THE STOCHSTIC CELL TRANSIMMISION MODEL BASED DYNAMIC JOURNEY TIME RELIABILITY ANALYSIS

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ABSTRACT

This paper attempts to assess the dynamic journey time reliability using the travel time reliability index which considers the dynamic journey time distribution. The stochastic cell transmission model (SCTM) is applied to propagate the dynamic stochastic traffic flows to generated cumulative in-/outflow distributions of each link which compose the route to be tested. The inputs into the SCTM are the stochastic fundamental diagrams and the stochastic demand profile along the path. Based on the dynamic cumulative flow distributions of each link, the Probability Mass Functions (PMF) of dynamic link exit (travel) time distribution is calculated using the First-In-First-Out (FIFO) principle and the 'sampling' technique, the journey time distribution is propagated through the weighted summation method. The model and the algorithm are tested against an empirical dataset of a freeway segment in California (PeMS database).

Keywords: Dynamic link exit (travel) time estimation, dynamic journey travel time distribution, the stochastic cell transmission model, the probability mass function, Buffer Time Index.

INTRODUCTION

Path (journey) travel time reliability, which describes the degree of stability of travel time, plays an important role in travelers' route choice and departure time choice behavior. The variation of journey time is caused by both of link demand uncertainties and random physical properties of the links that compose the route. The concept of travel time or journey time reliability has been analyzed mainly under the static modeling framework (see e.g. Asakura and Kashiwadani 1991;

and Walting 2008). However, the congestion or travel time variability in one time period may have impacts upon the network in other subsequent periods. Therefore, there is a strong need for the dynamic model with the stochastic elements to better evaluate the dynamic nature of transport network uncertainties.

To capture the randomness in both demand and supply sides, Sumalee *et al*. (2009) proposed the Stochastic Cell Transmission Model (SCTM) which extends the modified cell transmission model (MCTM) (Lebacque, 1996) to allow stochastic parameters of the fundamental flow-density diagram as well as the stochastic inflow travel demand. The supply uncertainties in SCTM are governed by the random parameters of the piecewise flow-density diagram, *i.e.*, free-flow speed, jam-density, and backward wave speed. The demand uncertainties are modeled as stochastic exogenous inputs to the SCTM. Sumalee *et al*. (2009) illustrated the applicability and performance of the proposed SCTM with the empirical dataset of a highway segment in California from the PeMS database. However, they did not propose a method to derive the travel time distribution from the SCTM.

For the deterministic CTM, the travel time for traffic entering a highway segment at each time period can be deduced to be the time that the cumulative outflow is equal to the cumulative inflow at the entering time following the First-In-First-Out (FIFO) principle. However, this approach cannot be directly applied to the SCTM since both of the cumulative in-flow and out-flow are stochastic (represented by statistical distributions). This paper, thus, proposes a method which extends the deterministic FIFO concept for calculating the travel time distribution based on the stochastic cumulative inflow and outflow curves. Each interval of potential exit time of vehicles entering the freeway section at time t will be evaluated the Probability Mass Function

(PMF) (Miller and Mahmassani, 2000) of its likelihood to be the exit time of the vehicles. Given the PMFs of all potential exit time intervals of vehicles entering the segment at time *t*, the distribution fitting technique can be used to calibrate the statistical distribution of the travel time. The distribution of travel time for traffic entering the link at each time period can then be used to analyze the travel time reliability of that highway segment. Several indicators were proposed for evaluating travel time reliability but in this paper we will adopt the buffer time index (which is similar to the concept of travel time budget).

The remainder of this paper is structured as follows. The next section introduces some preliminary concept of the SCTM and the index of travel time reliability used in this paper. Then, the third section describes the proposed method for calculating the statistical distribution of travel time based on the outputs from the SCTM. The fourth section presents the numerical test of the proposed method and SCTM with an empirical case study of a highway segment in California. The final section concludes the paper.

JOURNEY TIME ESTIMATION AND RELIABILITY INDEX

Evaluation of deterministic journey time

In a macroscopic deterministic dynamic traffic flow model, the flow propagation equation for a link l_m can be expressed as follows:

$$C_{in}^{l_m}(t) = C_{out}^{l_m}[\tau(t)], \tag{1}$$

where $\tau(t)$ is the exit time from link l_m for a vehicle that enters the link at time t. $C_{in}^{l_m}(t)$ and $C_{out}^{l_m}[\tau(t)]$ are the cumulative inflow volumes at time *t* and the cumulative outflow volumes at time $\tau(t)$, respectively. (1) is referred to as a time-flow consistency equation on the condition that FIFO holds. The link travel time $\eta_{l_m}(t)$ is then defined as

$$\eta_{l_m}(t) = \tau(t) - t. \tag{2}$$

Journey time of path *p* is affected by the travel time $\eta_{l_m}(t), \left(l_m \in p,\, p \in \mathrm{P}^{rs},\, m = 1, 2, \cdots N_l\right)$ of all the links that compose the route *p* which connects the OD pair *rs*. In the deterministic case, the journey time can be defined as:

$$\eta_p(t) = \tau_{l_{Nl}}(\cdots \tau_{l_m}(\cdots \tau_{l_1}(t))) - t. \tag{3}$$

Since the freeway is exposed to demand and supply uncertainties, the cumulative flows of a link are stochastic in nature which in turn yield stochastic travel time. To evaluate the stochastic travel time $\eta_{l_m}(t)$, we need to revise (1)-(3) to allow the stochasticity.

The Stochastic Cell Transmission Model

Preliminaries of the model

The stochastic cell transmission model (SCTM) was proposed by Sumalee *et al*. (2009). The SCTM is a stochastic macroscopic dynamic traffic flow model which is developed basing on the CTM (Daganzo, 1994), the MCTM (Lebacque, 1996) and the switching mode model (SMM)

(Munoz *et al.*, 2003).

For a certain route *p*, it can be divided into several segments according to the location of the detectors and the on-/off ramps (Fig. 2), while the SCTMs, which are developed for each specific segment will load in the dynamic demand profiles and stochastic physical parameters based on both of the historical and real-time data provided by the detectors on the boundaries of the link (Fig. 1), and propagate the dynamic traffic state distributions which can be used for evaluating the link time distribution and journey time distribution.

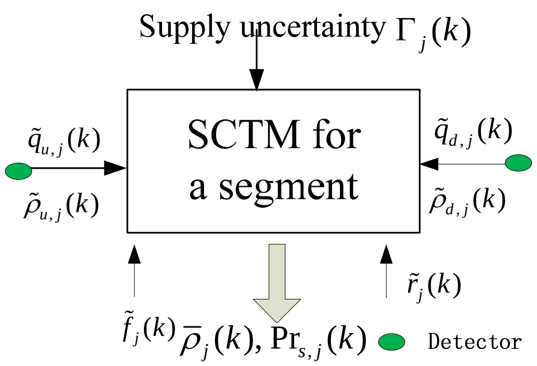


Figure 1.Schematic diagram of SCTM for segment *j*

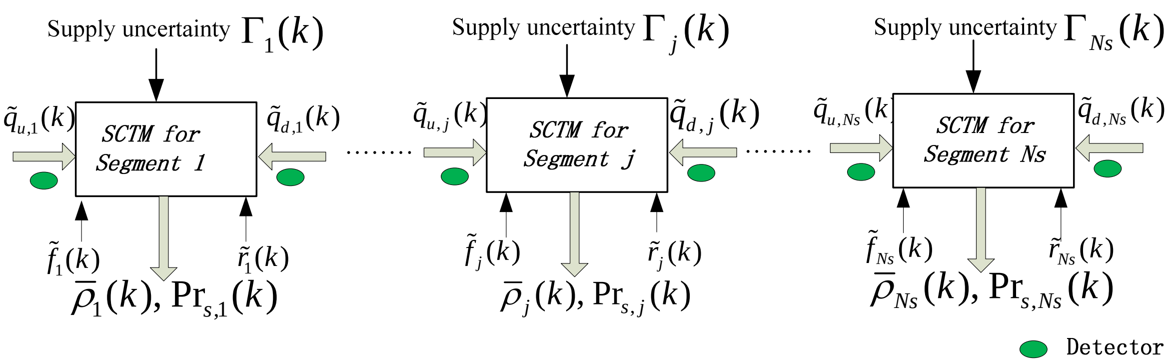


Figure 2.The schematic diagram of SCTM for route *p*

As depicted in Fig.4, a freeway segment with one on-ramp and one off-ramp is divided into *p* cells. Based on the triangular fundamental diagram (Fig. 3) and the “at most-one-wave front” assumption, where the wave front is the location that the congestion and free flow traffic states encounter, we categorize the freeway state into five modes: two steady state modes FF and CC, and three transient modes (CF, FC1, and FC2). FF denotes that all of the current cell densities are lower than their corresponding critical density ρ_c, while CC denotes that the densities are greater than their ρ_c. The three transient modes assumes that the one and only one wave front is located on the boundary between cells $l-1$ and l, the difference of FC1 and FC2 lies on the wave front’s moving direction. Since the traffic states are stochastic, it is obvious that there must be some traffic states beyond the assumption of five modes mentioned above, so the introduction of a new mode, which could represent the exceptional states, is rather necessary.

