EVOLUTIONARY FIRST-BEST ROAD PRICING SCHEME IMPLEMENTATION BASED ON STOCHASTIC TRAFFIC FLOW INFORMATION

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ABSTRACT

Traditionally, to implement the first-best road pricing scheme in a traffic network requires the information on the exact demand function or true origin-destination demand, which, however, is rarely available in practice. To overcome this dilemma, the trial-and-error method has been proposed to find the first-best pricing through an iterative process using the observed traffic volumes. This method guarantees the convergence of tolls and flows to the system optimal state based on the assumption of deterministic traffic conditions. However, in reality, it is very commonly seen that the travel demand and supply change from day to day that induces the variability of link flow and travel time. This paper aims to tackle the question that whether one can use the stochastic flow information to define the first-best marginal-cost toll. Meanwhile, an evolutionary implementation method that iteratively finds the optimal toll pattern according to the observed stochastic link flows is proposed. This algorithm only requests the statistical information of the observed link flows and travel time functions. The proof of the convergence of the iterative algorithm is given. The paper also analyzes the effect of the sampling error of the link flow data on the convergence of the algorithm and theoretically shows that the biases from the flow observation will not affect the convergence of the optimal toll and flow pattern. The numerical tests are provided for the illustration of the algorithm.

Keywords: road pricing, network uncertainties, stochastic network, traffic assignment, system optimum

INTRODUCTION

The fundamental principle of road pricing, based on the concept of marginal social cost, is to impose the tolls equivalent to the externality incurred by road users to realize the socially optimal flow pattern. For a single road, the optimal toll is shown to be equal to $v \cdot dt(v)/dv$, where v is the link flow and $dt(v)/dv$ is the derivative of the link travel time function. This principle is also proven to be applicable to the network case in which it is often referred to as the 'first-best' road pricing. To implement this ideal pricing scheme in a traffic network, the information on the exact demand function or true origin-destination (O-D) demand must be obtained. However, this is rarely available in practice in which most of the information on the demand function and O-D demand are often based on statistical estimations. In practice, on the other hand, the traffic volume data are observable and available in most cities. Li (1999, 2002) verified that the road pricing could go ahead on a trial-and-error basis without demand functions, which was first conjectured by Vickery (1993) and Downs (1993). The toll updating method developed by Li (2002) is just for a single road link, and it was soon extended to a general network by Yang et al. (2004), which proposed an efficient trial-and-error method to find the first-best pricing through an iterative process using the observed traffic volumes. The approach can guarantee the convergence of the tolls and flows to the system optimal state and successfully avoid the difficulty to acquire demand information. However, the method is based on the assumption of deterministic traffic environment. In reality, the travel demand and supply change from day to day that induces the variability of link flow and travel time. For instance, if one observe the link flow on a section of highway between a certain time of day and over a time period (e.g. one month), it is obvious that the link count data obtained will be random. In fact, the toll updating scheme for the ERP system in Singapore has recently been revised to consider the 85th percentile speed reflecting the need to include the flow and speed variability into the toll update scheme. To this end, the question is whether one can use this stochastic flow information to define the first-best toll.

The paper tackles this research question in which a trial-and-error method is again proposed to iteratively calculate the first-best marginal-cost toll based on the observable stochastic link flows. This paper postulates that the network uncertainty is caused by the stochastic day-to-day travel demand (e.g. Walting, 2002, Shao et al., 2006, Zhou and Chen, 2008). Hence the link flow and link travel time are both stochastic, too. Under this stochastic network (SN), the travelers' route choice decisions are assumed to follow the user equilibrium (UE) principle aiming to minimize their expected travel time. On the other hand, the system optimum (SO) is assumed to minimize the expected total travel time (for the fixed demand case). In order to distinguish them from those under the deterministic network, in this paper, they are referred to as SN-UE and SN-SO, respectively. The paper investigates the relationship between the SN-UE and SN-SO to establish the first-best marginal-cost pricing for SN (SN-MCP), which is different from the original MCP in deterministic network. In other words, we cannot obtain the SN-MCP by using the expected values to do substitution. In the newly proposed trial-and-error algorithm, each trial toll is determined by the SN-MCP expression. That algorithm only requires the information on the statistics of the observed link flows and travel time functions. The distribution parameters of stochastic O-D demand are not needed. The paper proves the convergence of the iterative algorithm and also analyzes the effect of the sampling error of the link flow data on the convergence of the algorithm, in which the observation period may not be sufficiently long to deduce the true mean and variance of the link flow. We show that the biases from the flow observation will not affect the convergence of the optimal toll and flow pattern.

The outline of the paper is as follows. First, we derive the general formulation of the SN-MCP and show a closed-form formulation/calculation of the SN-MCP for a special case. Then, the trial-and-error algorithm for evolutionary implementation of the first-best road pricing based on stochastic link traffic volumes is developed. The convergence properties are analyzed and proved. Following this, two numerical examples are examined to demonstrate the validity of the proposed algorithm and exhibit the difference of the SN-MCP and original MCP. Finally, some conclusions are given.

MARGINAL COST PRICING UNDER STOCHASTIC NETWORK

The MCP principle states that travelers using congested roads should pay a toll equal to the difference between the marginal social and private cost in order to minimize the total system cost if demands are fixed, or maximize the social welfare if demands are variable. In previous literatures, when all link flows, and thereby all link travel times, are deterministic, the MCP is expressed as below.

$$\mathrm{MCP}=\frac{\partial\left[\sum_{a\in A}v_a t_a\left(v_a\right)\right]}{\partial v_a}-t_a\left(v_a\right)=v_a\cdot\frac{dt_a\left(v_a\right)}{dv_a} \tag{1}$$

where v_a is the amount of traffic flow on link a, $t_a\left(v_a\right)$ is the private cost for traversing link a, and the derivative of the total system cost, $\sum_{a\in A}v_a t_a\left(v_a\right)$, with respect to v_a is the marginal social cost. All the costs here are in time unit.

In this paper, travel demand uncertainty is fully taken into account. The stochastic demands give rise to stochastic link flows and stochastic link travel times. In this situation, it is obvious that we need to modify the expression of the MCP in Eq. (1) to make it well defined. If using capital V_a to represent the stochastic link flow, one argument is that we can simply substitute the expected value of V_a (i.e. $E\left[V_a\right]$) into the original MCP, $v_a\cdot dt_a\left(v_a\right)/dv_a$. In this way, the expression of the original MCP changes to $E\left[V_a\right]\cdot dt_a\left(E\left[V_a\right]\right)/dE\left[V_a\right]$. However, noted that $t_a\left(E\left[V_a\right]\right)$ is not equal to $E\left[t_a\left(V_a\right)\right]$ (the mean link travel time) with any nonlinear link travel time function, such a substitution generally cannot provide us the real difference between the marginal social and private cost. In the following analysis, it will be further seen that even we take $E\left[t_a\left(V_a\right)\right]$ to replace $t_a\left(v_a\right)$ rather than $t_a\left(E\left[V_a\right]\right)$, that is, the original MCP changes to $E\left[V_a\right]\cdot dE\left[t_a\left(V_a\right)\right]/dE\left[V_a\right]$, it is not the real MCP under stochastic network. To obtain the

SN-MCP, we next develop the SN-UE and SN-SO models, respectively, and then explore the gap between them.

