MODELING AND OPTIMIZATION OF A TWO LEVEL LOGISTICS SYSTEM BY MEANS OF THE CONTINUUM APPROXIMATION

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ABSTRACT

This work introduces a model for the analysis of a logistic network. The network comprises two levels: a regional level and a local level. The products are transported from a central warehouse to a series of regional warehouses. From these warehouses, they are further distributed to the customers who are distributed over a number of zones within each region. The model considers a cost structure for the regional warehouses and for the transportation from the central warehouse to the final customers. The model allows finding the optimum configuration that minimises the total cost. This configuration is given by the number of zones per region and the number of regional warehouses. The methodology applied to solve the optimisation problem is based on replacing the set of customers by a continuum which is defined in terms of aggregated or averaged characteristics. Likewise, the costs of the routes from the warehouses to the customers are estimated by means of such a continuum approximation.

Keywords: Logistics, continuum approximation, route planning.

INTRODUCTION

In the resolution of the problems arising in the field of logistic systems planning, two different approaches can be distinguished. The conventional approach is based on the use of mathematical programming techniques together with detailed information of the problem. The opposite approach includes the methods based on continuous approximations. These methods are applicable to cases where the number of origin and/or destinations of the logistic chain are large and there is no detailed information regarding the parameters that define the network and the flow of goods through it. Under these conditions, the group of origins and destinations can be analysed as a continuum. Daganzo (1999) is the reference text for the employment of a continuous approximation in Logistic problems. The foundations of this approach go back to the works of Daganzo (1984) and Daganzo and Newell (1986). This line of investigation has been taken up again by Daganzo and Erera (1999) and more recently, several applications of the continuous approximation can be found in Dasci and Verter (2001), Ouyang and Daganzo (2003), Diana et al. (2006), Ouyang (2007), Pujari et al. (2008) and Li and Ouyang (2010). An early review of different continuous approximation applications can be found in Langevin et al. (1996). The model developed for this work is an extension of the model considered by Rosenfeld et al. (1992). In particular, more realistic travel times and cost structure have been considered.

MODEL LAY-OUT

The logistic network is distributed across a territory, which is in turn divided in a number of regions. The network has two levels and two type of routes. On a first level, the regional level, the products are distributed from a central warehouse to the regional warehouses. The corresponding routes are the regional routes. The second level is formed by local routes that depart from each regional warehouse to the zones in which the region has been divided. Figure 1 shows the network configuration.

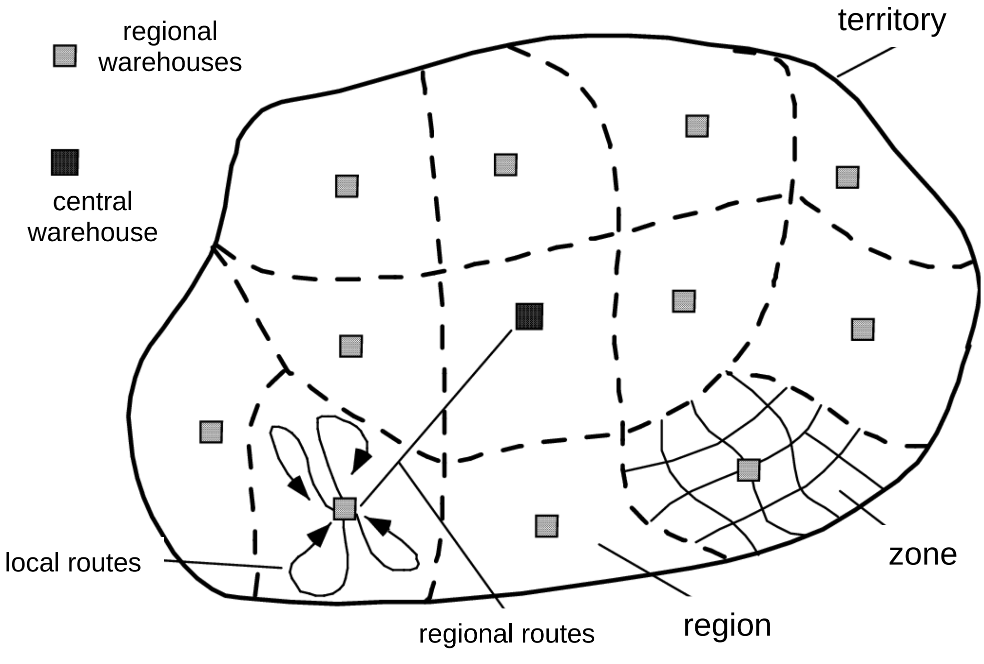

Figure 1 - Configuration of the two-level distribution network.