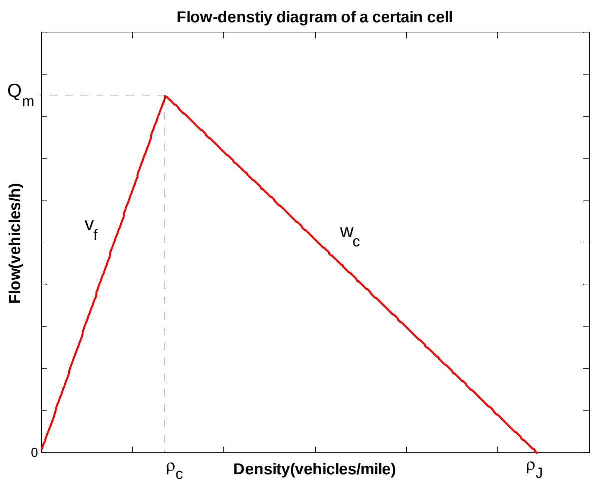
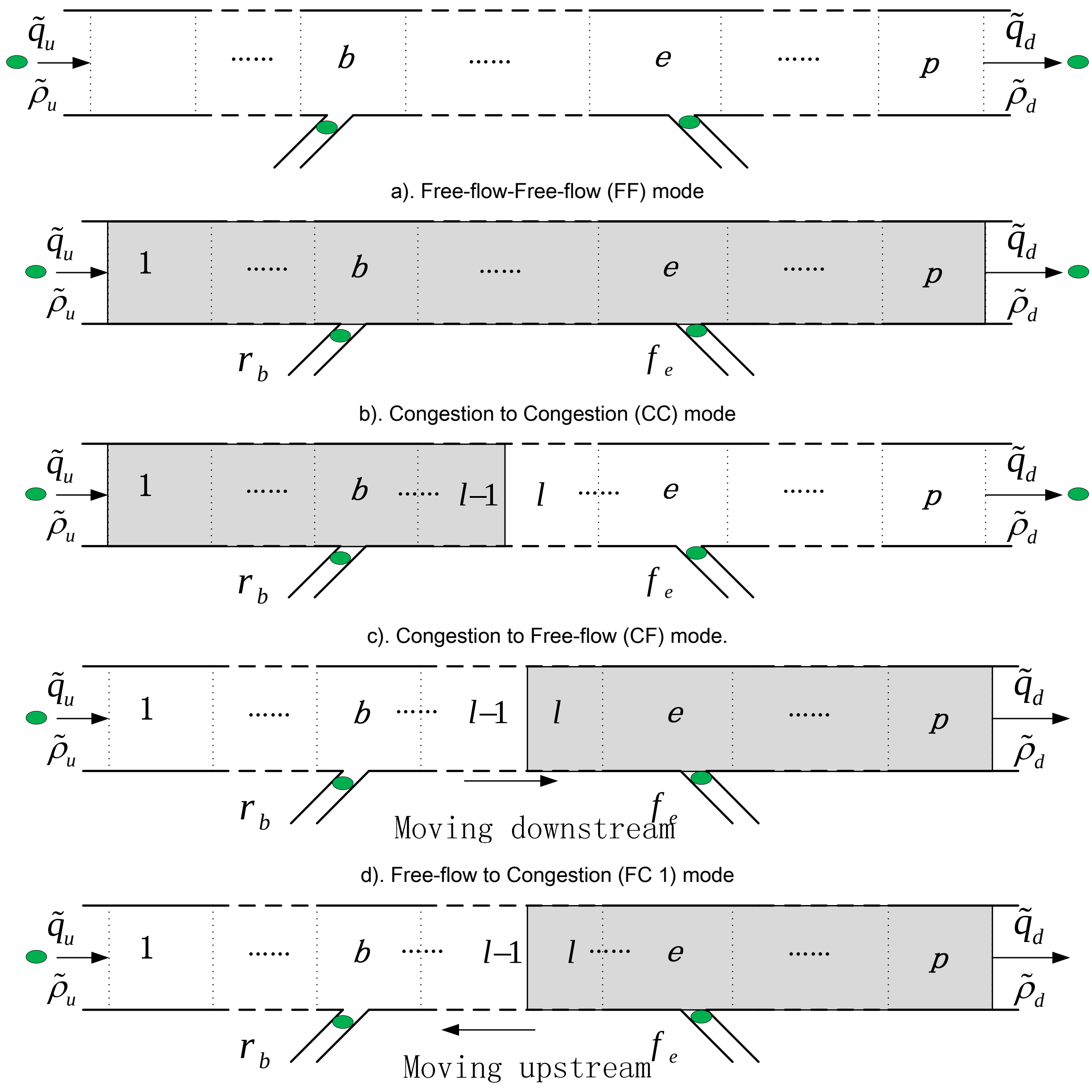


Figure 3.Triangular fundamental diagram of a cell



a). Free-flow-Free-flow (FF) mode

b). Congestion to Congestion (CC) mode

c). Congestion to Free-flow (CF) mode.

d). Free-flow to Congestion (FC 1) mode

e). Free-flow to Congestion (FC 2) mode

Figure4. Freeway segment with p cells

The flow chart of the dynamic stochastic traffic state propagation based on SCTM is depicted in Fig.5, where $P_{s,l}(k)$ $s \in \{FF, CC, CF, FC1, FC2, I\}$, $l = 2\cdots, p$, denotes the probability of each mode , $\rho_{s,l}(k)$ represents the density vector of mode *s* with wave front between $l-1$ and l (if necessary) at time step *k*. $\overline{\rho}_j(k)$ is the joint density of segment *j*, which has been updated as the finite mixture distribution of the five modes on the previous time step.

Specification of the prbability

The probabilities of $P_{s,l}(k)$ can be evaluated by:

FF mode: $P_{FF}(k) \triangleq \Pr\left(\tilde{\rho}_u(k) < \rho_{c,1} \cap \tilde{\rho}_d(k) < \rho_{c,p} \cap \overline{\rho}_i(k) < \rho_{c,i}\right), \left(i = 2, \cdots p-1\right)$

CC mode: $P_{CC}(k) \triangleq \Pr\left(\tilde{\rho}_u(k) \geq \rho_{c,1} \cap \tilde{\rho}_d(k) \geq \rho_{c,p} \cap \overline{\rho}_i(k) \geq \rho_{c,i}\right), \left(i = 2, \cdots p-1\right)$

CF mode: $P_{CF,l}(k) \triangleq \Pr\begin{pmatrix} \tilde{\rho}_u(k) \geq \rho_{c,1} \cap \tilde{\rho}_d(k) < \rho_{c,p} \\ \cap \overline{\rho}_{i1}(k) \geq \rho_{c,i1} \cap \overline{\rho}_{i2}(k) < \rho_{c,i2} \end{pmatrix}, \left(i1 = 2, \cdots l-1;\ i2 = l, \cdots p-1\right)$

FC1 mode: $P_{FC1,l}(k) \triangleq \Pr\begin{pmatrix} \tilde{\rho}_u(k) < \rho_{c,1} \cap \tilde{\rho}_d(k) \geq \rho_{c,p} \\ \cap \overline{\rho}_{i1}(k) < \rho_{c,i1} \cap \overline{\rho}_{i2}(k) \geq \rho_{c,i2} \\ \cap v_{f,l-1} \overline{\rho}_{l-1}(k) < w_{c,l}\left(\rho_{J,l} - \overline{\rho}_{c,l}(k)\right) \end{pmatrix}, \left(i1 = 2, \cdots l-1;\ i2 = l, \cdots p-1\right)$

