AN ACTIVITY-BASED APPROACH FOR ESTIMATION OF PASSENGER O-D TRIP MATRIX AND ACTIVITY PATTERNS

Yiliang XIONG and William H.K. LAM

Department of Civil & Structural Engineering

The Hong Kong Polytechnic University

Hong Kong, China

ABSTRACT

The trip-based traffic assignment models are commonly employed for solving the origin-destination (O-D) trip matrix estimation problems so as to reproduce the observed traffic counts. However, travellers may have different activity patterns that would affect their destination and path choices. These activity patterns would therefore have significant impacts on the O-D trip matrix estimation from traffic count data. In this paper, an activity-based transit network equilibrium model is proposed to ensure that the estimated passenger O-D trip matrices are consistent with the passenger activity/travel choices. A nested logit model is employed to capture simultaneously the passenger's behaviour on activity and travel choices in transit network. The parameters of the nested logit model and the passenger O-D matrix are estimated simultaneously using passenger count data and other relevant information. A sensitivity-based algorithm is proposed for solving this simultaneous estimation problem. Numerical experiments on a small transit network are used to illustrate the features and merits of the proposed model.

Keywords: activity pattern, trip chain, passenger O-D trip matrix estimation

1. INTRODUCTION

For estimation of an origin-destination (O-D) trip matrix from traffic counts, there are generally two sources of information required. The first is the outdated O-D trip matrix. If there is no such priori information, then travel survey is required to collect travel data and socioeconomic characteristics in the study areas. Subsequently, trip distribution models are calibrated to estimate each O-D trip matrix entry as a function of socioeconomic characteristics of these areas. The second source of information is the traffic counts. In fact, traffic counts impose linear constraints on the O-D trip matrix estimation problem. If congestion effects are considered, the relationship between O-D trip matrices and link flows will not be linear.

In the literature, there are a number of methods proposed for estimating the O-D trip matrix from traffic counts (Bell 1991; Maher 1983; Nie and Zhang 2008; Spiess 1987; Yang 1995; Yang et al. 2001). However, these methods did not reveal the behavioural relationship between activity and travel choices behind the O-D trip matrix estimation problems. In the past two decades, great advances have been made in activity-based approaches for travel demand modelling. Attention has also been given to simulation models that could incorporate the underlying activity/travel behaviour for estimation of travel demand.

Activity-based approaches treat travel as a demand derived from the desire to participate in spatially separated activities, while in the conventional O-D trip matrix estimation problem each trip is modelled as a desirable activity on its own right. Combining activity and travel choice behaviour would be a new avenue for solving the O-D trip matrix estimation problem.

The previous related studies adopted trip chain as the basic unit of analysis to bridge the gap between activities and trips. Lam and Huang (2002) modelled dynamics in activity choice and trip-chaining behaviour. Three typical activity patterns and the associated trip chains are considered in their model. Abdelghany and Mahmassani (2003) implemented a system to assign the corresponding trip chains of activity/travel patterns to the transportation network and determine the network conditions. Their network assignment process is basically a simulation approach. Maruyama and Harata (2005) formulated the trip-chaining behaviour as a convex nonlinear programming problem. Their trip chain choice model is based on the random utility theory, and the path choice model is developed on the basis of the user equilibrium (UE) principle.

Suppose that trip chain is considered as the bridge between activities and trips, each activity pattern can be represented by a set of trip chains and the corresponding O-D trip matrices and traffic flows on links are obtained by assigning these trip chains onto the network. The level of service on the network can affect the choice of activity patterns. This implies that the travellers might abandon certain activities due to the accessibility of their activity destination. These relationships are illustrated in Figure 1.

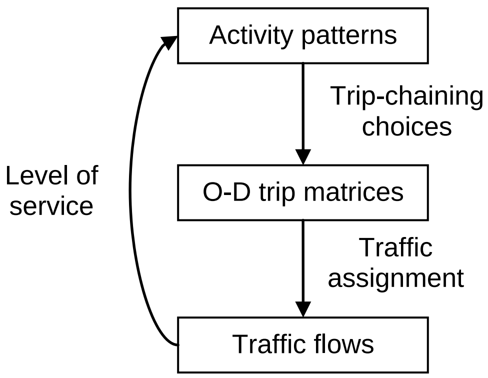

Figure 1 Relationships among activity patterns, O-D trip matrices and traffic flows

Although trip chains have been extensively employed in the activity-based travel demand models, such as those metioned above, the activity-based models are fundementally different from the trip-chaining models. The features of these two models are compared in Table 1. In general, the activity-based models are superior to trip-chaining models as the former is more complicated and realistic. Firstly, in activity-based models the accessibility of activity destination can affect the choice of activity pattern, while trip-chaining models only take account of the chaining relationship between individual trips. Secondly, beside the disutility of travel, the utility derived from activity is explicitly considered in the activity-based models so that travellers would schedule their activity/travel pattern to maximize their total utility. Thirdly, in activity-based models the set of trip chains are derived from activity demand and thereby consistent with the underlying activity patterns. Finally, the activity-based models can capture the household activity participation. The former three features of the activity-based models will be considered in the model proposed in this paper.

