A NEW MULTICLASS MULTIMODAL COMBINED MODEL FOR PASSENGER MARKET SHARE ESTIMATION IN ECONOMIC CIRCLES

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ABSTRACT

This paper presents a multiclass multimodal combined model for passenger market share estimation between main cities in an economic circle. An economic circle consists of more than one closely adjoining central cities and their influence zones, such as Yangtze River Delta in China. People may choose private car or public transit, such as intercity bus or train, to finish a trip. Mode choices of passengers are modelled in combination with flow assignment in a multimodal transportation network with deterministic travel demand. The multi-modal transportation network is composed of a highway network and a railway network. Travelers of a class perceive their generalized cost as a weighting of travel time and travel cost. Generalized travel cost model considers road congestion on the highway network as well as congestion and capacity effects on public transit network. Private cars and intercity buses run on the highway network with asymmetric cost interactions. We assume that stochastic user equilibrium governs the route choice of car users in the road network, while a deterministic user equilibrium principle governs which kind of public transport services will be chosen in the public transit network. And a logit model is used to determine passenger's choices of car or public transit. A variational inequality formulation is proposed to capture all the components of the proposed model in an integrated framework. The model provides an alternative to existing travel mode forecasting models. The MSA algorithm is presented to solve the model. A multimodal transportation network between two cities is presented to illustrate the proposed methodology. The results show the effectiveness of the proposed model and the solution algorithm.
Keywords: metropolitan travel modes forecasting, multiclass multimodal combined model, mode choice, traffic network equilibrium, variational inequality

1. INTRODUCTION

With the rapid development of China's economy, economic circles have been formed and developing in China, such as Yangtze River Delta economic circle, Bohai Sea rim economic circle, Pearl River Delta economic circle. An economic circle consists of more than one closely adjoining central cities and their influence zones. Economic circle is also called Metropolitan Area in some other countries. Neighboring cities in an economic circle are so close not only because of their physical distances but also because of their employment and commercial dependencies. People often commute between cities in an economic circle. Travel demand between neighboring cities has incurred in the economic circle dramatically. And the existing transportation network cannot satisfy the travel demand. So we need to construct new highways, new railways or update the existing transportation network. In order to configure transportation network in the economic circle scientifically and reasonably, traffic demand forecasting models for economic circle need to be developed. This paper presents a network equilibrium model for simultaneous prediction of mode choice and route choice for different users over multimodal transportation network of economic circle.

Researchers have done a lot of work about the design and optimization of transportation network. Since the first mathematical formulation of user-equilibrium assignment was proposed by Beckmann et al. (1956), studies on transportation network equilibrium have been developed rapidly. But they mostly focus on urban transportation network. In urban multimodal network context, important advances have been realized over the past 30 years in the formulation and analysis of multi-modal network equilibrium models (Florian, 1977; Florian and Spiess, 1983; Nagurney, 1984; Wong, 1998; Ferrari, 1999). Wu and Lam (2003) proposed a network equilibrium model with motorized and nonmotorized transport modes. As far as mode choice for trips between cities is concerned, logit type models were widely used in previous studies to predict the proportions of trips made among several competing transport modes (Vovsha, 1997; Hensher, 1998; Koppelman and Sethi, 2005; Monzon and Rodriguez-Dapena, 2006). The faults of these "pure" logit models are that they do not consider the configuration of network and how the flows are distributed over the network. The cost of a traveler choosing a mode is concerned with travel flow. So mode choice and route choice should be simultaneously predicted over economic circle transportation network, and one way is to use combined models. Combined models in a multimodal network setting are far from new. A synthesis and review of these models is presented by Boyce (1990) and Boyce (1998), whose work has made significant contributions in this field. The combined models can be formulated by using the equivalent optimization approach (Florian and Nguyen, 1978; Safwat and Magnanti, 1988; Lam and Huang, 1992; Abrahamsson and Lundqvist, 1999), variational inequality (VI) approach (Dafermos, 1982; Florian et al., 2002), or fixed-point approach (Bar-Gera and Boyce, 2003). Multi-class models refer to models that consider two or more classes of travelers with different behavioral or choice characteristics. The task of extending Beckmann's model to the multiple-class case was taken up by Dafermos (1972). And a lot of work has been done in this field.