FC2 mode: $P_{FC2,l}(k) \triangleq \Pr\begin{pmatrix} \tilde{\rho}_u(k) < \rho_{c,1} \cap \tilde{\rho}_d(k) \geq \rho_{c,p} \\ \cap \overline{\rho}_{i1}(k) < \rho_{c,i1} \cap \overline{\rho}_{i2}(k) \geq \rho_{c,i2} \\ \cap v_{f,l-1} \overline{\rho}_{l-1}(k) \geq w_{c,l}\left(\rho_{J,l} - \overline{\rho}_{c,l}(k)\right) \end{pmatrix}, \left(i1 = 2, \cdots l-1;\ i2 = l, \cdots p-1\right)$

and

$$P_I(k) = 1 - P_{FF}(k) - P_{CC}(k) - \sum_{l=2}^{p} P_{CF,l}(k) - \sum_{l=2}^{p} P_{FC1,l}(k) - \sum_{l=2}^{p} P_{FC2,l}(k)$$

where $\tilde{\rho}_u(k)$ and $\tilde{\rho}_d(k)$ are measured densities of the entry boundary and exit boundary of the whole segment, respectively, $\overline{\rho}_i(k)$ is the traffic density of cell *i* which is estimated at last time step, $\rho_{c,i}$ is the critical density of cell *i* in the segment. The case *I* assumes that the traffic state distribution remains unchanged, which represents the cases beyond the one wave front assumption.

The traffic state $\rho_{s,l}(k+1)$ of each mode *s* can be evaluated by a stochastic bilinear system.

$$\rho_{s,l}(k+1)=\left(A_{s,l,0}+\sum_{i=1}^{p}A_{s,l,i}\omega_{s,l,i}(k)\right)\overline{\rho}(k)+\left(B_{s,l,0}+\sum_{i=1}^{p}B_{s,l,i}\omega_{s,l,i}(k)\right)\lambda_{s,l}(k)+B_{s,l}u(k), \tag{4}$$

where $u(k)$ are the measured input signals, $\overline{\rho}(k)$ is the estimated joint traffic density which has been evaluated at last time index. $\rho_{s,l}(k+1)$ is the traffic density vector of mode *s* to be updated at current time index. $B_{s,l}, A_{s,l,i}, B_{s,l,i},\ i=0,\cdots,p$ are constant matrices with elements depending on the sample time T_s and the cell length l_i, $\omega_{s,l,i}(k),\ \lambda_{s,l}(k)$ are second-order mutually uncorrelated real-valued random processes which represent the supply uncertainties.

For example, assume *p*=4, *r*=2, *e*=3, $\overline{\rho}(k)=\left[\overline{\rho}_1(k),\ \overline{\rho}_2(k),\ \overline{\rho}_3(k),\ \overline{\rho}_4(k)\right]^T$,

and $u(k)=\left[q_u(k),\ r_2(k),\ f_3(k),\ q_d(k)\right]^T$,under the FF mode $\omega_{FF,i}(k)=v_{f,i}$, $\lambda_{FF}(k)=0_{4\times 1}$.

$$A_{FF,0}=I_{4\times 4}\ ,\ A_{FF,1}=\begin{bmatrix}-\frac{T_s}{l_1} & & & \\ \frac{T_s}{l_2} & 0 & & \\ & 0 & 0 & \\ & & 0 & 0\end{bmatrix},\ A_{FF,2}=\begin{bmatrix}0 & & & \\ 0 & -\frac{T_s}{l_2} & & \\ & \frac{T_s}{l_3} & 0 & \\ & & 0 & 0\end{bmatrix},A_{FF,3}=\begin{bmatrix}0 & & & \\ 0 & 0 & & \\ & 0 & -\frac{T_s}{l_3} & \\ & & \frac{T_s}{l_4} & 0\end{bmatrix},$$

$$A_{FF,4}=\begin{bmatrix}0 & & & \\ 0 & 0 & & \\ & 0 & 0 & \\ & & 0 & -\frac{T_s}{l_4}\end{bmatrix}, B_{FF}=\begin{bmatrix}\frac{T_s}{l_1} & & & \\ & \frac{T_s}{l_2} & & \\ & & 0 & \\ & & -\frac{T_s}{l_4} & 0\end{bmatrix}$$

As there is no wavefront in FF, l is not necseeary. The specification of other modes can be refered to Sumalee *et al*.(2009).

The mean and covariance matrix by finite mixture

The mean and auto correlation matrix of traffic density under each mode, that is $E\left(\rho_{s,l}(k+1)\right)$ and $Q_{s,l}(k+1)=E\left(\rho_{s,l}(k+1)\rho_{s,l}(k+1)^T\right)$ can be calcuated by a series of formulations (Sumalee *et al*. 2009) based on the the assumption that the demand and supply uncertainties are neither spatio nor temporal correlated.

The probability density function (PDF) of the joint traffic density $f\left(\overline{\rho}(k+1)\middle|\theta(k+1)\right)$ is defined as:

$$f\left(\overline{\rho}(k+1)\middle|\theta(k+1)\right)=\sum\nolimits_{s,l}P_{s,l}(k)f\left(\overline{\rho}(k+1)\middle|\theta_{s,l}(k+1)\right). \tag{5}$$

The expectation of joint density $E\left(\overline{\rho}(k+1)\middle|\theta(k+1)\right)$ is given by

$$E\left(\overline{\rho}(k+1)\mid\theta(k+1)\right)=\sum\nolimits_{s,l}P_{s,l}(k)E\left(\rho_{s,l}(k+1)\right). \tag{6}$$

Let $\mu_{s,l}(k+1)=E\left(\rho_{s,l}(k+1)\right)$ and $\mu(k+1)=E\left(\overline{\rho}(k+1)\middle|\theta(k+1)\right)$.

The covariance matirx of joint density is:

$$Var\left(\overline{\rho}(k+1)\middle|\theta(k+1)\right)=\sum\nolimits_{s,l}P_{s,l}(k)Q_{s,l}(k+1)-\mu(k+1)\mu^{T}(k+1). \tag{7}$$

The dynamic departure flow rate and arrival flow rate distributions can also be evaluated based on the fundamental diagram and finite mixture algorithm.

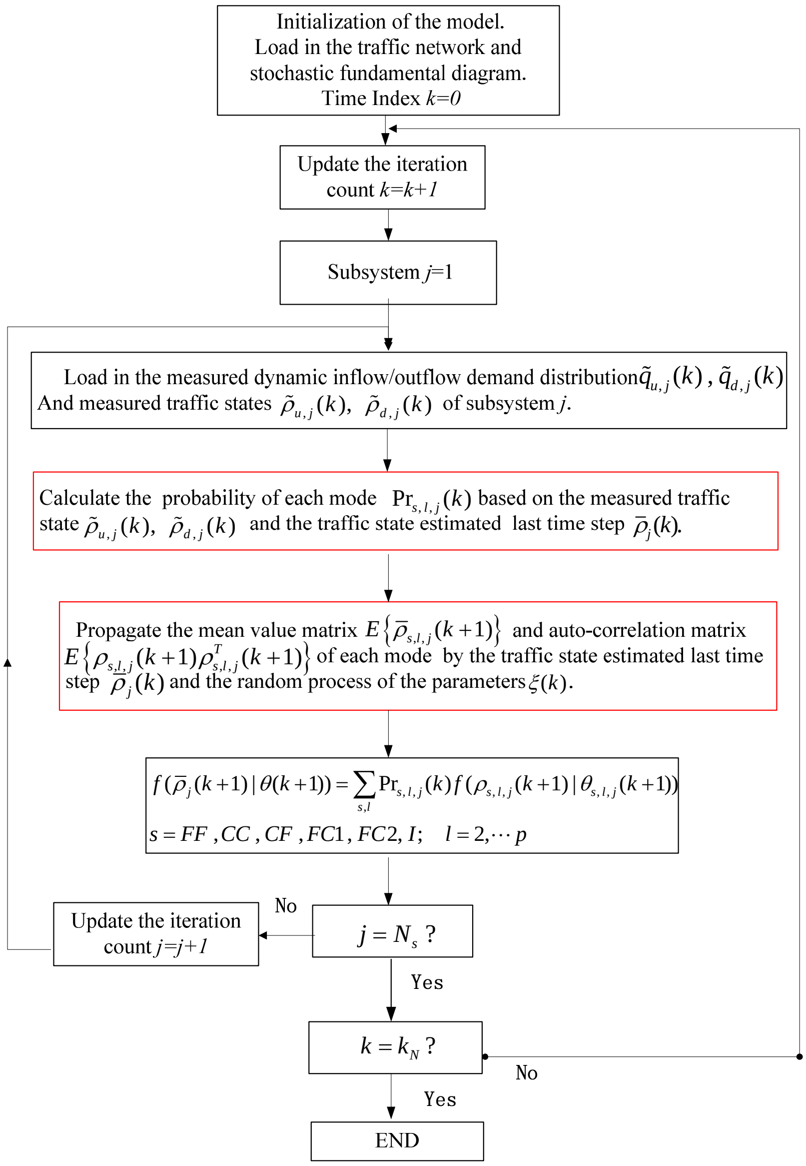


Figure 5. The flow chart of the propagation of dynamic stochastic traffic state based on SCTM

