Application of a Gumbel Multivariate Copula in a Competing Duration Hazard-based Vehicle Transaction Decision

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Abstract

Vehicle ownership has been the subject of many earlier studies, which have adopted a variety of econometric frameworks at both aggregate and disaggregate choice modeling level. Car manufacturers, oil companies, Metropolitan Planning Organizations (MPOs) and environmental agencies are among many organizations interested in using accurate aggregate and/or disaggregate vehicle ownership models. Among various modeling approaches, dynamic disaggregate models such as competing duration-based models, seem to provide better modeling fit with more effective forecasting capabilities relative to other vehicle ownership modeling frameworks. This is in large part due to their capability in jointly modeling transaction timing and type. Traditionally transaction timing has been overlooked in the commonly used static vehicle ownership models despite the significant role it plays in explaining vehicle ownership behavior. Transaction failure timing, like many other time related failures such as unemployment duration, is modeled by using duration risk based models. Additionally, the decision about transaction type is typically assumed to be independently made; therefore, independent timing models are developed for each transaction type. In other words, the inclusion of inter-correlation among the error terms of the vehicle transaction type decisions is typically ignored in the competing duration models due to the significant computational burden that it imposes on the model. This study aims to introduce a joint transaction type and timing model at the disaggregate level using a discrete competing proportional hazard model. The inter-correlation among multiple transaction types is modeled in this study by utilizing a copula density function to approximate the multivariate probability density function among the transaction types' error terms. Household transaction decisions are assumed to occur in discrete time intervals and are modeled as a generalization of the Han and Hausman (1990) formulation. More specifically, the formulation is extended to three transaction decisions instead of the previously introduced binary application. Additionally, unlike their work, the proportional hazard specification is directly estimated in this study while in their work, they approximated it with an ordered probit specification.

Introduction

A vehicle ownership model is a key element of travel demand modeling systems. Vehicle ownership can significantly affect all aspects of the modeling structure, however, the conventional 4-step travel demand models typically do not have an exclusive step for vehicle ownership or transaction behavior. Traditionally, vehicle ownership along with income and family size are considered as the main factors influencing personal trip production (Frank *et al.*, 2000, Cervero and Radisch, 1996; Gordon, 1994; Giuliano, 1993). The influence of vehicle ownership on mode choice does not need much elaboration. Vehicle availability can directly impact the formation of alternatives in the choice set as well as the selection of travel mode by the decision maker (Train 1986, Kitamura 2009 and Commins and Nolan 2010). The trip distribution and traffic assignment steps are also correlated with vehicle ownership models (Han 2001 and Dissanayake and Morikawa 2005).

In disaggregate activity-based travel demand modeling systems, there are several other choice elements (e.g., time-of-day, trip chaining, joint activities, etc.) that are directly influenced by vehicle availability. Therefore, vehicle ownership models are explicitly considered within activity-based modeling frameworks and they are modeled either exogenously or endogenously with activity and travel choices. In fact, the integration of vehicle transaction decisions with other household or personal decisions such as Vehicle Miles of Travel (VMT), vehicle utilization, emission, energy use and residential location search has received substantial attention in the literature, both from a substantive standpoint[1] and a methodological perspective[2]. Nonetheless, research on such integrated models is still in its infancy.

Disaggregate vehicle ownership models may take the form of either static or dynamic models. Static vehicle ownership models are developed using cross-sectional data while the taste evolution over time is not considered in them (Whelan 2007). Alternatively, a dynamic vehicle ownership model can capture households' taste changes over time. The taste evolution can be attributed to the household's previous decisions, variation in household's attributes over time, and the extent of information obtained from other sources including other vehicle owners (Mannering and Winston, 1985). Such variation in household's preferences over time consequently results in different transaction behavior. One can argue that changes to the household dynamics at each stage of its lifecycle can be the source of preference variation over time that can trigger a transaction. Major changes such as graduation, job change, household size reduction/increase and many other factors can magnify the tendency of making a transaction (de Jong and Kitamura, 2009). Once the household stress level reaches a specific threshold, a decision will be made and a transaction will be triggered. This dynamic interpretation of the

[1] Together, these decisions determine overall travel patterns, mobile source emissions, and greenhouse gas emissions; see Bhat and Sen, 2006, Cao *et al.*, 2006 and Kuwano *et al* 2009.