The following paper is organized as follows. The second section introduces basic considerations and notation. The third section gives the generalized cost for private and public networks with multiple user classes, respectively. The fourth section defines the equilibrium conditions used in this paper. The fifth section presents the VI formulation of this problem. Then a numerical example is followed. Finally, the conclusions are presented.

2. BASIC CONSIDERATIONS AND NOTATION

2.1. Basic considerations

Distances between two cities of economic circle is no more than 500km, travelers may complete their trips by auto, intercity bus or train, the travel choices are denoted by a, b and tr for short respectively. Intercity buses also run on the highways, and they share some same road segments with automobiles over the highway networks. Hence, asymmetric cost interactions between cars and intercity buses should be considered during the network analysis. In this paper, we consider that trip distribution is fixed and known for a given period, and assume that a traveler makes a trip from one city to another using a single mode, which means we do not consider travelers transfer between modes.

2.2. Notation

Consider a multimodal transportation network $G=(N,L)$, where N is the set of nodes and L is the set of links connecting nodes. The multimodal transportation network G in economic circle consists of the auto sub-network $G_a=(N_a,L_a)$, and the public transit sub-network $G_t=(N_t,L_t)$. The public transit sub-network includes intercity bus sub-network $G_b=(N_b,L_b)$ and train sub-network $G_{tr}=(N_{tr},L_{tr})$. Automobiles can change routes freely from the origin to the destination, so every physical links may be used by automobiles, and auto sub-network $G_a=(N_a,L_a)$ is the same as physical highway network. In comparison with automobiles, intercity buses have their own networks with some fixed routes, and some nodes and some physical links may not be included. We defined $N_b \subseteq N_a$, $L_b \subseteq L_a$. R is the set of origins, S is the set of destinations, $R \subset N, S \subset N$. q_{rs}^a denotes the travel demand by automobile between OD pair (r,s), $r \subset R$ and $s \subset S$. q_{rs}^b denotes the travel demand by intercity bus between OD pair (r,s), which is computed in passenger or person units, the passenger units can be transformed into the vehicular units by the seating capacity γ of an intercity bus. q_{rs}^{tr} denotes the travel demand by train between OD pair (r,s), which is also computed in passenger or person units. q_{rs}^t denotes the public transit demand, which is the sum of q_{rs}^{tr} and q_{rs}^b between OD pair (r,s). q_{rs} denotes travel demand between OD pair (r,s).

3. GENERALIZED TRAVEL COST

3.1. Generalized travel cost by automobile

Travel time is often used as the sole measure of travel cost because it is easier to be measured. However, ticket fare is also a very important factor which influences travelers' travel mode choices for an intercity trip. We can divide travelers into different classes with different utility functions to capture their specific travel choice characteristics.

Travel cost by the automobile mode is composed of two parts, travel time and fare. c_{rs}^{akp} is defined as generalized cost of path p for users of class k with automobile mode a ,which is formulated as,

$$c_{rs}^{akp} = \theta_1^k T_{rs}^{ap} \tau^k + \theta_2^k F_a + \theta_1^k C_{rs}^a$$

(1)

Where coefficients θ are weights . τ^k is the value of time for users of class k . F_a takes into account the fuel fares and the highway tolls. C_{rs}^a denotes travel time in city.

The travel time, T_{rs}^{ap} ,by automobile on route p from origin r to destination s can be given by the sum of the travel time on the links comprising this route, i.e.,

$$T_{rs}^{ap} = \sum_{l \in L_a} t^{al} \delta_{l,p}^{rs} \forall r, s, \ p \in P_{rs}^a, \ r \in R, \ s \in S$$

(2)

Where: $\delta_{l,p}^{rs}$ equals to 1 if link l is a part of route p from origin r to destination s , and 0 otherwise, L_a is the set of automobile links and P_{rs}^a is the set of all routes used by automobiles between OD pair (r,s) . t^{al} is the travel time on link l .

t^{al} can be computed by the following function.

$$t^{al} = t^{al}(x^{al}, v^{bl}) = t^{al(0)} \left[1 + \alpha_1 (\frac{x^{al}}{c^{al}})^{\beta_1} + \alpha_2 (\frac{v^{bl}}{c^{bl}})^{\beta_2} \right]$$

(3)

Where: $t^{al(0)}$ and c^{al} are the free-flow travel time and capacity of automobile link l respectively, c^{bl} are the free-flow travel time and capacity of intercity bus link l , α_1, α_2 , β_1 and β_2 are coefficients, x^{al} is the automobile flow on automobile link l v^{bl} is the intercity bus flow of intercity bus link l , and let $X^a = (..., x^{al}, ...)$.

