A MULTI-PERIOD INVESTMENT SELECTION MODEL FOR STRATEGIC RAILWAY CAPACITY PLANNING

Yung-Cheng (Rex) Lai, Assistant Professor, Department of Civil Engineering, National Taiwan University, Rm 313, Civil Engineering Building, No 1, Roosevelt Rd, Sec 4, Taipei, Taiwan, 10617, +886-2-3366-4243, Email: <u>vclai@ntu.edu.tw</u>

Mei-Cheng Shih, Graduate Research Assistant, Department of Civil Engineering, National Taiwan University, Rm 301, Civil Engineering Building, No 1, Roosevelt Rd, Sec 4, Taipei, Taiwan, 10617, +886-2-3366-4340, Email: shortmason@gmail.com

ABSTRACT

The potential growth of the railway traffic is expected to be substantial worldwide; hence railway companies and agencies are looking for better tools to allocate their capital investments on capacity planning in the best possible way. In this research we develop a Multi-period Investment Selection Model (MISM) by using robust optimization and multi-commodity flow techniques, and using Bender's Decomposition to solve the problem. Based on the estimated future demand, available budget and expansion options, MISM can determine the multi-period optimal investment plan regarding which portions of the network need to be upgraded with what kind of capacity improvements at each defined period in the decision horizon. Using this decision support tool will help railway companies or agencies maximize their return from capacity expansion projects and thus be better able to provide reliable service to their customers, and return on shareholders' investment.

Keywords: Railway Transportation, Capacity Planning, Decision Support

INTRODUCTION

The demand for railway transportation is expected to be significantly increased in the near future (AASHTO, 2007; MOTC, 2009). Therefore, the network capacity must be increased to meet the inevitable trend. Generally, network capacity can be improved through

operational changes or engineering upgrades. Changes in operational strategies should be considered first because they are usually less expensive and more quickly implemented than building new infrastructure. However, the projected increase in future demand is unlikely to be satisfied by solely changing the operating strategies, therefore railway capacity must be increased by upgrading the infrastructure (Lai & Barkan, 2008). Consequently, how to upgrade the infrastructure to accommodate the future demand becomes the main task in a long-term strategic capacity planning.

Lai and Barkan (2008) proposed a new capacity planning process to optimize the long-term capacity planning for the North American Freight Railroads (Figure 1). This process includes two modules: (1) the Alternatives Generator (AG), and (2) the Investment Selection Model (ISM). The former (AG) generates possible expansion alternatives on the basis of CN parametric model (Krueger, 1999) for each section of the network with their cost and capacity (Table 1) (Lai & Barkan, 2009). The latter (ISM) determines which portions of the network need to be upgraded with what kind of alternatives based on the estimated future demands, budget, and the available alternatives (i.e. the optimal investment plan). The original ISM was developed to determine the optimal investment plan for a particular timeframe; however, it did not take into account the sequence of the capital investments and the variation in demand throughout the horizon.



Figure 1 - Flowchart of the long-term capacity planning process

Section	Alternatives	Sidings	Signals /	Capacity	Cost (USD)
			spacing	(trains/day)	
1.2	1	+0	+0	+0	\$0
1.2	2	+0	+1	+3	\$1,000,000
1.2	3	+0	+2	+4	\$2,000,000
1.2	4	+1	+1	+6	\$6,570,000
•				•	:
•	•		•		•

Table 1 - An example of Alternatives Table

The decision horizon of a strategic railway capacity planning usually ranges from ten to twenty years; therefore, it is desired to obtain an optimal sequence for the capacity expansion projects with consideration of possible periodical budget constraint and

uncertainly in future demand. Consequently, in this paper, we focus on the development of a multi-period investment selection model (MISM) to determine how to allocate the capital investment for capacity expansion in the best possible way. Based on information of the current network and traffic, the available investment options, and stochastic future demands, the MISM can successfully determine the optimal solution regarding which subdivisions need to be upgraded and what kind of engineering options should be conducted at each defined period in the decision horizon.