Equilibrium models under stochastic network

Notations and assumptions

Consider a road network $\mathbf{G} = (\mathbf{N}, \mathbf{A})$ with $\mathbf{N}$ being the set of nodes and $\mathbf{A}$ being the set of links, respectively. Let $\mathbf{W}$ be the set of all O-D pairs and $\mathbf{P}_w$ the set of all paths for O-D pair $w \in \mathbf{W}$. In the following, for consistency, random variables are expressed in upper-case letters and lower-case letters are used for the mean values.

Q_w	travel demand between O-D pair $w \in \mathbf{W}$;
q_w	mean travel demand between O-D pair $w \in \mathbf{W}$;
ε_w^q	variance of demand between O-D pair $w \in \mathbf{W}$;
F_k^w	traffic flow on path $k \in \mathbf{P}_w$;
f_k^w	mean traffic flow on path $k \in \mathbf{P}_w$;
$\mathbf{f}$	vector of mean path flow, $\mathbf{f} = \left\{ f_k^w \right\}$;
$\varepsilon_k^{w,f}$	variance of traffic flow on path $k \in \mathbf{P}_w$;
V_a	traffic flow on link $a \in \mathbf{A}$;
v_a	mean traffic flow on link $a \in \mathbf{A}$;
$\mathbf{v}$	vector of mean link flow, $\mathbf{v} = \left\{ v_a \right\}$;
ε_a^v	variance of traffic flow on link $a \in \mathbf{A}$;
T_a	travel time on link $a \in \mathbf{A}$;
t_a	mean travel time on link $a \in \mathbf{A}$;
$\mathbf{t}$	vector of mean link travel time, $\mathbf{t} = \left\{ t_a \right\}$;
C_k^w	travel time on path $k \in \mathbf{P}_w$;
c_k^w	mean travel time on path $k \in \mathbf{P}_w$;
$\mathbf{c}$	vector of mean path travel time, $\mathbf{c} = \left\{ c_k^w \right\}$;
π_w	minimum mean travel time between O-D pair $w \in \mathbf{W}$;
$\delta_{k,a}^w$	indicator variable, 1 if path $k \in \mathbf{P}_w$ contains link $a \in \mathbf{A}$, 0 otherwise;
TT	total travel time, $TT = \sum_{a \in \mathbf{A}} V_a T_a$.

The following basic assumptions are made throughout the paper, which are common in the literature of traffic equilibrium assignment with stochastic demand.

A1. The O-D travel demands are assumed independently distributed. VMR_w is referred to as the variance to mean ratio of the stochastic demand in which $VMR_w = \varepsilon_w^q / q_w$.

A2. The path flow is assumed to be the product of path choice proportion and the O-D travel demand, i.e., $F_k^w = p_k^w Q_w$, where $p_k^w = f_k^w / q_w$ that can be obtained from the model's output (Lam et al., 2008). Then, it follows from A1 that F_k^w is also an independent random variable and follows the same statistical distribution as the O-D demand.

A3. The *VMR*s of path flows are assumed equal to that of the corresponding O-D demand (Zhou and Chen, 2008).

SN-UE model

Due to the uncertainty of travel time travelers may consider the trade-off between the mean and variance of travel time when making their route choice decision (Bell and Cassir, 2002). Different travelers with different risk-taking attitude will consider different weight between the mean and variance. If the weight on variance is positive (negative), it indicates that travelers are “risk-averse” (“risk-prone”). If travelers are concerned with the mean travel time only, i.e., the weight on variance is zero, they are regarded as “risk-neutral”. In this study, we consider the case with the risk-neutral traveler only. The case with the risk prone/averse traveler can be extended in future.

The SN-UE proposed here is analogous to the Wardropian principle. That is, the equilibrium reaches when no risk-neutral traveler can change his/her route unilaterally to reduce his/her average travel time which he/she experiences from day to day. This SN-UE condition can be mathematically stated as:

$$\begin{cases} c_k^w = \pi_w, & \text{if } f_k^w > 0 \\ c_k^w \geq \pi_w, & \text{if } f_k^w = 0 \end{cases} \quad \forall k \in \mathbf{P}_w,\ w \in \mathbf{W} \tag{2}$$

Such complementary conditions can be further reformulated as a Variational Inequality (VI) problem: for any $\mathbf{f} \in \Omega_\mathbf{f}$, find $\mathbf{f}^* \in \Omega_\mathbf{f}$ such that

$$\left(\mathbf{f} - \mathbf{f}^*\right)^{\mathrm{T}} \mathbf{c}^* \geq 0 \tag{3}$$

where $\Omega_\mathbf{f}$ is the feasible set of mean path flows defined as below:

$$\Omega_\mathbf{f} = \left\{ \mathbf{f} \,\middle|\, \mathbf{f} \geq \mathbf{0}; \sum_{k \in \mathbf{P}_w} f_k^w = q_w, \forall w \in \mathbf{W} \right\}$$

Note that the link flow is the sum of the flows on all paths using the link, which gives

$$v_a = E\left[V_a\right] = E\left[\sum_{w \in \mathbf{W}} \sum_{k \in \mathbf{P}_w} \delta_{k,a}^w F_k^w\right] = \sum_{w \in \mathbf{W}} \sum_{k \in \mathbf{P}_w} \delta_{k,a}^w E\left[F_k^w\right] = \sum_{w \in \mathbf{W}} \sum_{k \in \mathbf{P}_w} \delta_{k,a}^w f_k^w \tag{4}$$

In addition, the path travel time is the sum of the travel times on all links comprising the path, which gives

$$c_k^w = E\left[C_k^w\right] = E\left[\sum_{a \in A} \delta_{k,a}^w T_a\right] = \sum_{a \in A} \delta_{k,a}^w E\left[T_a\right] = \sum_{a \in A} \delta_{k,a}^w t_a \tag{5}$$

More compactly, Eqs. (4) and (5) can be written as $\mathbf{v} = \boldsymbol{\Delta}\mathbf{f}$ and $\mathbf{c} = \boldsymbol{\Delta}^{\mathrm{T}}\mathbf{t}$, respectively, where $\boldsymbol{\Delta} = \left\{\delta_{k,a}^w\right\}$ is the link-path incidence matrix. Substituting them into Eq. (3) we have

$$\left(\mathbf{v} - \mathbf{v}^*\right)^{\mathrm{T}} \mathbf{t}^* \geq 0 \tag{6}$$

where the feasible region changes to $\Omega_{\mathbf{v}}$ as well.

$$\Omega_{\mathbf{v}} = \left\{ \mathbf{v} \,\middle|\, \mathbf{v} = \boldsymbol{\Delta}\mathbf{f}, \mathbf{f} \geq \mathbf{0}; \sum_{k \in \mathbf{P}_w} f_k^w = q_w, \forall w \in \mathbf{W} \right\}$$

SN-SO model

For consistency, we also consider a risk-neutral system manager here first. Let the expected total travel time be the measure of system performance. To achieve the best state, the SN-SO targets to minimize the expected total travel time. Based on the Beckmann's formulation (Beckmann et al., 1956), the following mathematical program (MP) for SN-SO is defined:

$$\min_{\mathbf{v} \in \Omega_{\mathbf{v}}} \; E\left[TT\right] = E\left[\sum_{a \in \mathbf{A}} V_a T_a\right] \tag{7}$$

The optimality conditions of a MP can also be written as a VI problem if the objective function is continuously differentiable and the feasible region is closed and convex (Nagurney, 1999). These two conditions are obviously satisfied by MP (7). Therefore, it can be reformulated as a VI problem: for any $\mathbf{v} \in \Omega_{\mathbf{v}}$, find $\mathbf{v}^* \in \Omega_{\mathbf{v}}$ such that

$$\left(\mathbf{v} - \mathbf{v}^*\right)^{\mathrm{T}} \nabla_{\mathbf{v}} \mathbf{E}\left[TT^*\right] \geq 0 \tag{8}$$

where $\nabla_{\mathbf{v}} \mathbf{E}\left[TT^*\right] = \left\{\partial E\left[TT^*\right] / \partial v_a^*\right\}$.