[2] To consider interdependencies in these choice processes; see Mannering and Winston, 1985, Dargay J, Gately, 1997, Fang 2008, and Rashidi *et al.* 2009.

household vehicle transaction behavior promotes the application of hazard-based models in which the probability of changing the current state can be increased by the time elapsed since the last transaction.

Intuitively, a complete vehicle transaction decision can be broken into several sub-decisions among which the most important decisions are transaction timing and transaction type (Rashidi *et al.*, 2009 and Brownstone *et al*., 1996). From a behavioral perspective, people initially decide to make a transaction at the time when their perceived utility of making such transaction reaches a specific threshold, and then conditional on the transaction decision, other choices can be modeled. The consequent decisions after transaction timing and type decisions will be elaborated later in this paper.

Transaction type in a general classification can be identified by two major decision types of *acquisition* and *sell* or dispose of a car. Nonetheless, *trade* (replacement) is also usually considered as a complimentary transaction type in addition to purchase and sell (Yamamoto *et al.* 1999). Other than these three basic transaction types, combinations of them have also been considered in several studies. That is, selling two vehicles, purchasing more than one vehicle or trading a vehicle while adding a new vehicle can be defined and considered as the transaction types (Brownstone *et al.* 1996).Without losing the generality, this study only considers the three major transaction types that can be selected by a household including *acquisition*, *trade* and *disposing of* a vehicle .

This study presents a behavioral, disaggregate and dynamic model for the household transaction decision with three potential transaction type alternatives. The transaction type and timing selections are modeled using a competing duration model for which the interdependency among the competing transactions is constructed using the theory of copulas. More explanation about the application of copulas and competing hazard formulation are provided in the next sections.

The rest of the paper is organized as follows: First, a brief literature review is presented and the contributions of the current study are discussed. Second the panel data used in this study is presented and the variables that were used for modeling are defined. Then the process of developing a competing hazard duration model with multivariate copula distribution is discussed along with the derivation of formulation and objective functions. Following that, experimental results for different competing hazard duration models are presented. Finally, conclusions and future research directions are discussed in the last section.

Background and Contributions

A behavioral dynamic vehicle transaction model comprises several choice decisions. These decisions include transaction timing, transaction type, and detailed attributes of the selected vehicle. The later one can be broken into several sub-decisions including: whether the vehicle is used or new, leased or owned, whether it is an alternative-fuel or hybrid vehicle, what is the vehicle class (e.g., sedan, SUV, coupe, etc), how old is the car, and many other attributes of the vehicle that should be considered. While it may be appealing to consider modeling all these decisions jointly, the dimensions of such model and the fact that decisions can be made at different points in time would make it practically infeasible to adopt a joint modeling approach. Besides, there is the question of whether individuals and households are truly able to absorb the many choice dimensions at the same time and make joint decisions, or whether a bounded-rational approach is considered where some level of hierarchy in the choice process is adopted (Lerman and Ben-Akiva, 1976). It is quite typical in dynamic vehicle transaction models to use some form of hierarchy in the choice dimensions. A commonly used decision hierarchy is to start with the transaction timing decision, followed by a transaction type choice and the detailed vehicle type and vintage choices can be modeled afterwards. In this framework, the attributes of the purchased/disposed vehicle are conditional on the transaction type. Typically, transaction timing is modeled for different transaction types separately (Yamamoto *et al.*, 1999, Rashidi and Mohammadian 2008). Hazard-based duration models provide a suitable platform to model transaction timing and type together in a unified competing formulation. In previous studies reported in the literature, this task has been left as a future research task. The current study attempts to address this shortcoming of the previous studies.

One burden before developing a dynamic vehicle-holding duration model is that developing such a model requires using panel data. There is no such a need for developing static vehicle ownership models where cross sectional data is satisfactory. Panel data also enable the modeler to incorporate the impact of time varying covariates in the model which can considerably enhance the dynamic nature of the model (Yamamoto *et al.* 2004).

In the existing literature, discrete choice models are typically used in static vehicle ownership models (Bhat and Pulugurta 1998 and Mohammadian and Miller 2003) while hazard-based duration models are used in dynamic vehicle holding and transaction models (de Jong and Pommer, 1996, Yamamoto *et al.*, 1999, Mohammadian and Rashidi 2007). Application of hazard-based models in fields other than transportation, such as politics, medicine, economics and sociology has a long history (Cox 1959 and 1972, Lancaster 1979, Han and Hausman 1990 and Bhat 1996a). Han and Hausman (1990) illustrated a discrete competing hazed model with two competing outcomes of unemployment duration. They incorporated a bivariate normal distribution estimated with a Monte-Carlo experiment to approximate the joint error terms of the competing hazards. This paper adopts their formulation and extends it to the case of three competing vehicle transaction outcomes (trade, acquisition and dispose), while utilizing a flexible trivariate copula distribution instead of a trivariate normal distribution. Furthermore, Han and Hauman approximated the hazards specification with an ordered probit while this study directly estimates the competing proportional hazards specification.