The regional routes are direct and join the central warehouse with the regional warehouses. These routes are served by vehicles know as “regional”. Likewise, “local” vehicles are used for the local routes. From each regional warehouse, a route departs for each one of the zones assigned to that warehouse. Therefore, the number of local routes is equal to the number of zones.

The number of destinations is in principle fixed and they are distributed uniformly throughout the entire territory. Then, it is possible to approximate the set of destinations by a continuum with a known density of destinations. The aforementioned scenario is characterised by the following variables: A= territory area; N= number of destinations in the entire territory; T= time available for each vehicle to serve the customers located in one zone; V=average vehicle speed in the territory; Z= number of zones in the entire territory (= number of local routes); R=number of regions the territory is divided into (=number of regional warehouses). The final goal is obtaining the number of regional warehouses and the number of zones that minimize the transportation cost.

DELIVERY TIME CONSTRAINT

The time constraint forces to deliver the goods from the central warehouse to its final destination in a maximum time T. On the other hand, the available time for the product delivery from the central warehouse to any destination is the sum of the following times:

$$T = T_C + T_T + T_R + T_Z + T_S \tag{1}$$

The first time T_C is the time necessary to travel the distance between the central warehouse and each one of the regional warehouses. In average, this time corresponds to the average distance from the central warehouse to the regional warehouses divided by the average speed V in this route, that is:

$$T_C = \frac{k_C}{V}\sqrt{A} \tag{2}$$

The coefficient k_C will fundamentally depend on the territory shape and the relative location of the central warehouse.

The time T_T is the time required to carry out the transhipment operations of the goods from the regional vehicles to the local vehicles. The time T_R + T_Z is the time required to serve the destinations located in a zone. These destinations are served by a local route comprising two stretches. The first one is the access stretch to the area, going from the regional warehouse to the area's outskirts. The second one is the zone stretch which corresponds to the part of the route that visits the destinations within the zone boundary. Figure 2 shows a local route and its two differentiated stretches. If T_R is the time necessary to travel the access stretch, then:

$$T_R = \frac{k_R}{V}\sqrt{\frac{A}{R}} \tag{3}$$

The above expression is similar to (2). In this case, the average distance from the regional warehouse (beginning and end of the route) to the zone will depend on the region's average size which is $\sqrt{A/R}$. In a similar way, k_R will be a coefficient which value will fundamentally depend on the region shape and the regional warehouse location. Additionally, T_Z is the time employed in the zone stretch. In accordance to the local route optimum scheme suggested by Daganzo (1984), the average length of the route's zone will be:

$$T_Z = \frac{k_Z}{V}\frac{N}{Z}\sqrt{\frac{A}{N}} = \frac{k_Z}{ZV}\sqrt{AN} \tag{4}$$

since N/Z is the number of destinations by zone and A/N is the density of destinations in the territory. The coefficient kZ will depend on the road network's geometry. It is important to keep in mind that by defining TR and TZ as in (3) and (4) respectively, we are considering that all the zones are served simultaneously, which requires having at least Z available vehicles.

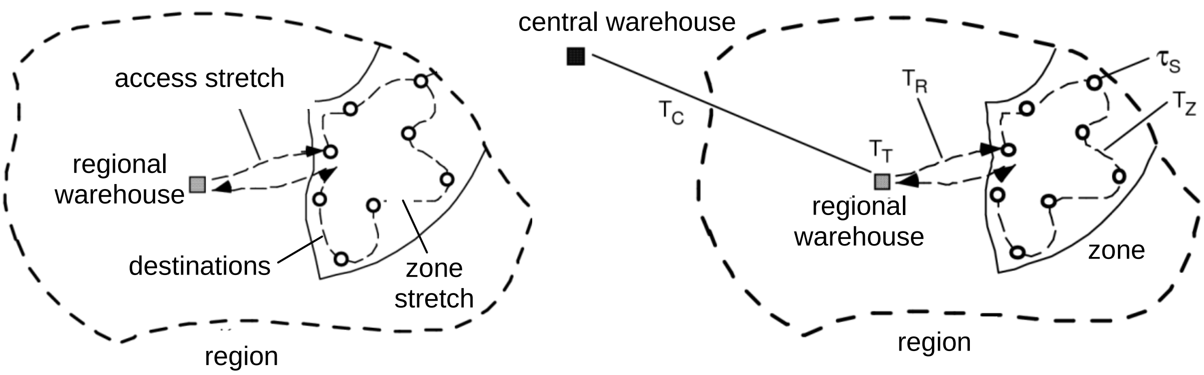

Figure 2 – Left: local route and the access stretch and zone stretch. Right: time interval required at each stage.