The SN-MCP and its calculation

Derivation of the SN-MCP

Comparing Eqs. (6) and (8), if travelers can realize that their travel cost is $\partial E\left[TT\right]/\partial v_a$ but not t_a, the traffic flow pattern of SN-UE and SN-SO will be the same. However, the marginal external cost, i.e., the gap between $\partial E\left[TT\right]/\partial v_a$ and t_a, is often ignored by travelers. Thus, as in the deterministic network, to force travelers to notice such a cost, a toll that equates the gap on each link is charged.

Now we specifically focus on the link travel time function in a polynomial form:

$$T_a = t_a\left(V_a\right) = \sum_{j=0}^{m} b_{ja} V_a^j, \quad \forall a \in \mathbf{A} \tag{9}$$

where b_{ja} is the coefficient associated with V_a^j. The power-law form of the commonly used Bureau of Public Roads (BPR) functions are a special case of Eq. (9). For other functional forms, a polynomial Taylor series approximation may be used to obtain the link travel time function in the form of Eq. (9).

Based on Eq. (9), the mean travel time on link $a \in \mathbf{A}$ is

$$E\left[T_a\right] = E\left[\sum_{j=0}^{m} b_{ja} V_a^j\right] = \sum_{j=0}^{m} b_{ja} E\left[V_a^j\right] \tag{10}$$

and the expected total travel time is

$$E\left[TT\right] = E\left[\sum_{a\in\mathbf{A}} \sum_{j=0}^{m} b_{ja} V_a^{j+1}\right] = \sum_{a\in\mathbf{A}} \sum_{j=0}^{m} b_{ja} E\left[V_a^{j+1}\right] \tag{11}$$

Therefore, the SN-MCP can be given as below.

$$\text{SN-MCP} = \frac{\partial E\left[TT\right]}{\partial v_a} - E\left[T_a\right] = \sum_{j=0}^{m} b_{ja}\left(\frac{\partial E\left[V_a^{j+1}\right]}{\partial v_a} - E\left[V_a^j\right]\right) \tag{12}$$

Note that

$$E\left[V_a^j\right] = \frac{\partial\left(E\left[V_a^j\right]E\left[V_a\right]\right)}{\partial v_a} - E\left[V_a\right]\frac{\partial E\left[V_a^j\right]}{\partial v_a} \tag{13}$$

Then replacing the term $E\left[V_a^j\right]$ in Eq. (12) with (13), we have

$$\begin{aligned}
\text{SN-MCP} &= \sum_{j=0}^{m} b_{ja}\left(\frac{\partial E\left[V_a^{j+1}\right]}{\partial v_a} - \frac{\partial\left(E\left[V_a^j\right]E\left[V_a\right]\right)}{\partial v_a} + E\left[V_a\right]\frac{\partial E\left[V_a^j\right]}{\partial v_a}\right) \\
&= v_a\frac{\partial E\left[T_a\right]}{\partial v_a} + \sum_{j=0}^{m} b_{ja}\left(\frac{\partial E\left[V_a^{j+1}\right]}{\partial v_a} - \frac{\partial\left(E\left[V_a^j\right]E\left[V_a\right]\right)}{\partial v_a}\right) \\
&= v_a\frac{\partial E\left[T_a\right]}{\partial v_a} + \sum_{j=0}^{m} b_{ja}\frac{\partial \mathrm{Cov}\left[V_a^j, V_a\right]}{\partial v_a}
\end{aligned} \tag{14}$$

where the first term of the right-hand-side of Eq. (14) is referred to as the average MCP and the second term is what the average MCP ignores. Obviously, as long as the covariance of V_a^j and V_a is a strictly monotone function of v_a, that is, $\partial \mathrm{Cov}\left[V_a^j, V_a\right] / \partial v_a > 0$, the average MCP underestimates the real SN-MCP.

Calculation of the SN-MCP for a special case

To specifically quantify the SN-MCP, the exact value of each derivative in Eq. (14) should be calculated. In this subsection, we show how to derive the closed-form formulation of Eq. (14) for a special case with log-normal demands and an assumption of constant VMR across all O-D pairs.

The log-normal distribution is a positive and asymmetric distribution. The assumption that O-D demands follow log-normal distribution was adopted in many studies (see e.g., Zhao and

Kockelman, 2002; Zhou and Chen, 2008). Moreover, Uno et al. (2009) used the empirical travel time data to validate that the path travel times in their specific case study follow the log-normal distribution. To obtain the log-normal stochastic travel time in the proposed SN, the assumption of the log-normal O-D demands is required.

Note that according to **A2**, the path flows follow the log-normal distribution as well. Following Fenton (1960), the summation of the log-normal random variables can be still estimated by a log-normal distribution. It shows that the link flow, which is the sum of related path flows, also follows the log-normal distribution, i.e., $V_a \sim LN\left(\mu_a^v, \sigma_a^v\right)$, where the distribution parameters μ_a^v and σ_a^v are as below.

$$\mu_a^v = \ln\left(v_a\right) - \frac{1}{2}\ln\left(1 + \frac{\varepsilon_a^v}{\left(v_a\right)^2}\right) \text{ and } \left(\sigma_a^v\right)^2 = \ln\left(1 + \frac{\varepsilon_a^v}{\left(v_a\right)^2}\right) \tag{15}$$

Any *j*-th moment of the log-normal link flow V_a exists and can be calculated via the moment generating function. The general expression is given by

$$E\left[V_a^j\right] = \exp\left(j\mu_a^v + \frac{j^2\left(\sigma_a^v\right)^2}{2}\right) \tag{16}$$

In addition to the assumption on statistical distribution of demands, the other assumption that the VMR of demands are the same for all O-D pairs is required to allow for the closed-form calculation of the proposed SN-MCP, in which $E\left[V_a^j\right]$ and $\mathrm{Cov}\left[V_a^j, V_a\right]$ can be derived as a function of the mean link flow v_a.