Table 1 Features of trip-chaining and activity-based travel demand models

	Travel pattern	Individual's behaviour	Generation of trip chains	Household interaction
Trip-chaining models	Trips in the same trip chain influence each other in terms of total travel time and mode choice.	The individuals choose a sequence of paths with least total travel time.	The choice set of trip chains is pre-determined. It is generated without modelling the underlying activity.	Each individual is modelled independently.
Activity-based models	The attributes of travel pattern, like travel time, may affect the choice of activity pattern.	The individuals evaluate the overall utility of a daily activity/travel schedule.	The generation and choice of trip chains are derived from the activity demand.	Interaction between household members can be explicitly modelled.

Recently, Chan et al. (2007) and Ouyang et al. (2008) incorporated the activity/destination choice model into the O-D trip matrix estimation problem. Their model implicitly considered the trip-chaining behaviour on pedestrian networks and road networks. Following this line of research, we estimate simultaneously the passenger O-D trip matrix and their activity patterns during the evening rush hour with explicit consideration of the trip-chaining and activity behaviour in transit network.

In this paper, the combined activity and travel choice problem is formulated as a nested logit model, which is actually a variant of the combined travel demand model (Safwat and Magnanti 1988; Oppenheim 1995). It can also be considered as an extension of the model proposed by Maruyama and Harata (2005), yet it is governed by the stochastic user equilibrium (SUE) principle and is closer to the travel choice behaviours in reality. The equivalent convex optimization problem is formulated and presented in this paper. A sensitivity-based solution algorithm is adapted to estimate simultaneously the passenger O-D matrices and the parameters of the nested logit model.

The reminder of the paper is organized as follows. Section 2 presents the activity-based transit network equilibrium model together with its equivalent optimization problem. In Section 3, a bilevel programming model is proposed to estimate the O-D trip matrix and calibrate the activity-based model by using passenger counts and other related information. It follows with a numerical example for illustration and discussion of the insightful results and merits of the proposed model. Finally, conclusions are given together with suggestions for further studies. The definitions of notations are appended at the end of the paper.

2. ACTIVITY-BASED TRANSIT NETWORK EQUILIBRIUM MODEL

The activity-based travel demand forecasting models generally have hierarchical structures. These complicated structures can be divided into two levels (Abdelghany and Mahmassani 2003). The upper level specifies activity participation, sequence of activities, etc., and produces a set of activity patterns. The lower level is to assign these activity patterns onto a transit network, involving activity/trip chain/path choices.

The trip chain $h \in H^p$ is defined as a set of ordered nodes representing the activity destinations, i.e. h: workplace → shopping mall → home. Each activity pattern is associated with several trip chains. And each trip chain defines the activity destinations for all activities in the activity pattern. Thus, the trip chain choice is equivalent to the destination choice in this paper. The following example network is presented to illustrate the concepts of activity pattern and trip chain choices together with their relationship.

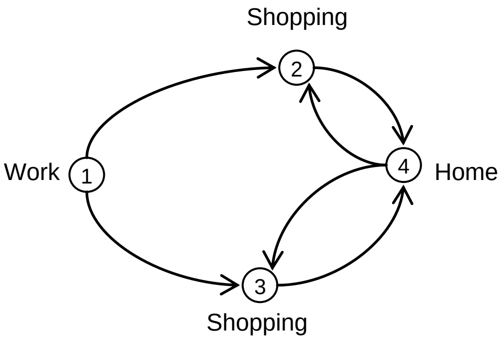

Figure 2 A 4-node example network

In the above network, people work at node 1 and travel to node 4 (home) in the evening rush hour. The choice of going home directly is denoted as activity pattern p_1, the choice of doing shopping on the way to home is denoted as activity pattern p_2, and the choice of going home first then going out and shopping is denoted as activity pattern p_3. These three types of activity patterns can be illustrated as follows,

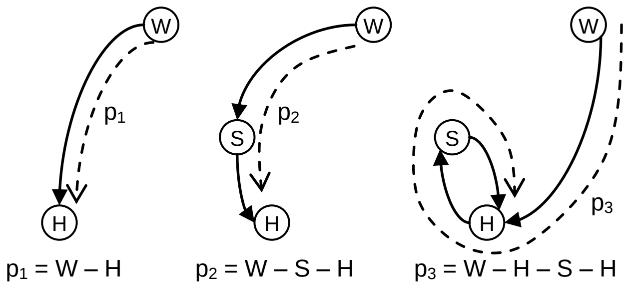

Figure 3 Three typical activity patterns

Activity pattern p_1 is associated with the only trip chain 1 → 4. Activity pattern p_2 is associated with two trip chains: 1 → 2 → 4, 1 → 3 → 4, since there are two shopping destinations in the network. And activity pattern p_3 is associated with trip chains 1 → 4 → 2

→ 4 and 1 → 4 → 3 → 4. As illustrated above, each activity pattern is associated with a bundle of trip chains. In the following subsections, the choice probability for each trip chain and the related paths is derived on basis of random utility theory.