THE MULTI-PERIOD CAPACITY EXPANSION PROBLEM

The multi-period capacity expansion problem is a task to determine which portions of the network need to be upgraded with what kind of expansion options at each defined period in the planning horizon according to the future demand (i.e. the optimal multi-period investment plan). An "optimal multi-period investment plan" is different from an "optimal investment plan" by introducing additional information regarding the sequence of expansion projects with consideration of the periodical budget constraint and demand increase rate. Figure 2 shows the input and output of a MISM model. The optimal multi-period investment plan is the one fulfils the estimated demand with minimum cost throughout the decision horizon. Compared to the single-period capacity expansion problem, solving this multi-period problem must take into account the periodical budget constraint and the demand fluctuation. Furthermore, the unfulfilled demand should also be taken into account since the infrastructure upgrade may not always be able to keep up with the demand at every period in the planning horizon.



Figure 2 - Inputs and outputs of multi-period investment plan

For a long term capacity planning process, the expected traffic demand for each defined period may not be a fixed number. Therefore, instead of using a fixed pattern for future demand, we adopt a scheme assuming that the demand for a following period would be more than the previous period by a predefined range (e.g. $2\% \sim 4\%$). This range can also be different for different time period since we usually have a better idea regarding what's going to happen in five years compared to in ten years. This information is usually provided by the marketing department through demand forecasting process.

In this study, a series of possible demands of every defined period in planning horizon is called a demand pattern. For example, Table 2 lists some of the possible demand patterns for a planning task with five periods in the decision horizon. Two types of decisions should be made in the planning process: (1) the optimal investment plan determines which portions of the network need to be upgraded with what kind of capacity improvements at each period; and, (2) the routing of trains at each period (Figure 2). There is a trade-off between infrastructure investment and flow cost. The investment cost is from the expansion project, and the flow cost is associated with running trains, which is the summation of transportation cost and maintenance of way (MOW) cost. More capacity on a crucial link can reduce the flow cost but would also result in a higher infrastructure investment cost. Therefore, the developed MISM should consider both components in order to determine the most cost-effective investment plan.

Demand pattern	Demand increase rate				
(w)	Period 1	Period 2	Period 3	Period 4	Period 5
1	2%	2%	2%	2%	2%
2	2%	3%	3%	3%	2%
3	2%	4%	3%	4%	3%
4	3%	2%	2%	2%	4%
5	3%	3%	4%	2%	3%
			•	•	•
	•	•			

Table 2 – Example of demand pattern

MULTI-PERIOD INVESTMENT SELECTION MODEL

Railway capacity planning can be categorized as a resource allocation problem; and, it is usually formulated as a mixed integer network design model (Magnanti & Wong, 1984; Minous, 1989; Ahuja et al., 1993). This type of formulation can determine the optimal solution for a one-time investment decision; however, it is not able to take into account the uncertainty in future demand. In order to deal with multi-period decision process with stochastic demand, we need to introduce a different optimization framework to formulate this problem. Robust optimization was chosen in this study to help capacity planner make robust investment decision at different epoch according to possible demand patterns (Mulvey et al., 1995). In addition, during the period of upgrading the infrastructure, it is possible to have unfulfilled demand (i.e. deficit) due to insufficient budget and construction schedule; therefore, it is essential to incorporate this possibility (supply < demand) in the model framework. In this section, we first introduce a deterministic model framework to address the problem with unfulfilled demand followed by the robust optimization model accommodating demand fluctuation.