Again utilizing the relationship between the link and path flows, we have

$$\varepsilon_a^v = \mathrm{Var}\left[V_a\right] = \mathrm{Var}\left[\sum_{w\in\mathbf{W}}\sum_{k\in\mathbf{P}_w}\delta_{k,a}^w F_k^w\right] = \sum_{w\in\mathbf{W}}\sum_{k\in\mathbf{P}_w}\left(\delta_{k,a}^w\right)^2 \mathrm{Var}\left[F_k^w\right] = \sum_{w\in\mathbf{W}}\sum_{k\in\mathbf{P}_w}\delta_{k,a}^w \varepsilon_k^{w,f} \tag{17}$$

With **A3** and the constant VMR assumption, $\varepsilon_k^{w,f} = VMR \cdot f_k^w$. Therefore, it follows from Eqs. (17) and (4) that the variance of link flow can be further reduced to

$$\varepsilon_a^v = VMR \cdot \sum_{w\in\mathbf{W}}\sum_{k\in\mathbf{P}_w}\delta_{k,a}^w f_k^w = VMR \cdot v_a \tag{18}$$

Now let us combine Eqs. (15), (16) and (18) together, and perform some manipulations, then it yields that

$$E\left[V_a^j\right] = v_a^j\left(\sqrt{1 + VMR/v_a}\right)^{j^2 - j} \tag{19}$$

and

$$\begin{aligned}\mathrm{Cov}\left[V_a^j, V_a\right] &= E\left[V_a^{j+1}\right] - E\left[V_a^j\right]E\left[V_a\right] \\ &= v_a^{j+1}\left(\sqrt{1 + VMR/v_a}\right)^{j^2 + j} - v_a^{j+1}\left(\sqrt{1 + VMR/v_a}\right)^{j^2 - j}\end{aligned} \tag{20}$$

Using notation $s_a = \sqrt{1 + VMR/v_a}$ to simplify the formulation, we have

$$\frac{\partial \mathrm{Cov}\left[V_a^j, V_a\right]}{\partial v_a} = \left(j+1\right)v_a^j\left[s_a^{j^2+j} - s_a^{j^2-j}\right] + \left(\frac{1 - s_a^2}{2s_a^2}\right)v_a^j\left[\left(j^2 + j\right)s_a^{j^2+j} - \left(j^2 - j\right)s_a^{j^2-j}\right] \tag{21}$$

Furthermore,

$$\frac{\partial E\left[T_a\right]}{\partial v_a}=\sum_{j=0}^{m} b_{ja}\frac{\partial E\left[V_a^j\right]}{\partial v_a}=\sum_{j=0}^{m} b_{ja} v_a^{j-1} s_a^{j^2-j}\left[j+\left(j^2-j\right)\cdot\frac{1-s_a^2}{2s_a^2}\right] \tag{22}$$

By substituting Eqs. (21) and (22) into Eq. (14), the value of the SN-MCP can be determined as long as the mean link flows under the SN-SO are known. The closed-form formulation can also be derived from other statistical distributions of travel demand. In the numerical test part, the case with normal distribution will be checked.

TRIAL-AND-ERROR IMPLEMENTATION OF SN-MCP

As mentioned, the trial-and-error methods developed before are all based on the assumption of deterministic traffic state. The precise deterministic UE link flows are assumed observable and are used to adjust the toll levels in each trial. However, only considering the observation errors, such an assumption cannot be truly realized in practice, not to mention the variability of link flows and travel times due to stochastic demand. If one observe the link flow on a section of highway between a certain time of day and over a time period (e.g., one month), it is obvious that the link count data obtained will be random.

Due to the uncertainty of traffic flow information, we, of course, cannot arbitrarily pick up one day's observation data to refer to in the toll updating scheme. Instead the average of the data collected over a time period is preferred to be utilized. From the collected data, the mean of UE link flows could be obtained approximately by applying some suitable data fitting method. However, it should be noted that when the average values (or say mean values) are used in the toll updating scheme, the MCP determination formula as shown in Eq. (14) (i.e. SN-MCP) is different with the original MCP replaced by the mean value. Therefore, in this section, we are intent to revise the trial-and-error procedure presented by Yang et al. (2004) to allow it to proceed in the studied SN case as well.

Here assume that the stochastic travel demand follows some statistical distribution, but the distribution parameters are unknown and not easy to be estimated. In other words, the mean and variance of each O-D demand are not available. Under this situation, it is impossible to find the equilibrium flow pattern and thereby obtain the marginal-cost tolls by directly solving the related mathematical problem. The obstacle that we meet now is the same as that in the case of deterministic network. To remove the influence of the limited information, we turn to the trial-and-error idea again to propose a method that iteratively calculates the marginal-cost tolls based on the observable stochastic link flows. The detailed implementation procedure is presented below.

Step 0. (*Initialization*) Let $\left\{v_a^{(0)}, a\in\mathbf{A}\right\}$ be an initial set of feasible mean link flows. Set $k=0$.

Step 1. (*Estimate link tolls*) For each link $a\in\mathbf{A}$, calculate the current link toll $\tau_a^{(k)}$ by

$$\tau_a^{(k)}=v_a^{(k)}\frac{\partial E\left[T_a^{(k)}\right]}{\partial v_a^{(k)}}+\sum_{j=0}^{m} b_{ja}\frac{\partial \mathrm{Cov}\left[\left(V_a^{(k)}\right)^j, V_a^{(k)}\right]}{\partial v_a^{(k)}} \tag{23}$$

Step 2. (*Observe link flows*) After imposition of the link tolls given by Eq. (23) on a network, observe and collect the link count data over a period of time. Then, compute the mean of the collected link flows via data fitting. Let $\left\{\bar{v}_a^{(k)}, a \in \mathbf{A}\right\}$ denote the estimated mean value.

Step 3. (*Check convergence*) If $\left\|\bar{\mathbf{v}}^{(k)} - \mathbf{v}^{(k)}\right\| / \left\|\mathbf{v}^{(k)}\right\| < \varepsilon$, then stop. Otherwise, go to Step 4.

Step 4. (*Update link flows*) Set

$$v_a^{(k+1)} = v_a^{(k)} + \alpha^{(k)}\left(\bar{v}_a^{(k)} - v_a^{(k)}\right), \quad a \in \mathbf{A} \tag{24}$$

and $k := k+1$, go to Step 1.

In above procedure, '$\|\cdot\|$' denotes the Euclidean norm; ε is a positive number of convergence tolerance; $\left\{\alpha^{(k)}\right\}$ is a sequence of predetermined step sizes and it must satisfy the following three conditions:

$$0 < \alpha^{(k)} \le 1, \quad \sum_{k=1}^{\infty} \alpha^{(k)} = +\infty, \quad \sum_{k=1}^{\infty} \left(\alpha^{(k)}\right)^2 < +\infty \tag{25}$$

A typical sequence of $\alpha^{(k)}$ is $\alpha^{(k)} = 1/k$.

Convergence of the trail-and-error implementation

In order to prove the convergence of the above presented trial-and-error method, we provide the following two important propositions, which are similar as that in Yang et al. (2004).