2.1 Model Assumptions

Unlike the dynamic model developed by Lam and Huang (2002), the activity-based transit network equilibrium model proposed in this paper is basically a static model as it is intended for long-term travel demand estimation and strategic planning purposes. With this in mind, the following assumptions are adopted in this paper:

1. The headways of the transit lines are assumed to be constant in the study period. The frequency-based transit assignment model is employed to obtain the passengers flows (Spiess and Florian 1989) for long-term strategic planning.
2. The disutility of travel consists of in-vehicle travel time and passenger waiting time at bus stop. The congestion effect is modelled by a generalized travel time function. The access walking time and transfer penalty are ignored for clarity, but they can be incorporated easily.
3. The capacity of the transit vehicle is assumed to be infinite, and thereby the overload delays at bus stops in the congested transit network are ignored. This assumption is often adopted in frequency-based transit assignment models (Spiess and Florian 1989; Wu et al. 1994) for strategic planning purpose. However, it can be extended by incorporating the capacity constraints into the model (Lam et al. 1999).
4. The set of activity patterns is assumed to be pre-specified and given. It implies that the generation of activity patterns is not considered in this paper as only major activity patterns would be considered for long-term strategic planning. This assumption has been adopted in the previous related studies, such as Abdelghany and Mahmassani (2003), and Maruyama and Harata (2005).
5. It is assumed that people would choose their activity/travel pattern by evaluating the overall utility of activity minus the disutility of travel. The higher the net utility, the activity/travel pattern is chosen.
6. The prior information contains an outdated passenger origin-destination (O-D) trip matrix and passenger counts at some transit line segments. The utility of each activity is assumed to be given and obtained from the previous travel survey data. With these limited data, the passenger O-D trip matrix can be updated together with the activity choice model parameters for long-term strategic planning purposes.

2.2 Transit assignment model

In the past decades, considerable research has already been conducted on the frequency-based transit network equilibrium problem (Spiess and Florian 1989; Wu et al. 1994; Lam et al. 2002). We do not require any specific or complicated transit assignment model here. It is only required to compute the route section cost t_a in the transit network. In order to facilitate the presentation of the essential ideas, a simplified frequency-based transit assignment model is adopted in this paper.

In the frequency-based transit assignment models, a path consists of a sequence of route sections and a route section is connected between each pair of bus stops. The passenger travel time on a route section is the sum of two components: the waiting time at bus stop for the first bus to arrive, and the in-vehicle time travelling on the line segment of the route section concerned. The total travel time of route section is specified as the sum of these two components

$$t_a(x_a) = \frac{1.0}{\sum_{l \in L_a} z_l} + \frac{\sum_{l \in L_a} t_l(x_l) \cdot z_l}{\sum_{l \in L_a} z_l} \quad \forall a \in A \tag{1}$$

The passenger flows x_l on line segment l of route section a is proportional to the frequency of the corresponding transit line

$$x_l = x_a \cdot \frac{z_l}{\sum_{l \in L_a} z_l} \tag{2}$$

To capture the effect of crowding on the activity/travel choice of passengers, the generalized travel time of line segment is assumed to be an increasing function of passenger flows on the line segment

$$t_l(x_l) = t_l^0 \cdot \left(1.0 + \rho \cdot \left(\frac{x_l}{\kappa_l \cdot z_l} \right)^n \right) \quad \forall l \in L_a, a \in A \tag{3}$$

where t_l^0 is the in-vehicle time of the line segment with no crowding impact, ρ and n are the parameters specifying the congestion effects or crowding impact on the line segment. And the generalized travel time on path j of trip chain h is given by

$$t_{hj}^p = \sum_{a \in A} \delta_{phj,a} \cdot t_a(x_a) \quad \forall j \in J_h^p, h \in H^p, p \in P \tag{4}$$

2.3 Activity/trip chain/path choice behaviour

The model presented in this section is a variant of the combined travel demand model. In short, we consider the combined choice of trip chain and path, and trip chain choice is equivalent to destination choice of activity. The paths of a trip chain connect each activity destination in the trip chain. Precisely the choices of trip chain and path are determined simultaneously in the decision-making process, which is illustrated by a hierarchical structure in Figure 4.

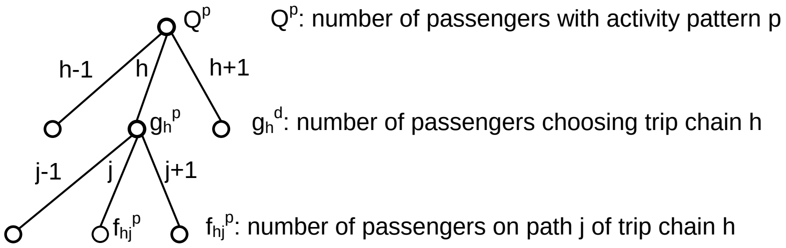

Figure 4 Hierarchical structure of the activity/trip chain/path choice model

The sum of activity utility of a trip chain is given by

$$V_h^p = \sum_{n \in N} \sigma_{ph,n} \cdot V_n \quad \forall h \in H^p, p \in P \tag{5}$$

where V_n is the utility of participating activity at node n. According to the random utility theory, the total utility derived from path j of trip chain h is the sum of activity utility of the trip chain minus the disutility of travel

$$U_{hj}^{p} = V_h^p - \alpha \cdot t_{hj}^{p} + \varepsilon_h + \varepsilon_{hj} \quad \forall j \in J_h^d, h \in H^p, p \in P \tag{6}$$

where α is the equivalent utility of unit travel time, ε_h's and ε_{hj}'s are assumed to be independent random variables with extreme value distribution. The first term in equation (6) is specified to trip chain choice, while the second term is specified to the combined path choice. That is, the first term differs among the trip chains but is common to all passengers choosing the same trip chain. The second term depends on the attributes of path.