Bhat (1996b) generalized Han and Hausman’s formulation by introducing a joint discrete choice model and a duration model. Bhat’s formulation obviates the application of bivariate probit distribution approximation for the joint probability density function by substituting it with a bivariate extreme value distribution. Additionally, Bhat’s generalized formulation can be easily extended to the case of multiple competing outcomes while Han and Hausman’s formulation become very cumbersome if one generalizes it to multiple competing outcomes because it requires computation of multivariate integrals (Sueyoshi 1992). Nonetheless, Han and Hausman's formulation is still widely used in different fields (e.g., Abbring and Van den Berg, 2003, Honore and Lleras-Muney 2007 and Horny and Picchio, 2010) because of its perspective about the competing nature of the outcomes. Foremost, in Han and Hausman's formulation hazard functions of different outcomes compete with each other based on their covariate and failure outcome timing while in Bhat's formulation different outcomes compete with each other via a discrete choice model. Secondly, the duration of outcome survival (the failure timing) assumed to be conditional on the type of the outcome in Bhat's formulation while there is no such an assumption in Han and Hausman's formulation. Therefore, both formulation approaches can reflect the reality of the competing natures of the outcomes. While this paper does not intend to examine the superiority of one approach over the other one, it attempts to propose an alternative view for the generalization of Han and Hausman's competing hazard formulation.

Han and Hausman approximated the joint distribution with a bivariate probit distribution. This normality assumption itself seems to be a major assumption. Furthermore, by increasing the number of competing outcomes, computation of the integral becomes very cumbersome due to the fact that the normal multivariate distribution can only be estimated using a simulation estimation procedure. Other than multivariate normal distribution applications for tackling the correlation among error terms, copulas have also been a popular multivariate modeling tool in many fields (including finance, economics, and biomedical studies) where multivariate normal dependency is questionable. For example, Rosenberg and Shuermann (2004) studied an application of copula to integrate risk management for financial institutions for aggregating risk types (market, credit, and operational). In economics, Patton (2006) considered an extension of the theory of copulas to model asymmetric exchange rate dependency. In Biomedical studies, Kim *et al.* (2008) also utilized a copula method for modeling directional dependence of genes.

Recently, in the transportation related studies, the application of copula distributions has been attracting a great deal of attention. Bhat and Eluru (2009) used a bivariate copula model to examine the correlation between residential neighborhood choice and daily household vehicle miles of travel (VMT). Bhat and Sener (2009) also demonstrated a bivariate copula model for accommodating spatial dependency in data indexed by geographic location. Nonetheless, the application of copula in transportation is still relatively new, and the effectiveness of using copula approach for different applications needs to be explored.

In short, Han and Hausman's formulation provides a starting point for the mathematical formulation of the model that is presented in this paper. Their bivariate formulation is initially generalized to a trivariate formulation. Then the assumption of normal distribution dependency between the error terms is released and replaced with a Gumbel copula distribution which results in a closed-form likelihood formula. To the best of authors' knowledge, application of a multivariate copula in a competing vehicle transaction duration model that is one of the contributions of this study has not been studied before.

Data

The main database used in this study was extracted from a survey of household car ownership in Toronto, Canada. The survey was completed at the University of Toronto in 1998 (Roorda et al. 2000). The database includes information on the characteristics of over 900 households, individual members, and their vehicle information, as well as information about their residential, employment, and lifestyle changes over time. The survey covers a 9-year period from 1990 to 1998 and any vehicle transaction record beside this period is censored. Information about the duration failures (i.e., transactions), which is normally reported at yearly intervals, is available at monthly intervals in this data. There are several monetary variables in the set of utilized explanatory variables. These variables cannot be directly used in the model because they belong to different years of the panel data, and the variation in the dollar values of different years has not accounted for inflation. Therefore these variables are adjusted to a comparable dollar value of the base year (1998) using the historical Canadian inflation rates.