Proposition 1. *If $\left\|\bar{\mathbf{v}}^{(k)} - \mathbf{v}^{(k)}\right\| = 0$ at the convergent point, then $\left\{v_a^{(k)}, a \in \mathbf{A}\right\}$ is the mean SN-UE link flow pattern and $\left\{\tau_a^{(k)}, a \in \mathbf{A}\right\}$ is the corresponding optimal link toll pattern.*

Proof. According to above algorithm, after the toll pattern $\boldsymbol{\tau}^{(k)} = \left\{\tau_a^{(k)}, a \in \mathbf{A}\right\}$ as shown in Eq. (23) is implemented, the revealed mean link flow pattern, $\bar{\mathbf{v}}^{(k)} = \left\{\bar{v}_a^{(k)}, a \in \mathbf{A}\right\}$, is the solution of the following VI problem:

$$\left(\mathbf{v} - \bar{\mathbf{v}}^{(k)}\right)^{\mathrm{T}} \left(\bar{\mathbf{t}}^{(k)} + \boldsymbol{\tau}^{(k)}\right) \ge 0, \quad \forall \mathbf{v} \in \Omega \tag{26}$$

If $\left\|\bar{\mathbf{v}}^{(k)} - \mathbf{v}^{(k)}\right\| = 0$, then $\bar{v}_a^{(k)} = v_a^{(k)}$ for all link a. Hence

$$\bar{\mathbf{t}}^{(k)} + \boldsymbol{\tau}^{(k)} = \mathbf{t}^{(k)} + \boldsymbol{\tau}^{(k)} = \nabla_{\mathbf{v}} \mathbf{E}\left[TT^{(k)}\right]$$

which means $\bar{\mathbf{v}}^{(k)}$, or say $\mathbf{v}^{(k)}$, is already the optimal solution of VI problem (8). That is, the toll pattern $\boldsymbol{\tau}^{(k)}$ now is the optimal one in the sense of first-best pricing.

Proposition 2. *If the mean link travel cost $\mathbf{t}$ is strongly monotone with respect to the mean link flow $\mathbf{v}$, i.e., there exists a positive constant ρ such that, for any distinct $\mathbf{v}_1 \in \Omega_{\mathbf{v}}$ and $\mathbf{v}_2 \in \Omega_{\mathbf{v}}$, we have*

$$\left(\mathbf{t}_1 - \mathbf{t}_2\right)^{\mathrm{T}} \left(\mathbf{v}_1 - \mathbf{v}_2\right) \ge \rho \left\|\mathbf{v}_1 - \mathbf{v}_2\right\|^2 \tag{27}$$

then the vector $\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)$ *is a feasible descent direction of the objective function of MP* (7) *at* $\mathbf{v}^{(k)}$.

Proof. Note that the gradient of objective function (7) is given as below

$$\nabla_{\mathbf{v}}\mathbf{E}\left[TT^{(k)}\right]=\mathbf{t}^{(k)}+\boldsymbol{\tau}^{(k)} \tag{28}$$

Therefore,

$$\begin{aligned}\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\nabla_{\mathbf{v}}\mathbf{E}\left[TT^{(k)}\right]&=\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\left(\mathbf{t}^{(k)}+\boldsymbol{\tau}^{(k)}\right)\\&=\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\left(\mathbf{t}^{(k)}-\overline{\mathbf{t}}^{(k)}\right)+\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\left(\overline{\mathbf{t}}^{(k)}+\boldsymbol{\tau}^{(k)}\right)\end{aligned} \tag{29}$$

From Eq. (27), the first term of Eq. (29) is less than or equal to $-\rho\left\|\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right\|^2$. From Eq. (26), the second term of Eq. (29) is non-positive. Thus,

$$\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\nabla_{\mathbf{v}}\mathbf{E}\left[TT^{(k)}\right]\le-\rho\left\|\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right\|^2 \tag{30}$$

It is obvious that as long as $\overline{\mathbf{v}}^{(k)}\neq\mathbf{v}^{(k)}$, $\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\nabla_{\mathbf{v}}\mathbf{E}\left[TT^{(k)}\right]<0$, which indicates that the vector $\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)$ is a descent direction.

Now let us illustrate that the proposed iterative procedure does converge, i.e., $\mathbf{v}^{(k)}\to\mathbf{v}^*$ and $\boldsymbol{\tau}^{(k)}\to\boldsymbol{\tau}^*$ when $k\to\infty$. Since the separable link travel time function $t_a\left(V_a\right)$ is assumed, the Hessian matrix of objective function (7) must be a diagonal matrix. Let $E''_a\left[TT\right]$ represent its diagonal element, then

$$\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)^T\nabla^2_{\mathbf{v}}\mathbf{E}\left[TT\right]\left(\overline{\mathbf{v}}^{(k)}-\mathbf{v}^{(k)}\right)=\sum_{a\in\mathbf{A}}E''_a\left[TT\right]\left(\overline{v}_a^{(k)}-v_a^{(k)}\right)^2 \tag{31}$$

The right-hand-side of Eq. (31) is always bounded for the bounded link travel time functions. Combining this conclusion with Proposition 2 and the conditions for choosing the step size sequence, all convergence requirements regarding the method of successive averages (MSA) are satisfied (Powell and Sheffi, 1982). Thus, the convergence of the proposed trial-and-error algorithm is followed.

Analysis the sampling error on the trail-and-error implementation

In above, we present a theoretical proof on the convergence of the proposed trial-and-error procedure. All the propositions are shown to hold with the true mean link flows under SN-UE. In real application, however, the true mean link flows also cannot be exactly obtained. When the trial-and-error procedure is to be implemented in practice, what we can do is just observe and collect the link flow data over a limited period of time (e.g. one month). Hence, the mean link flows calculated from the limited sampling data are generally not equal to their actual true values. In other words, at each iteration, only an approximate mean link flow pattern can be used to update the trial tolls. Then a question emerges: will such an approximation affect the convergence of the trial-and-error procedure or the correctness of the convergent result?

Fortunately, the iterative algorithm here is in essential the method of successive averages. The convergence requirements of such a method can guarantee the sequence of solutions to converge to the minimum even if the search direction is a descent vector only on the average. For example, when the MSA is applied to solve the SUE problem, it sometimes requires a probit stochastic network loading via using Monte-Carlo simulation to determine the descent direction. If such a technique is resorted to, then no matter how many times the simulation procedure is repeated, the resulting average link flows are not the actual ones. The direction vectors that are produced by such simulations are thereby random direction. However, even in this case, the MSA algorithm still can converge as long as an unbiased estimate of the direction is utilized.

Now we encounter the same situation when simulating the mean values of the link flows. As mentioned before, regardless of how long the observation period is allowed and how many the sampling data are collected, the true mean value cannot be accurately computed but an estimate. Therefore, each time after the trial toll charge being imposed on the network, the estimated mean SN-UE link flow pattern $\overline{\mathbf{v}}^{(k)}$ is in fact a random variable. That is to say, the descent direction vector $\left(\overline{\mathbf{v}}^{(k)} - \mathbf{v}^{(k)}\right)$ is random as well.