The following notation is used in MISM: *i* and *j* are the indexes referring to node as stations, and (*i*, *j*) represents the arc from node *i* to *j*; $\delta^+(j)$ represents the station *j* serving as the

departure station, and $\delta(j)$ represents the station *j* serving as arriving station. Both *p* and *t* are the indexes for predefined period in the decision horizon. *k* corresponds to the origindestination (OD) pairs of nodes (o_1, e_1) , (o_2, e_2) , ..., (o_k, e_k) in which o_k and e_k denote the origin and destination of the k^{th} OD pair; *q* represents the capacity expansion options; B^p is the budget available at period *p*; d^{pk} is the demand of k^{th} origin-destination pair at period *p*; c_{ij} stands for the cost of a unit train passing arc (i, j); h_{ij}^{q} indicates the cost of each arc (i, j) associated with option *q* and U_{ij} represents for the capacity increasing amount of option *q* at arc (i, j). α and β are the weights related to the planning horizon. τ is the penalty of unfulfilled demand which is also the penalty weight to infeasibility.

There are three sets of decision variables in the MISM. The first variable is denoted by x_{ij}^{pk} which is the number of trains running on section (*i*, *j*) from the k^{th} OD pair at period *p*. The second variable is a binary variable which determining the chosen engineering option *q* in section (*i*, *j*), denoted by y_{ij}^{pq} . The third variable a^{pk} is the supply of k^{th} OD pair at period *p*.

A multi-period, deterministic, integer programming model for MISM is given by:

min
$$\alpha \sum_{i} \sum_{j} \sum_{p} \sum_{q} h_{ij}^{q} y_{ij}^{pq} + \beta \sum_{i} \sum_{j} \sum_{p} \sum_{k} c_{ij} x_{ij}^{pk} + \tau \sum_{p} \sum_{k} (d^{pk} - a^{pk})$$
(1)

subject to:

$$\sum_{i}\sum_{j}\sum_{t\in(t\leq\rho)}\sum_{q}h_{ij}^{q}y_{ij}^{tq}\leq\sum_{t\in(t\leq\rho)}B^{t}\qquad\forall\rho$$
(2)

$$\sum_{p} \sum_{q} \mathcal{Y}_{ij}^{pq} \leq 1 \qquad \forall (i,j), \ i < j \tag{3}$$

$$\sum_{k} (X_{ij}^{pk} + X_{ji}^{pk}) \le U_{ij} + \sum_{t \in (t \le p)} \sum_{q} U_{ij}^{q} Y_{ij}^{tq} \qquad \forall (i, j), \ i < j, \ p$$
(4)

$$\sum_{j\in\delta^{+}(j)} \mathbf{x}_{ij}^{pk} - \sum_{j\in\delta^{-}(j)} \mathbf{x}_{ji}^{pk} = \begin{cases} \mathbf{a}^{pk} & i\in\mathbf{0}_{k} \\ -\mathbf{a}^{pk} & i\in\mathbf{e}_{k} \\ \mathbf{0} & otherwise \end{cases}$$
(5)

$$\boldsymbol{a}^{pk} \leq \boldsymbol{d}^{pk} \quad \forall \boldsymbol{p}, \boldsymbol{k} \tag{6}$$

and

$$x_{ij}^{\ \ pk} \& a^{pk} \ge 0, \ x_{ij}^{\ \ pk} \& a^{pk} \in \text{ integer}, \ y_{ij}^{\ \ pq} \in \{0,1\}$$
 (7)

The objective function, equation (1), aims to minimize the total cost and the unfulfilled demand over the decision horizon. The total cost is the summation of costs associated with "construction cost", and "flow cost". As mentioned earlier, the construction cost is from the expansion project, and the flow cost the summation of transportation cost and maintenance

of way (MOW) cost. The relative importance of the construction cost (α) and flow costs (β) depends on the planning horizon. The longer the planning horizon, the more the railroads would be willing to invest due to larger reductions in flow cost over time. τ is the penalty assigned to avoid unfulfilled demand. The value of this penalty is set to be the largest possible flow cost for a train from its origin to its destination among all OD pairs to ensure trains will be dispatched whenever there is available capacity.

