OPTIMIZATION OF PRICING POLICY FOR LOW-CARBON-ORIENTED MULTI-MODAL URBAN PASSENGER TRANSPORT SYSTEM

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ABSTRACT

Multilevel optimization programming problem is developed to obtain optimal pricing scheme for low-carbon-oriented multi-modal urban passenger transport system. The model system provides least generalised cost to realise the target modal split in the upper level, while the target modal split is determined in the middle level under the constraints of the limitations of carbon emission, energy consumption, and government investment at the lowest environmental cost. The lower level is the joint traffic flow assignment models of automobile and bus. The workability of the model system is confirmed by a hypothetical simple numerical example.

Keywords: multilevel programming problem, modal share, carbon emission, energy constraint

INTRODUCTION

Transport emissions, energy consumption, and traffic congestion have become crucial problems in many cities, especially under the rapid urbanization occurring in developing

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countries. It has been highlighted to how to reduce the carbon emission in the urban transportation system, because in most cases, it is more difficult to meet the limitation of carbon emission than to meet the limitation of capacity of the network. The most effective solutions are considered to be changing the passenger transport mode share, developing public transport systems, and controlling the desire to use private cars (Kenworthy et al. 1992). On the other hand, the use of urban passenger transport systems is directly affected by the user's travel mode choice behaviour, which, in turn, is strongly influenced by the travel cost (May et al. 2000).

Traditional pricing optimization studies usually concentrate on the effect of road pricing on the road networks. Furthermore, they focus only on the road tolls for automobiles as a means of alleviating traffic congestion. The public transport system usually is not considered, and neither is the relationship between the eco-factors and congestion. In addition, implementation is affected by the skill of the implementers and the uncertainty of social acceptance. Studies on transport environment conducted by Carslaw and Beevers (2002) and Shrestha et al. (2005) have only focused on the supply side regardless of the users, and have not considered the changes in travel demand that could occur due to changes in transport service pricing resulting from changes in the costs of the services. Furthermore, these studies usually have not considered the costs of travel time and congestion. In fact, subways and traditional vehicles can show significant differences in travel time and congestion level. Moreover, flow variations in the network can influence carbon emission factors. Eco-city design principles have been highlighted in studies by Register (1987), Roseland (1997) etc., but they do not propose specific methods for applying these principles to urban transport system.

This paper aims to find an optimal mode share scheme for urban passenger transport system that meets the limitations of carbon emission, energy consumption, and government investment at the lowest environmental cost (EC). This optimal mode share scheme could be used as a suggested modal-split target for a low carbon eco-city. This paper also proposes a method for changing traveler's mode-choice behaviour by applying an optimal pricing policy to each traffic mode. This method meets the former modal-split target at the lowest general cost (GC) for the networks.

METHODOLOGY

Multiple decision-makers interact in the transport systems, and Stackelberg game theory can be used as the modelling approach to represent such an interaction (Benson 1989; Vicente et al. 1996; Feng et al. 2010). These decision-makers are present in different layers of the overall decision-making system. The middle layer, consisting of government or transport system management agencies, decides what kind of modal split should be achieved under the eco-constraints. The resulted modal split becomes the constraint for the layer below. In this lower layer, the users of the transport system also can implement decision-making rights to choose a preferred travel mode as well as a preferred travel route on the network according to the generalized travel cost. Obviously, the users have various issues to

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consider during their decision-making process. Furthermore, the pricing policy is made by the upper layer, also consisting of government or transport system management agencies, to make sure the target modal share would be realized by the user's mode-choice behaviour. As a result, the transaction is handled by different modelling sections, as described below.

Optimal share of low-carbon-oriented passenger transport

The various modes clearly generate different levels of carbon emission and consume different levels of energy (Penic and Upchurch 1992). Thus we can calculate the optimal modal split when considering the environmental impact of different traffic modes. On the other hand, the different traffic modes cost different construction fee. Basically, it is very difficult to describe and estimate every eco-footprints in one modelling, so the effects of transport on the eco-system could be generalized by how much money could be spent on the transport system. Moreover, the money which could be used on a city's transport by the government isn't limitless neither. So the money constraint should be an investment constraint or an environmental capacity constraint. These two aspects should be the main factors for the government to consider when planning an optimal low-carbon-oriented modal share.