The dynamic stochastic journey time estimation

Relative frequency and PMF of the link exit (travel) time

Given the distributions of the inflow rate $q_{in}^{l_m}(k)$ and outflow rate $q_{out}^{l_m}(k)$, the corresponding cumulative inflow and outflow can be evaluated. Assume the cumulative inflow volume $C_{in}^{l_m}(k) \cong f_{Cin}\left(\overline{C}_{in}^{l_m}(k), \sigma_{C_{in}^{l_m}}(k)\right)$ where $f_{Cin}(\cdot)$ denotes a certain random distribution, $\overline{C}_{in}^{l_m}(k)$ denotes the mean of $C_{in}^{l_m}(k)$, and $\sigma_{C_{in}^{l_m}}(k)$ is the standard deviation. The corresponding cumulative arrival volumes $C_{out}^{l_m}(k) \cong f_{Cout}\left(\overline{C}_{out}^{l_m}(k), \sigma_{C_{out}^{l_m}}(k)\right)$ follows a similar notation.

In traffic engineering, "sampling" in conjunction with the probability mass function (PMF) is usually applied to construct the distributions of traffic volume and speed. In Miller-Hooks and Mahmassani, (2000) and (1998), the PMF approach is applied to discretize the cumulative distribution function (CDF) of the stochastic travel time. Based on these two methods, we propose the following definition:

Definition 1 For a vehicle enters the link l_m at time index *k* (Entry time index $ETI = k$), the probability of the link exit time to be *k'* is defined as $P'_{k'|k} = Pr\left(-\varepsilon \leq C_{out}^{l_m}(k') - C_{in}^{l_m}(k) \leq \varepsilon \mid ETI = k\right)$, with a pre-defined $\varepsilon \in R^+$. Define the matching error $e_k(k') = C_{out}^{l_m}(k') - C_{in}^{l_m}(k)$, which represents the difference between the cumulative inflow at time index *k* and cumulative out flow at time index *k'*. ε is set to be a small integer.

Since the SCTM is developed for any second-order random processes with wide-sense stationary noise sequences (random demands and parameters), we can handle any cumulative flows which are described by second-order random processes with the same assumptions of the SCTM. Under this definition, we define our "sampling" process as depicted in Fig. 6.

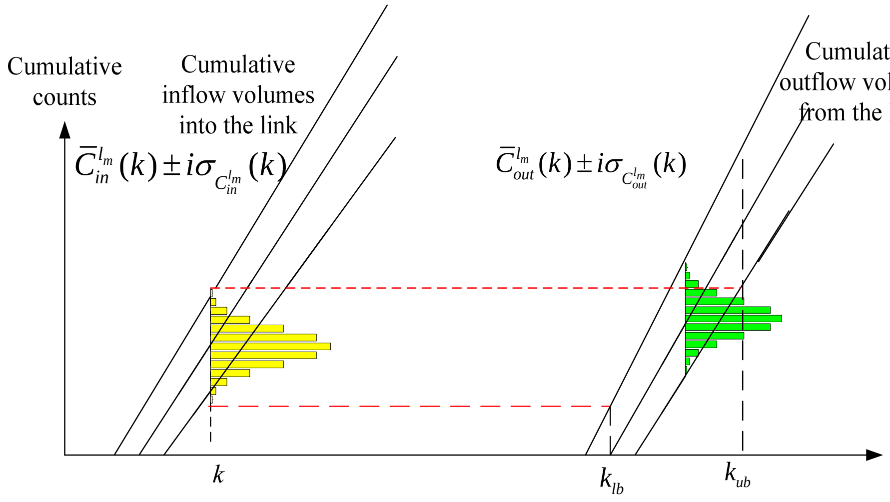


Figure 6. An illustration of selection of the "sampling" region

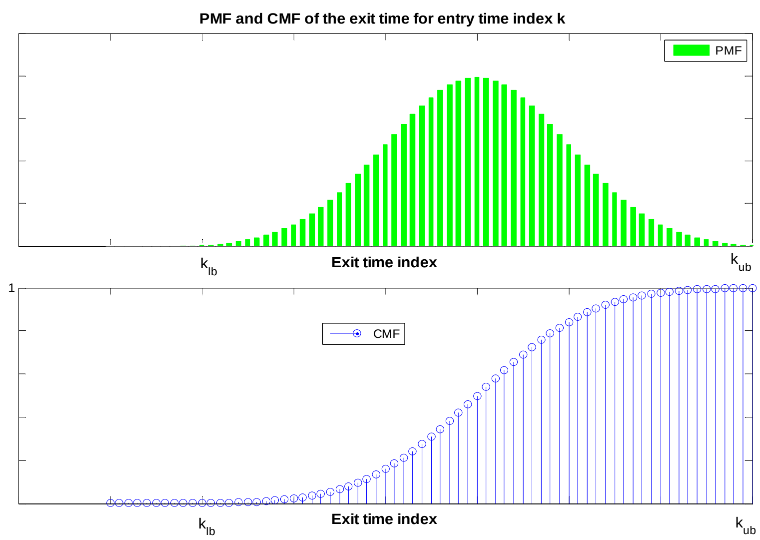


Figure 7.The PMF of the relative frequency and the corresponding CMF respect to exit time index.

Definition 2 The “sampling” time interval $[k_{lb}, k_{ub}]$ is defined as:

$$k_{lb} = \min\left\{k' : \left\|\left(\overline{C}_{in}^{l_m}(k) - i\sigma_{C_{in}^{l_m}}(k)\right) - \left(\overline{C}_{out}^{l_m}(k') + j\sigma_{C_{out}^{l_m}}(k)\right)\right\| \leq \varepsilon_s\right\},$$

$$k_{ub} = \min\left\{k' : \left\|\left(\overline{C}_{in}^{l_m}(k) + i\sigma_{C_{in}^{l_m}}(k)\right) - \left(\overline{C}_{out}^{l_m}(k') - j\sigma_{C_{out}^{l_m}}(k)\right)\right\| \leq \varepsilon_s\right\},$$

Where ε_s is a small positive number, and *i* ,*j* are positive integers which can be adjusted such that there is no overlapping between the two curves $\overline{C}_{in}^{l_m}(k) - i\sigma_{C_{in}^{l_m}}(k)$ and $\overline{C}_{out}^{l_m}(k') + j\sigma_{C_{out}^{l_m}}(k)$ (which in turn implies *k'*>*k*).

By this “sampling” technique described as definition 1 and definition 2, for an entry time *k*, we can obtain a series of $P'_{k'|k}$ which describe the likelihood of the exit time index to be equal to *k'(*link travel time index to be equal to *k'-k)*. Notice that for an entry time *k*, the summation of the probabilities of all possible travel time $\sum_{k'} P'_{k'|k}$ may not be equal to 1, while the following definition of relative frequency can solve this problems.

Definition 3 For a vehicle entering link a at time *k*, the relative frequency $P_{k'|k}$ of the link travel time to be *k'*-*k* is defined as:

$$P_{k'|k} = \frac{P'_{k'|k}}{\sum_{k'=k_{lb}}^{k'=k_{ub}} P'_{k'|k}}, \forall k' \in [k_{lb}, k_{ub}]. \tag{8}$$

Remark 1 By definitions 1-3, we can construct the PMF of the link travel time for traffic entering the link at time *k*. An illustration of the construction process of the PMF of the link travel time is shown in Fig.7. Each bar on the upper plot represents $P_{k'|k}$. The solid line with dots on the lower plot denotes the corresponding cumulative mass function (CMF).

