Exploring the social structure of cities with an agent-based model

Rémi Lemoy[1,2], Charles Raux[1,2], Pablo Jensen[1,2,3]

[1]LET (Transport Economics Laboratory), [2]IXXI (Complex Systems Institute), [3]ENS Lyon Physics Laboratory

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Abstract

The standard Urban Economics model of Alonso, Muth, Mills, describes analytically an equilibrium location of households in urban areas. We present an agent-based model, using simple interactions between agents, and able to reach this equilibrium in a dynamic way. We calibrate a Muth version of this model with realistic values of parameters. The agent-based model allows us to simulate the development of a city by combination of heterogeneous agents, travel time cost and the introduction of several job centers. This tool allows the addition of a wide variety of features to the Urban Economics model to study their effects.

Keywords: agent-based model, urban economics, location choice, travel time, polycentric city

Introduction

Agent-based models are widely used to simulate traffic at a microscopic level. The goal of the work presented here is to use this tool in Urban Economics to deal with research questions regarding urban systems, for instance the location of households with respect to their income.

There are in the literature numerous analytical works on the Urban Economics model of Alonso, Muth, Mills (AMM model), studying the factors which explain the location choices of households within this model (Bruckner et al. [1999], Gofette-Nagot et al. [2000]).

This work uses a simulation tool which is more and more widely used in social sciences: agent-based programming. It allows us to study economic agents living in a predefined simulation space. These agents interact in a simple way, and from their interactions, collective behaviours emerge, which are difficult to predict in an intuitive way or to compute analytically. Agent-based models have three main components: agents, an environment and rules of behaviour. The agent has internal states, some fixed and others that can change, preferences for instance, and rules of behaviour. The environment is a two-dimensional space supporting resources and can also be a communication network. It is a device that is separated from the agents and that interacts with them. Rules of behaviour determine the interactions between agents, between agents and the environment and within the environment. For this work we use the multi-agent programming language NetLogo and the integrated modeling environment that bears the same name.

The study presented here is based on a model which has been widely studied, the standard Urban Economics model developed by Alonso, Muth, Mills. This model is exposed in section 1. Numerous works have been led on this model, theoretical works and also empirical ones, so that this model is interesting to reproduce with agent-based systems. The first stage of this work is the reproduction of the classical results of the AMM model. To build an agent-based model, the analytical model has to be discretized: there is a finite number of agents who interact individually. This is a difference with the analytical model, which is continuous, but it can be argued that social systems are indeed built on individual interactions of a finite number of agents. This even led some authors to study discrete versions of the Urban Economics model, but with different simplifications in order to be able to solve the model analytically. A comparison between the results of the simulations and those of the continuous analytical model is presented in section 2, along with a calibration of the model with realistic values of the parameters.

The use of agent-based models allows us to handle easily agents' states, rules of behaviour and environment. Sets of agents such as neighborhoods can be used, so that it is easy to introduce neighborhood effects. And individual and collective behaviours can be monitored in a simple way. In addition, the simulations and the model are dynamic, which is not the case for the AMM model and for most analytical economic models, which are equilibrium models. This time dependence allows us to see the equilibrium emerge from the interactions between agents, which is described in section 3, or to study out of equilibrium dynamics. The emergence of an equilibrium can have no relationship with the historical evolution of a real city, but the interactions between agents can also be used to study specifically features of cities which are linked to their historical evolution. As the results of the agent-based model are validated by the comparison with the analytical model, the model can be made more complex by adding different ingredients, and firstly income groups. This allows us to explore phenomena which are difficult or impossible to treat analytically. This work is presented in section 4 with the introduction of transport time costs and the modelization of a city with several job centers.

1 Description of the model

1.1 Urban Economics model

The AMM model was developed to study the location choices of economic agents in a urban space, with agents competing for housing. Agents have transport costs, monetary and (depending on the versions of the model) time cost, to commute for work. Their workplace is located in a central business district (CBD), which is represented by a point in the urban space. Amenities can be introduced in certain versions of the model to study their influence on the location of agents (Wu and Plantinga [2003]). Agents usually represent single workers, but they can also be used to describe households, which can be made more complex in further versions of the model. Their housing is rented by landowners who rent to the highest bidder: this mechanism introduces a competition for housing between agents.

We consider two versions of the AMM model here: the first one, which we will call

Alonso's version, is simpler because building construction is not studied. Land and housing surfaces are not distinguished, there is no vertical housing. In the second version studied here, following the contribution of Muth [1969], building construction is managed by firms of the housing industry, using land and capital. These firms have a production function F which is assumed to have constant returns to scale. Let q be the quantity of housing service consumed by an agent: the production function of firms has here a Cobb-Douglas form and can be written $q = F(s, k) = As^a k^b$, with s and k the quantity of land and capital used by the housing industry to produce a quantity q of housing service. A, a and b are parameters characterizing this production function, with $a + b = 1$ to ensure the concavity of F in s and k.

Land rent and housing rent are distinguished. Firms compete for land use and seek to maximize their profit $\Pi = p_H q - ps - k$, where p_H is the price of housing and p the price of land (both per unit surface). Land can also be used for agriculture, providing an agricultural rent R_a per unit surface to the landowners. The competition between firms for land implies that land rents p within the city must be higher than the agricultural rent, and that the profit of firms is zero at the equilibrium.

Agents have a utility function which has here a log-linear form $U = \alpha \ln z + \beta \ln q$, where z is a composite good representing all consumer goods except housing and transport, q is the surface of housing, α and β are parameters describing agents' preferences for composite good and housing surface. These last parameters are chosen so that $\alpha + \beta = 1$, without loss of generality. This log-linear form of the utility is often used and gives the same results as a Cobb-Douglas function (the exponential of this log-linear function). Agents also have a budget constraint $Y = z + tx + p_H q = z + tx + ps + k$ (zero profit of firms), where Y is their income, t the transport cost per unit distance, and x their distance to the CBD.

Alonso's version of the model is somewhat simpler: housing and land surfaces are not distinguished, so we denote them by s_A. Agents have a utility function $U = \alpha \ln z + \beta \ln s_A$, and a budget constraint $Y = z + tx + ps_A$ (land and housing prices are not distinguished either). Production of housing is not studied, so that the behaviour of housing firms can be ignored (see Fujita [1989] for a more detailed description of both versions of the model).

The analytical model reproduced in this work with agent-based simulations is a closed city model: the number of agents N in the city is chosen exogenously and remains constant during a simulation. Both versions (Muth and Alonso) of this model can be solved analytically in a two-dimensional space if $R_a = 0$ (see Fujita [1989]). For $R_a > 0$ it can be solved numerically for one income group. With a population divided into several income groups, one needs to build a specific algorithm for the resolution of the model, whose general form is described in Fujita [1989]. We did not write such an algorithm, so that the results of the agent-based model will be compared quantitatively to the results of the standard model only with one income group. Though, simulations are performed also on a model with two income groups.

1.2 Agent-based implementation

In the agent-based system, the simulation space is a two-dimensional grid where each cell can be inhabited by one or several agents, or used for agriculture. The CBD is represented by a point at the center of the space.