Table 1 Mean and standard deviation of the explanatory variables*

Variable Names	Dispose	Acquisition	Trade
Transaction Duration	59.96(38.86)	32.27(18.85)	66.11(39.79)
Purchased Vehicle New	-	0.33(0.47)	0.52(0.5)
Purchased Vehicle MPG	-	27.68(5.52)	27.02(5.12)
Purchased Vehicle Weight	-	2914.46(607.04)	3055.62(563.14)
Purchased Vehicle Price	-	9.39(7.83)	12.06(6.57)
Purchased vehicle Age	-	4.28(4.46)	2.38(3.62)
Disposed Vehicle Age	3.01(4.26)	-	2.22(3.41)
Disposed Vehicle New	0.5(0.5)	-	0.53(0.5)
Disposed Vehicle MPG	28.77(6.32)	-	27.94(6.37)
Disposed Vehicle Wieght	2721.03(539.75)	-	2860.19(598.18)
Average Price of Fleet	5.69(4.3)	7.11(4.63)	5.67(4.59)
Average Age of Fleet	6.81(3.47)	5.64(3.99)	7.44(3.69)
Average Length of Ownership	4.05(2.62)	2.81(1.94)	4.89(2.86)
Average Fuel Cost	1.51(0.75)	1.21(0.76)	1.3(0.64)
Average Dipreciation Cost	0.95(0.75)	1.08(0.85)	0.88(1)
Average Age of people in DMU	47.01(67.62)	38.12(45.03)	41.34(60.33)
Average Parking Cost	3.05(9.72)	2.43(9.33)	3.68(12.18)
DMU Income	44.22(22.07)	49.81(21.27)	47.47(22.67)
Tenure	0.25(0.43)	0.22(0.41)	0.23(0.42)
Number of Vehicles in DMU	1.89(0.83)	1.46(0.87)	1.58(0.66)
Number of People in DMU	2.85(1.34)	3.74(1.26)	3(1.35)
Number of Children in DMU	0.61(0.97)	0.87(1.04)	0.84(1.05)
Number of Males in DMU	1.37(0.8)	1.76(0.83)	1.49(0.91)
Number of Jobs in DMU	1.39(0.77)	1.76(0.87)	1.39(0.81)
Number of Licenses in DMU	1.85(0.93)	2.44(1.03)	1.88(0.71)
Number of Retired in DMU	0.21(0.49)	0.17(0.44)	0.17(0.45)
Number of Full-Time Jobs in DMU	1.17(0.76)	1.51(0.81)	1.21(0.76)
Number of Part Time Degree in DMU	0.05(0.23)	0.07(0.28)	0.03(0.18)
Number of Non-Educated in DMU	0.35(0.7)	0.51(0.87)	0.44(0.76)
Number of College Education in DMU	0.45(0.67)	0.48(0.67)	0.43(0.62)
Number of Bachelors Education in DMU	0.44(0.66)	0.54(0.79)	0.46(0.68)
Number of Graduate Education in DMU	0.28(0.54)	0.3(0.64)	0.23(0.52)
Number of Parking Spot Available	3.08(2.28)	3.59(2.21)	2.88(1.98)
Age of the Owner	45.46(15.84)	-	47.96(44.64)
Gender of the Owner	0.67(0.47)	-	0.65(0.48)

*numbers in parenthesis are standard deviations

Several explanatory variables are tested in the competing duration hazard of this paper, ranging from decision making unit (DMU) and individual socio-demographic and economic attributes to vehicle characteristics. In this study DMU is considered as the unit of observation for modeling purposes. The decision making unit is defined as a set of individuals within a household that make vehicle ownership decisions in conjunction with each other. It is assumed that a household may consist of one or more decision making units, and that a decision making unit may be comprised of one or more persons (Roorda et al. 2000). A list of the mean and standard deviation of these explanatory variables along with the transaction duration period is presented in Table 1. It should be noted that not all of these variables are necessarily kept in the final model as some are dropped because they were not found to be statistically significant in the model.