Equation (2) is the budget constraint with the possibility to roll over unused budget from previous period to the next. The constraint can be easily modified if a roll-over scheme is not possible. Equation (3) ensures each arc will select only one capacity expansion option. Equation (4) is the link capacity constraint total flow on arc (*i*, *j*) is less than or equal to the current capacity, plus the increased capacity due to upgraded infrastructure. Equation (5) is the flow conservation constraint guaranteeing that the outflow is always equal to the inflow for trans-shipment nodes; otherwise, the difference between them should be equal to the supply of the k^{th} OD pair. Equation (6) is the supply constraint, it guarantees that supply is less than or equal to demand. Combining the equations (5) and (6) enables the possibility to handle cases without sufficient ability to fulfil demand. Finally, equation (7) indicates the type of variables.

As mentioned earlier in this section, the deterministic MISM can tackle problems with deterministic demand pattern. To incorporate the uncertainty in future demand, we therefore introduce the concept of Robust Optimization into this deterministic model (Paraskevopoulos et al., 1991; Malcolm & Zenios, 1994; Mulvey et al., 1995; Soteriou & Chase, 2000; Yu & Li, 2000). This robust optimization framework considers both the quality and robustness of the solution simultaneously. According to the robust optimization framework, we introduce a set of demand scenarios *w* for the uncertain traffic demands { d^{pk} } into the MISM formulation and revise it into a robust optimization model. The stochastic traffic demand is expressed as d^{pkw} , with the probability of entering demand pattern *w* as ρ_w . The scenario-dependent variables are set to be control variables. They are the flow on each link and the supply of each OD pair, which are denoted by x_{ij}^{pkw} and a^{pkw} . The deterministic MISM given above is then reformulated into the following stochastic MISM:

min
$$\alpha \sum_{i} \sum_{j} \sum_{p} \sum_{q} h_{ij}^{q} y_{ij}^{pq} + \beta \sum_{w} \rho_{w} \left(\sum_{i} \sum_{j} \sum_{p} \sum_{k} c_{ij} x_{ij}^{pkw} \right) + \tau \sum_{w} \rho_{w} \left[\sum_{p} \sum_{k} (d^{pkw} - a^{pkw}) \right]$$
(8)

subject to:

(2), (3), and

$$\sum_{k} (x_{ij}^{pkw} + x_{ji}^{pkw}) \le U_{ij} + \sum_{t \in (t \le p)} \sum_{q} U_{ij}^{pq} y_{ij}^{tq} \qquad \forall (i, j), \ i < j, \ p$$
(9)

$$\sum_{j\in\delta^{+}(j)} X_{ij}^{pkw} - \sum_{j\in\delta^{-}(j)} X_{ji}^{pkw} = \begin{cases} a^{pkw} & i \in O_k \\ -a^{pkw} & i \in e_k \\ 0 & otherwise \end{cases}$$
(10)

$$\boldsymbol{a}^{pkw} \leq \boldsymbol{d}^{pkw} \quad \forall \boldsymbol{p}, \boldsymbol{k}, \boldsymbol{w} \tag{11}$$

and

The objective function of the Robust Optimization MISM (RO-MISM) formulation has two parts. The first part is the expected total cost of the solution. It is the summation of the values that obtained from multiplying the probability of entering demand pattern w to the objective value of that pattern and the objective value of each demand pattern. The second part penalizes the unfulfilled demand, which is regarded as the infeasibility. Equations (9), (10), (11), and (12) are similar to equations (4), (5), (6), and (7). The only difference between them is the presence of w in control variables and parameters with uncertainty. This stochastic MISM is a NP-hard problem; therefore, in the following section we proposed a solution process to tackle this problem.