Objective function

The objective of the model is to help the government determine an optimal modal share scheme for the urban passenger transport system that meets environmental constraints for carbon emissions, energy consumption and governmental investment at the lowest environmental cost. The total cost includes carbon emission treatment costs for various types of emissions (e.g., CO, CO₂), energy prices for various types of fuel consumption (e.g. gasoline, diesel, LPG, CNG, and electricity), and the construction costs for the different transport modes. The objective function can be expressed as follows.

$$\min \sum_{i} \sum_{b} \sum_{k} P_{ki}^{G*} x_{ik}^{b} F^{G}(\overline{V_{i}^{b}}) + P_{ki}^{E*} x_{ik}^{b} F^{E}(\overline{V_{i}^{b}}) + x_{ik}^{b} d_{ik}^{b}$$
(1)

Where $P_{i_i}^{G^*} x_{i_k}^b F^G(\overline{V_i^b})$ is the cost of carbon emission price according to emission market, $P_{i_i}^{G^*}$ is the cost of carbon emission price per passenger kilometre for mode *i* and emission type *k*, $x_{i_k}^b$ is passenger kilometre of mode *i* and energy type *k* in zone *b*, $F^G(\overline{V_i^b})$ is carbon emission factor for mode *i* and energy type *k* in zone *b*, it is a function of the average speed $\overline{V_i^b}$ in zone *b* (Barth et al. 1996, Pattas 1994, Nagurney 2000). The function of carbon emission factor is given as

$$F^{G}(\overline{V_{i}^{b}}) = \frac{\alpha_{i}^{k} \exp(\beta_{i}^{k} \overline{V_{i}^{b}})}{\gamma_{i}^{k} \overline{V_{i}^{b}}}$$
(2)

where α_i^k , β_i^k , γ_i^k are parameters for carbon emission factor of mode *i* and energy mixed type *k* from regression method, and the emission factor for every mode is calculated under the consideration of average vehicle with coefficient of this mode (US EPA, 1994).

Similarly, $P_{ki}^{E*} x_{ik}^{b} F^{E}(\overline{V_{i}^{b}})$ is the cost of energy. λ_{i}^{k} , ψ_{i}^{k} , ξ_{i}^{k} are parameters for energy consumption factor of mode *i* and energy type *k* (US EPA, 1994). They are also calculated under the consideration of average vehicle with coefficient of this mode. $x_{ik}^{b} d_{ik}^{b}$ is construction fee for different traffic mode in different district with using different energy. d_{ik}^{b} is construction fee of per passenger kilometre of mode *i* and energy mixed type *k* in zone *b*. The function of energy consumption factor is given as

$$F^{E}(\overline{V_{i}^{b}}) = \frac{\lambda_{i}^{k} \exp(\xi_{i}^{k} V_{i}^{b})}{\psi_{i}^{k} \overline{V_{i}^{b}}}$$
(3)

Carbon emissions constraint

Carbon emission by passenger transport in the planning year must be less than or equal to the upper limit G_k , which is decided by the environmental consideration or by national/local environmental regulations.

$$\sum_{i}\sum_{b}x_{i}^{b}F^{G}(\overline{V_{i}^{b}}) \leq G_{k}$$
(4)

Where $\overline{V_i^b}$ is average speed in zone *b* of mode *i*. The average speed can be obtained from the lower level of modeling structure, which is introduced in the next section.

Energy consumption constraint

Similarly, energy consumption of type k by passenger transport in the planning year must be less than or equal to the upper limit E_k , which is decided by the national/local energy consumption strategy.

$$\sum_{i}\sum_{b}x_{i}^{b}F^{E}(\overline{V_{i}^{b}}) \leq E_{k}$$
(5)

Investment constraint

Investment on transport facilities of different modes should be less or equal to the upper limit *I*, which is decided by a specific situation of a given city.

$$\sum_{i}\sum_{b}\sum_{k} x_{ik}^{b}d_{ik}^{b} \leq I$$
(6)