Remark 2 By definition 1 the probability of the exit time index to be *k'*-*k* can be converted to be $P'_{k'|k} = Pr\left(-\varepsilon \le e_k(k') \le \varepsilon \mid ETI = k\right)$, where $e_k(k') = C_{out}^{l_m}(k') - C_{in}^{l_m}(k)$.

The PDF of $e_k(k')$ is required to solve $P'_{k'|k}$.

$$\begin{aligned} e_k(k') &= C_{out}^{l_m}(k') - C_{in}^{l_m}(k) \\ &= \sum_{x=1}^{x=k'} q_{out}^{l_m}(x)T_s - \sum_{x=1}^{x=k} q_{in}^{l_m}(x)T_s \\ &= \sum_{x=k+1}^{x=k'} q_{out}^{l_m}(x)T_s - (-\sum_{x=1}^{x=k} q_{out}^{l_m}(x) + \sum_{x=1}^{x=k} {}_{in}^{l_m}(x))T_s \\ &= \sum_{x=k+1}^{x=k'} q_{out}^{l_m}(x)T_s - \sum_{i=1}^{p_{lm}} \overline{\rho}_i^{l_m}(k)l_i \end{aligned}$$

Assume that $q_{out}^{l_m}(x),\ x = k+1,\cdots k'$ and $\overline{\rho}_i^{l_m}(k), i = 1,\cdots p_m$ are independent, where p_m is the number of the cells involved in the link l_m, the mean and variance of the distribution $e_k(k')$ can be evaluated by the properties of normal distribution.

Remark 2 avoids the issue in which the variance of $e_k(k')$ may increase linearly with time, and the computing cost becomes smaller since for the flow rates before time *k* is not required for the calculation.

Journey time distribution

Assume the whole journey starts from link l_1 and ends at l_{N_l} (Fig.8), the relative frequency of exit time index to be k^m for entry time index k^{m-1} on link l_m is defined as $P_{l_m}(k^m \mid k^{m-1})$, $m = 1,\cdots l_N,\, k^m > k^{m-1} > 0,\, and\ k^m = k_{k^{m-1},lb},\, k_{k^{m-1},lb} + 1, \cdots,\ k_{k^{m-1},ub}$.

The probability of journey exit time to be k^{Ns} for the vehicle enters the path at *k* is calculated as the sequence, say link after link :

$$
\begin{aligned}
&P(k^1 \mid k) = P_1(k^1 \mid k)\\
&P(k^2 \mid k) = \sum_{k^{1,x}=k_{k,lb}}^{k^{1,x}=k_{k,ub}} P_2(k^2 \mid k^{1,x}) P_1(k^{1,x} \mid k)\\
&\cdots\\
&P(k^m \mid k) = \sum_{k^{m-1,x}=k_{k_{m-2},lb}}^{k^{m-1,x}=k_{k_{m-2},ub}} P_m(k^2 \mid k^{m-1,x}) P_{m-1}(k^{m-1,x} \mid k)\\
&\cdots\\
&P_p(k^{Ns} \mid k) = \sum_{k^{m-1,x}=k_{k_{Ns}-2,lb}}^{k^{m-1,x}=k_{k_{Ns}-2,ub}} P_m(k^{Ns} \mid k^{Ns-1,x}) P_{Ns-1}(k^{Ns-1,x} \mid k)
\end{aligned}
\tag{9}
$$

Given the PMF of the journey time distribution evaluated by (9), the journey time distribution respect to entry time index *k* is solved.

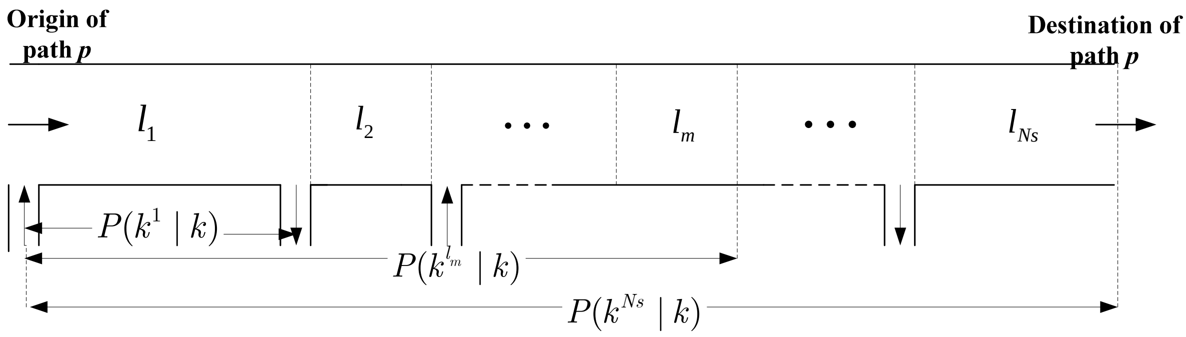


Figure 8. Path p with N_l links

Figure 9.The fitted distribution respect to link exit time index

Distribution fitting and dynamic stochastic journey (link) time

After obtaining the PMF of the whole route, various random distribution fitting techniques can be applied to obtain the distribution that best fits the PMF. Define the distribution of the dynamic link travel time as follows:

Definition 4 A distribution $g\left(\overline{\beta}(k), \sigma_{\beta(k)}, \alpha\right)$ is the distribution of the dynamic link travel time at time *k*, *i.e.*, $T_{rs}(k) \cong g\left(\overline{\beta}(k), \sigma_{\beta(k)}, \alpha\right)$, if $g\left(\overline{\beta}(k), \sigma_{\beta(k)}, \alpha\right)$ best fits the PMF and/or the corresponding CMF of $\beta(k)$, where α is the shape parameter which reflects the skewness of the distribution (Fig.9).

The "sampling" process we defined above is similar to the idea used in Miller-Hooks and Mahmassani, (2000), Miller-Hooks and Mahmassani, (1998) which discretized the CDF to obtain the PMF of dynamic travel time. From the literature, the link travel time under the free-flow condition usually follows a normal distribution. On the other hand, under the congested condition the link travel time may follow a skewed distribution[1] (Lint *et al*.,2008, Kharoufeh and Gautam 2004). In this article, we use MATLAB to fit the distribution of the stochastic dynamic link travel time by skew normal distribution, see Azzalini and Capitanio (1999).

Index of Travel Time Reliability

Buffer time is one of the most widely used travel time reliability indices, in which the buffer time represents the extra time that travelers must add to their average travel time when planning trips to ensure his on-time arrival (Lomax *et al.* 2003). This concept is similar to the safety margin measure proposed in Hall (1983) and recently adopted in Lam *et al* (2008). In this paper, the probability of arriving on-time will be set to the 95th percentile which can be translated as "can be late to work 1 day a month without getting into too much trouble", that is 95% of travel time

[1] If $\alpha > 0$, the distribution is positive skewed, $\alpha < 0$, it's negative skewed, else it is normal distribution.

observations can be found below this criteria. Buffer time index can then be calculated by equation (10):

$$Buffer\ Time\ Index = \left[\frac{95th\ Pencentile\ Travel\ Time - Average\ Travel\ Time}{Average\ Travel\ Time} \times 100\%\right] \quad (10)$$

It is obvious that the journey time distribution is necessary to evaluate the BTI of the whole journey.

EMPIRICAL EXPERIMENT

Test site description and data preparation

The case study is a section of Interstate 210 West, approximately two miles in length, shown in Fig. 10. This section, located in Los Angeles, stretches from S Myrtle Ave (A) through W. Huntington Dr(B) to N Santa Anita Ave(C), and contains 2 on-ramps and 2 off-ramps. The section is instrumented with single-loop inductance detectors, which are embedded in the pavement along the mainline, HOV lane, on-ramps, and off-ramps. In this empirical study, we divide the route into 4 cells as shown in the schematic diagram, composing a SCTM with 4 cells in one segment, and choose cell 1 and cell 2 as the first link, cell 3 and cell 4 as the second link . During the dynamic stochastic traffic states' propagating progress, we assume the detector between cell 2 and cell 3 doesn't provide the real time traffic state, aiming at comparing the estimated dynamic distributions with the actual ones.