BENDER'S DECOMPOSITION

The RO-MISM cannot be solved by the commercial solver due to its complexity; therefore, we applied "Bender's Decomposition (Bender, 1962)" technique to tackle this problem. This technique partitions the problem into multiple smaller problems (i.e. master and sub problems) instead of considering all decision variables and constraints simultaneously. Master and sub problems handle different sets of decision variables, respectively. For our RO-MISM formulation, the master problem at each iteration determines the location, engineering option, and period to upgrade the infrastructure (y_{ij}^{pq}). These values are then fixed and transferred to the sub problem to determine the he number of trains running on section (*i*, *j*) from the *k*th OD pair at period *p* (x_{ij}^{pk}), and the supply of of *k*th OD pair at period *p*

 (a^{pk}) . If the solution of the sub problem is unbounded, the dual variables of the sub problem will be used to construct "feasibility cut" constraint in the master problem to eliminate infeasible solutions. If the solution is feasible, an "optimality cut" constraint will be added to the master problem to improve the quality of the solution of the master problem. In the next

iteration, the master problem generates a new set of y_{ij}^{pq} and feeds them into the sub problems again. After several iterations, the process will converge to an optimal solution. The followings are the master problem of RO-MISM:

$$\min \alpha \sum_{i} \sum_{j} \sum_{p} \sum_{q} h_{ij}^{q} y_{ij}^{pq} + \sum_{w} \rho_{w} \theta_{w}$$
(13)

subject to (2), (3) and:

$$\theta_{w} \geq \sum_{i} \sum_{j} \sum_{p} \pi_{pij}^{Cw} [U_{ij} + \sum_{t \in (t \leq p)} \sum_{q} U_{ij}^{q} y_{ij}^{tq}] + \sum_{p} \sum_{k} \pi_{pk}^{Sw} (d^{pkw}) + \rho_{w} \tau \sum_{p} \sum_{k} d^{pkw} \quad \forall w$$
(14)

$$\boldsymbol{y}_{ij}^{pq} \in \left\{0,1\right\}$$

The objective (equation (13)) of master problem aims to minimize the investment cost and the expected value of sub problems. θ_w represents the objective of sub problems in the master problem. Equation (14) is the optimality cut; it is constructed by the value of dual variables in the sub problem. π_{pij}^{Cw} and π_{pk}^{sw} are dual variables which corresponding to capacity constraint, and supply constraint , respectively; both of them can be calculated by maximizing the dual sub problem. The optimality cut can improve the quality of solution in the master problem. For our problem (RO-MISM), there is no feasibility cut in the master problem because the deficit term inside the objective function eliminates the possibilities to have infeasible solutions.

A sub problem model is constructed for each demand pattern:

min
$$\rho_w \beta \sum_i \sum_j \sum_p \sum_k c_{ij} x_{ij}^{pkw} + \rho_w \tau \sum_p \sum_k (d^{pkw} - a^{pkw})$$
 (16)

subject to (10), (11) and :

$$\sum_{k} (x_{ij}^{pkw} + x_{ji}^{pkw}) \le U_{ij} + \sum_{t \in (t \le p)} \sum_{q} U_{ij}^{q} \hat{y}_{ij}^{tq} \qquad \forall (i, j), \ i < j, \ p, \ w$$
(17)

$$x_{ij}^{pkw} \& a^{pkw} \ge 0, x_{ij}^{pkw} \& a^{pkw} \in \text{ integer}$$
 (18)

The objective (equation (16)) of this sub problem minimizes the total flow cost and deficit for each demand pattern. Equation (17) works as the capacity constraint, but \hat{y}_{ij}^{tq} is fixed according to the result from the master problem.

To obtain the dual variables in the sub problem to construct the optimality and feasibility cuts, the sub problem must be transformed into dual sub problem (equations (19~22) for each demand pattern *w*. Besides π_{pij}^{Cw} and π_{pk}^{Sw} , π_{pki}^{Bw} was also added as a dual variable corresponding to balance constraint,

$$\max \sum_{i} \sum_{j} \sum_{p} \pi_{pij}^{Cw} (U_{ij} + \sum_{t \in (t \le p)} \sum_{q} U_{ij}^{q} \hat{y}_{ij}^{tq}) + \sum_{p} \sum_{k} \pi_{pk}^{Sw} (d^{pkw}) + \rho_{w} \tau \sum_{p} \sum_{k} d^{pkw}$$
(19)

subject to:

$$\pi_{piJ}^{Cw} + \pi_{pki}^{Bw} - \pi_{pkj}^{Bw} \le \rho_w \beta c_{ij} \qquad \forall (i < j), p, k, w$$

$$-\sum_{i \in o_k} \pi_{pki}^{Bw} + \sum_{i \in e_k} \pi_{pki}^{Bw} + \pi_{pk}^{Sw} \le -\rho_w \tau \qquad \forall p, k, w$$
(20)
(21)

$$\pi_{\scriptscriptstyle pij}^{\scriptscriptstyle Cw}$$
 and $\pi_{\scriptscriptstyle pki}^{\scriptscriptstyle Sw}$, $\pi_{\scriptscriptstyle pki}^{\scriptscriptstyle Bw} \le 0$ (22)

With this formulation, the master problem determines the multi-period investment plan at each iteration, and then the sub problems examine this plan by optimizing dual sub problems. The dual variables of the sub problems will then be used to construct optimality cuts in the master problem as feedback. This process will eventually converge to an optimal solution.

CASE STUDY

To demonstrate the potential use of the proposed model, we applied it to solve a network capacity planning problem in North America. Figure 3 shows the selected network in which the nodes represent stations or yards, and the arcs represent the connecting rail lines. There are two types of links in this network, mainline (denoted by solid lines) and secondary line (dotted lines).



Figure 3 - The railway network in the case study

The decision horizon for this case study was set to be 10 years with five two-year periods. The demand increase at the last period was expected to be ranging from 10 % to 20 % of the current traffic volume. Also, the range of demand increase was predicted as 2% to 4% more than the demand of the previous time period at each period. Table 3 shows the current demand (trains/day) of all OD pairs. To prepare the input data for RO-MISM, we discretized the demand growth rate by 1% and create 243 demand patterns (3 possible increase rates for each period, i.e. 2%, 3%, and 4%) as the stochastic information provided to the RO-MISM model. The total available budget was set to be 50 million and was allocated as 10 million for each period.

1	Table 5 - OD Pails and Current Deman				
	OD-pair	O-D	Demand		
	(index)	(nodes)	(trains/day)		
	1	1-9	4		
	2	3-6	6		
	3	3-13	9		
	4	3-15	15		
	5	4-6	8		
	6	4-13	5		
	7	6-3	6		
	8	6-15	12		
	9	10-3	2		
	10	13-3	8		
	11	15-3	18		
	12	15-6	6		

Table 3 - OD Pairs and Current Demand

The alternatives table (Table 1) listing all possible options to upgrade the infrastructure was also supplied to RO-MISM. It was generated by the alternatives generator developed in Lai and Barkan (2008) according to the traffic, plant, and operating parameters in the network.

The life of railroad infrastructure is usually assumed to be 20 years approximately. Therefore, for a 10-year decision horizon, α was set to be 0.5 considering the residual value of the infrastructure. β was determined based on the number of days in a planning period, which is 730 for this case study. The penalty τ was set to be the largest possible flow cost for a train from its origin to its destination among all OD pairs; this value ensures the model to fulfill the demand as much as possible.

The process was coded in GAMS and Matlab to solve this problem iteratively. TABLE 4 shows the optimal investment plans for the robust optimization model.

Arc (i, j)	Capacity increased (trains/day)	Construction time (period)	Sidings added	Signals added
3.	5	3	2	2	28
3.	11	3	4	0	8
5.	6	7	3	2	14
6.	9	3	1	0	30
7.	13	3	2	2	44
9.	11	3	2	0	14
11.	12	7	1	1	8
11.	13	4	4	1	22
13	14	4	5	1	11
Total investment cost (million USD) 40.98					

Table 4 – Optimal multi-period investment plan of RO-MISM

To show the benefit of using RO-MISM, the problem is also solved by using the deterministic MISM (DE-MISM) model according to the average rate of demand increase (=3% per period). Table 5 is the comparison between two scenarios. Although RO-MISM results in higher the investment cost and flow cost, the expected penalty cost is considerably lower than the DE-MISM scenario.