Other constraints

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The total supply of all modes of urban passenger transport should be equal to the demand forecast for planning year D, and the supply of mode i in zone b should also be less than or equal to the upper limit S_{iMax}^{b} of the passenger demand that can be offered by mode i in zone b in the planning year. At the same time, in order to avoid wasting existing equipment and facilities of zone b, the supply of mode i in zone b should also be greater than or equal to the lower limit S_{iMin}^{b} of the passenger demand that must be offered by the mode in the planning year.

$$\sum_{i} \sum_{k} \sum_{b} x_{ik}^{b} = D$$

$$S_{iMin}^{b} \le x_{i}^{b} \le S_{iMax}^{b}$$
(7)

As a result, the model can be rewritten as follows.

$$\min \sum_{i} \sum_{b} \sum_{k} P_{ki}^{G*} x_{ik}^{b} F^{G}(\overline{V_{i}^{b}}) + P_{ki}^{E*} x_{ik}^{b} F^{E}(\overline{V_{i}^{b}}) + x_{ik}^{b} d_{ik}^{b}$$

$$F^{G}(\overline{V_{i}^{b}}) = \frac{\alpha_{i}^{k} \exp(\beta_{i}^{k} \overline{V_{i}^{b}})}{\gamma_{i}^{k} \overline{V_{i}^{b}}}, F^{E}(\overline{V_{i}^{b}}) = \frac{\lambda_{i}^{k} \exp(\xi_{i}^{k} \overline{V_{i}^{b}})}{\psi_{i}^{k} \overline{V_{i}^{b}}}$$

$$\begin{cases} \sum_{i} \sum_{b} x_{i}^{b} F^{G}(\overline{V_{i}^{b}}) \leq G_{k} \\ \sum_{i} \sum_{b} x_{i}^{b} F^{E}(\overline{V_{i}^{b}}) \leq E_{k} \\ \sum_{i} \sum_{b} \sum_{k} x_{ik}^{b} d_{ik}^{b} \leq I \\ \sum_{i} \sum_{b} \sum_{k} x_{ik}^{b} d_{ik}^{b} \leq I \\ \sum_{i} \sum_{k} \sum_{b} x_{ik}^{b} = D \\ S_{iMin}^{b} \leq x_{i}^{b} \leq S_{iMax}^{b} \end{cases}$$

$$(8)$$

Optimal pricing scheme of passenger transport system realizing the modal share

In general, the model can consider the main travel modes in a city, for example, pedestrian, bicycle, private automobile, taxi, bus, urban rail transit, and motorcycle. Since the pedestrian and bicycle modes are not motorized, they clearly differ from the other passenger transport modes in terms of travel cost. Furthermore, it is not realistic for a city to have an extreme type of motorized-vehicle transport system that does not allow pedestrians and bicycles. Therefore, certain minimum stocks for pedestrians and bicycles can be set according to the study of 45 international cities by Kenworthy et al (1999). In this study, just two traffic modes are considered to test the workability of the model structure, they are automobile and bus.

Objective function

The objective function of the pricing-scheme optimization model is to achieve an optimal pricing policy for different traffic modes—at least in terms of the general cost (GC) of the

networks—that meets the modal split target obtained from the model of the optimal lowcarbon-oriented passenger transport modal share. The decision variable is P^* , the pricing level of the selected mode, in this study, including that of bus system and energy tax. The objective function can be expressed as follows.

$$\min\sum_{n} GC(P^*TT, P^*D, P^*T)$$
(9)

where P*TT is the total cost of travel time of all the users in the network, P*D is the total cost of energy consumption of all the users in the network, and P*T is total price of public transport tickets all users pay in the network.

Constraints

$$P_{in} = \frac{X_i}{\sum X_i} A_i = \frac{e^{V_{in}}}{\sum_{j \in A_n} e^{V_{jn}}}$$

$$U_{in} = W_{in} + \varepsilon_{in}$$

$$P^*T \ge B_{asic} P^*T$$
(10)

And when the multinomial logit approach is used to formulate utility functions, the utility functions for motobile (U_{n3}) and bus (U_{n5}) are as follows.

$$U_{n3} = W_{n3} + \varepsilon_3 = \theta_1 TT + \theta_2 \left(P^{G^*} + P^{GT^*} \right)_k D + \theta_6 I + \varepsilon_3$$

$$U_{n5} = W_{n5} + \varepsilon_5 = \theta_1 TT + \theta_5 T + \theta_6 I + \varepsilon_5$$
(11)