Methodology and Mathematical Estimations

Consider Han and Hausman's (1990) competing duration risk-based formulation in the case of two failure types of trade (*Tra*) and dispose (*Dis*):

$$P_{Tra}^{t} = P[Tra = 1\, and\ Dis = 0\ at\ (t-1, t)] = \int_{\delta_{t-1}^{1} - x\beta_1}^{\delta_t^1 - x\beta_1} \int_{m}^{\infty} f(\varepsilon_1, \varepsilon_2) d\varepsilon_2 d\varepsilon_1 \quad (1)$$

Where $f(.,.)$ is the joint probability density function of the error terms, δ_t^1 is the logarithm of the integrated baseline hazard of failure type 1 (assumed to be "*Tra*" here) during period *t*, β_1 is the coefficient vector of the covariates, *x* is the vector of covariate variables and *m* is estimated such that it is assured that failure time of type 2 (assumed to be "*Dis*" here) occurs after type 1. It is assumed in Equation 1 that failure type 1 had been observed and therefore, failure type 2 should have occurred sometime after the failure time of type 1. To make sure the formulation represents the order of the failures, Han and Hausman introduced t^* as a latent failure time of the observed failure type. Therefore the inside integration of Equation 1 can be broken into two parts: one part from the latent failure time to time *t* and then from time *t* to infinity as:

$$P_{Tra}^{t} = \int_{\delta_{t-1}^{1} - x\beta_1}^{\delta_t^1 - x\beta_1} \left(\int_{m_{t^*}}^{\delta_t^1 - x\beta_1} f(\varepsilon_1, \varepsilon_2) d\varepsilon_2 + \int_{\delta_t^1 - x\beta_1}^{\infty} f(\varepsilon_1, \varepsilon_2) d\varepsilon_2 \right) d\varepsilon_1 \quad (2)$$

The first integral of the inside integrals in Equation 2 is dismissed in this study because the transaction failures are observed in the monthly intervals in the utilized retrospective panel data. It is argued that one month is too short a period for a household in which to change the transaction decision. In other words, the simplifying assumption of ignoring the first integral of the internal integral of Equation 2 is not controversial to the household transaction decision behavior because if a household has already decided to make a specific transaction type in the current month, the probability of any other transaction type in the same month would be extremely small. For example, changes to the household dynamics or adjustments in the economic conditions may affect household's condition and motivate the household to dispose of a car in a given month. However, those changes within less than a month are less likely to be in such an extent to change the household decision by motivating another type of transaction. Eventually, by excluding the first term of the internal integral of equation 2, the binary competing hazard formulation that is utilized in this study becomes:

$$P_{Tra}^{t} = \int_{\delta_{t-1}^{1} - x\beta_1}^{\delta_t^1 - x\beta_1} \int_{\delta_t^1 - x\beta_1}^{\infty} f(\varepsilon_1, \varepsilon_2) d\varepsilon_2 d\varepsilon_1 \quad (3)$$

Han and Hausman defined m_{t^*} to make sure that the implied failure time of type 2 (which has not actually been observed to fail) is greater than the implied failure time of type 1. Logically, this implied failure time of type 2 can be in the same time interval in which type 1 failure is observed but sometime after t^*. Practically, in the case of unemployment duration failures with annual time steps that was studied by Han and Hausman, it is very probable that if a person who had not accepted one type of employment till a given time t^*, would have accepted another employment type resulting in unemployment duration failure. . This is because many external factors may change during a year; including those that can result in making an individual select another type of employment as time passes. However, the probability of selecting another type of transaction, other than the one already chosen, in an interval of one month during which household dynamics would seldom change is small and negligible. Therefore, omitting the first internal integral of the equation 2 in the case of this study for which time intervals are very small is intuitive and reasonable. This constructs the major simplifying assumption of the study that should be clearly understood.

Therefore, in a case of three transaction types: dispose (*Dis*), acquisition (*Acq*) and trade (*Tra*), the probability of occurrence of trade in the *t* period would be:

$$P_{Tra}^{t} = P[Tra = 1, \text{Acq} = 0 \text{ and Dis} = 0 \text{ at } (\text{t - 1, t}) =$$

$$\int_{\delta_{t-1}^{Tra}-x\beta_{Tra}}^{\delta_t^{Tra}-x\beta_{Tra}} \int_{\delta_t^{Acq}-x\beta_{Acq}}^{\infty} \int_{\delta_t^{Dis}-x\beta_{Dis}}^{\infty} f(\varepsilon_{Tra}, \varepsilon_{Acq}, \varepsilon_{Dis}) d\varepsilon_{Dis} d\varepsilon_{Acq} d\varepsilon_{Tra} \tag{4}$$

Similar probability density functions can be derived for acquisition and dispose decisions. The presented probability density function of Equation 4 can be approximated by using numerical approaches or a copula multivariate density function that can result in a closed-form formulation.