3.2. Generalized travel cost by public transit

For every pure public transportation mode m , capacity constraints should be considered. As the travel demand increases, passengers will feel uncomfortable because it is crowded in vehicles. So the generalized cost functions by public transit mode are composed of the travel time, ticket fare and an additional penalty considering the capacity of modes.

c_{rs}^{mk} defined as the generalized cost for users of class k with mode m from origin r to destination s by public transit mode, can be expressed as,

$$c_{rs}^{mk} = a_m^k (\frac{q_{rs}^m}{Q_{rs}^m})^{c_m^k} + \theta_1^k \tau^k T_{rs}^m + \theta_2^k F_{rs}^m + \tau^k \theta_1^k C_{rs}^m \quad , \quad m = b, tr$$

(4)

Where: a_m^k and c_m^k are coefficients which need to be defined according to realistic data. q_{rs}^m is the passenger volume of mode m . Q_{rs}^m is the capacity of transit line between r and s . F_{rs}^m is the ticket fare of m from r to s .Unlike automobile mode, travel time by public transit mode is composed of the time outside the vehicle (including the access time from origin to station, waiting time at station, egress time from station to the final destination) and the in-vehicle travel time. C_{rs}^{m} denotes time outside the vehicle, which is used to measure the convenience of mode m.

Travel times of intercity buses depend on the flows of both intercity buses and private vehicles.The in-vehicle travel time, T_{rs}^{bp} ,by intercity bus on route p from origin r to destination s can be given by the sum of the travel time on the links comprising this route, i.e.,

$$T_{rs}^{bp} = \sum_{l \in L_b} t^{bl} \delta_{l,p}^{rs} \forall r, s,\ p \in P_{rs}^{b},\ r \in R,\ s \in S$$

(5)

Where: $\delta_{l,p}^{rs}$ equals to 1 if link l is a part of route p from origin r to destination s, and 0 otherwise, L_b is the set of intercity bus links and P_{rs}^{b} is the set of all routes used by intercity buses between OD pair (r,s).

t^{bl} is expressed as ,

$$t^{bl} = t^{bl}(v^{bl}, x^{al}) = t^{bl(0)} \left[1 + \alpha_3 (\frac{v^{bl}}{c^{bl}})^{\beta_3} + \alpha_4 (\frac{x^{al}}{c^{al}})^{\beta_4} \right]$$

(6)

Where: $t^{bl(0)}$ is the free-flow travel time of intercity link l, α_3, α_4, β_3, and β_4 are coefficients.

Railway transport is assumed to be congestion free, and to operate at a constant speed through the corridor. So the in-vehicle travel time T_{rs}^{tr} is a constant.

4. EQUILIBRIUM CONDITIONS

We assume that when a multimodal network reaches an equilibrium state, the route choices of auto drivers in the auto networks follows the stochastic user-equilibrium (SUE) conditions, travel choice decisions of travelers in the public transit networks satisfy the deterministic user-equilibrium (DUE) condition, and the modal split between public transit mode and auto is governed by the logit formulation. The definition of multimodal transportation network equilibrium in economic circle mentioned in this paper is described further as follows.

4.1. The choice of route for the pure auto mode

Travel choice decisions of auto users on route satisfy the SUE condition, which is stated as: for each OD pair, no motorist can improve his perceived cost by unilaterally changing routes. The SUE condition for the pure auto mode can mathematically be expressed as，

$$f_{rs}^{akp} = q_{rs}^{ak} \frac{\exp(-\alpha^k c_{rs}^{akp})}{\sum_{p' \in P_{rs}^{a}} \exp(-\alpha^k c_{rs}^{akp'})} \quad r \in R\,, s \in S \qquad (7)$$

Where: f_{rs}^{akp} is the automobile flow on path p between OD pair (r,s) and α^k is a given parameter which is used to measure the different degree of traveler’s knowledge of class k about the path travel cost.