At the initialization, the population N is fixed and agents are placed at random locations. Land prices are equal to the agricultural rent $p_0 = R_a$. Following the resolution of Muth model presented in Fujita [1989], firms of the housing industry are left apart and agents choose land and capital inputs themselves. Instead of competing for housing (bidding on p_H), they compete for land and bid on land price p. At a given location, they choose the quantities of land and capital which provide them with the higher possible utility with price p: $s = a\beta\frac{Y-tx}{p}$ and $k = b\beta(Y - tx)$. This determines the quantity of composite good they consume and also their utility. These expressions for the optimal land and capital quantities can be obtained by differentiating with respect to land surface s and capital k the utility function, at fixed location and price (see appendix A). The quantity of housing service they consume is then given by the production function of firms $q = F(s, k) = As^a k^b$, and the price of housing is given by the condition of zero profit of firms $p_H = (ps - k)/q$.

Agent-based implementation of Alonso's version of the model is similar, without the distinction between land and housing surfaces and prices. At a given location, the optimal surface chosen by an agent is $s_A = \beta\frac{Y-tx}{p}$ (see appendix A).

1.2.1 Dynamics of moves

The main feature of the model consists in agent-based dynamics of moves in the urban space. The rules defining agents' moves are suggested by the competition for land in the analytical model.

Agents move with no cost. Let us describe an iteration n of a simulation, changing the variables from their value at step n to their value at step $n + 1$. An agent which will be candidate to a move and a cell are chosen randomly. The price of this cell, located at a distance x of the center, is p_n at step n. The optimal land and capital quantities that the agent can choose in the candidate cell are $s = a\beta\frac{Y-tx}{p_n}$ and $k = b\beta(Y - tx)$, which allows us to compute his composite good consumption and the utility that he would get thanks to the move and to compare it to his current utility.

If the agent candidate can gain utility $\Delta U > 0$ by moving into the candidate cell, then he moves. In this purpose he raises the price of the candidate cell by proposing a bid $p_{n+1} = p_n(1 + \epsilon\frac{s_{\text{occ}}}{s_{\text{tot}}}\frac{\Delta U}{U})$, where ϵ is a parameter that we introduce to control the magnitude of this bid. Prices evolve quicker if this parameter is high. s_{occ} is the surface of land occupied by other agents in the cell and s_{tot} the total land surface of the cell. The factor $\frac{s_{\text{occ}}}{s_{\text{tot}}}$ makes the bid higher if the cell is more occupied, that is to say, more attractive. The first agent to move in an empty cell does not raise the price. The price is a price per unit surface and it is a variable linked to a cell. When an agent bids higher, the price is changed immediately for all agents in the cell. Their consumption of land is also changed according to $s = a\beta\frac{Y-tx}{p_{n+1}}$ and their utility is computed again.

This feature of the model defines a competition for land between agents, as in the standard analytical model. But specific situations arise which do not appear in an equilibrium (static) model. For instance, an agent may want to move into a candidate cell that is already full. This case is described in the next paragraph.

1.2.2 Displacement of agents, time decreases of prices and utility

In the case where the candidate cell is already full, the rule of behaviour which is given to agents is again suggested by the analytical model. To reach the analytical equilibrium rapidly, the housing market needs to be as fluid as possible, so that the rules of behaviour need to prevent market frictions. Of course, an important drawback of the reproduction of the analytical model is that agents' rules of behaviour are quite unrealistic, more than they could be in agent-based models. Market frictions need to be introduced later to study their effect on the system, and the agent-based modelization should be a very convenient tool to study such effects.

To make the dynamics of moves and prices very fluid, one or more agents are chosen at random within the candidate cell and are sent "to the hotel", until there is enough space for the agent candidate to move in. These agents which have been displaced are then the next candidates for a move. And they stay priority candidates until they have all found a new location. While they are searching for another cell to live in ("at the hotel"), their utility decreases exponentially following $U_{n+1} = U_n - (U_n - U_{\min})/T_e$, where U_n is the initial utility, U_{n+1} the decreased utility, $U_{\min}$ the minimal utility in the city and T_e a parameter governing the speed of this decrease. Thanks to this mechanism, agents at the hotel find a new cell to live in.

There is another comparable mechanism of decrease in this model: the time decrease of prices of vacant cells. Indeed, with the bid mechanism presented before, where agent and cell are chosen at random, the price of a cell where several agents move in successively can increase so much that it reaches a value which makes the cell unattractive. In this case, agents living there will progressively leave the cell for more attractive locations. Therefore we choose to decrease exponentially the price of cells which are not completely full[1]. The decrease formula is $p_{n+1} = p_n - (p_n - R_a \times 0,9)\frac{\Delta U}{T_p}\frac{s_{\text{free}}}{s_{\text{tot}}}$, where $\Delta U = (U_{\max} - U_{\min})/U_{\max}$ measures the homogeneity of the utility in the city, T_p is a parameter determining the speed of decrease of prices, $s_{\text{free}} = s_{\text{tot}} - s_{\text{occ}}$ and s_{tot} are the non occupied surface and the total surface of the cell. If no agent moves in, the price decreases according to this formula until it reaches the agricultural rent, where the decrease stops: the cell is then used for agriculture. The factor $\frac{s_{\text{free}}}{s_{\text{tot}}}$ makes this time decrease quicker as the cell is emptier and thus less attractive. The ΔU factor makes the decrease slower as the utility becomes more homogeneous in the city, that is, when the system approaches the equilibrium.

1.2.3 Parameters

Most of the work presented here has been realized on the simpler version of the model (Alonso), with a number of agents which is relatively small (some hundreds or thousands). This simple version of the model can not be calibrated, even roughly, on a real city, because of the absence of vertical housing. All agents live on the ground, so that densities can not come close to real densities. The city size is also unrealistic as a result. Thus we use a much heavier simulation of Muth version of the model to calibrate it roughly on a real city, which is described in paragraph 2.2. The other results presented in this work are obtained with Alonso's version of the model, with parameters given in table 1. Their default value has been chosen according to several criteria.

First for the parameters of the model itself: α, β, Y, t, N, R_a, s_{tot}. Their value has been chosen mainly for technical reasons regarding the comparison between the (continuous) analytical model and the (discrete) agent-based model, but naturally other values could have been chosen, without changing the qualitative behaviour of the model. For instance, a higher population N could have improved the agreement between the analytical and the agent-based model, but it would have slowed down the simulations (see paragraph 2.2).

Parameters ϵ, T_e and T_p are specific to the agent-based model. Their values have been

[1]With two income groups, it is difficult to determine whether a cell is full or not: we choose to let the price decrease if the mean surface of housing s_{mean} of agents there is smaller than the free surface of the cell s_{free}.

chosen such that the competition between agents on the housing market is efficient and the system reaches the equilibrium in the whole city. The agent-based model has different behaviours and for instance does not reach an equilibrium (the utility of agents does not become completely homogeneous across the city) for certain values of these parameters, but the study of these different behaviours is beyond the scope of the present paper.