Using the simplest step size sequence that satisfies conditions (25), $\left\{\alpha^{(k)} = 1/k\right\}$, then

$$\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \frac{1}{k}\left(\overline{\mathbf{v}}^{(k)} - \mathbf{v}^{(k)}\right) \tag{32}$$

This relation can be rewritten as follows as its name, the method of successive averages, that is

$$\mathbf{v}^{(k+1)} = \frac{1}{k}\sum_{l=1}^{k}\overline{\mathbf{v}}^{(l)} \tag{33}$$

Thus the variance of each component of $\mathbf{v}^{(k+1)}$ is given by

$$\mathrm{Var}\left(v_a^{(k+1)}\right) = \frac{1}{k^2}\sum_{l=1}^{k}\mathrm{Var}\left(\overline{v}_a^{(l)}\right) \tag{34}$$

At each iteration, since the revealed link flow $\overline{v}_a^{(l)}$ is always below the sum of all O-D flows, the variance of $\overline{v}_a^{(l)}$ is bounded by some value, i.e., $\mathrm{Var}\left(\overline{v}_a^{(l)}\right) < \sigma^2 < \infty,\ \forall l$. Therefore,

$$\mathrm{Var}\left(v_a^{(k+1)}\right) < \frac{1}{k^2}\sum_{l=1}^{k}\sigma^2 \tag{35}$$

Obviously, the right-hand-side of Eq. (35) approaches zero as *k* grows, meaning that the variance of $v_a^{(k+1)}$ approaches zero as the algorithm progresses. This further indicates that the variance of trial toll pattern $\boldsymbol{\tau}^{(k+1)}$ updated by using $\mathbf{v}^{(k+1)}$ approaches zero, too. In terms of these properties, it can be ensured that the convergence of the proposed iterative procedure will not be affected no matter how accurate the simulation is at each iteration.

NUMERICAL TESTS

Example 1

The test network for Example 1 is taken from Yang et al. (2004) as shown in Figure 2, which has 7 nodes, 11 links and 4 O-D pairs.

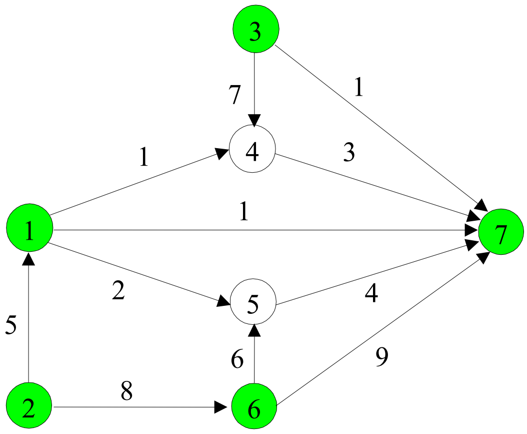

Figure 1 – Road network used for Example 1

In this example, the BPR-type link travel time function, that is,

$$t_a(V_a) = t_a^0 + b_a\left(\frac{V_a}{c_a}\right)^{n_a} \quad (36)$$

is adopted with $n_a = 4$ for all link a. The other link cost parameters (e.g., the free-flow travel time t_a^0 and the capacity c_a of each link) are given in Table 1.

Table 1 – Link cost parameters for Example 1

Link	t_a^0	b_a	c_a
1	6	0.90	200
2	5	0.75	200
3	6	0.90	200
4	7	1.05	200
5	6	0.90	100
6	1	0.15	100
7	5	0.75	150
8	10	1.50	150
9	11	1.65	200
10	11	1.65	200
11	15	2.25	200

For each O-D pair, the mean of the log-normal demand is shown in Table 2. It is worth noting that the reason we give the mean values here is that, in numerical experiments, the SN-UE flow pattern cannot be observed. Thus, the mean values are purely used for generating the observed link flows, which is not necessary for updating link tolls in the real implementation of trial-and-error method.

Table 2 – Mean of the log-normal demand for Example 1

O-D Pair	q_w
1 → 7	420
2 → 7	350
3 → 7	350
6 → 7	280

In this example, we examine three VMR levels, i.e., VMR=0, 20 and 40. When VMR=0, it means no uncertainty on the network, that is, deterministic network. The true mean link flows and tolls calculated by the MSA at different VMR levels are summarized in Table 3. Table 4 further shows the expected total travel time at different VMR levels and under different toll schemes (i.e., toll free, SN-MCP, Average-MCP and Original-MCP). From these two tables, it is seen that the link tolls and the expected total travel time both increase with the increase of VMR, which means the higher the uncertainty, the more the travelers will spend. Moreover, in Table 4, the "Improv." column shows the percentage of improvement in the expected total travel time from the SN-UE compared to the SN-SO case, i.e.,

$$\text{Improv.}(case) = \frac{E(TT_{SN-UE}) - E(TT_{case})}{E(TT_{SN-UE}) - E(TT_{SN-SO})} \times 100\% \tag{37}$$

Figure 2 plots the percentage improvements shown in Table 4. From the figure, it is clear that the improvement of the expected total travel time made by the average MCP and the original MCP schemes are lower than that of the SN-MCP scheme. In particular, when the network is highly uncertain (e.g. VMR=40 in this example), the origin MCP schemes instead increases the expected total travel time compared to the no toll case.

Table 3 – True mean link flows and tolls under SN-SO

Link	VMR = 0		VMR = 20		VMR = 40	
	Mean Flow v_a	Toll τ_a	Mean Flow v_a	Toll τ_a	Mean Flow v_a	Toll τ_a
1	212.2	4.6	207.9	9.0	204.8	16.9
2	119.7	0.4	121.9	1.4	123.6	4.0
3	301.7	18.6	300.7	31.6	299.3	50.9
4	305.4	22.8	306.0	39.1	306.1	63.6
5	158.5	22.7	153.4	54.9	147.7	117.0
6	185.7	7.1	184.0	16.2	182.6	33.2
7	89.5	0.4	92.8	2.1	94.5	7.2
8	191.5	16.0	196.6	39.6	202.3	86.3
9	285.8	27.5	292.6	52.6	299.7	93.7
10	260.5	19.0	257.2	33.7	255.5	58.1
11	246.6	20.8	243.5	38.2	239.4	65.6

Table 4 – Comparison of the expected total time under different VMR levels

VMR	SN-UE (Toll Free)		SN-SO (SN-MCP)		Average-MCP		Original-MCP	
	$E(TT)$	Improv.	$E(TT)$	Improv.	$E(TT)$	Improv.	$E(TT)$	Improv.
0	29098	0.0%	28919	100.0%	28919	100.0%	28919	100.0%

20	40994	0.0%	40838	100.0%	40848	93.8%	40873	78.0%
40	65752	0.0%	65593	100.0%	65666	53.6%	65793	-25.8%

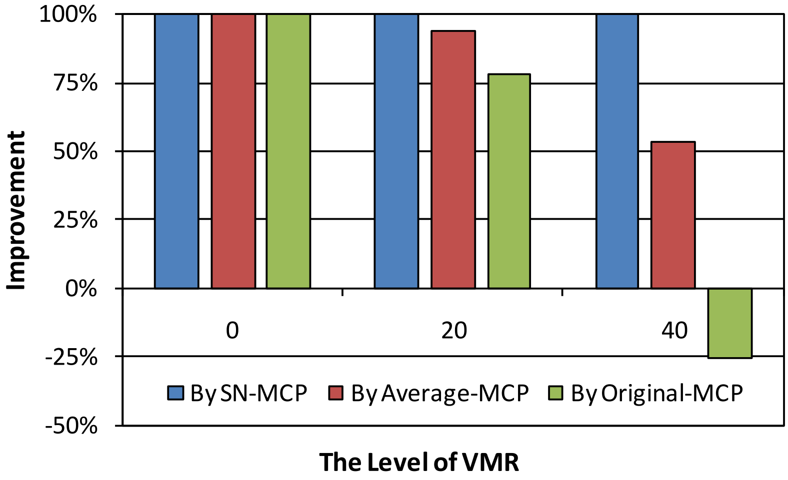

Figure 2 – Improvement of system performance by different MCP schemes

The estimated mean link flows and SN-SO tolls obtained through the trial-and-error algorithm are summarized in Table 5. The iterative procedure initially starts from a uniform toll scheme, i.e., $\tau_a = 15$ for all link a. The step size: $\alpha^{(k)} = 1/k$ and convergence tolerance: $\varepsilon = 0.001$ are utilized in the trial-and-error algorithm. For different VMR levels, Figure 3 illustrates the nice convergence of the iterative procedure.

