \hat{H}_{22}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_1 \partial K_1}] \geq (\leq)\ \hat{H}_{12}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_1 \partial K_2}]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \hat{K}_2}{\partial c_1} &= \frac{1}{|\hat{H}|}\{\hat{H}_{11}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_1 \partial K_2}] - \hat{H}_{21}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_1 \partial K_1}]\} \\ &\geq (\leq)\ 0 \text{ iff } \hat{H}_{11}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_1 \partial K_2}] \geq (\leq)\ \hat{H}_{21}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_1 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_1 \partial K_1}].\end{aligned}$$

Differentiating (45) with respect to c_2 and evaluating at $K^* = (K_1^*,\ K_2^*)$ yields

$$\hat{H} \cdot \begin{bmatrix} \frac{\partial \hat{K}_1}{\partial c_2} \\ \frac{\partial \hat{K}_2}{\partial c_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_2 \partial K_1} \\ \frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_2 \partial K_2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\begin{aligned}\frac{\partial \hat{K}_1}{\partial c_2} &= \frac{1}{|\hat{H}|}\{\hat{H}_{22}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_2 \partial K_1}] - \hat{H}_{12}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_2 \partial K_2}]\} \\ &\geq (\leq)\ 0 \text{ iff } \hat{H}_{22}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_2 \partial K_1}] \geq (\leq)\ \hat{H}_{12}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_2 \partial K_2}]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \hat{K}_2}{\partial c_2} &= \frac{1}{|\hat{H}|}\{\hat{H}_{11}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_2 \partial K_2}] - \hat{H}_{21}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_2 \partial K_1}]\} \\ &\geq (\leq)\ 0 \text{ iff } \hat{H}_{11}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_2} + \frac{\partial^2 \Delta_2}{\partial c_2 \partial K_2}] \geq (\leq)\ \hat{H}_{21}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial c_2 \partial K_1} + \frac{\partial^2 \Delta_1}{\partial c_2 \partial K_1}].\end{aligned}$$

(v) Differentiating (44) with respect to b and evaluating at $K^* = (K_1^*,\ K_2^*)$ produces

$$\hat{H} \cdot \begin{bmatrix} \frac{\partial \hat{K}_1}{\partial b} \\ \frac{\partial \hat{K}_2}{\partial b} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_1} + \frac{\partial^2 \Delta_1}{\partial b \partial K_1} \\ \frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_2} + \frac{\partial^2 \Delta_2}{\partial b \partial K_2} \end{bmatrix}.$$

By Cramer's rule, we have

$$\begin{aligned}\frac{\partial \hat{K}_1}{\partial b} &= \frac{1}{|\hat{H}|}\{\hat{H}_{22}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_1} + \frac{\partial^2 \Delta_1}{\partial b \partial K_1}] - \hat{H}_{12}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_2} + \frac{\partial^2 \Delta_2}{\partial b \partial K_2}]\} \\ &\geq (\leq)\ 0 \text{ iff } \hat{H}_{22}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_1} + \frac{\partial^2 \Delta_1}{\partial b \partial K_1}] \geq (\leq)\ \hat{H}_{12}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_2} + \frac{\partial^2 \Delta_2}{\partial b \partial K_2}]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \hat{K}_2}{\partial b} &= \frac{1}{|\hat{H}|}\{\hat{H}_{11}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_2} + \frac{\partial^2 \Delta_2}{\partial b \partial K_2}] - \hat{H}_{21}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_1} + \frac{\partial^2 \Delta_1}{\partial b \partial K_1}]\} \\ &\geq (\leq)\ 0 \text{ iff } \hat{H}_{11}[\frac{\partial^2 \pi_2^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_2} + \frac{\partial^2 \Delta_2}{\partial b \partial K_2}] \geq (\leq)\ \hat{H}_{21}[\frac{\partial^2 \pi_1^*(K_1^*,\ K_2^*,\ 0)}{\partial b \partial K_1} + \frac{\partial^2 \Delta_1}{\partial b \partial K_1}].\end{aligned}$$

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