Where *TT* is the travel time, $(P^{G^*} + P^{GT^*})_k D$ is the distance price, which equals the energy price, P^{G^*} , plus the energy tax price, P^{GT^*} , for fuel mix type *k*, *T* is ticket price, *I* is the income. θ_1 , θ_2 , θ_5 , θ_6 are parameters, and ε_1 , ε_2 are error terms following Gumbel distribution. Ticket price is also constrained so as to be equal to or higher than the minimum unit price for the network.

$$P^*T \ge B_{asic}P^*T \tag{12}$$

Therefore, the model can be rewritten as follows:

$$\min \sum_{n} GC(P^*TT, P^*D, P^*T)$$

$$s.t.\begin{cases}
P_{in} = \frac{x_i}{\sum x_i} A_i = \frac{e^{V_{in}}}{\sum_{j \in A_n} e^{V_{jn}}} \\
U_{in} = W_{in} + \varepsilon_{in} \\
P^*T \ge B_{asic} P^*T
\end{cases}$$

(13)

Joint traffic flow assignment on the network

Classic UE equilibrium based automobile traffic flow assignment model and Non-UE transit flow assignment method, that means the flow assignment of transit line is according to the transit vehicle departure frequency, are jointly used to obtain traffic flow on every link (Sheffi 1985). Objective function of UE equilibrium can be expressed as:

$$\min_{y} \sum_{a} \int_{0}^{y_{a}} t_{a}(y) dy$$

$$\int_{k} \sum_{k} f_{k}^{rs} = \frac{x_{3}}{\sum_{i} x_{i}} D^{rs}, \quad \forall r, s$$

$$f_{k}^{rs} \ge 0, \quad \forall k, r, s$$

$$y_{a} = \sum_{r} \sum_{s} \sum_{k} f_{k}^{rs} \delta_{a,k}^{rs}$$
(12)

where, for the automobile, y_a is traffic flow of automobile on link *a*, t_a is travel time on link *a*, f_k^{rs} is traffic flow on route *k*. Mean while, we can obtain the average speed $\overline{V_3^b}$ of zone *b* as an output by:

$$\overline{V_3^b} = \frac{VMT_3^b}{VHT_3^b} = \frac{\sum_{a \in b} l_a y_a}{\sum_{a \in b} t_a y_a}$$
(13)

Where VMT_i^b is vehicle miles travelled in zone *b* of mode *i*, VHT_i^b is vehicle hours travelled in zone *b* of mode *i*. l_a is length of link *a*. They can be received from the lower level of modelling structure, which is introduced in the next section. And it is assumed that the

average speed of mode bus $\overline{V_5^b}$ equal to $0.65\overline{V_3^b}$ in the same zone. Here the parameter of 0.65 is just a temtative value, so it could be adjusted according to specific situation of a certain city.

FRAMEWORK OF MULTI-LEVEL MODELING

Designing a multi-mode passenger transport system using pricing scheme optimization is the first level of the model structure. In this level, the government and public agencies are tied together using pricing policy because pricing policy is set by managers. Pricing policy directly influences the mode choice behaviour of users. The optimal eco-oriented passenger transport modal share is the second level of the model structure and reflects government's considerations regarding the type of mode-share scheme that should be a target for the pricing policy. The first and second levels form the main body of the multi-level model. The third level is joint traffic flow assignment on the network. It supplements the second level and reflects user considerations. Calculations begin with this level using actual data for actual urban situations, and there is a loop between the second and third levels. A change in mode share would influence the traffic flow assignment, which would produce new speed factors at every link and, consequently, new emission factors, thereby creating a new optimal mode-share scheme. The difference between sequential iterations should gradually decrease until a convergence criterion is satisfied.

The resulting optimisation procedure is shown in Figure 1.