Under the above specifications and the assumption about the distribution of ε's, the number of passengers with activity pattern p choosing path j of trip chain h is given by the following nested logit model

$$f_{hj}^{p} = Q^p \cdot \frac{\exp\left(\beta \cdot \overline{V}_h^p\right)}{\sum_{h' \in H^p} \exp\left(\beta \cdot \overline{V}_{h'}^p\right)} \cdot \frac{\exp\left(\theta \cdot \left(V_h^p - \alpha \cdot t_{hj}^p\right)\right)}{\sum_{j' \in J_h^p} \exp\left(\theta \cdot \left(V_h^p - \alpha \cdot t_{hj'}^p\right)\right)} \quad \forall j \in J_h^d, h \in H^p, p \in P \tag{7}$$

where $\overline{V}_h^p$ is the expected utility of trip chain h,

$$\overline{V}_h^p = \frac{1}{\theta} \cdot \ln\left(\sum_{j \in J_h^p} \exp\left(\theta \cdot \left(V_h^p - \alpha \cdot t_{hj}^p\right)\right)\right) \quad \forall h \in H^p, p \in P \tag{8}$$

or

$$\overline{V}_h^p = V_h^p - \frac{1}{\theta} \cdot \ln\left(\sum_{j \in J_h^p} \exp\left(\theta \cdot \alpha \cdot t_{hj}^p\right)\right) \quad \forall h \in H^p, p \in P \tag{9}$$

The second term of equation (9) is the expected minimum path travel time. It can be evaluated directly without path enumeration by calculating the link probability using Dial's algorithm (Akamatsu 1997). Note that as $\theta \to \infty$, the second term of equation (9) approaches to the least path travel time, and then this model degenerates to the user equilibrium (UE) model proposed by Maruyama and Harata (2005).

Finally the number of passengers with activity pattern p choosing trip chain h is expressed in terms of the marginal choice probability

$$g_h^p = Q^p \cdot \frac{\exp\left(\beta \cdot \overline{V}_h^p\right)}{\sum_{h' \in H^p} \exp\left(\beta \cdot \overline{V}_{h'}^p\right)} \quad \forall h \in H^p, p \in P \tag{10}$$

2.4 The equivalent optimization problem

Generally, there are two approaches for computing the resultant passenger flows at equilibrium. We can formulate the problem as a system of variational inequalities or an equivalent optimization problem. The former approach becomes primary tool of analysis for modelling travel demand. But with latter approach (i.e. the optimization formulation), the existence and uniqueness of the equilibrium solution can be easily obtained. Furthermore, the sensitivity analysis of nonlinear programming problem can be used to develop solution algorithm for solving the passenger O-D trip matrix estimation problem. For these reasons, the latter approach is adopted in this paper.

The model specified in the previous section can be formulated as the following equivalent optimization problem,

$$\min_{\mathbf{f},\mathbf{g}} \sum_{a\in A}\int_0^{x_a} t_a(\omega)d\omega + \frac{1}{\theta}\sum_{p\in P}\sum_{h\in H^p}\sum_{j\in J_h^p} f_{hj}^p \cdot \ln\frac{f_{hj}^p}{g_h^p} + \frac{1}{\alpha\cdot\beta}\sum_{p\in P}\sum_{h\in H^p} g_h^p \cdot \ln g_h^p - \frac{1}{\alpha}\sum_{p\in P}\sum_{h\in H^p} V_h^p \cdot g_h^p \quad (11)$$

subject to

$$\sum_{j\in J_h^p} f_{hj}^p = g_h^p \quad \forall h\in H^d, p\in P \quad (12)$$

$$\sum_{h\in H^p} g_h^p = Q^p \quad \forall p\in P \quad (13)$$

$$f_{hj}^p \ge 0 \quad \forall j\in J_h^p, h\in H^p, p\in P \quad (14)$$

$$g_h^p \ge 0 \quad \forall h\in H^p, p\in P \quad (15)$$

with definition equations

$$x_a = \sum_{p\in P}\sum_{h\in H^p}\sum_{j\in J_h^p} \delta_{phj,a}\cdot f_{hj}^p \quad \forall a\in A \quad (16)$$

$$q^{rs} = \sum_{p\in P}\sum_{h\in H^p} \lambda_{ph,rs}\cdot g_h^p \quad \forall rs\in W \quad (17)$$

Equations (12) and (13) are the flow conversation constraints stating that the number of passenger trips on all possible paths for a particular trip chain is equal to the number of passengers choosing the trip chain, and that the number of passengers choosing all possible trip chains associated with an activity pattern is equal to the number of passengers with that activity pattern. Inequalities (14) and (15) restrict all the flow variables to nonnegative values. The equilibrium passenger flows $\{f_{hj}^p\}$, $\{g_h^p\}$ are the solution of the optimization problem (11)-(15), and the corresponding route section flows and O-D trips can be obtained by using equations (16) and (17). The four terms in the objective function (10) are similar to those defined in other combined travel demand models (Florian et al. 1975; Boyce et al. 1988).
The existence and uniqueness of the optimization problem (10) can be proved easily, as the equivalence of the solution satisfies the Karush-Kuhn-Tucker (KKT) conditions. For given number of passengers $\{Q^p\}$ and parameters θ, β, the partial linearization method can be used to solve the above activity/travel choice model (Evans 1976).