Parameters	Description	Default value
α, β	Preferences for composite good and housing	$0,9; 0,1$
Y	Income	300
t	Transport cost (unit distance)	5
N	Population	700
R_a	Agricultural rent	10
s_{tot}	Surface of a cell	12
ϵ	Bidding parameter	20
T_e	Time decrease of the utility of displaced agents	7000
T_p	Time decrease of the price of non-full cells	15

Table 1: Parameters of the model

1.3 Socioeconomic outcome

The goal of this work is to study the urban social structure and the socioeconomic outcomes of the models developped here. Thus we study especially some variables of the model, which characterize these outcomes. Our benchmark is a reference simulation of a monocentric city with two income groups with the same transport cost. Then we compare the values of the socioeconomic variables in the reference simulation and in more complex models to observe the effects of the modifications which are introduced in the standard model.

The variables which we find most relevant to describe the outcomes of the models are the utility of individuals, which is associated to their welfare and gives an economic outcome of the models, the cumulated distances of agents' trips to work, which give the environmental outcome of the models, and the social inequalities, which are given by observing the difference in the utility of individuals of different income groups.

The use of agent-based systems allows naturally an easy access to any individual or global variable of the model, so that price effects for instance could also be studied.

2 Comparison with the analytical model and calibration

2.1 Alonso's model

The simulations allow us to reach the equilibrium of the AMM model. This equilibrium corresponds to a configuration where no agent can raise his utility by moving, and therefore no agent has an incentive to do so. In each income group, individuals have an identical utility across the city. With two income groups, the utility of "rich" agents is still higher than that of "poor" agents, because they do not have the same exogenous parameters (they have different incomes). The cells which are occupied are those closest to the city center (CBD), the prices at the border of the city are equal to the agricultural rent and prices increase when the distance to the center decreases. The surfaces of housing increase when the distance to the center increases.

The results of the simulations do not match exactly the analytical results because of the effects of the discretization (which leads for instance to a border of the city which is not exactly at the same distance from the center all around the city). The discrete character of the simulation appears in particular on the density curve, which is like a step function in the simulations and a continuous function in the analytical results.

The results of the model can thus be compared to the analytical results when these ones exist: figure 1 presents such a comparison for a city with only one income group. It shows the density, rent and surface curves as functions of the distance to the center

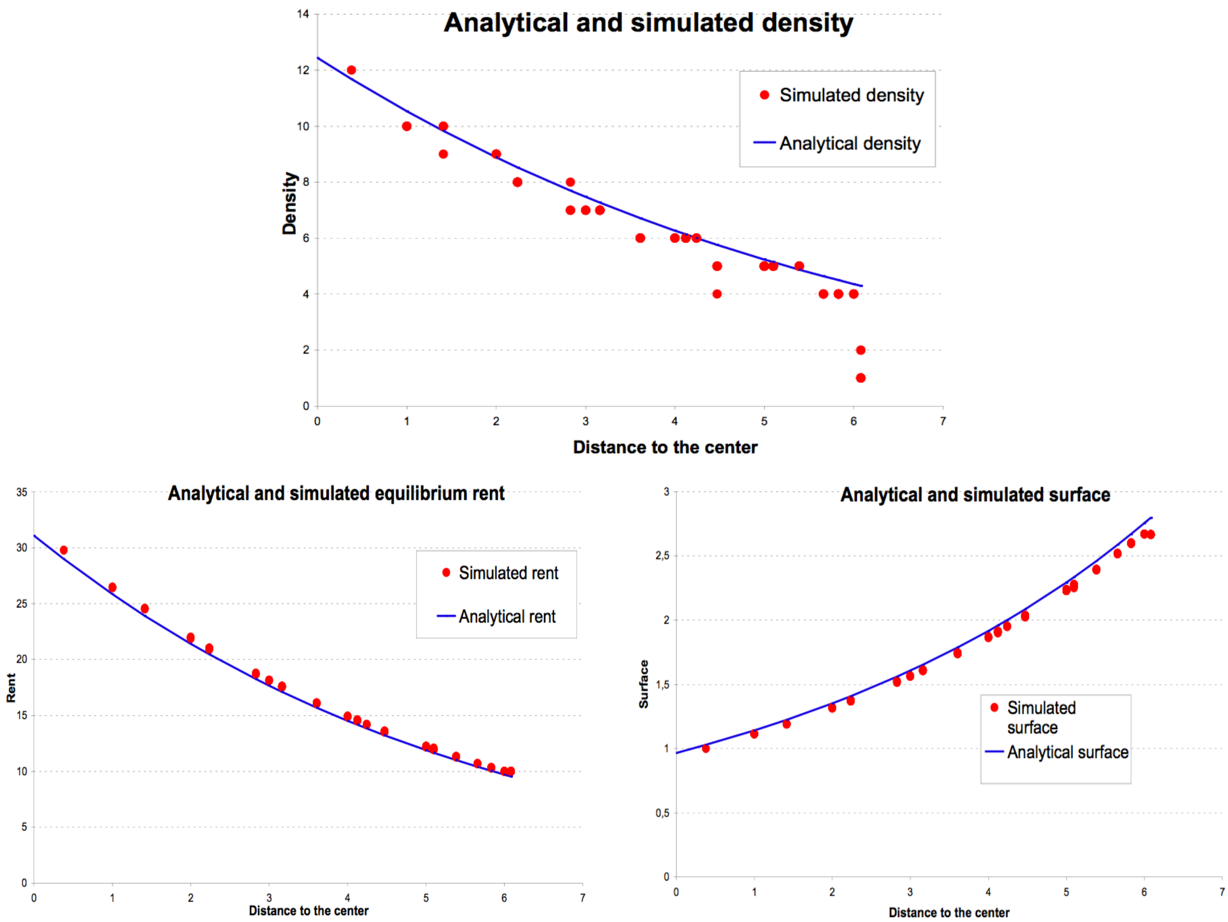


Figure 1: Comparison between the results of the agent-based model and the analytical results: rents and surface as functions of the distance to the center.

for the analytical and the agent-based model. Because of the discretization, cells are not completely full in the agent-based model and the density is in general lower than the analytical density. The city is slightly more spread, rents are slightly higher, surfaces slightly lower and the equilibrium utility is slightly lower.

It would be interesting to compare the results of the simulations with the results of discrete models of Urban Economics, which the simulations should reproduce exactly. However there are few or no analytical results concerning discrete models of circular cities. Analytical works deal mainly with continuous models or discrete models of linear (one-dimensional) cities.