Table 5 – Estimated mean link flows and tolls under SN-SO

Link	VMR = 0		VMR = 20		VMR = 40	
	Mean Flow v_a	Toll τ_a	Mean Flow v_a	Toll τ_a	Mean Flow v_a	Toll τ_a
1	213.4	4.5	207.9	9.0	203.8	16.9
2	117.7	0.4	121.2	1.4	123.3	4.0
3	301.9	18.7	299.7	31.5	299.1	51.1
4	303.8	22.8	306.0	39.1	307.1	63.9
5	158.2	22.7	154.7	54.9	147.8	116.7
6	185.4	7.1	184.6	16.2	183.9	33.3
7	88.3	0.4	91.7	2.2	95.3	7.2
8	192.1	15.9	195.4	39.7	202.4	86.4
9	286.5	27.7	291.0	52.6	298.6	93.3
10	261.3	19.0	258.1	33.6	254.4	57.8
11	246.7	20.8	245.4	38.4	240.3	65.4

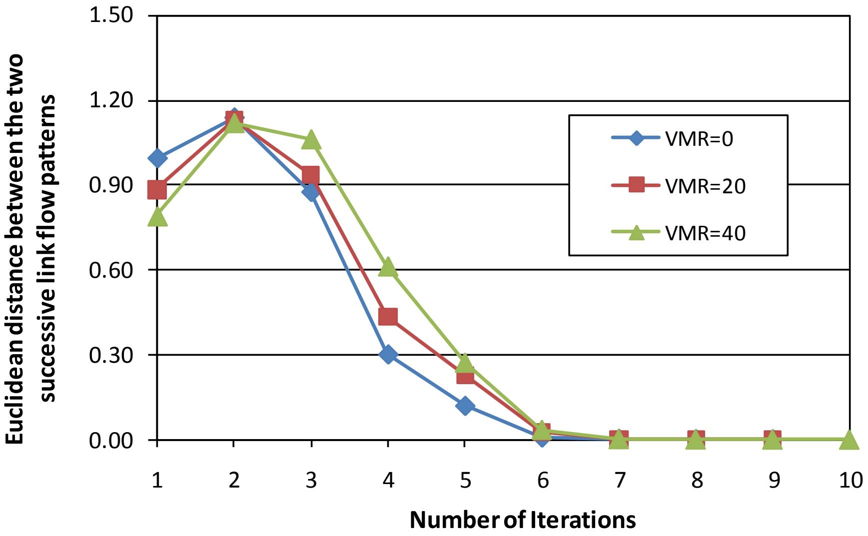

Figure 3 – Convergence of the trial-and-error procedure for SN-MCP with different VMR levels

If we do not use the SN-MCP formula to calculate the trial tolls in Step 1 as shown in Eq. (23) but refer to the average MCP or the original MCP, then the trial-and-error procedure cannot converge to the true optimal tolls that achieves SN-SO. Figure 4 shows this phenomena for VMR=20.

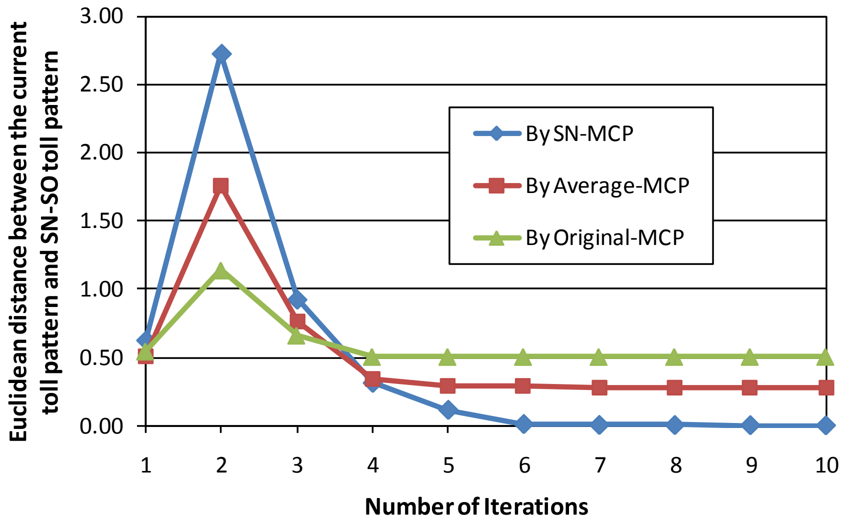

Figure 4 – Convergence of the trial-and-error procedure by different MCP schemes

Example 2

The test network for Example 2 is taken from Sumalee et al. (2006) as shown in Figure 5, which has 7 nodes, 18 links and 6 O-D pairs. This example again takes the BPR-type link travel time function (36) with $n_a = 4$ for all link a. Table 6 gives the other link cost parameters adopted in this example. The mean value of each stochastic O-D demand is shown in Table 7.

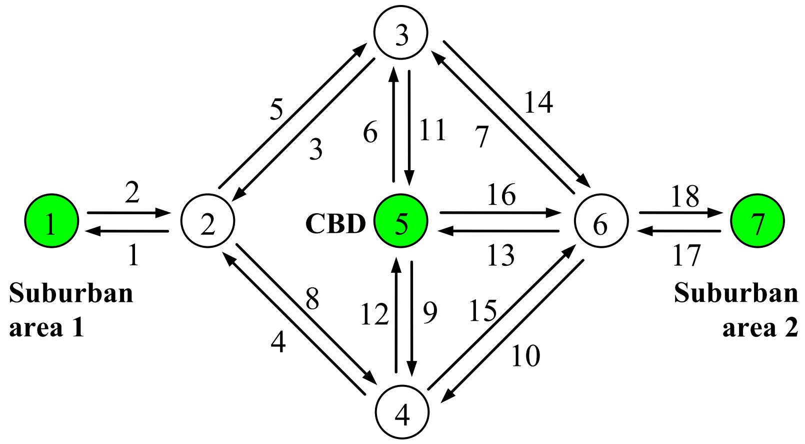

Figure 5 – Road network used for Example 2

Table 6 – Link cost parameters for Example 2

Link	t_a^0	b_a	c_a
1	1.2500	0.0253	1800
2	1.2500	0.0253	1800
3	9.1667	6.2610	1100
4	9.1667	6.2610	1100
5	9.1667	6.2610	1100
6	2.5000	1.7075	1100
7	7.5000	1.0866	1100
8	9.1667	6.2610	1100
9	2.5000	1.7075	1100
10	7.5000	1.0866	1100
11	2.5000	1.7075	1100
12	2.5000	1.7075	1100
13	2.0000	1.3660	1100
14	7.5000	1.0866	1100
15	7.5000	1.0866	1100
16	2.0000	1.3660	1100
17	1.2500	0.0253	1800
18	1.2500	0.0253	1800

Table 7 – Mean of the normal demand for Example 2

Origin	Destination		
	1	5	7
1	-	600	400
5	500	-	600
7	375	800	-

Here the numerical test is based on another statistical distribution of travel demand, normal distribution, which is also widely adopted in the literature (e.g. Waller et al., 2001; Chen et al., 2003; Lam et al., 2008). From this example, it is found that some of mean link flows could be zero when arriving at the equilibrium, for example, the SN-UE flow pattern with no toll. The log-normal distribution however excludes this instance. Under the assumptions that the travel demand follows a normal distribution and the path flow has the same kind of distribution as the demand, the link flow, which is the sum of path flows, also follows normal distribution, i.e., $V_a \sim N\left(v_a, \varepsilon_a^v\right)$.