4.2. The choice of different pure public mode

The equilibrium flows over the public network are assumed to satisfy UE conditions. At user equilibrium, for each user class, the travel cost on all used mode are equal. Thus, at equilibrium, the flows and travel times over the public network are such that,

$$c_{rs}^{mk} \begin{cases} = u_{rs}^{tk} & q_{rs}^{mk} > 0 \\ > u_{rs}^{tk} & q_{rs}^{mk} = 0 \end{cases} \tag{8}$$

Where: u_{rs}^{tk} is the minimum generalized travel cost between OD pair (r,s) for users of class k with public transit mode.

4.3. The choice of the auto and the public transit mode

The travelers' choice of auto and the public transit mode is governed by a logit-type formula, expressed as,

$$p_{rs}^{ak} = \frac{\exp\left[-\beta^k (u_{rs}^{ak} - \varphi_{rs}^{ak})\right]}{\exp\left[-\beta^k (u_{rs}^{ak} - \varphi_{rs}^{ak})\right] + \exp\left[-\beta^k (u_{rs}^{tk} - \varphi_{rs}^{tk})\right]}$$

(9)

Where φ_{rs}^{ak} and φ_{rs}^{tk} represent the bias parameter of commuters on auto and public mode. β^k describes the importance of travel disutility perception in the mode choice decision.(Fernandez et al., 1994; Garcia and Marin, 2005).

u_{rs}^{ak} is taken as the expected maximum utility of routes set P_{rs}^{a}:

$$u_{rs}^{ak} = -\frac{1}{\alpha^k} \ln\left[\sum_{p \in P_{rs}^a} \exp(-\alpha^k C_{rs}^{akp})\right]$$

(10)

5. VARIATIONAL INEQUALITY MODEL FORMULATION

Since travel time and flow interactions between the inter-city bus and automobile modes are asymmetric, the problem under consideration cannot be formulated and solved as an equivalent minimization program. The problem is thus formulated as a variational inequality. The equivalent VI formulation for the network equilibrium conditions presented in the previous section is given as below.

$$\sum_{rs}\sum_{p \in P_a}\sum_{k}(c_{rs}^{akp}(f^{akp*}, q^{b*k}) + \frac{1}{\alpha_{rs}} \ln f_{rs}^{akp*})(f_{rs}^{akp} - f_{rs}^{akp*}) + \sum_{rs}\sum_{m \in t} c_{rs}^{mk}(q_{rs}^{mk*})(q_{rs}^{mk} - q_{rs}^{mk*})$$
$$+\sum_{rs}\sum_{k} (\frac{1}{\beta_{rs}^k} \ln \frac{q_{rs}^{ak*}}{q_{rs}^k} - \varphi_{rs}^{ak})(q_{rs}^{ak} - q_{rs}^{ak*}) + \sum_{rs}\sum_{k}(\frac{1}{\beta_{rs}^k} \ln \frac{q_{rs}^{tk*}}{q_{rs}^k} - \varphi_{rs}^{tk})(q_{rs}^{tk} - q_{rs}^{tk*}) \geq 0$$

(11)

Subject to

$$\sum_{k} q_{rs}^{ak} + \sum_{k} q_{rs}^{tk} = q_{rs}, \forall r, s$$

(12)

$$\sum_{m\in t} q_{rs}^{mk} = q_{rs}^{tk}$$

(13)

$$\sum_{p\in P_{rs}^{a}} f_{rs}^{apk} = q_{rs}^{ak}, \forall r,s$$

(14)

$$f_{rs}^{ap} \geq 0, \forall r,s,p \in P_{rs}^{a}$$

(15)

$$q_{rs}^{mk} \geq 0, \forall r,s,m \in t$$

(16)

$$x^{al} = \sum_{rs}\sum_{k}\sum_{p\in P_{rs}^{a}} f_{rs}^{apk}\delta_{l,p}^{rs}, \forall r,s$$

(17)

$$x^{bl} = \sum_{rs}\sum_{k}\sum_{p\in P_{rs}^{b}} f_{rs}^{bpk}\delta_{l,p}^{rs}, \forall r,s$$

(18)

$$v^{bl} = \frac{x^{bl}}{\gamma}, \forall r,s$$

(19)

Where x^{bl} is the passenger flow on the intercity bus link l and let $X^{b} = (...,x^{bl},...)$. Equation (12) and (13) are the modal demand conservation constraint. Equation (14) represents a set of flow conservation constraints. Equation (15) and (16) are nonnegative constraints. Equation (17) and (18) are the relationship between link flow and path flow for the automobile mode and intercity bus mode respectively. Equation (19) transforms the passenger units into the vehicular units by the seating capacity γ of an intercity bus.