2.2 Calibration of a model with building construction

In this paragraph, we present a rough calibration of Muth version of the model on the Urban Community of Lyon, the city where this work has been done. This Urban Community is composed by the city of Lyon and some of its suburbs, with a total of 1.2 million inhabitants. With a mean of 2.3 persons per household in France, we perform a simulation

Parameters	Description	Value
α, β	Preferences for composite good and housing	$0,7$; $0,3$
a, b	Land and capital exponents of the production function	$0,2$; $0,8$
A	Multiplicative of the production function	$0,025$
Y	Income	3×10^4€/year
t	Transport cost	0,3€/m/year
N	Population	5×10^5 inhabitants
R_a	Agricultural rent	8€/m^2/year
s_{tot}	Surface of a cell	200m $\times$ 200m $= 4 \times 10^4$m^2

Table 2: Parameters of the calibrated Muth model

with 0.5 million households. This simulation runs for a much longer time than the other simulation presented in this work before it reaches the equilibrium. The simulated city is a grid of square cells which are considered to be 200 m long. Only one income group is considered, to allow the comparison with the analytical results. The income of each households is fixed at 30 k€ per year (2500 € per month). The transport cost is composed of a monetary and of a time cost. The monetary cost is taken as $0,30$ €/km (for a one way trip), which is realistic for car travel. And the time cost is taken as 8 € per hour, which gives $0,32$ €/km at a speed of 25 km/h. With 250 roundtrips per year, the global transport cost is 30 0€ per kilometer and per year. The agricultural land rent is taken as 8 € per square meter per year ($0,67$ €/m^2/month): this is the estimated land rent at the border of the urban community. All values of parameters are given in table 2. The model only considers the residential use of land. As in reality only a given share of urban land is dedicated to housing, this should be introduced in the agent-based model. Here we use 25% as an estimated value of the mean share of land which is used for housing in the Urban Community of Lyon.

On figure 2 are given the results of this calibrated model. Only the density curve is compared to the analytical curve, the other curves coincide with the corresponding analytical curves. The surface of the Urban Community of Lyon is approximately 500 km^2, so that its radius would be of nearly 13 km if its shape were a disc. The radius of the simulated city is approximately 9 km. This model gives values of variables which are coherent in their evolution with the distance to the CBD, and of the right order of magnitude. Though it is a perspective of this work to compare them precisely to real data and understand the differences. The share of land which is used for housing is taken as constant on the whole territory, which is unrealistic because land is more urbanized near the center of the city than in the suburbs. This contributes to explaining why the density curve is not as steep as it is in reality, because the density is underestimated near the center and overestimated at the periphery.

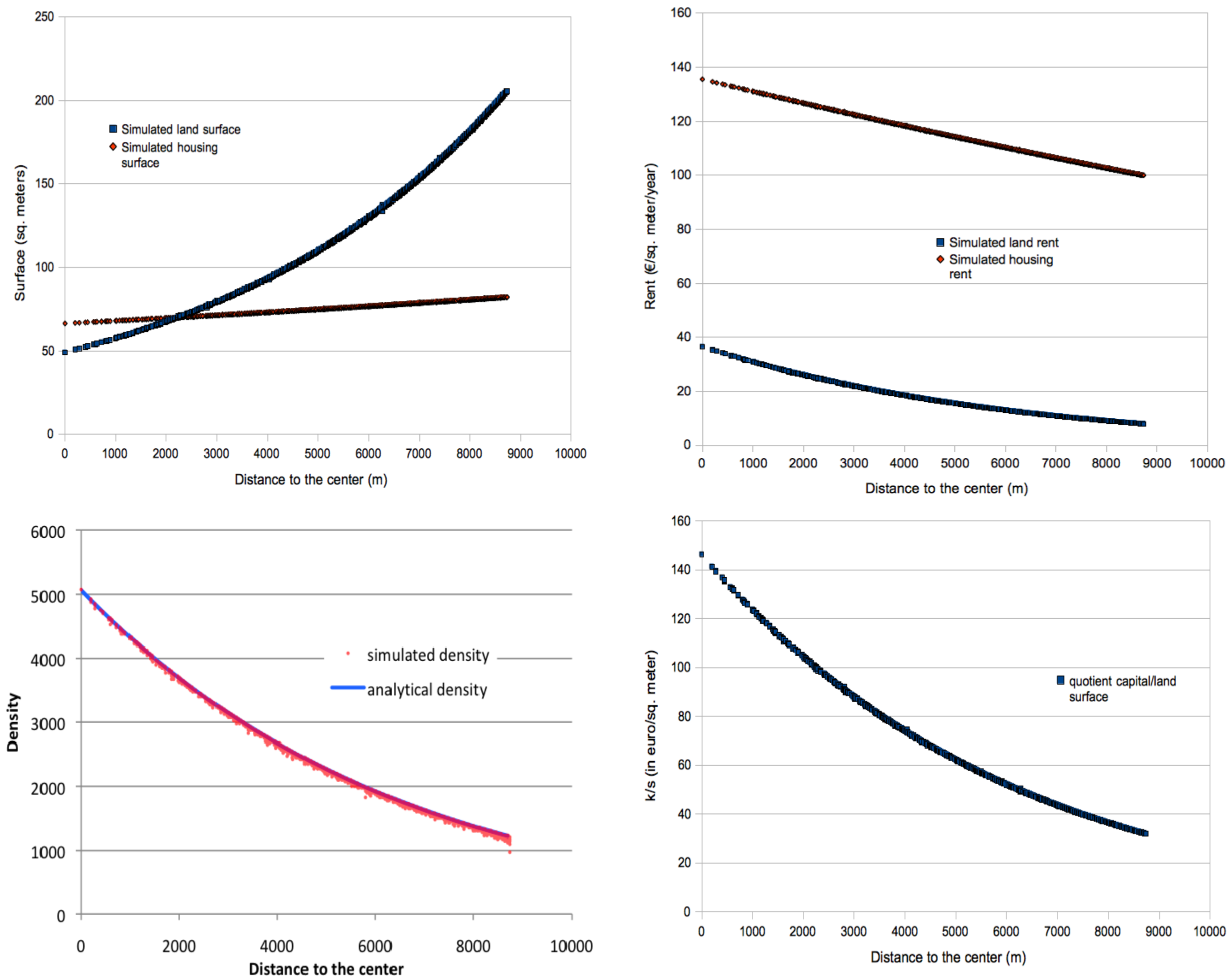


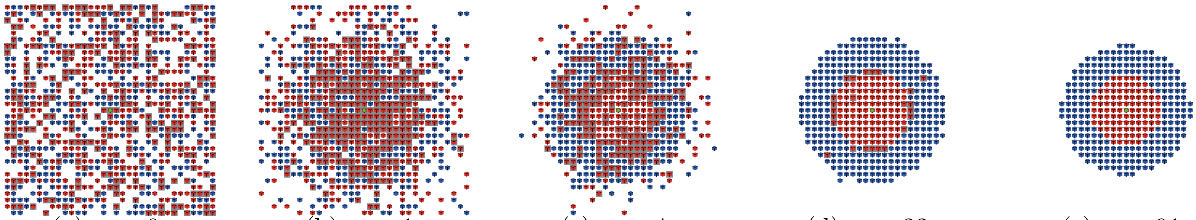
Figure 2: Results of the calibrated model: surfaces (in m^2), rents (in €/m^2/year), density (in inhabitants/km^2) and capital-land ratio (in €/m^2), as functions of the distance to the center (in m).