Figure 1. The resulting optimisation procedure

ALGORITHM FOR MULTI-LEVEL MODELLING

- 1. Step one. Calculate the initial values for the carbon emissions factors using the assignment results of the multi-modal network, which is based on OD data and the current modal-split for the city.
- 2. Step two. Apply the carbon emissions factors to the second level of the model and use the resulting initial optimal-modal split as input for the third level of the model.
- 3. Step three. Use the modal-split results from step 2 to again compute the UE assignment on the multi-modal network and compute new emissions and energy-consumption factors for the next loop.
- 4. Step four. If the objective function of the second level does not improve, go to step 5. Otherwise, repeat from step 2.
- 5. Step five. Calculate A_i from the performance of multi-modal network of the last loop. Use the modal split results from step 2 and the assignment results from step 3 to solve the disaggregate choice model of the first level and obtain the optimal pricing plan.

A NUMERICAL EXAMPLE

A hypothetical simple network is used to test the workability of the proposed modelling approach. For the sake of simplicity, the network has only two modes—automobile and bus—and two links, as shown in Figure 2 below. Moreover, only CO and gasolion are considered as emission and fuel.



Figure 2. A simple network

Input for calculation

In this network, the flow assignment of the bus line is determined by the bus departure frequency. The following data is given as the input for calculation:

- Existing OD flow between r and s is Q_{rs}^1 and equal to 100
- Current modal split (*Auto:Bus*)¹ is 3:2
- Link performance function is $t = 10 + 0.2 y_a^2$
- Bus departure frequency ratio f_{bus1} : f_{bus2} = 1:3
- Investment intensive $d_5 = 1.7d_3 = 17$ monetary unit
- CO emissions factor for automobile and bus are $a_{1,3} = 8a_{1,5} = 3.3963 \frac{e^{0.01456\overline{V}_3}}{10000\overline{V}_3}$ (Yafeng et

al. 1999)

- $\theta_1 = -0.006958$, $\theta_2 = -0.000187$, $\theta_5 = -0.000232$, $\theta_6 = 0.000036$, *IP* = 2468 monetary unit. They came from past actual projects of Beijing city (Jiao and Lu 2005)
- Forecasted OD flow between *r* and *s* in the planning year is $Q_{rs} = 120$, passenger kilometre of passenger overturn is D = 30.
- The planning year's G_1 is equal to $0.8G_1^0$, E_1 is equal to $0.8E_1^0$, I is equal to I^0 , G_1^0, E_1^0, I^0 are the CO emission quantity, energy consumption quantity and transport invest of the existing situation, so they are calculated from the associated existing data.
- Threshold for the ending of the loop between level two and level three is set as that when the improvement of EC of level tow is less than or equal to 2%.

Results of calculation

The iterative calculation between the second level and third level got convergence after three iterations, and the optimised modal split meeting with the eco-constrains at the planning year is $(Auto:Bus)^4 = 43:57$ Output at each interation is shown in Table 1. The results indicate that the modal split of automobile and bus in the planning year should shit from current 60:40 to 43:57, with the CO₂ mitigation to 80% of current level and reduction of energy consumption to 80% of current level, as well as with no increase of government investment for transport construction.

	Q_{rs}	$\overline{V_3}$	Auto:Bus	EC	EC improve rate
Existing situation	100	48.61	60:40	758142	
Iteration 1	120	50.92	49:51	697263	8.03%
Iteration 2	120	51.21	45:55	675020	3.19%
Iteration 3	120	51.47	43:57	663004	1.78%

Table1. Output of the optimization at each interation

The decision variable in the first level is the pricing scheme for bus tickets and the energy tax for gasoline. To meet the modal share target of 43:57, the optimized price policy is determined as follows.

- Bus ticket TP =1 unit of money per kilometre
- Energy tax DP = 1.3481 unit of money per kilometre
- Minimum GC = 49599

CONCLUSIONS AND FUTURE RESEARCH

Stackelberg game theory based multi-level modelling has the following advantages in the application to optimise the modal share and price policy for low-carbon-oriented urban passenger transport system:

- Different objectives can be analysed simultaneously during the decision-making process.
- Multi-value criteria for transport management and planning, usually by the government and users, could be treated as more realistic, and the interaction between them can be described properly.
- Modular structure of the modeling provides a good flexibility for every module and allows the entire structure to be applied in different cities with varying contexts.