3. THE ESTIMATION PROBLEM

There are few studies concerning with both the estimation of O-D trip matrix and calibration of parameters of travel demand model from traffic counts. Shihsien and Fricker (1996) proposed a two-stage method: in the first stage they estimated the O-D matrix in uncongested network with a fixed parameter; in the second stage they calibrated the travel-cost parameter by maximum likelihood method. Yang et al. (2001) proposed a successive quadratic-programming algorithm for solving the simultaneous estimation problem. In these two studies, a logit-based model is used to model the path choice behaviour.
In view of the logit-based path choice models adopted in the above two studies, this paper proposes to estimate the O-D trip matrix and calibrate the parameters of a nested logit model simultaneously. Since we can formulate the activity-based model as an equivalent optimization problem, the sensitivity analysis of nonlinear programming problem is employed to develop solution algorithm for the simultaneous estimation problem.

Another approach is proposed by García-Ródenas and Marín (2009) for simultaneous estimation of O-D trip matrix and calibration of the parameters of nested logit model. They developed a heuristic column generation algorithm for calibrating the parameters of a network equilibrium model with combined modes as well as updating of the O-D trip matrix.

3.1 The simultaneous estimation problem

In this paper, a bilevel programming model is formulated to estimate simultaneously the parameters β, θ and the numbers of passengers with different activity patterns $\{Q^p\}$, which are the decision variables in the upper-level problem. For given parameters and number of passengers, a unique set of line segment flows **x** and passenger O-D trips **q** can be obtained by solving the lower-level optimization problem (11)-(17). Let M(·) denote this one-to-one correspondence. The formulation of the simultaneous estimation problem is

$$\min_{\beta,\theta,\mathbf{Q}} \gamma_1 \cdot F_1\left(\mathbf{x},\mathbf{x}\right) + \gamma_2 \cdot F_2\left(\mathbf{q},\mathbf{q}\right) \tag{18}$$

subject to

$$\left(\mathbf{x},\mathbf{q}\right) = M\left(\beta,\theta,\mathbf{Q}\right) \tag{19}$$

$$Q^q \geq 0 \quad \forall p \in P \tag{20}$$

$$\beta,\theta \geq 0 \tag{21}$$

The objective functions F_1, F_2 consist of certain metric distant measurements. γ_1, γ_2 are the weights assigned to these distant measurements. The above objective function (18) can be adapted to any available information if one or some of these target flows are not available.
Generalized Least Square (GLS) estimators are commonly employed as the distant measures. If the dispersion matrix of the passenger counts is W and the priori passenger O-D trip matrix has an error with variance-covariance matrix Z, the GLS estimator is

$$\min_{\beta,\theta,\mathbf{Q}} \frac{\gamma}{2} \cdot \left(\mathbf{x}-\mathbf{x}\right)^T W^{-1}\left(\mathbf{x}-\mathbf{x}\right) + \frac{1-\gamma}{2} \cdot \left(\mathbf{q}-\mathbf{q}\right)^T Z^{-1}\left(\mathbf{q}-\mathbf{q}\right)$$

and subject to the same set of constraints as (18). The variance-covariance matrices W and Z are commonly assumed to be unit matrices for simplicity or lack of such information.

3.2 Sensitivity-based algorithm

A solution algorithm based on sensitivity analysis is adapted to solve the simultaneous estimation problem (Yang 1995). We describe as follows the explicit expressions of the derivatives of model variables with respect to perturbations of input variables and parameters of the nested logit model, on the basis of work of Yang and Chen (2009). The explicit expressions of the derivatives are developed as below, and then it follows with the presentation of the solution algorithm.
The equilibrium flow variables of the activity-based model satisfy the KKT conditions of optimization problem (11)-(15). Since all variables in the activity-based model should be positive at equilibrium, there is no need to consider the nonnegative constraints (14)(15). The equilibrium flow variables are characterized by the following equations

$$\frac{\partial L}{\partial f_{hj}^p} = t_{hj}^p + \frac{1}{\theta} \cdot \left(\ln\left(f_{hj}^p / g_h^p\right) + 1\right) - \mu_h^p = 0 \quad \forall\, j \in J_h^p, h \in H^p, p \in P \tag{22}$$

$$\frac{\partial L}{\partial g_h^p} = -\frac{1}{\theta} + \frac{1}{\alpha \cdot \beta} \cdot \left(\ln g_h^p + 1\right) - \frac{1}{\alpha} \cdot V_h^p + \mu_h^d - \nu^p = 0 \quad \forall h \in H^p, p \in P \tag{23}$$

$$\frac{\partial L}{\mu_h^p} = g_h^p - \sum_j f_{hj}^p \quad \forall h \in H^d, p \in P \tag{24}$$

$$\frac{\partial L}{\partial \nu^p} = Q^p - \sum_h g_h^p \quad \forall p \in P \tag{25}$$