3 Emergence of a city

We now describe how a city emerges from the interactions between individuals during a simulation. Initially, all agents are located randomly and all rents are equal to the agricultural rent. Then agents move and bid higher, and the rent curve evolves from a flat rent to the equilibrium rent, and the density curve evolves towards the equilibrium density. Figure 3 shows how the shape of the city evolves during a simulation. In this simulation two income groups are present, "poor" agents in red and "rich" agents in blue. Paragraph 4.1 describes the equilibrium configuration in a detailed way.

At the beginning of the simulation (figures 3(a) and 3(b)), few people are displaced (as described in paragraph 1.2.2), agents gather at the city center without competing much for land, because many cells close to the center are still not full. But when all agents are concentrated around the center (from figure 3(d) on), most moves result in one or several agents being displaced, for few cells have a sufficient free surface to allow an agent to move in with an interesting utility. This feature of the model arises because the vacancy rate of the standard Urban Economics model we reproduce is zero.

The variable which shows the proximity to the equilibrium is the homogeneity of the utility of agents. To describe this homogeneity, we use the variable $\Delta U = (U_{\max} - U_{\min})/U_{\max}$, which gives the relative inhomogeneity of the utility. With two income groups,



(a) $n = 0$ (b) $n = 1$ (c) $n = 4$ (d) $n = 22$ (e) $n = 91$

Figure 3: Evolution of the shape of the city during a simulation. The CBD is a green point. Cells whose background is grey indicate that poor and rich agents live there: at the equilibrium, the city is completely segregated and there are no more such cells. n indicates the mean number of moves per agent since the beginning of the simulation.

we use the maximum of this variable in both income groups. Initially, this variable has a quite high value as all agents are located randomly, and it decreases during the simulations. We choose to stop the simulations when the relative variations of utility in the income groups are smaller than 10^{-6}, which means that this variable has decreased by approximately five orders of magnitude.

4 Results of the simulations

4.1 Two income groups: model 1

On figure 4 we give the shape of the city with two income groups, and the rent as a function of the distance to the center. The values of the parameters are those used in table 1. The

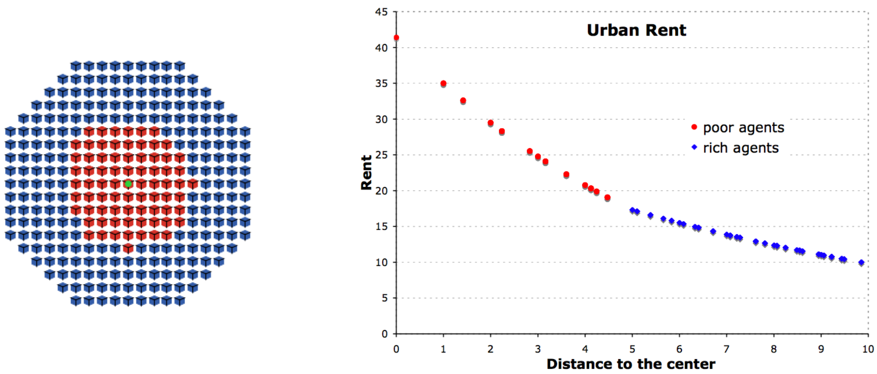


Figure 4: City with two income groups: rich in blue and poor in red. The CBD is a green dot. Right panel: equilibrium rent as a function of the distance to the center.

city population is composed of two groups of 700 individuals each: "poor" agents (in red) with income $Y_p = 300$ and "rich" agents (in blue) with income $Y_r = 300 \times 1,6 = 480$. As in the analytical equilibrium, rich agents are located at the periphery of the city, where they pay lower prices and have higher housing surfaces, but also with higher transport costs.

On figure 5 are shown the density and the housing surface as functions of the distance to the center. The agent-based model with two income groups has the same behaviour as the analytical model for the city shape, density curve (discretized), rent and surface

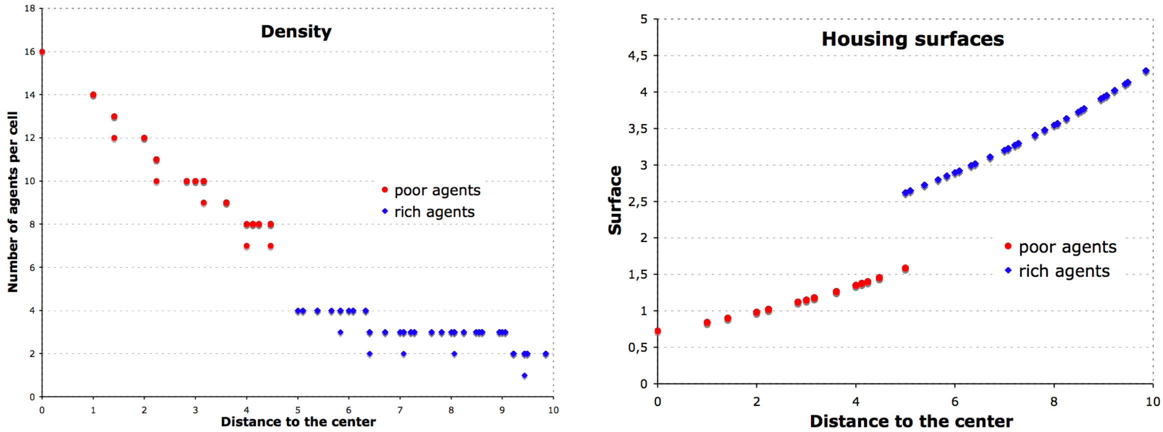


Figure 5: Population density (number of agents per cell) and housing surface as functions of the distance to the center.

curves. But as said previously we did not build an algorithm for the resolution of the analytical model in this case, so that we can not compare quantitatively the results.

4.2 Value of time: model 2

The equilibrium of the standard Urban Economics model we just presented gives a configuration where rich households live in the periphery of the city and poorer households near the center, which is in agreement with empirical results in most North American cities, but many European cities have an inverse configuration (see Bruckner et al. [1999]). One feature of the Urban Economics model that could account for this difference is the introduction of a difference in transport time cost. Rich households have a higher value of time than poorer households, and thus a higher global transport cost per unit distance.

This can be introduced in the model by adding a transport time cost (per unit distance) $v_t/\mathbf{s}$, where v_t is the value of time and $\mathbf{s}$ the transport speed. The global transport cost can therefore be written

$$T(x) = tx + \frac{v_t}{\mathbf{s}}x$$

where x is the distance to the center and t the monetary cost. Analytical treatment and

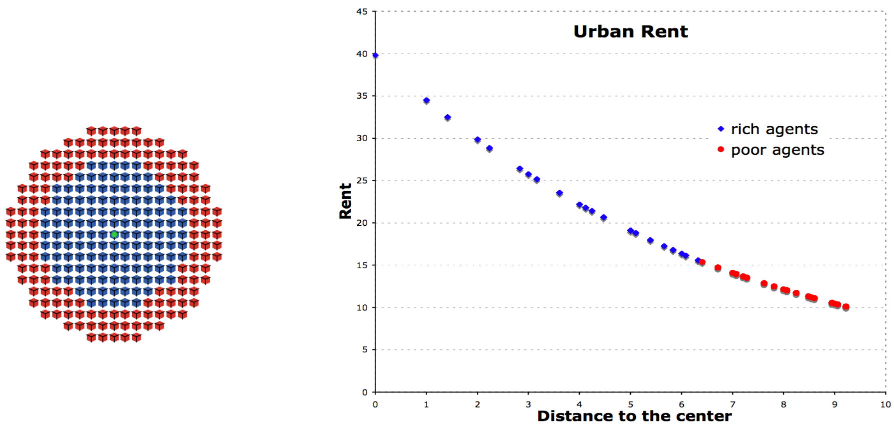


Figure 6: Shape of the city and equilibrium rent with a transport time cost much higher for rich agents than for poor agents.