Competing Hazard Function with a Copula Dependency

Let ε_{Dis} , ε_{Acq} and ε_{Tra} be three random variables with marginal distribution functions of $F(\varepsilon_{Dis})$, $F(\varepsilon_{Acq})$ and $F(\varepsilon_{Tra})$ and also let C_θ be a tri-dimensional copula with the cumulative distribution function of $C_\theta(F(\varepsilon_{Dis}), F(\varepsilon_{Acq}), F(\varepsilon_{Tra}))$. This copula function is used to substitute the joint probability density function of Equation 4. From Sklar's theorem (Sklar 1959), it is known that for a joint density function of the triple vector of scalar error terms, there exists a copula function of C_θ such that,

$$F(\varepsilon_{Dis}, \varepsilon_{Acq}, \varepsilon_{Tra}) = C_\theta(F(\varepsilon_{Dis}), F(\varepsilon_{Acq}), F(\varepsilon_{Tra})) \tag{5}$$

Under the assumption that the marginal distributions are continuous, the copula is unique. In Equation 5, the parameter θ is a dependency parameter which jointly represents the inter-dependency among the marginal distributions. The differentiated form of Sklar's theorem can be used to formulate the probability density function of the error terms based on the marginal distributions. It can be shown that $c_\theta(u,v,w) = \partial C_\theta(u,v,w) / \partial u \partial v \partial w$, therefore the joint probability density function becomes,

$$f(\varepsilon_{Dis}, \varepsilon_{Acq}, \varepsilon_{Tra}) = f(\varepsilon_{Dis}) f(\varepsilon_{Acq}) f(\varepsilon_{Tra}) \times c_\theta(F(\varepsilon_{Dis}), F(\varepsilon_{Acq}), F(\varepsilon_{Tra})) \tag{6}$$

The probability density function of Equation 6 is then substituted in Equation 4 and the triple integral of Equation 4 is calculated assuming an extreme value form for the transaction types' error terms with distribution function given by;

$$G(z) = 1 - e^{-e^{z}} \tag{7}$$

Among all the generated copulas there is a wide range of applications for Archimedean copulas. Archimedean copulas can be easily constructed and they have many useful properties (Nelson 1999). In this study we utilized the Gumbel copula of the Archimedean Family.

$$C_\theta(F(\varepsilon_{Dis}),F(\varepsilon_{Acq}),F(\varepsilon_{Tra}))=e^{-[(-\ln(F(\varepsilon_{Dis}))^\theta+(-\ln(F(\varepsilon_{Acq}))^\theta+(-\ln(F(\varepsilon_{Tra}))^\theta]^{\frac{1}{\theta}}} \quad \text{`(8)}$$

By integrating Equations 6, 7 and 8 into equation 4, the probability of trading in period t can be re-written as,

$$\begin{aligned}P^t_{Tra}&=\int_{\delta^{Tra}_{t-1}-X\beta_{Tra}}^{\delta^{Tra}_{t}-X\beta_{Tra}}\int_{\delta^{Acq}_{t}-X\beta_{Acq}}^{\infty}\int_{\delta^{Dis}_{t}-X\beta_{Dis}}^{\infty} f(\varepsilon_{Tra},\varepsilon_{Acq},\varepsilon_{Dis})d\varepsilon_{Dis}d\varepsilon_{Acq}d\varepsilon_{Tra}=\\ &\psi(\xi^t_{Tra},\xi^t_{Dis},\xi^t_{Acq})+\psi(\xi^t_{Tra})-\psi(\xi^t_{Tra},\xi^t_{Acq})-\psi(\xi^t_{Tra},\xi^t_{Dis})-\\ &[\psi(\xi^{t-1}_{Tra},\xi^t_{Dis},\xi^t_{Acq})+\psi(\xi^{t-1}_{Tra})-\psi(\xi^{t-1}_{Tra},\xi^t_{Acq})-\psi(\xi^{t-1}_{Tra},\xi^t_{Dis})]\end{aligned} \quad (9)$$

where:

$$\psi(u)=e^{-(u^\theta)^{\frac{1}{\theta}}},\psi(u,v)=e^{-(u^\theta+v^\theta)^{\frac{1}{\theta}}},\psi(u,v,w)=e^{-(u^\theta+v^\theta+w^\theta)^{\frac{1}{\theta}}}$$

$$\xi^t_l=-\ln[G(\delta^t_l-X\beta_l)],l=\textit{Tra, Acq and Dis, and } \delta^t_l=\int_0^t h^l_0(u)du$$

where h^l_0 is the baseline hazard of transaction type l

In this study, other than the well-known monotonic Weibull baseline hazard, a non-monotonic log-logistic baseline hazard is also tested in the duration model.