Finally, T_S represents the time necessary to carry out the loading/unloading and pickup/delivery operations in all the zone destinations. This time is:

$$T_S = \tau_S \frac{N}{Z} \tag{5}$$

where τ_S represents the same time for just one destination. Figure 2 right illustrates the meaning of each one of the times that intervene in the transportation and distribution processes. Introducing the expressions (2) to (5) in expression (1) and working out the number of zones, we get finally:

$$Z = \frac{\tau_S N + (k_Z/V)\sqrt{AN}}{T - (k_C/V)\sqrt{A} - T_T - (k_R/V)\sqrt{A/R}} \tag{6}$$

NETWORK TOTAL COST

The network distribution cost includes the maintenance and amortization expenses of the regional warehouses plus the drivers´ salaries and the vehicles amortization. The cost of a regional warehouse may have two components: a fixed cost C_F plus a variable cost C_P which is assumed to be proportional to the warehouse's size. Specifically, we will assume the cost of a warehouse to be:

$$C_A = C_F + \frac{C_P}{R} \tag{7}$$

where we have assumed the variable cost to be inversely proportional to the number of regions since the bigger the number of regions, the smaller the region that the warehouse should serve. Based on this reasonable assumption, the sum of costs of all the warehouses will be formed by an term proportional to the number of regions and by a fixed term:

$$C_A R = C_F R + C_P \tag{8}$$

In conclusion, the size-proportional cost becomes the territory's fixed cost and the regional warehouse's fixed cost represents a cost proportional to the number of regions. Therefore, the cost C_P can be left out from the analysis since it entails a total fixed cost.

On the other hand, the regional routes cost will be proportional to the cost of the driver's working hour for the amount of time employed in all the regional routes including transhipments, plus the cost of the regional vehicles which will be proportional to the kilometres travelled in all the regional routes. Likewise, the local routes cost will also be proportional to the driver's cost plus the local vehicle cost and the time employed in all the local routes. Consequently, the total cost *C* is given by:

$$C = C_F R + (C'_C + C'_V V) T_C R + C'_C T_T R + (C_C + C_V V)(T_R + T_Z + T_S) Z \tag{9}$$

where:

C_F = fixed maintenance and amortization cost of a regional warehouse;
C'_C = cost of a regional vehicle driver's working hour;
C'_V = cost of a regional vehicle in accordance to the travelled kilometres;
C_C = cost of a local vehicle driver's working hour;
C_V = cost of a local vehicle in accordance to the travelled kilometres;

DETERMINATION OF THE OPTIMUM NETWORK

After introducing expression (6) in the expression of the total cost (9) we get the cost as a function exclusively of the number of regions. The resulting expression is:

$$C = C_R R + \frac{T'(C_C + C_V V)(\tau_S N + (k_Z/V)\sqrt{AN})}{T' - (k_R/V)\sqrt{A/R}} \tag{10}$$

where

$$C_R = C_F + (C'_C + C'_V V) T_C + C'_C T_T = C_F + (C'_C + C'_V V)\frac{k_C}{V}\sqrt{A} + C'_C T_T \tag{11}$$

is the cost directly proportional to the number of regions and

$$T' = T_R + T_Z + T_S = T - T_C - T_T = T - \frac{k_C}{V}\sqrt{A} - T_T \tag{12}$$

is the time employed in the route from a regional warehouse to its destinations. It is possible to find the optimum number of regions that minimise the cost since expression (10) may be written as follows:

$$C = C_R a^2 \left[\lambda^2 + \frac{2\alpha}{1 - 1/\lambda}\right] \tag{13}$$

where *a*, *b* and α are dimensionless coefficients related to the physical variables by:

$$a = \frac{k_R\sqrt{A}}{T'V} \qquad b = \frac{C_C + C_V V}{2C_R}\left(\tau_S N + \frac{k_Z}{V}\sqrt{AN}\right) \qquad \alpha = \frac{b}{a^2} \tag{14}$$