agent-based simulations agree on the results of such a change in the model: to have rich agents located in the center and poor ones at the periphery, the quotient of the global transport costs per unit distance of rich and poor agents must be superior to the quotient of their incomes (see appendix B):

$$T_r/T_p > Y_r/Y_p \tag{1}$$

with T_r and T_p the global transport costs per unit distance of rich and poor agents respectively. This situation is represented on figure 6, where the monetary cost of transport has a value $t = 2$ for both income groups, and transport time cost has a value $v_t^p/\mathbf{s} = 2$ for poor agents and $v_t^r/\mathbf{s} = (v_t^p/\mathbf{s}) \times 2,4 = 4,8$ for rich agents. Then we have $T_r/T_p = 1,7 > Y_r/Y_p = 1,6$.

The condition (1) for the inversion of income groups can also be written as $T_r/Y_r > T_p/Y_p$, which can be more intuitive. This means that the income group which will be located near the center of the city is the one for which the global transport cost per unit distance represents a higher share of the income.

Discussion

To study the effect of the value of time on the model, we can suppose that time cost is the most important part of the global transport cost (which amounts to neglecting monetary cost). Then the value of time needs to increase with income quicker than proportionnally to have condition (1) true, that is, the income elasticity of the value of time needs to be greater than one. For instance, $v_t(Y) = a_1 Y^{a_2}$, with $a_1 > 0$ and $a_2 > 1$ constants (note that if $v_t(Y) = a_0 + a_1 Y^{a_2}$ with the same a_1 and a_2, and $a_0 > 0$ another constant, then condition (1) is less likely to be true). We do not have data to test this hypothesis and fit the evolution with income of the value of time, but a quick calculation (such as what is done in paragraph 2.2) shows that monetary and time costs are rather of the same order of magnitude. In Urban Economics studies, the income elasticity of the value of time is usually supposed to be smaller than one, but models often take it as a given share of the income per hour (which implies an income elasticity equal to one).

Empirically, it can be observed that the value of time does rise with income (de Palma and Fontan [2001]), but not enough to have condition (1) true if the monetary cost t and speed $\mathbf{s}$ are kept identical for both income groups (that is to say, if all agents use the same mode of commuting, a car for instance). In this model, the value of time alone can not account for the difference of location of households depending on their income between European and North American cities.

Introducing different tranport modes

Another feature to test consists then in letting monetary cost t and speed $\mathbf{s}$ of commuting vary across income groups. In our case, it correspond to the introduction of two different transport modes, which we label as h: $(t_h, \mathbf{s}_h)$ (higher speed) and l: $(t_l, \mathbf{s}_l)$ (lower speed). Let us suppose that high income households use the faster transport mode, which has a higher monetary cost than the slower transport mode used by low income households: $t_h > t_l$ and $\mathbf{s}_h > \mathbf{s}_l$. This hypothesis has two opposite effects on the global transport cost of rich agents, compared to that of poor agents: it increases the monetary cost of travel and decreases time cost.

For rich agents to use the faster transport mode, another assumption has to be made. This transport mode needs to have a lower global cost for them than the slower one:

$t_h + v_t^r/\mathbf{s}_h < t_l + v_t^r/\mathbf{s}_l$ (and conversely $t_h + v_t^p/\mathbf{s}_h > t_l + v_t^p/\mathbf{s}_l$ for poor agents). However, it can be shown that this condition is not compatible with condition (1) if the income elasticity of the value of time is smaller than or equal to 1, which is usually supposed in urban economics works. So even the introduction of different transport modes can not explain the central location of rich households within this model. This result is directly linked to the log-linear expression of the utility used in this work (which is equivalent to its exponential, the Cobb-Douglas function). This utility function is widely used because it is very convenient for computations and it allows to determine easily the share of income which is used on different goods, but it has an income elasticity of the demand for housing equal to one (Chung [1994]), which is presumably higher than the income elasticity of marginal transport cost. This problem led LeRoy and Sonstelie [1983] to suppose in their work that the utility function is such that the income elasticity of the demand for housing is smaller than that of marginal transport cost. Thus with only one transport mode, rich agents are located in the center of the city ("European" city). The introduction of two transport modes allows them to explain the social structure of American cities under certain hypotheses.

Effect of congestion

The introduction of congestion within this model decreases transport speed and increases as a consequence the time share of global transport cost, which tends to make the monetary cost negligible. Though we saw that to have condition (1) true in this case, the income elasticity of the value of time needs to be greater than one, which seems unrealistic.

Numerous analytical works deal with the question of the value of time and its influence on the location of households in the AMM model: the importance of this factor is still debated in the literature (see Gofette-Nagot et al. [2000], Bruckner et al. [1999], LeRoy and Sonstelie [1983], Wheaton [1977]).

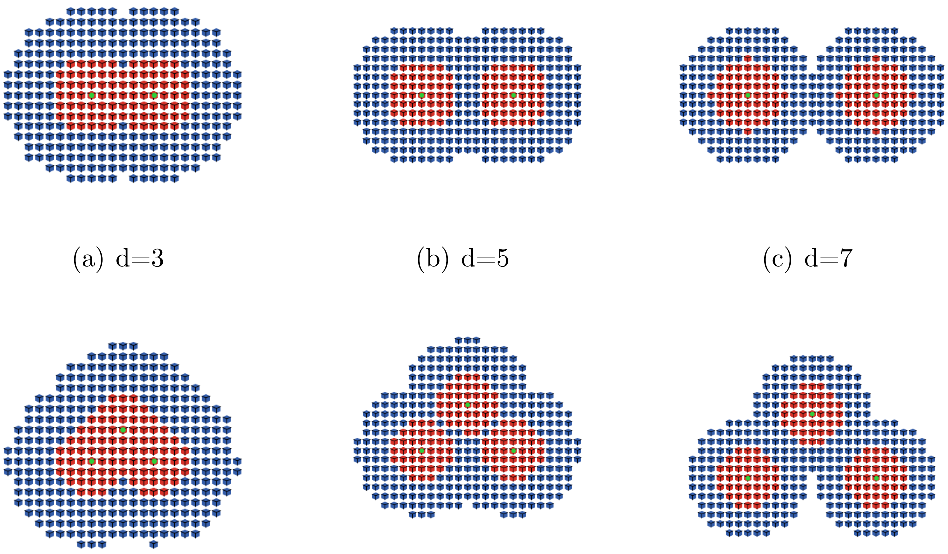
4.3 Polycentric city: model 3

The agent-based mechanism introduced in this work to reproduce the results of the AMM model is robust enough for us to introduce effects which are difficult to treat analytically. For instance, several centers can be introduced, which to our knowledge has not been done analytically for a circular city. Agents work at the center which is the closest to their housing, and as a consequence, can change jobs as they move. This last feature seems unrealistic but allows to prevent market frictions and reach more rapidly the equilibrium. The results of a such model are given on figure 7.