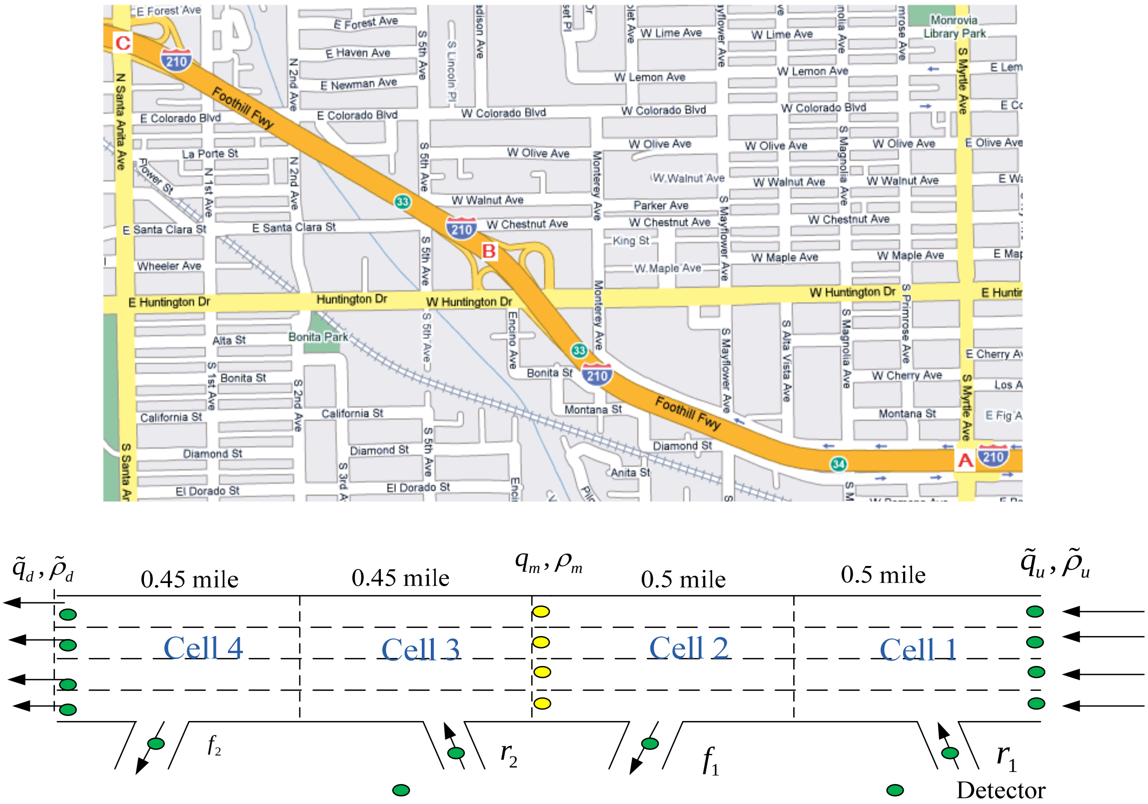


Figure 10. Map and Section of I210-W divided into 2 cells and its detector configuration

The traffic flow data of 8 hours (4:00 am-12:00 am) collected on Tuesday, Wednesday and Thursday of April, 2008 and 2009 from the Performance Measurement System (PeMS) is used in this test. Each loop detector provides the data on traffic volume (veh/time-step) and occupancy measurements of the corresponding lane for every 30 seconds. The densities can be computed for each lane using the occupancy divided by the *g*-factor, where the g-factor is the effective vehicle length (in miles), provided by the system. The time step should be less than 30 seconds to satisfy the requirement of the SCTM. Thus a $zero^{th}$ -order interpolation is applied to the PeMS data to yield the data with $T_s = 5$ sec interval in order to ensure that $v_{f,i} \cdot T_s \leq l_i$ for almost all the time. The calibration of the stochastic triangular fundamental diagram is conducted for the four cells using the historical data over two months period. The results are shown in Table 1 and Fig.11. The notations with the hat symbol denote the mean values of the parameters.

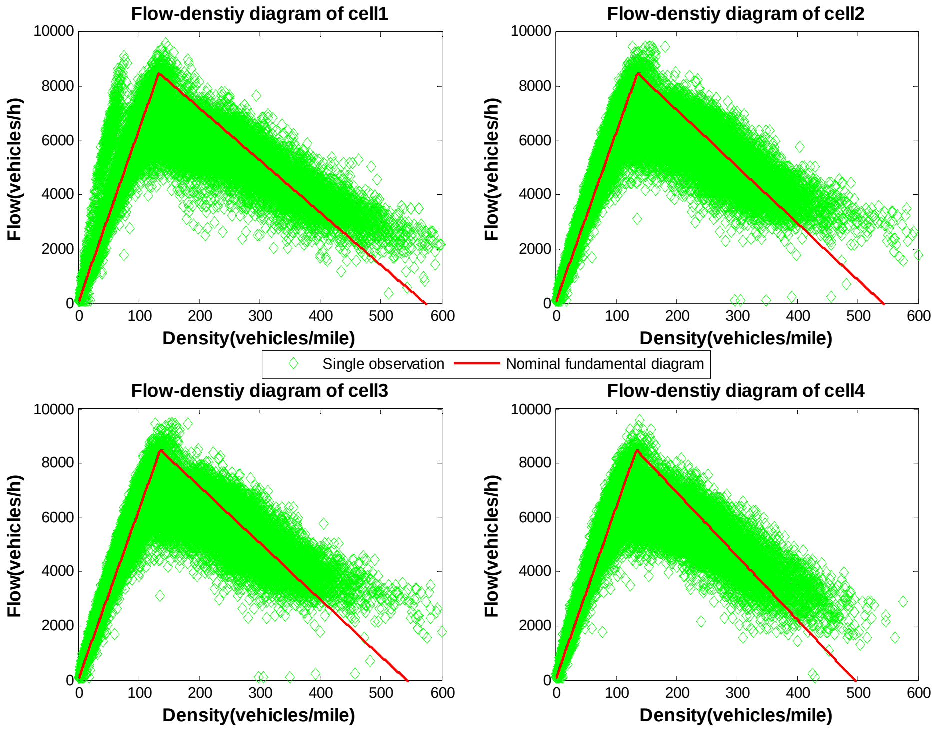


Figure 11. Fundamental diagram of the 4 cells

Table1. Calibration results of the 4 cells

	$\hat{v}_f$	$\sigma_{\hat{v}_f}$	$\hat{w}_c$	$\sigma_{\hat{w}_c}$	$\hat{\rho}_c$	$\hat{\rho}_J$	$\sigma_{\hat{\rho}_J}$	$\hat{Q}_m$	$\sigma_{\hat{Q}_m}$
Cell 1	63.56	8.88	19.34	5.85	133.74	573.29	146.48	8500	1188
Cell 2	62.40	6.53	20.86	6.83	136.21	543.79	149.42	8500	889.61
Cell 3	62.40	6.53	20.86	6.83	136.21	543.79	149.42	8500	889.61
Cell 4	63.12	6.73	19.55	6.00	126.74	535.87	138.17	8000	852.65

Overall description of the test results

Fig. 12 depicts the mean values of the simulated densities and its 68 percent confidence interval $[\overline{\rho}-\sigma_{\overline{\rho}},\overline{\rho}+\sigma_{\overline{\rho}}]$, generated by the SCTM against the measured traffic densities. As we have mentioned, the density and flow rate of the boundary between cell 2 and cell 3 are assumed to be unknown during the SCTM simulation. Compared with the estimated density of the two cells with the measured one, the results reflect that the traffic state distribution can be estimated accurately.

Fig.13 depicts the probability distributions of the five modes over time. During 4:00AM - 5:30AM, the FF mode dominates the stochastic traffic states. Then, the transient modes become active between 5:30 – 6:30. During 6:30-10:00 the CC mode becomes the dominant state. The order of the dominant states is then in reverse after the peak period (i.e. after 10:00). The probability of the case *I* is very small, suggesting that the one wavefront assumption is still valid for this case since is not very long highway segment.

Fig. 14 show the estimated dynamic link travel time distributions by the proposed method for every 5 second against the measured travel time (every 5 minute). Fig. 14 (1) presents the travel time on the section composed by the first 2 cells, and Fig.14 (2) presents the result of last 2 cells. Fig. 15 depicts the overall journey time distribution. Compared with the travel time recorded by PeMS, both the link travel time and journey time can capture the travel time trend, and the confidence interval is credible.