where μ_h^p and v^p are the Lagrangian multipliers of conversation constraints (12) and (13).
The Jacobian matrix of equations (22)(23)(24)(25) with respect to the vector of flow variables and the Lagrangian multipliers, $\mathbf{y}$ = ($\mathbf{f}$, $\mathbf{g}$, $\boldsymbol{\mu}$, $\mathbf{v}$), is denoted as $J_{\mathbf{y}}$. The Jacobian matrix of equations (22)(23)(24)(25) with respect to the decision variables of the upper-level problem, $\boldsymbol{\varepsilon}$ = (β, θ, $\mathbf{Q}$), is denoted as $J_{\boldsymbol{\varepsilon}}$. In other words, $\boldsymbol{\varepsilon}$ is the perturbation of the lower-level problem.
Using the sensitivity analysis method described in (Yang and Chen 2009), we obtain the gradient of $\mathbf{y}$ with respect to $\boldsymbol{\varepsilon}$

$$\nabla_{\boldsymbol{\varepsilon}} \boldsymbol{y} = -J_{\mathbf{y}}^{-1} \cdot J_{\boldsymbol{\varepsilon}} \tag{26}$$

Using the gradient (26), the implicit function, $\mathbf{y}(\boldsymbol{\varepsilon})$, can be linearly approximated at point $\boldsymbol{\varepsilon}^0$

$$\mathbf{y}(\boldsymbol{\varepsilon}) \approx \mathbf{y}\left(\boldsymbol{\varepsilon}^0\right) + \nabla_{\boldsymbol{\varepsilon}} \mathbf{y}\Big|_{\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^0} \cdot \boldsymbol{\varepsilon} \tag{27}$$

Then the bilevel problem is approximated as a quadratic programming problem, which can be solved efficiently as follows.

1. Suppose that an initial vector of passenger number $\mathbf{Q}^0$ is given. The initial value of β, θ is set to any positive numbers. Set n ← 0.
2. Solve the lower-level problem based on current solution $\boldsymbol{\varepsilon}^n$ to find the equilibrium flow variables $\mathbf{x}^n$, $\mathbf{q}^n$, $\mathbf{f}^n$, $\mathbf{g}^n$. Save the data necessary for calculating the Jacobian matrices $J_{\mathbf{y}}$ and $J_{\boldsymbol{\varepsilon}}$.
3. The gradient of $\mathbf{y}$ with respect to $\boldsymbol{\varepsilon}$ is calculated by using equation (26).
4. Based on the linear approximation (27), the upper-level problem is solved to obtain a new solution $\boldsymbol{\varepsilon}^{n+1}$.
5. Check convergence criteria. If so, stop; otherwise, n ← n+1, return to step 2.

The input data are the observed passenger counts and target passenger O-D matrix. The decision variables are the estimated number of passengers $\mathbf{Q}$ and the parameters β, θ.

4. NUMERICAL EXAMPLES

A small transit network is designed to demonstrate the application of the proposed model and solution algorithm for simultaneous estimation of passenger O-D matrix and calibration of the parameters of the activity-based model (see Figure 5).
In this example, there are three activity patterns for passengers commuting between workplace W and home H in the evening rush hours (as shown in Table 2). Each activity pattern is associated with one or more trip chains. The actual numbers of passengers per hour with respect to the three activity patterns (given in Table 2) are 600, 1000, and 400, while the true parameters of the activity model are assumed as θ = 0.20, β = 0.10.

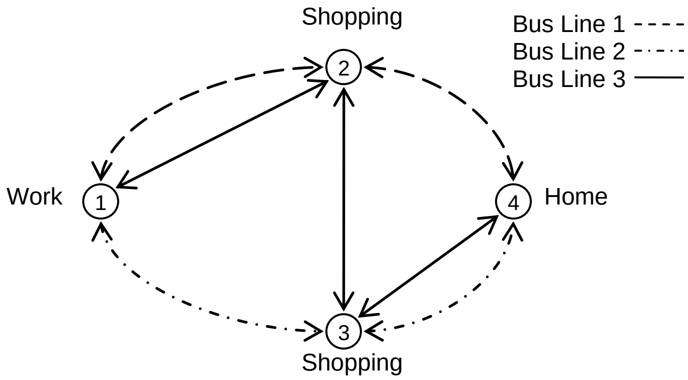

Figure 5 The example network with 3 transit lines

Table 2 Activity patterns and trip chains

Activity patterns	Trip chains	
W – H	① – ④	
W – S – H	① – ② – ④	① – ③ – ④
W – H – S – H	① – ④ – ② – ④	① – ④ – ③ – ④

There are 3 transit lines in this example network. Table 3 gives the basic input data of transit lines in the example network. The vehicle capacity is assumed to be 120 passengers per vehicle for all transit vehicles. The headways of the three transit lines are 10 min, 10 min and 12 min, respectively. The parameters of the generalized travel time function (3) is set to $n = 4$, $\rho = 0.15$.

Table 3 Travel time of transit line segments

Line segment	(1, 2)	(1, 3)	(2, 3)	(2, 4)	(3, 4)
Travel time (min)	12	15	5	5	5

The actual O-D matrix and the passenger counts are obtained by assigning the activity patterns onto network with the activity-based transit equilibrium model. The prior O-D matrix is obtained by introducing errors to the actual passenger O-D matrix. The actual passenger O-D matrix and the prior O-D matrix are shown in Table 4 and 5.