Rents, housing surface and density as functions of the distance to the nearest center give curves which are similar to those of figures 4 and 5.

Table 3 allows us to see the evolution of different variables for this polycentric model, such as agents' utility (actually the exponential of this utility, corresponding to a Cobb-Douglas function, whose variations are more significant), the mean commuting distance for each income group, the total commuting distance, the total rent, the total surface of the city and the mean density in the city, compared with the reference configuration with two income groups from paragraph 4.1.

Raising the number of centers amounts to raising the surface available at a given commuting distance in the city, and to reducing as a consequence the competition for housing. Agents have greater housing surfaces, smaller commuting distances and a higher utility. The total rent increases, which can seem surprising but can be explained by the



(a) d=3 (b) d=5 (c) d=7

(d) d=3 (e) d=5 (f) d=7

Figure 7: Cities with two centers separated by $2d$ cells (first line) and cities with three centers located at $(-d;0)$, $(d;0)$ and $(0;d)$, for different values of d. Centers are indicated by green dots, and agents work in the center closest to their housing.

fact that housing surfaces are greater. The mean density decreases while housing surfaces increase. These effects are more pronounced when the centers are further away from one another.

Thus economic and environmental outcomes of the introduction of several centers in this model are naturally positive: agents' utility increases and commuting distances decrease. Agents' utility increases when the distance between centers increases, but the effect on commuting distances is more complex (see table 3). The social outcome is less obvious: poor agents' utility increases more than rich agents'. With two centers, this utility gap is smaller when the centers are closer. With three centers, the utility gap remains constant when centers are further away.

Table 3: Comparison between the different polycentric models. Variables are rich and poor agents' utility U_r and U_p, their difference, rich and poor agents' mean commuting distances D^r_{mean} and D^p_{mean}, the total commuting distance D_{tot}, total rent R_{tot}, mean unit surface price p_{mean}, total surface of the city S_{tot} and mean density in the city ρ_{mean}.

Model	U_r	U_p	$U_r - U_p$	D^r_{mean}	D^p_{mean}	D_{tot}	R_{tot}	p_{mean}	S_{tot}	ρ_{mean}
2 income groups (§ 4.1)	100	100	100	100	100	100	100	100	100	100
2 centers $d=3$ fig. 7(a)	101,6	102,6	99,0	83,1	78,1	81,7	101,3	94,9	106,7	93,8
2 centers $d=5$ fig. 7(b)	102,2	103,1	99,1	78,2	80,3	78,8	101,5	92,9	109,7	91,2
2 centers $d=7$ fig. 7(c)	102,6	103,4	99,3	78,8	81,9	79,7	101,4	88,5	114,5	87,4
3 centers $d=3$ fig. 7(d)	102,1	103,4	98,7	77,4	67,0	74,4	101,8	94,3	108,0	92,7
3 centers $d=5$ fig. 7(e)	103,1	104,4	98,7	70,6	69,2	70,2	102,1	88,8	114,9	87,0
3 centers $d=7$ fig. 7(f)	103,9	105,3	98,7	66,7	71,3	68,0	102,3	83,8	122,1	82,0
100% $d=2$ fig. 8(a)	100,0	100,49	99,5	87,2	120,5	96,8	100,2	100	99,5	100,6
100% $d=6$ fig. 8(b)	100,8	102,0	99,8	72,0	122,8	86,6	100,9	99,0	102,0	98,1
100% $d=10$ fig. 8(c)	101,4	103,1	98,4	70,1	124,4	85,8	101,0	92,5	109,3	91,5
80% $d=2$ fig. 8(d)	100,4	100,9	99,6	90,8	97,2	92,7	100,5	100	101,1	98,9
80% $d=6$ fig. 8(e)	101,3	102,4	98,9	79,9	91,0	83,2	101,2	96,1	105,2	95,1
80% $d=10$ fig. 8(f)	101,9	103,2	98,8	75,5	95,3	81,3	101,3	91,8	110,3	90,8

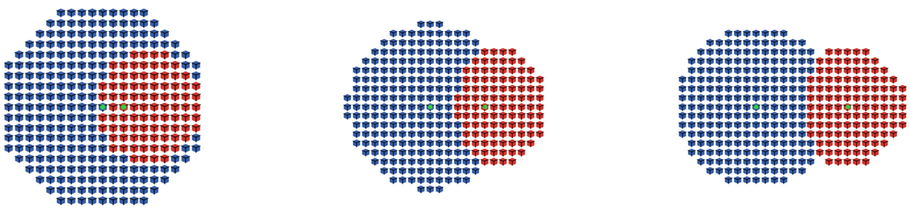
4.4 Constrained polycentric city: model 4

It is also possible to ask agents to choose a center at the beginning of the simulation and to keep it. The computation of the equilibrium in this configuration on a two-dimensional city has not been done to our knowledge (neither in a discrete nor in a continuous model), but the bid mechanism used here allows us to find this equilibrium.

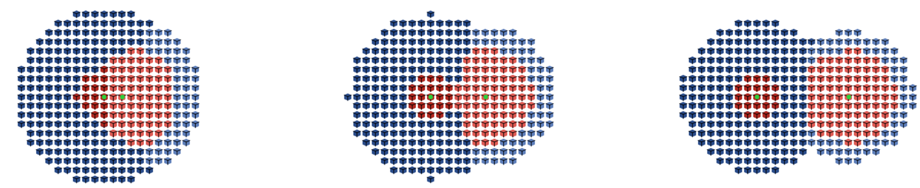
For instance, all rich agents work in a certain center and all poor ones in another center at another location. The result of a such model is given on the first line of figure 8, where the center on the right can be seen as a center with low-skill jobs (or an industrial zone) in the east of the city, and the center of the left, a center with high-skilled workers on the west of the city.

It is also possible to have only a part of each income group working at each center, that is to say, to suppose that centers are not completely specialized. In this case, as agents in a certain income group have different constraints, their utility is not homogeneous within an income group. Utility is homogeneous among agents of the same income group working at the same center.

This is done on the second line of figure 8, with two centers which are not indifferent for agents (contrarily to what is done in model 3) and two income groups, that is, four (2×2) utility groups at the equilibrium.



(a) d=2 (b) d=6 (c) d=10



(d) d=2 (e) d=6 (f) d=10

Figure 8: First line: cities with two centers where poor agents work in the east center and rich agents in the west center. Second line: 80% of poor agents work in the east center and 20% in the west center, and conversely for rich agents. The distance d between both centers is indicated. On the second line, agents of the same income group working in different centers have different colors.

As can be seen on table 3, the global effect of the introduction of centers with constraints for agents is quite similar to the effect of centers without constraints: the surface available at a given commuting distance in the city is increased, and the utility of agents increases. The economics outcome is positive: agents' utility increases when the distance between centers increases.