Buffer Time Index of journey time is then illustrated in Fig.16. The index maintains at a low level before 6:00, and rise to 30% during 6:00-6:30. The BTI is then at the high level until 11:00. The result reflects that after the congestion period even the congestion already dissolved an 30% extra time is still required to ensure the on-time arrival with 95% confidence interval.

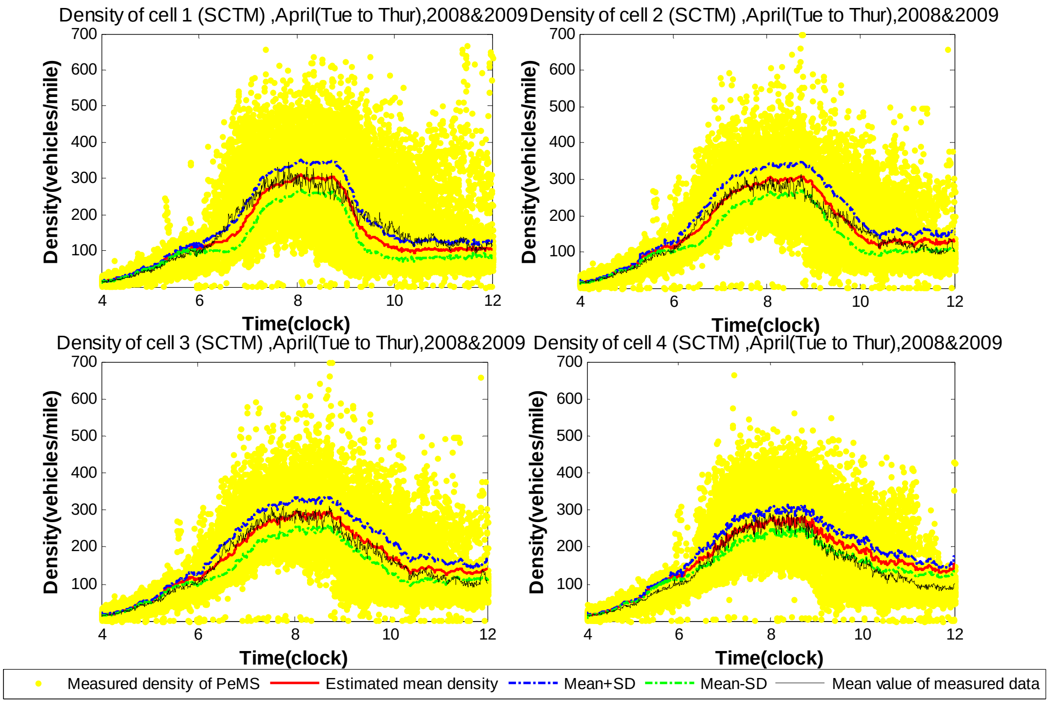


Figure 12. The estimated density distribution of the 4 cells.

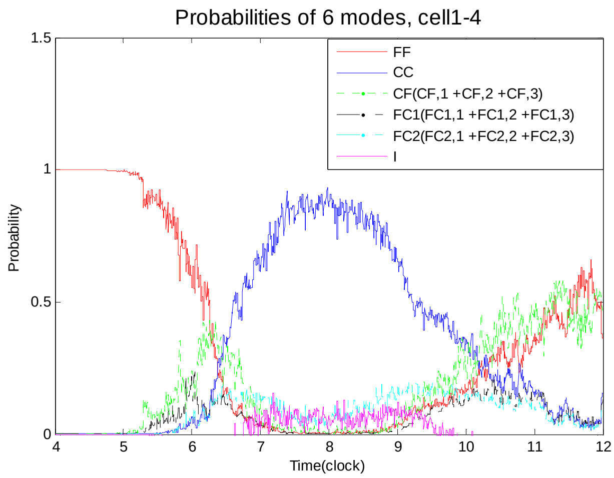


Figure 13. The probability of five modes of the freeway segment.

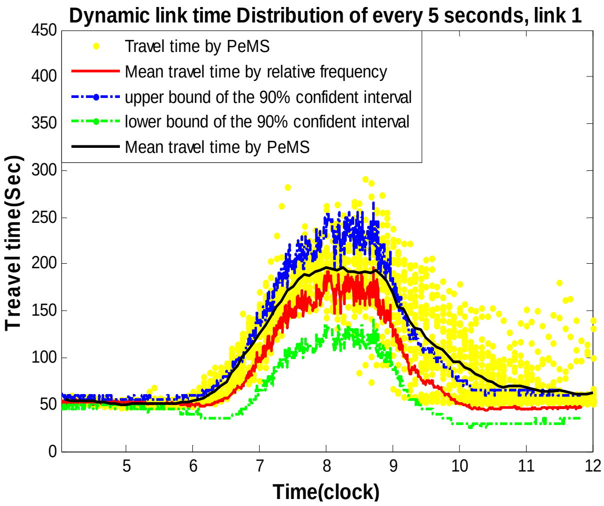


Figure 14(1). Dynamic travel time of the link cell1+cell2

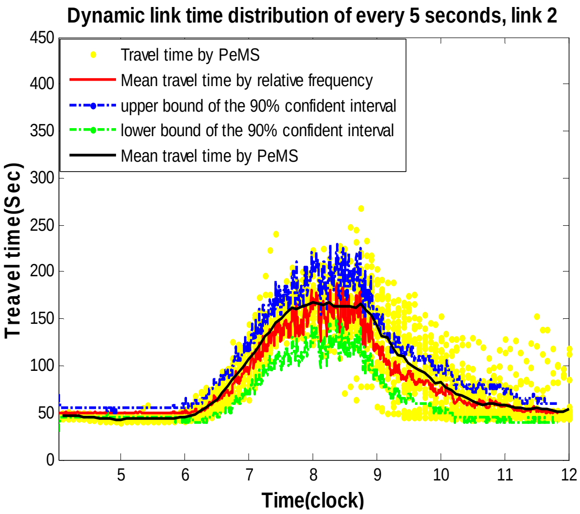


Figure14(2) . Dynamic travel time of the link cell3+cel4

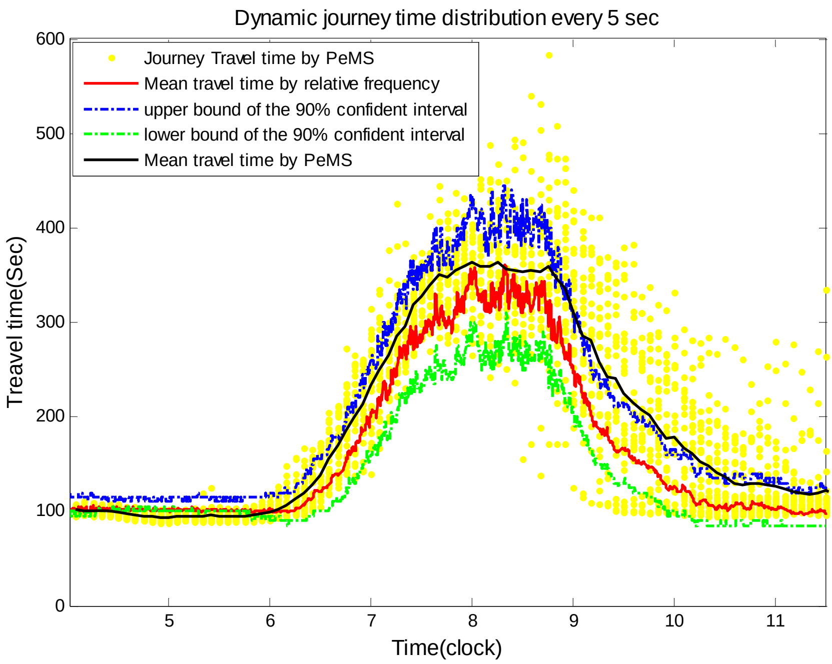


Figure 15. Dynamic journey time distribution

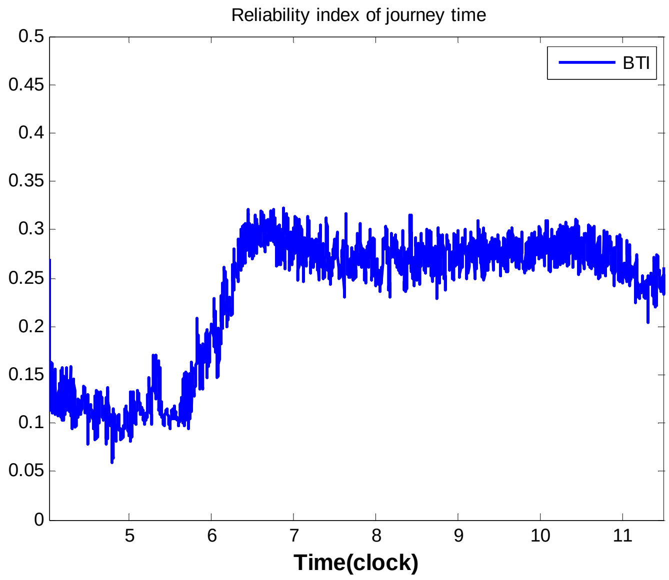


Figure 16. Buffer time index of journey time distribution

Discussion of the results

Sensitivity

The corresponding mean absolute percent error (MAPE) of the 2-mile journey by different ε and flow mapping regions (adjusted by *i* and *j*) are given in Table 2. As shown in the table, the MAPE is not sensitive to the value of ε nor to the mapping interval.