$$h^l_0(u)=\begin{cases}\textit{Weibull} & \gamma_l t^{\gamma_l-1}\\ \qquad\qquad or & \\ LL & \dfrac{\frac{\beta_l}{\alpha_l}(\frac{t}{\alpha_l})^{\beta_l-1}}{1+(\frac{t}{\alpha_l})^{\beta_l}})\end{cases}$$

and α_l, β_l and γ_l are baseline hazard parameters.

Equation 9 can be easily re-written for the occurrence of acquisition and dispose cases. Finally the likelihood function for estimating the parameters can be written as,

$$L=\prod_{i=1}^{N}\prod_{t=1}^{T}[(P^t_{Tra})^{D_i^{Tra}}\times(P^t_{Acq})^{D_i^{Acq}}\times(P^t_{Dis})^{D_i^{Dis}}]^{y_{it}} \quad (10)$$

where y_{it} is equal to 1 if the individual makes a transaction in the interval t and 0 otherwise, N is the number of individuals and T represents the number of intervals. In this equation, $D_i^{Tra}+D_i^{Acq}+D_i^{Dis}=1$, and each term is equal to 1 if that type of transaction has occurred and zero otherwise.

It is worth noting that logically, there is no competing behavior among the transaction types for the households with no auto during the current time interval for which only vehicle acquisition can be conceived of. The likelihood function of Equation 10 is finally utilized to estimate the model parameters. The results of this estimation are presented in the next section.

Competing Hazard Function with No Dependency

Unlike the formulations presented in the previous section, it can also be assumed that there is no dependency exists amongst the transaction decisions. In other words, it can be assumed that the three transaction decisions of trade, dispose and acquisition can be made independently by a DMU. In that case, the joint probability density function of Equation 4 can be substituted by the product of the marginal transaction probability distributions that can be easily rewritten as (under the assumption of trade occurrence):

$$P_{Tra}^{t} = e^{-\sum_{l=Acq\,and\,Dis} e^{\delta_l^t - X\beta_l} - e^{\delta_{Tra}^{t-1} - X\beta_{Tra}}} - e^{-\sum_{l=Acq,\,Dis\,and\,Tra} e^{\delta_l^t - X\beta_l}} \tag{11}$$

The occurrence probability for acquisition and disposal in the (t, $t+1$) period can also be written similar to Equation 11. These probabilities are then utilized to form a likelihood function similar to the one presented in Equation 10, which is then maximized in order to estimate the model parameters.

Results and Analysis

The proposed competing hazard formulation is utilized to develop models of vehicle transaction timing and type decision using the dataset from Toronto metropolitan area. The results are then compared with the results of a competing hazard model with independent transaction timing and type decisions. To do so, initially a comparison between the likelihood values at convergence for two cases of *with-copula* and *without-copula* models with alternative baseline hazards is presented. Table 2 presents the results of such comparison that confirms the importance of considering the dependencies among the three decisions. As shown in Table 2, considering the interdependencies among the transaction decisions can considerably reduce the Bayesian information criterion (BIC) statistic, resulting in model improvement. It is also observed from Table 2 that in the case of presence of copula dependency the Weibull baseline hazard provides a better model fit compared to the log-logistic baseline hazard.

Table 2 Comparison of BIC with various models

Model Type	Likelihood at Convergence	Number of Parameters	BIC
Gumble Copula with Weibull Baseline	-4026.71	30	4126.70
Gumble Copula with Log-logistc Baseline	-4059.96	33	4169.94
No Copula With Weibull Baseline	-4486.34	29	4582.99
No Copula with Log-logistic Baseline	-4188.13	32	4294.78

BIC= - ln(L_C)+0.5 p ln(N)
ln(L_C) is the log-likelihood value at convergence
p is the number of parameters
N is the number of samples

It can be discerned from Table 2 that *with-copula* models can considerably dominate the *without-copula* models. This finding supports the intuitive of the interdependency among the transaction decisions. In other words, if the interdependencies among the transaction types are included in the modeling formulation, it is expected that the model better explain the household and individual's behaviors.