One of the key factors of the modal share problem is the values of parameters of the model. It would be given more descriptions in the future research. Actually, the performance of a modal share strategy is very sensitive to variations in specification, and particularly to changes in service fares and charges for car use. Topics for future research include developing more complex and realistic examples and a more sensitive analysis. Since this study considered only the passenger transport modes, future studies also should consider freight transport options. And not surprisingly the optimal modal share strategy differs considerably between cities, and is dependent in particular on the inherited levels of infrastructure provision, subsidy for public transport, and congestion. All these also merit further research.

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REFERENCES

Barth, M., F. An, J. Norbeck and M. Ross (1996). Modal emission modeling: a physical approach. Transportation Research Record, 1520, 81-88.

Benson, H. P. (1989). On the structure and properties of a linear multilevel programming problem. Journal of Optimization Theory and Applications, 60, 353-373.

- Carslaw, D. C. and S. D. Beevers (2002). The efficacy of low emission zones in central London as a means of reducing nitrogen dioxide concentration. Transportation Research Part D, 7, 49-64.
- Feng, T., J. Zhang, A. Fujiwara and H. J. P. Timmermans (2010). An integrated model system and policy evaluation tool for maximizing mobility under environmental capacity constraints: A case study in Dalian City, China. Transportation Reasearch Part D, 15, 263-274.
- Kenworthy, J. R., F.B. Larbe, P. W. G. Newman, P. A. Barter, T. Raad, C. Poboon and B. Guia (1999). An International Sourcebook of Automobile Dependence in Cities 1960-1990. University of Colorado Press, Boulder.
- Kenworthy, J. R., P. W. G. Newman and T. J. Lyons (1992). The ecology of urban driving Imethodology. Transportation Research Part A, 26, 263-272.
- May, A. D. and M. Roberts (1995). The design of integrated transport strategies. Transport Policy, 2, 97-105.
- May, A. D., S. P. Shepherd and P. M. Timms (1999). The specification of sustainable urban transport strategies. International Journal of Sustainable Development and World Ecology, 6, 293-304.
- May, A. D., S. P. Shepherd and P. M. Timms (2000). Optimal transport strategies for European cities. Transportation, 27, 285-315.
- Nagurney A. (2000). Congested urban transportation netwoks and emission paradoxes. Transportation Research Part D, 5, 145-151.
- Pattas, K. N., N. A. Kyriakis and Z. C. Samaras (1994). Actual emissions of vehicles of the N1 category. Science of The Total Environment, 146, 191-199.
- Penic M. A. and J. Upchurch (1992). TRANSYT-7F, Enhancement for fuel consumption, pollution emissions and user costs. Transportation Research Record, 1360, 104-111.
- Jiao, P. and H. Lu (2005). Study on disaggregate model based on stated preference data. Journal of Highway and Transportation Research and Development. 22, 114-117 (in Chinese).
- Pi, X.-L., T. Yamamoto and T. Morikawa (2010). Optimization of mode share for low-carbon oriented urban passenger transportation system. Proceedings of The Seventh International Conference on Traffic and Transportation Studies (forthcoming).
- Register, R. (1987). Ecocity Berkeley: Building Cities for a Healthy Future, North Atlantic Books, Berkeley.
- Roseland, M. (1997). Dimensions of the future: an eco-city overview. In: Eco-City Dimensions: Healthy Communities, Healthy Planet (M. Roseland, ed.), pp. 1-12. New Society Publishers, Gabriola Island.
- Sheffi, Y. (1985). Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods. Prentice-Hall, Englewood Cliffs.
- Shrestha, R. M., G. Anandarajah, S. Adhikari, K. Jiang and Z. Songli (2005). Energy and environmental implications of NOx emission reduction from the transport sector of Beijing: a least-cost planning analysis. Transportation Research Part D, 10, 1-11.
- U.S. Environmental Protection Agency (1994). Office of Mobile Sources, User's Guide to MOBILE 5. EPA-AA-TEB-94-01.

- Vicente, L., G. Savard and J. Judice (1996). Discrete linear bilevel programming problem. Journal of Optimization Theory and Applications, 89, 597-614.
- Yin, Y. and H. Lu (1999). Traffic equilibrium problems with environmental concerns. Journal of the Eastern Asia Society for Transportation Studies, 3(6), 195-206.