Table 4 Actual passenger O-D matrix

From\To	1	2	3	4	Total
1	0	562	438	1000	2000
2	0	0	0	682	682
3	0	0	0	718	718
4	0	120	280	0	400
Total	0	682	718	2400	3800

Table 5 Prior passenger O-D matrix

From\To	1	2	3	4	Total
1	0	500	500	1050	2050
2	0	0	0	650	650
3	0	0	0	750	750
4	0	150	250	0	400

Total	0	650	750	2450	3850

In general, prior O-D matrix and passenger counts are the information available for estimation of updated O-D matrix and calibration of the parameters in the nested logit-based activity model. The utility of activity is set to zero due to the lack of socioeconomic information. The equivalent utility of unit travel time α is 2.0. The GLS estimator with unit variance-covariance matrices is adopted in this example and the weight γ is set to 0.8.
In total, there are 17 line segments in this example network as listed in Table 6. Some of the line segments in the transit network do not carry passenger flows (e.g. line segment (2, 1) and (3, 1)), and therefore, they are omitted in Table 6. Four sets of passenger counts are given in Table 6 and used for assessing the performance of the proposed model. These sets of passenger counts are different in the counting locations or the number of counting sites.

Table 6 Passenger counts associated with the actual O-D matrix in Table 4

Line	Line segment	Passenger counts			
		Set 1	Set 2	Set 3	Set 4
1	(1, 2)	–	–	–	408.4
	(1, 4)	–	–	–	263.2
	(2, 4)	448.9	–	448.9	448.9
	(4, 2)	–	73.8	73.8	73.8
2	(1, 3)	–	–	–	275.7
	(1, 4)	–	–	–	263.2
	(3, 4)	–	482.7	482.7	482.7
	(4, 3)	–	–	–	173.7
3	(1, 2)	340.4	–	340.4	340.4
	(1, 3)	–	–	–	229.8
	(1, 4)	–	–	–	219.3
	(2, 3)	–	–	–	69.5
	(2, 4)	374.0	–	374.0	374.0
	(3, 2)	–	–	–	8.3
	(3, 4)	–	–	–	402.3
	(4, 2)	–	–	–	61.5
	(4, 3)	–	144.7	144.7	144.7

Each set of these passenger counts is used as input for the O-D trip matrix estimation in the example network. The results estimated by the trip-based model will be compared against that of the activity-based model proposed in this paper. To evaluate the accuracy of the estimation, the sum of the squared deviations between the estimated O-D matrix **q** and the actual O-D matrix $\bar{\mathbf{q}}$ (shown in Table 4) is adopted as an index

$$S = \left(\mathbf{q}-\bar{\mathbf{q}}\right)^T \cdot \left(\mathbf{q}-\bar{\mathbf{q}}\right) \tag{28}$$

When the sum of the squared deviations is smaller, the estimated O-D matrix is then closer to the actual one. As shown in Table 7, the activity-based model performs better than the trip-based model for all the four sets of passenger count data. The S values of Set 1 and 2 indicate that the counting locations have significant effects on the accuracy of the O-D estimation. Similarly, it was found from the S values of Set 2 and 3 that the number of counting sites also plays an important role in the O-D estimation as well. Finally, if passenger counts are available on all line segments within the example network, the O-D estimation can achieve a very high level of accuracy (refer to the S values of Set 4).

Table 7 The sums of the squared deviations for different sets of passenger counts

Lower-level problems	Passenger counts			
	Set 1	Set 2	Set 3	Set 4
Activity-based model	6984.1	5061.8	4681.0	2365.0
Trip-based model	14106.1	12737.1	10071.1	3236.3

The following paragraphs offer the detailed estimation results for the third set of passenger counts. Both the activity patterns and passenger O-D matrix are estimated. The activity/travel choice behaviour behind passenger O-D matrix estimation is shown in the example as well.
The activity patterns are estimated by using the activity-based transit network equilibrium model. Table 8 presents the estimated number of passengers with different activity patterns and the total travel times associated with these activity patterns. There are 642 passengers returning home directly from workplace. The passengers with activity pattern W – S – H tend to prefer shopping location ○,2 than ○,3, but the passengers with activity pattern W – H – S – H prefer to do shopping at ○,3. The reason is that the travel time for shopping at ○,2 is shorter than shopping at ○,3 in the former activity pattern while the absolute difference of travel times for shopping at these two locations is reversed in the latter case.

Table 8 Estimated number of passengers with different activity patterns

Activity patterns	Trip chains	Number of passengers	Total travel time (min)
W – H	○,1 – ○,4	642	21.5
W – S – H	○,1 – ○,2 – ○,4	534	30.0
	○,1 – ○,3 – ○,4	455	31.3
W – H – S – H	○,1 – ○,4 – ○,2 – ○,4	150	47.0
	○,1 – ○,4 – ○,3 – ○,4	258	42.7

The passenger O-D matrix is estimated by using two different lower-level models. With the activity-based model in the lower level, the following results are obtained and displayed in Table 9. Note that the number of trips originating at ○,2 and ○,3 is equal to the number of trips terminating at these two intermediate destinations, because in the evening rush hours that each activity pattern begins at workplace and ends at home (or residential location).