Coefficient *a* is the relation between the territory average size and the maximum possible distance between a zone and its regional warehouse. Coefficient *b* is the relation between the cost of the route zone stretches and the effective cost per warehouse. Finally, the dimensionless variable λ is:

$$\lambda = \frac{\sqrt{R}}{a} \tag{15}$$

We get the optimum value of λ by differentiating in (13) and making the derivative equal to zero. Then, the optimum value λ_0 satisfies the equation:

$$\lambda_0(\lambda_0 - 1)^2 = \alpha \tag{16}$$

From this equation, only the solutions higher than 1 make sense since the number of zones, which is given by the expression (6), depend on λ in the following manner:

$$Z = \frac{\tau_S N + (k_Z/V)\sqrt{AN}}{T'(1 - 1/\lambda)} \tag{17}$$

and the denominator of (17) must be positive. Equation (16) always has at least one real solution greater than the unit. For $\alpha < 4/27$, there are three real solutions for λ, but two of them are less than one and therefore invalid. The root higher than the unit has been represented in the left vertical axis of figure 3 as a function of $\alpha^{1/3}$.

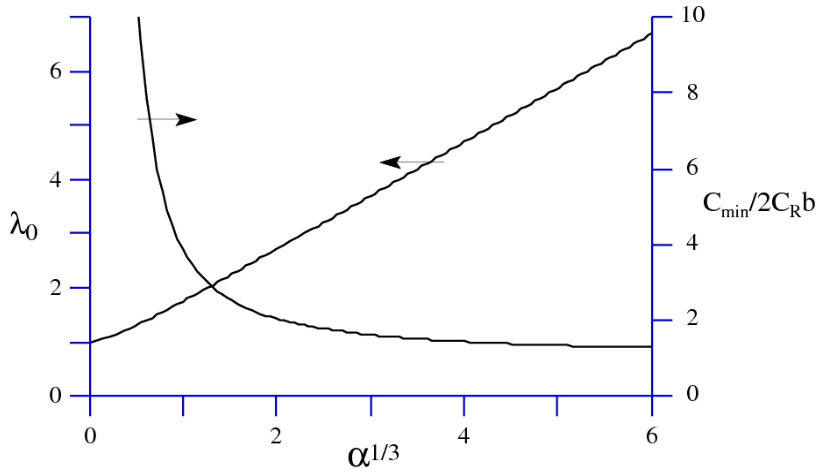

Figure 3 - Optimum value of λ and minimum cost.

For $\alpha > 1$ the following approximation yields an error less than 5%:

$$\lambda_0 \approx \frac{2}{3} + \sqrt[3]{\alpha} \tag{18}$$

For greater α values, the approximation is practically exact. The minimum cost is given by:

$$C_{min} = C(\lambda_0) = C_R a^2 \left[\lambda_0^2 + \frac{2\alpha}{1 - 1/\lambda_0}\right] = C_R b \frac{\lambda_0(2\lambda_0 - 1)}{(\lambda_0 - 1)^2} \geq 2C_R b \tag{19}$$

The right vertical axis in figure 3 represents the dimensionless minimum cost $C_{min}/2\, C_R\, b$ as a function of $\alpha^{1/3}$. It can be noted that as α increases, the minimum cost tends to an absolute minimum whose value is:

$$C_{min} \to 2C_R b = (C_C + C_V V)\left(\tau_S N + \frac{k_Z}{V}\sqrt{AN}\right) \tag{20}$$

On the contrary, for $\alpha < 1$ the cost grows without limit. Another important quantity that helps interpreting the nature of the optimum solution is the optimum number of zones per warehouse. This quantity, in accordance with (15), (16) and (17), is given by:

$$\frac{Z_0}{R_0} = \frac{\tau_S N + (k_Z/V)\sqrt{AN}}{a^2 T' \lambda_0(\lambda_0 - 1)} = \frac{(\tau_S N + (k_Z/V)\sqrt{AN})(\lambda_0 - 1)}{bT'} \tag{21}$$