Table 5 – Comparison between RO-MISM and DE-MISM Scenarios

Investment	Expected flow	Expected	Expected
cost	cost	penalty cost	objective value
(USD million)	(USD million)	(USD million)	(USD million)
40.98	6,566	231	6,817
39.08	6,558	258	6,836
	Investment cost (USD million) 40.98 39.08	Investment Expected flow cost cost (USD million) (USD million) 40.98 6,566 39.08 6,558	InvestmentExpected flowExpectedcostcostpenalty cost(USD million)(USD million)(USD million)40.986,56623139.086,558258

The real objective value of each demand pattern (sub problems) from both RO-MISM and DE-MISM are shown in Figure 6. In this figure, the greater the index number of the demand pattern, the higher the total demand. DE-MISM scenario performs slightly better than RO-MISM at lower demand, but RO-MISM significantly outperform DE-MISM for cases with medium to large demand increase. This result shows that RO-MISM can significantly reduce the risk in strategic capacity planning subject to uncertainty in demand forecast.



Figure 7 - The comparison between "DE-MISM" and "RO-MISM" scenarios

CONCLUSION

Railroads are approaching the limits of practical capacity due to the substantial future demand. Therefore, we developed a "Robust Multi-period Investment Selection Model (RO-MISM)" in this research to assist their long term strategic capacity planning. Bender's decomposition technique was also used to solve the complex problem. Based on the estimated future demand, available budget and expansion options, the proposed model can determine which portions of the network need to be upgraded with what kind of expansion options at each defined period in the planning horizon. Using this tool can help railway agencies maximize their return from capacity expansion projects and able to provide reliable service to their customers.

REFERENCE

- Ahuja, R.K., T.L. Magnanti, and J.B. Orlin (1993). Network Flows: Theory, Algorithms, and Applications. Prentice Hall, New Jersey.
- American Association of State Highway and Transportation Officials (AASHTO). (2007). Transportation - Invest in Our Future: America's Freight Challenge. AASHTO, Washington, DC.
- Benders, J. F (1962). Partitioning procedures for solving mixed-variables programming problems, Numerische Mathematik, Vol. 4.
- Krueger, H (1999). Parametric modeling in rail capacity planning. Proceedings of Winter Simulation Conference, Phoenix, AZ.
- Lai, Y.C., and C.P.L. Barkan (2008). A quantitative decision support framework for optimal railway capacity planning. Proceedings of 8th World Congress on Railway Research, Seoul, Korea.
- Lai, Y.C., and C.P.L. Barkan (2009). Enhanced parametric railway capacity evaluation tool. Transportation Research Record - Journal of Transportation Research Board, 2117, 33-40.
- Magnanti, T.L., and R.T. Wong (1984). Network design and transportation planning: models and algorithms. Transportation Science, 18 (1), 1-55.
- Malcolm, S., and S. A. Zenios (1994). Robust optimization for power capacity expansion planning. Journal of Operations Research Society, 45, 1040-1049.
- Ministry of Transportation and Communications (MOTC) (2009). Monthly Statistics of Transportation & Communications, MOTC, Taiwan

www.motc.gov.tw/mocwebGIP/wSite/lp?ctNode=160&mp=1. Accessed July 28.

- Minoux, M. (1989). Network synthesis and optimum network design problems: models, solution methods, and applications. Networks, 19, 313-360.
- Mulvey, J. M., J.R. Vanderbei and S.A. Zenios (1995). Robust optimization of large-scale systems. Operations Research, 43 (2), 264-281
- Paraskevopoulos, D., E. Karakitsos and B. Rustem (1991). Robust capacity planning under uncertainty. Management Science, 37, 787-800.
- Soteriou, A.C., and R.B. Chase (2000). A robust optimization approach for improving service quality. Manufacturing and Service Operations Management, 2 (3), 264–286.
- Yu, C., and H. Li (2000). A robust optimization model for stochastic logistic problems. International Journal of Production Economics, 64, 385–397.