Table 9 Passenger O-D matrix estimated by the activity-based model

From\To	1	2	3	4	Total
1	0	533.5	455.1	1049.6	2038.2
2	0	0	0	683.8	683.8
3	0	0	0	712.6	712.6
4	0	150.3	257.5	0	407.8
Total	0	683.8	712.6	2446.0	3842.4

$\theta = 0.1984$, $\beta = 0.0634$

However, when the trip-based SUE assignment is used in the lower level, inconsistencies are found in the results of the estimated passenger O-D matrix, even if the passenger counts are accurate and consistent. Table 10 shows the passenger O-D matrix estimated by the trip-based model. The number of trips originating at ○,3 is 746.6, while the number of trips terminating at ○,3 is 766.8. This means that there are 20.2 passengers entering into ○,3 for shopping but without leaving it. It is not consistent with the three typical activity patterns in the evening rush hours and this inconsistency introduces significant errors into the estimated passenger O-D matrix.

Table 10 Passenger O-D matrix estimated by the trip-based model

From\To	1	2	3	4	Total
1	0	499.7	501.1	1049.6	2050.4
2	0	0	0	649.8	649.8
3	0	0	0	746.6	746.6
4	0	137.1	265.7	0	402.8
Total	0	636.8	766.8	2446.0	3849.6

$\theta = 0.2221$

Both the activity-based model and the trip-based model are used to estimate the dispersion parameter for path choice θ. When using the former model, the error between the estimated value and the actual value of the parameter is 0.0016, which is smaller than the error by using the latter model, 0.0221. To be consistent with the random utility theory, the dispersion parameter for path choice θ needs to be greater or equal to the dispersion parameter for trip chain choice β. This constraint is not explicitly considered in the estimation problem, but the estimated values turn out to satisfy this constraint, i.e. θ (=0.1984) > β (=0.0634).

5. CONCLUSIONS

In this paper, an activity-based transit network equilibrium model is proposed to capture the passenger's activity and travel choice behaviour in transit network. The interaction between the choice of trip chain and path travel time is explicitly considered. The proposed model has several special features for simultaneous estimation of passenger O-D matrix and calibration of activity choice model parameters. Firstly, the nested logit model is used for the combined activity/travel choice. Hence, the results are consistent with the random utility theory. Secondly, the proposed model can be successfully formulated as an equivalent convex optimization problem. As such, the uniqueness and the equivalence of the model solution can be proved easily. Thirdly, a sensitivity-based solution algorithm can be adapted for simultaneously estimating the passenger O-D trip matrices and calibrating the parameters of the nested logit model. Finally, the resultant passenger O-D trip matrices are consistent with the trip-chaining behaviour captured by the activity-based model while the model parameters can also be updated simultaneously using the available passenger counts.

Further work should be conducted to consider the impact of crowding effects at activity destinations on the activity/travel choice and the optimal locations of counting sites. Further study is also required to improve the computation of the partial derivatives of the decision variables with respect to perturbations of the lower-level problem. In addition, over 90 percent of transit passengers are using smart cards (so called the Octopus card) for payment of transit fares in Hong Kong. When the smart card data over a week or month can be available, it is feasible to estimate easily the hourly average passenger flow by transit line and activity pattern as the activity patterns of the same passenger can be extracted for updating the pre-specific activity patterns.

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Notations

N: set of activity destinations
W: set of origin-destination (O-D) pairs rs for trips
P: set of activity patterns
H^p: set of trip chains associated with activity pattern p, $p \in P$
J_h^p: set of paths connecting each activity destinations in trip chain h, $h \in H^p$, $p \in P$
A: set of route sections
L_a: set of line segments in route section a, $a \in A$

$\delta_{phj,a}$: if route section a is on path j of trip chain h, $\delta_{phj,a}$ = 1; otherwise $\delta_{phj,a}$ = 0
$\sigma_{ph,n}$: if activity destination n is included in trip chain h, $\sigma_{ph,n}$ = 1; otherwise $\sigma_{ph,n}$ = 0
$\lambda_{ph,rs}$: if rs are adjacent activity destinations in trip chain h, $\lambda_{ph,rs}$ = 1, otherwise $\lambda_{ph,rs}$ = 0
x_l: estimated passenger flows on line segment l, $\mathbf{x} = (\ldots, x_l, \ldots)$
$\hat{x}_l$: observed passenger flows on line segment l, $\hat{\mathbf{x}} = (\ldots,\hat{x}_l, \ldots)$
κ_l: the passenger capacity of line segment l
t_l: generalized travel time on line segment l
z_l: frequency of transit line segments l
t_a: total travel time of route section a
t_{hj}^p: generalized travel time on path j of trip chain h
P_h^p: the marginal probability of choosing trip chain h
P_{hj}^p: the probability of travelling on path j of activity h
Q^p: estimated number of passengers with activity pattern p, $\mathbf{Q} = (\ldots, Q^p, \ldots)$
g_h^p: estimated number of passengers choosing trip chain h, $\mathbf{g} = (\ldots,g_h^p, \ldots)$
f_{hj}^p: estimated passenger flows on path j of trip chain h, $\mathbf{f} = (\ldots,f_{hj}^p, \ldots)$
q^{rs}: estimated passenger trips between O-D pair rs, $\mathbf{q} = (\ldots, q^{rs}, \ldots)$
$\hat{q}^{rs}$: priori passenger trips between O-D pair rs, $\hat{\mathbf{q}} = (\ldots,\hat{q}^{rs}, \ldots)$
θ: the dispersion parameter for path choice
β: the dispersion parameter for trip chain choice