The range $\alpha >> 1$ corresponds to the situation where the available time for the local routes is much longer than the average time necessary to travel across the territory, or to the case where the cost proportional to the number of warehouses is much smaller than the cost of the zone routes. Under these circumstances, the cost is basically determined by the routes zone stretches in the *N* destinations. In this situation, the optimum number of warehouses grows with α and in accordance with (21). The optimum number of zones for each warehouse also grows, and therefore, the optimum number of zones grows even faster. Specifically regarding (15), (18) and (21), the following proportionality relations are obtained when $\alpha > 1$:

$$R_0 \propto \lambda_0^2 \qquad Z_0 \propto \lambda_0^3 \propto \alpha \tag{22}$$

The infimum cost is determined by (20) and will be reached when the cost proportional to the number of warehouses is zero, or when the available time for the routes from the warehouses is infinite. Under these ideal conditions, the infimum cost would simply be the cost of the routes zone stretches.

In the range $0 < \alpha < 1$, the minimum cost blows up since in this situation, the time constraint for the local routes or the cost proportional to the warehouses is very relevant. Therefore in order to complete the service within the required time, it is necessary to reduce the number of zones per warehouse, as indicated by (21). In the limit when α tends towards zero, λ_0 gets closer to the unit and the number of zones per warehouse gets closer to zero. Obviously, the least feasible value of λ_0 corresponds to the case $Z_0 = R_0$, in which each warehouse has a single route or zone. Figure 4 shows the optimum number of regional warehouses for different values of *a* and α .

CONCLUSIONS

This work has presented a two-level distribution network model. The model is based on approximating the set of destinations by a continuum. This allows calculating the routes length without the need of knowing the destinations locations. The model is a generalisation of Rosenfield´s model (Rosenfield et al., 1992). These authors ignore the time constraint for the regional routes. In other words, they consider $T_C = T_T = 0$, which allows them unlinking the regional problem from the local problem. The model proposed in this work allows dealing with a more complete cost structure and it further shows that it is possible to optimise the network by applying a continuous approximation. Finally, employing this approximation allows identifying the main parameters that determine the solution.

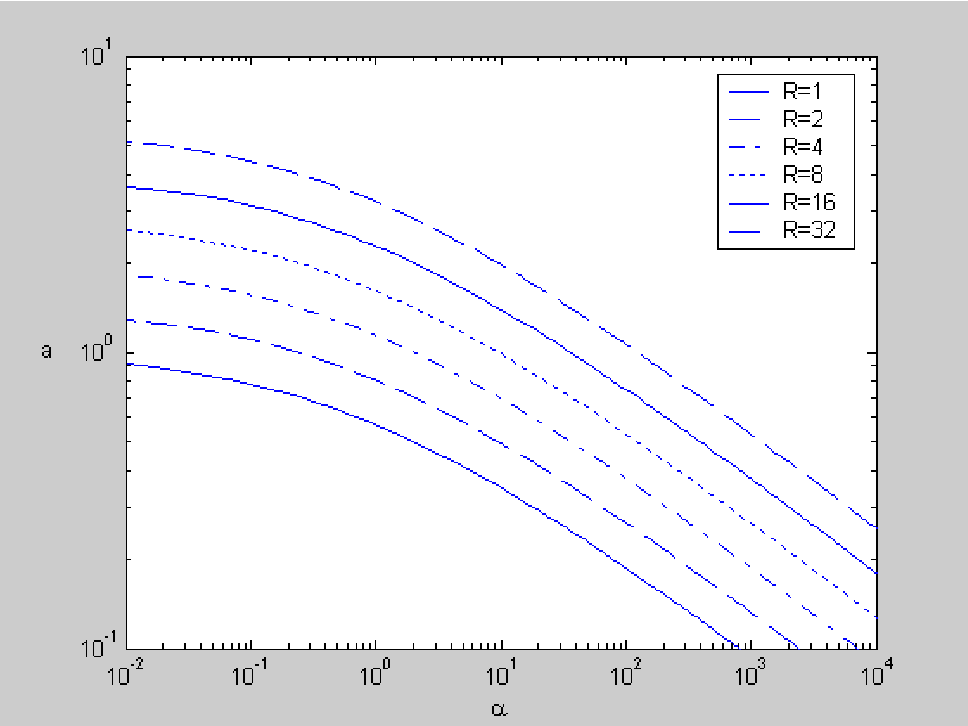

Figure 4 - Optimum number of regional warehouses (R) for given values of *a* and α .

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