We can prove that the proposed VI formulation (11) lead to equilibrium conditions (7) – (9) according to the KKT conditions of VI formulation.

6. SOLUTION ALGORITHM

The MSA algorithm is one of methods that can solve the above variational inequality program. Detailed description of the MSA algorithm can be found in the literature (Powell and Sheffi, 1982; Huang and Li, 2007). A description of the solution algorithm is as follows:

Step 0: Initialization. Find a feasible link flow pattern vector. Set n=1.

Use the modal split function, equation (9), to determine $q_{rs}^{ak(1)}$, $q_{rs}^{tk(1)}$ with flow equaling to 0 for each class. Perform SUE assignment for automobile sub-network based on $t^{al} = t^{al}(0,0)$. This yields $\left\{x^{al(1)}\right\}$.Perform all-or-nothing assignment for transit network based on the initial modal demands. This yields $\left\{x^{tl(1)}\right\}$.

Step 1: Update travel times. Set travel times for automobile and inter-city bus based on the current travel pattern $\left\{x^{al(n)}\right\}$ and $\left\{x^{tl(n)}\right\}$.

Step 2: Find auxiliary travel patterns. Use equation (9) to determine $p_{rs}^{ak(n)}$ and $p_{rs}^{tk(n)}$ with current link flow. Perform SUE assignment for automobile sub-network based on current link flow to obtain $\left\{y^{al(n)}\right\}$. Perform user equilibrium assignment for transit network to obtain $\left\{y^{tl(n)}\right\}$.

Step 4: Updating.

Set $x^{al(n+1)} = x^{al(n)} + \dfrac{1}{n+1}(y^{al(n)} - x^{al(n)})$; $x^{tl(n+1)} = x^{tl(n)} + \dfrac{1}{n+1}(y^{tl(n)} - x^{tl(n)})$; $q_{rs}^{a(n+1)} = q_{rs}^{a(n)} + \dfrac{1}{n+1}(p_{rs}^{a(n)} - q_{rs}^{a(n)})$; $q_{rs}^{t(n+1)} = q_{rs}^{t(n)} + \dfrac{1}{n+1}(p_{rs}^{t(n)} - q_{rs}^{t(n)})$.

Step 5: Convergence test. If a certain convergence criterion is satisfied, stop; otherwise, set $n = n+1$ and go to step 1.

7. APPLICATION EXAMPLE

This section applies the model to solve a simple example. Guangzhou and Shenzhen are important cities in Pearl River Delta economic circle in China, with a distance of 130km. As shown in figure 1, there are four surface transportation modes between two cities. They are ordinary train, intercity express train, intercity bus and auto. Congestion effects due to the limited capacity of road links and the transit lines are considered. Two user classes are considered. The total demand between Guangzhou and Shenzhen are 20000 passengers/h. The attributes (e.g. travel time, price and capacity) of the four surface modes in this example are given in Table 1. The weight parameters corresponding to travel time, price, and convenience are: θ_1^1 =0.7, θ_2^1 =0.3, θ_1^2 =0.2, θ_2^2 =0.8, $\tau^1 = 60$ Yuan/h, $\tau^2 = 30$ Yuan/h, $\beta^1 = \beta^2$ =0.1, φ^{a1} =15, φ^{t1} =10, φ^{a2} =0, φ^{t2} =40, $a_b^1 = a_{tr}^1$ =20, $c_b^1 = c_b^2 = c_{tr}^1 = c_{tr}^2 = 1$, $a_b^2 = a_{tr}^2 = 10$.The seating capacity of an intercity bus is supposed to be 60 passengers. Other model parameters are: $\alpha_1 = 0.15$, $\alpha_2 = 0.1$, $\alpha_3 = 0.2$, $\alpha_4 = 0.15$, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 4$.