The housing surfaces increase, and they increase when the distance between centers increases. However, the simulation presented on figure 8(a) is an exception: the city surface is reduced and the mean density is higher than in the reference configuration.

Partial or total segregation of rich and poor agents in job locations decreases in fact the competition for housing between both income groups: poor agents are less pushed toward the center by their competition with rich agents, and rich agents are less pushed toward the outskirts of the city. Two effects appear on commuting distances. This decrease of the competition between income groups for housing raises poor agent's commuting distances and decreases those of rich agents. And the increase in the surface available at a given commuting distance decreases all commuting distances. So the environmental outcome is positive, as commuting distances decrease globally when the distance between centers increases. Yet the effects on each income group are more complex, as can be seen on table 3.

The social outcome is also globally positive. The utility gap between rich and poor agents decreases. When the segregation linked to employment is total, the effect of increasing the distance between centers is not monotonous (see table 3). When this segregation is partial, social inequities decrease when the distance between centers increases. Though it must be remembered that within each income group, a new disparity has appeared.

Conclusion

From a methodological point of view, this work shows the interest of agent-based systems in the study of collective phenomena carried out by social sciences. With the example of the standard Urban Economics model, we use this simulation tool to reproduce the results of an equilibrium model. To this end, we introduce an interaction between agents which allows to lead the system towards the equilibrium. This can be seen as an improvement of the equilibrium model because simple interaction mechanisms can be studied in this way. And as this simulation model is dynamic, it can be an interesting tool to study the dynamics of urban location as a perspective of this work.

In addition, these agent-based simulations allow us to modelize phenomena which are difficult to deal with analytically, like the introduction of polycentrism in a circular closed city with two income groups. More complex features can be added to the model, like non-central amenities breaking the circular symmetry of the city, more realistic interactions between agents etc. The effects of such features are multiple, on the shape of the city, on rents, surfaces of housing, density and commuting distances. So that the overall effect can be difficult to predict. The use of an agent-based system is convenient in such cases, because it allows to handle agents easily and to have an access to individual or global variables characterizing the state of the system.

Thanks to this model we explore research questions such as the influence of a value of time on the location of households depending on their income. We show that the Urban Economics model with a log-linear (or Cobb-Douglas) utility function can not explain the location of households in "European" cities (with richer households living near the center and poorer at the periphery). Even by introducing a value of time which increases with income, the model leads to a "North American" city configuration if this value of time has an income elasticity which is not higher than one, as is usually supposed in Urban Economics works. The introduction of different transport modes is not sufficient to have an inversion of the "North American" city configuration with this utility function.

The introduction of several centers in this model has a positive impact on agents' utility and on commuting distances, but even the effect of such a simple feature is not trivial when both income groups are compared for instance.

The use of agent-based systems on calibrated urban models could also allow to test the effect of different urban policies, and to have a global view of their effect on the urban system. In this goal a calibration of the dynamic evolution of the model is an interesting perspective of research.

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A Computation of the optimal land and capital quantities

Muth model

Price of land p and distance to the center x are fixed. We compute the derivatives of agents' utility U with respect to s and k to find the optimal quantities of land and capital inputs for an agent:

$$\begin{aligned}
U &= \alpha \ln \big(Y - tx - ps - k\big) + a\beta \ln s + b\beta \ln k + \beta \ln A \\
\frac{\partial U}{\partial s} &= \frac{-\alpha p}{Y - tx - ps - k} + \frac{\beta a}{s} \\
\partial U/\partial s &= 0 \text{ if } \alpha ps = \beta a(Y - tx - ps - k) \\
\frac{\partial U}{\partial k} &= \frac{-\alpha}{Y - tx - ps - k} + \frac{\beta b}{k} \\
\partial U/\partial k &= 0 \text{ if } \alpha k = \beta b(Y - tx - ps - k)
\end{aligned}$$

We have a system of two equations in s and k:

$$\begin{cases} \alpha ps + \beta a(ps + k) = \beta a(Y - tx) \\ \alpha k + \beta b(ps + k) = \beta b(Y - tx) \end{cases}$$

Adding these two equations, knowing that $\alpha + \beta = a + b = 1$, gives $k + ps = \beta(Y - tx)$. Hence the expression of the optimal land and capital quantities:

$$\begin{cases} s = \beta a(Y - tx)/p \\ k = \beta b(Y - tx) \end{cases}$$

Alonso's model

We consider price p and distance to the center x fixed, and compute the derivative of U with respect to surface s_A:

$$U = \alpha \ln \big(Y - tx - ps_A\big) + \beta \ln s_A$$
$$\frac{\partial U}{\partial s_A} = \frac{-\alpha p}{Y - tx - ps_A} + \frac{\beta}{s_A}$$
$$\frac{\partial U}{\partial s_A} = 0 \text{ if } \alpha ps_A = \beta(Y - tx - ps_A)$$

Hence the expression of the optimal surface $s_A = \beta(Y - tx)/p$, with $\alpha + \beta = 1$.

B Elasticity of the demand for housing and of the marginal transport cost

To find a condition determining which income group is located in the center or in the outskirts of the city, we compare here the income elasticities of the demand for housing and of the marginal transport cost. Indeed, demand for housing and transport cost are the two opposing forces determining the location of agents in Urban Economics models. Their evolution with income is studied thanks to income elasticities, as is usually done in the litterature (see Fujita [1989] for instance). Let us consider two income groups with incomes Y_p and Y_r respectively, with $Y_p < Y_r$: we use arc elasticities defined as

$$\epsilon(y,x) = \frac{\Delta y}{\Delta x}\frac{\bar{x}}{\bar{y}}$$

where $\epsilon(y,x)$ is the arc elasticity of variable y with respect to variable x, Δy (Δx respectively) is the variation of y (x) between the two points considered, $\bar{y}$ ($\bar{x}$) is the mean of y (x) between the two points.

The demand for housing in the model of Alonso with a log-linear (or Cobb-Douglas) function is given by equation: $s = \beta(Y - Tx)/p$ (see appendix A). The income arc elasticity of the demand for housing is then

$$\epsilon(s,Y) = \frac{Y_r - Y_p - (T_r - T_p)x}{Y_r + Y_p - (T_r + T_p)x} . \frac{Y_r + Y_p}{Y_r - Y_p}$$

where p and r subscripts indicate "poor" and "rich" agents.

The income arc elasticiy of marginal transport cost T is

$$\epsilon(T,Y) = \frac{T_r - T_p}{T_r + T_p} . \frac{Y_r + Y_p}{Y_r - Y_p}$$

To have rich agents located in the center of the city, the elasticity of marginal transport cost must be greater: $\epsilon(T,Y) > \epsilon(s,Y)$, which can be written

$$\frac{T_r - T_p}{T_r + T_p} > \frac{Y_r - Y_p - (T_r - T_p)x}{Y_r + Y_p - (T_r + T_p)x}$$

and reduced to condition (1): $T_r/T_p > Y_r/Y_p$, or in an equivalent way $T_r/Y_r > T_p/Y_p$.