Table 2. MAPE with different ε and mapping interval respect to the journey time

ε / i, j \ MAPE	0. 5	1	2	5	10
$[-\sigma,\sigma]$	15. 67%	15. 67%	15. 67%	15. 67%	15. 68%
$[-2\sigma,2\sigma]$	15. 21%	15. 21%	15. 23%	15. 24%	15. 26%
$[-3\sigma,3\sigma]$	15. 21%	15. 21%	15. 21%	15. 22%	15. 24%
$[-4\sigma,4\sigma]$	15. 21%	15. 21%	15. 21%	15. 21%	15. 23%

Skewness

It can be observed that the gap between the upper bound of 90% confidence interval with the mean value and the gap between the lower bound of the interval are not equivalent, which is not consistent with the properties of normal distribution. The difference is caused by the skewness of the distribution. It is reported that the travel time are always skewed under congestion conditions (Lint *et al*., 2008):

• Free-flow condition: the mean travel times are low and the spread of the distribution is small. The distribution of travel times is approximately symmetric.

• Congestion onset: the mean travel times are increasing, but the distribution is skewed to the left.

•Congestion: the mean travel times are high, while the travel time distribution is wide and either symmetric or slightly right skewed.

• Congestion dissolve: the mean travel times are decreasing, and the distribution is skewed to the left again.

Similar results can be observed in the distribution of the dynamic travel time presented in this paper. During the onset of congestion and congestion dissolve periods (Fig.17), the distribution is positive-skewed, in which the distribution is left skewed . During the free flow period and congestion period (around 8:20 am) (Fig. 18), the distribution is either normal or slightly right skewed. The results are generally consistent with the statistical results reported in Lint *et al* (2008). The comparisons between the PMF and PDF as well as the comparisons between CDFs prove that the distribution fitting method is accurate. In addition, this empirical example illustrates that the dynamic stochastic travel time can be approximated by the skew normal distribution.

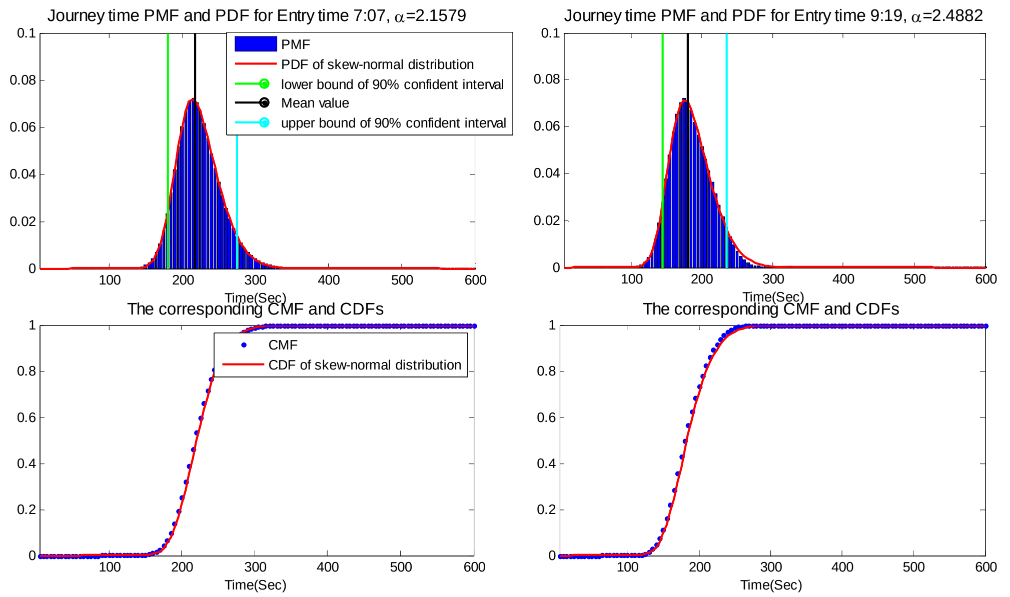


Figure 18.The fitted positive-skewed normally distributed travel time and its CDF

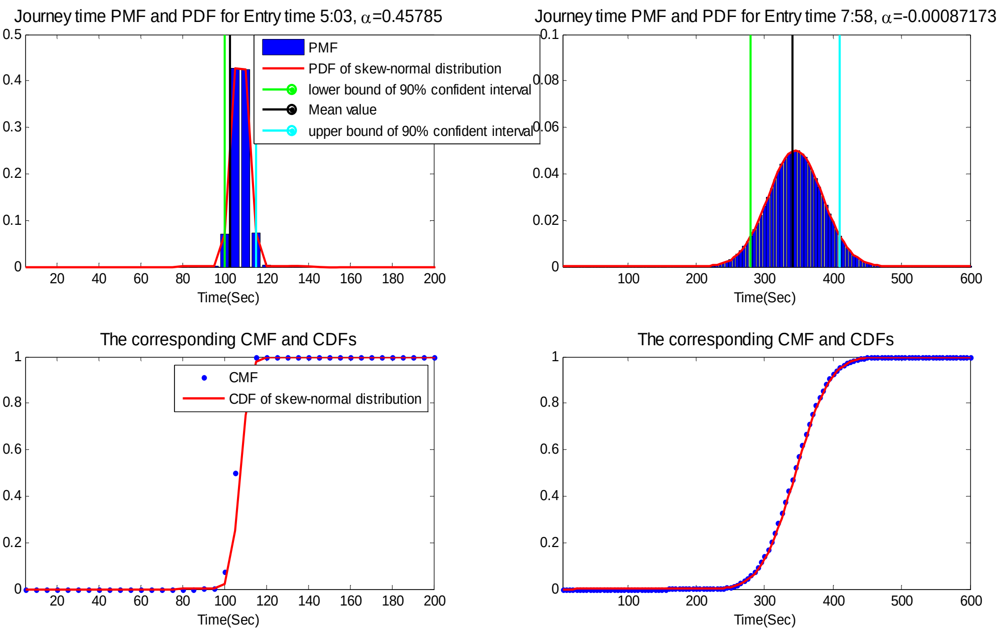


Figure 19. The fitted normally distributed travel time and its CDF.

CONCLUSION

This paper proposed a method to derive the statistical distribution of journey (link) time from the outputs of the SCTM which is a stochastic dynamic traffic flow model. The proposed method and framework allows the analysis of travel time reliability from the static to dynamic paradigm. The proposed algorithm extends the FIFO principle (which is only applicable to the deterministic cumulative in-flow and out-flow curve) to the case with stochastic cumulative in-flow and out-flow. The concept of internal sampling to estimate the likelihood of the exit time for each entry time on a highway segment based on the FIFO concept is used. The likelihood of each possible travel time can then be used to construct the PMF of the travel time. The paper then assessed the journey time reliability based on the dynamic journey time distribution. First, the SCTM is certified once again to be able to perform satisfactory in replicating stochastic traffic states under demand and supply uncertainties efficiently and accurately. Second, the PMF based link travel time estimation method is confirmed to be capable of describing the distribution of travel time instead of providing only mean and variance. The algorithm is not sensitive to parameter variations. The skewness of the estimated travel time is also consistent with the statistical observation in the other study. A possible extension of this model is to introduce the journey time reliability into the dynamic traffic assignment involving route choice and departure time choice decisions in a full traffic network. The SCTM and the proposed travel time estimation algorithm should also be improved to consider the multi-lane and weaving effect to better represent the traffic dynamics.

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