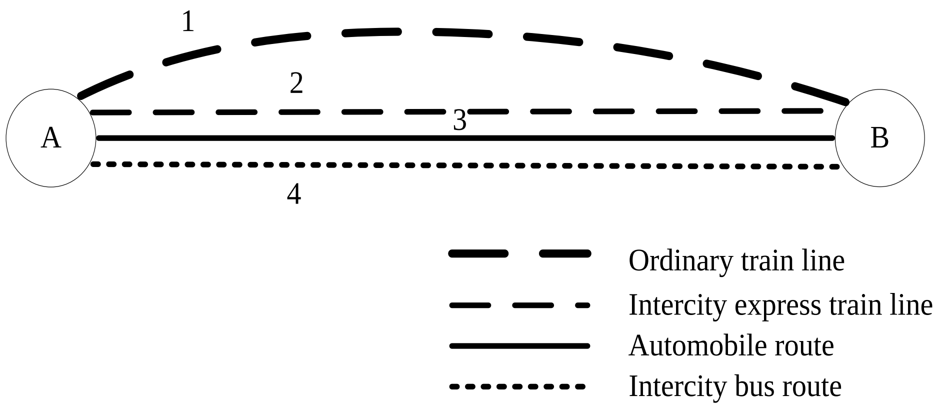

Figure 1 –Test network between Guangzhou and Shenzhen

The equilibrium demand for different modes is: q^a =3780, q^b =7707, q^{tr1} =1597(q^{tr1} means ordinary train), q^{tr2} =6916(q^{tr2} means intercity express train), respectively.

Table I – The attributes of four surface transportation modes between Guangzhou and Shenzhen

mode	Travel time(h)	fare(Yuan)	convenience(h)	Capacity(passengers)

auto	1.23	113	0.33	8000
Intercity bus	1.53	55	0.5	3400
Ordinary train	1.67	24	1	354
Intercity express train	1	75	1	4368

Next, an examination of change fare of intercity bus is conducted. Figure 2 shows how the passengers' mode choices change when intercity bus fare changes and ordinary train fare and intercity express train remain unchanged. It can be found that the intercity bus and intercity express train are main travel modes between cities. As the intercity bus fare decreases, the number of passengers who travel by intercity bus increases and the number of passengers who travel by intercity express train decreases, and the competition between intercity bus and intercity express train is intensive. The ordinary train demand is lower because of poor level of service. Trips made by car are lower too, because class 2 users whose value of time is lower are the majority and the generalized cost of auto are too high for them.

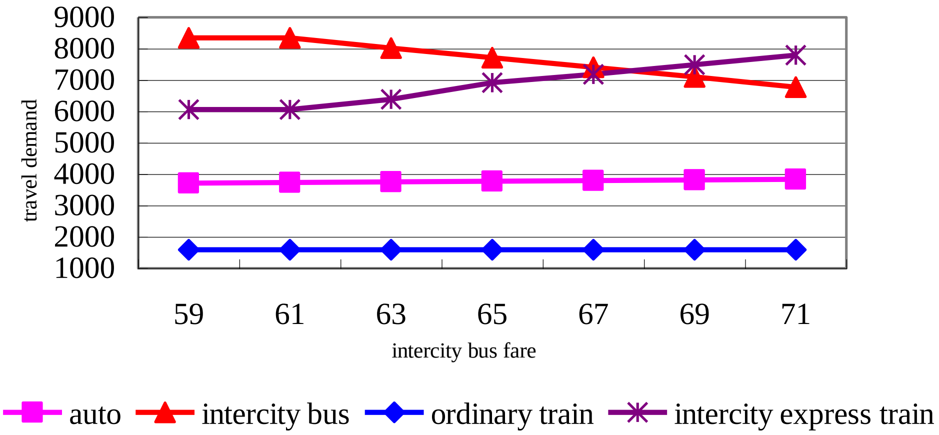

Figure 2 –Travel demand of passengers by different travel mode

CONCLUSIONS

In this paper, the model extends previous works in the multi-modal transportation networks. It is a combined modal split and assignment model for multi-class in multimodal transportation network. The proposed model can simultaneously predict mode choice and route choice over multimodal transportation network of economic circle with determinate travel demand. It is assumed that the choices of routes for automobiles satisfy stochastic user equilibrium conditions, and passenger flows over the public network are assumed to satisfy UE conditions. In addition, the travelers' choice of auto and transit is governed by a logit-type formula. The multimodal network equilibrium problem has been formulated as a variational inequality formulation. The findings have shown that travel mode demand can be obtained when the multi-modal network reaches an equilibrium state, the intercity bus and intercity express train are main competition modes in passenger transport market in economic circle in China, as well as how the fare change effect passengers' choice of travel modes.

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