Using the method of moment generating function, the mean link travel time can be expressed explicitly in terms of the mean and variance of the stochastic link flow. And further noting the relationship between the mean and variance as shown in Eq. (18), we have

$$\begin{aligned} E\left[T_a\right] &= t_a^0 + \frac{b_a}{c_a^4} E\left[V_a^4\right] = t_a^0 + \frac{b_a}{c_a^4}\left[v_a^4 + 6v_a^2\varepsilon_a^v + 3\left(\varepsilon_a^v\right)^2\right] \\ &= t_a^0 + \frac{b_a\left[v_a^4 + 6VMRv_a^3 + 3VMR^2v_a^2\right]}{c_a^4} \end{aligned} \tag{38}$$

Similarly, we can compute $E\left[TT\right]$, which is

$$E\left[TT\right] = t_a^0 v_a + \frac{b_a\left[v_a^5 + 10VMRv_a^4 + 15VMR^2v_a^3\right]}{c_a^4} \tag{39}$$

Therefore,

$$\frac{\partial E\left[TT\right]}{\partial v_a} = t_a^0 + \frac{b_a\left[5v_a^4 + 40VMRv_a^3 + 45VMR^2v_a^2\right]}{c_a^4} \tag{40}$$

Then,

$$\text{SN-MCP} = \frac{\partial E\left[TT\right]}{\partial v_a} - E\left[T_a\right] = \frac{b_a\left[4v_a^4 + 34VMRv_a^3 + 42VMR^2v_a^2\right]}{c_a^4} \tag{41}$$

Again, in this test, we use a uniform toll scheme, i.e., $\tau_a = 15$ for all link a to initiate the trial-and-error procedure, and take the step size: $\alpha^{(k)} = 1/k$ and convergence tolerance: $\varepsilon = 0.001$ in the algorithm. The optimal toll charges under stochastic demands with VMR=100 are now estimated. The VMR value used here is some larger than the previous example is due to the higher mean value of the total travel demand in this test. The estimated results as well as the true results are both shown in Table 8 for comparison. Figure 6 again exhibits the convergence of the iterative procedure.

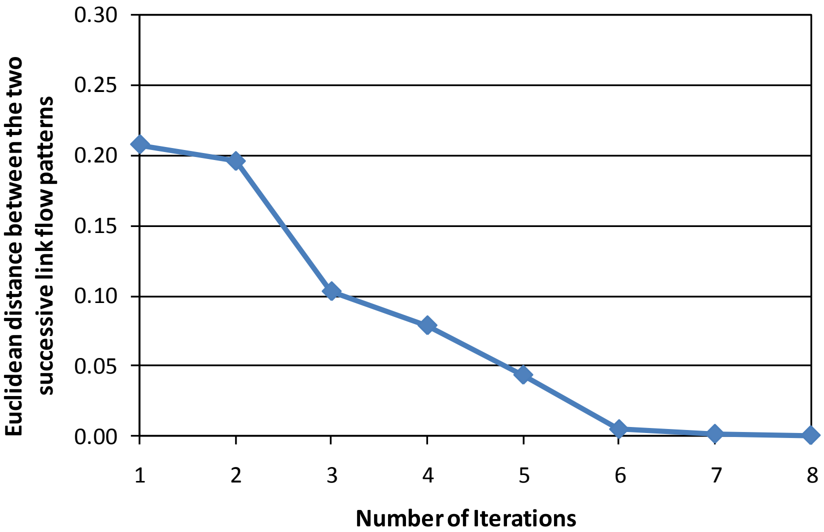

Figure 6 – Convergence of the trial-and-error procedure for SN-MCP

Table 8 – Comparison between the true and estimated mean link flows and tolls under SN-SO

Link	True Results		Estimated Results	
	Mean Flow v_a	Toll τ_a	Mean Flow v_a	Toll τ_a
1	875.0	0.01	875.0	0.01
2	1000.0	0.02	1000.0	0.02
3	437.5	2.19	437.5	2.19
4	437.5	2.19	437.5	2.19
5	500.0	3.34	500.1	3.34
6	250.0	0.11	250.0	0.11
7	187.5	0.03	187.5	0.03
8	500.0	3.34	500.0	3.34
9	250.0	0.11	250.0	0.11
10	187.5	0.03	187.5	0.03
11	352.2	0.31	352.2	0.31
12	352.2	0.31	352.1	0.31
13	800.0	3.40	800.0	3.40
14	147.9	0.02	147.8	0.02
15	147.9	0.02	147.8	0.02
16	704.3	2.22	704.3	2.22
17	1175.0	0.03	1175.0	0.03
18	1000.0	0.02	1000.0	0.02

CONCLUSIONS

This study discussed the first-best marginal-cost pricing and its trial-and-error implementation under a stochastic network that explicitly considers the demand uncertainty. In the paper, we derived the SN-MCP by re-investigating the real gap between the marginal social and private costs in terms of stochastic traffic flows. There are two terms involved in the SN-MCP when travelers are assumed risk-neutral. The first term can be regarded as the simple modification of the original MCP, i.e., the originally deterministic link flows and travel times are substituted by their mean values. The second term is the additional term related to the variability of travel demand, which is often ignored.

Given the specific statistical distribution of random demands and the assumption of constant VMR across all O-D pairs, the closed-form formulation of SN-MCP can be calculated with the information of true mean values of demands. In reality, when this information is not available, a revised trial-and-error method was proposed for practical implementation. The trial tolls are computed based on the formula of SN-MCP at each step, but not the original MCP. Though the mean link flows cannot be precisely obtained due to the limited sample size, it has been shown that the sampling error won't affect the convergence of the proposed trial-and-error procedure. The convergence and effectiveness of this evolutionary implementation was not only proved theoretically, but also examined by two numerical tests. The first one was carried out under the assumption of log-normal demand, while the second one assumed the normal demand.

This paper only investigated the traffic assignment and road pricing with the case of inelastic demand noting that the mean of stochastic demand is fixed. If the level of average demand is influenced by the travel cost of the trip, i.e., with the case of elastic demand, the SN-MCP we derived in this study is still applicable to drive the link flow pattern towards a SN-SO, where the SN-SO aims to maximize the expected social welfare. At the same time, the convergence of the revised trial-and-error algorithm can be guaranteed as well. The proof analysis is very similar as that given in this study if assuming the demand function incorporates the uncertainty in an additive fashion. The other possible extensions are to also include the stochastic link capacity degradation in the analysis of the SN-MCP and to relax the strict assumption about constant VMR across all O-D pairs.

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