Knowing that *with-copula* models are more reliable, after maximizing the likelihood function of Equation 10, Table 3 presents the results of the competing duration model with Weibull baseline hazard while Table 4 presents the competing duration model with log-logistic baseline hazard. It can be discerned from these two tables that Weibull distribution parameters are generally greater than one, which indicates that the baseline hazard increases monotonically while the log-logistic baseline hazard is non-monotonic because the beta values are greater than one. It should be noted that the two competing models have estimated parameters with identical signs, although the model with Weibull baseline hazard provides better modeling fit if the likelihood function at convergence is considered. Constant variable values are all significant in the models; nonetheless, their values are considerably smaller in the models with log-logistic baseline hazards where a smaller value is more desirable.

It should be also noted that the effect of covariates in the model is facilitated by incorporating a negative sign for parameters. Therefore, a negative coefficient can result in the increased value of the hazard function that suggests a decrease in duration. Starting from the top of Tables 3 and 4 in the same order as the variables are reported, if a DMU is willing to purchase a vehicle with a higher mileage per gallon (i.e., more fuel efficient), this tendency affects its acquisition behavior by accelerating the decision making process. The indicator showing the household's willingness to purchase a new car is found to be significant in both acquisition and trading decisions with a positive sign implying that if the household is willing to buy a new car it may postpone the transaction time. It is also shown that the tendency to buy a more expensive car or buy an old car may increase the chance of an earlier trade transaction.

Table3 Competing vehicle transaction model with copula distribution and Weibull baseline hazard

	Trade		Acquisition		Dispose	
	Parameter	t-value	Parameter	t-value	Parameter	t-value
Constant	3.986	13.85	2.380	18.94	3.650	4.83
Gamma	1.515	36.48	1.219	33.82	1.524	30.69
Alpha						
Betha						
Vehicle Attributes						
Purchased Vehicle MPG			-0.008	-3.45		
Purchased Vehicle New/Used	0.355	6.59	0.474	6.23		
Purchased Vehilce Pirce	-0.038	-10.73				
Purchased Vehicle Age	-0.018	-2.60				
Disposed Vehicle Age	-0.012	-10.51			-0.123	-9.40
Disposed Vehicle Weight /1000	-0.123	-3.98				
Disposed Vehicle MPG	-0.027	-2.92				
DMU Attributes						
No. of Licenses in DMU			-0.099	-2.64		
No. of Graduate Education Degrees in DMU	0.073	2.51				
No. of Vehicles in DMU	0.118	3.91	0.316	4.77		
No. of Jobs in the DMU			-0.033	-1.04		
No. of Parking Spots Available for DMU			-0.028	-1.95		
No. of Children					0.025	1.38
No. of Members with College Degree or Higher					0.035	1.87
DMU Anuual Income /100					0.216	2.72
Average Price of Fleet in DMU	-0.044	-3.76			-0.037	-1.81
Average Cost of Depreciation of DMU			-0.103	-3.15		
Copula Dependency Parameter	5.025	27.57				
Likelihood at Convergence	-4026.71					

The next set of explanatory variables refers to the attributes of the vehicle that will be disposed of the DMU’s fleet. Age and weight of the disposed vehicle are statistically significant in the trade model with a negative sign indicating that older and/or heavier vehicles can be traded sooner.

Household socio-demographic attributes are also found to be significant in all three transaction decisions. Findings in the competing hazard model reinforce the intuitive understanding that DMUs with a greater number of licensed drivers are likely to purchase more cars. In other words, if the number of licensed members increases, the household will tend to purchase an automobile.

Table 4 Competing vehicle transaction model with copula distribution and log-logistic baseline hazard

	Trade		Acquisition		Dispose	
	Parameter	t-value	Parameter	t-value	Parameter	t-value
Constant	1.993	12.06	1.304	8.34	1.733	13.54
Gamma						
Alpha	3.762	20.29	2.128	14.16	3.215	18.88
Betha	2.327	29.37	2.440	22.56	2.570	24.03
Vehicle Attributes						
Purchased Vehicle MPG			-0.010	-3.92		
Purchased Vehicle New/Used	0.340	6.50	0.513	6.89		
Purchased Vehilce Pirce	-0.043	-12.13				
Purchased Vehicle Age	-0.025	-3.25				
Disposed Vehicle Age	-0.013	-10.62			-0.131	-10.39
Disposed Vehicle Weight /1000	-0.173	-5.61				
Disposed Vehicle MPG	-0.035	-9.57				
DMU Attributes						
No. of Licenses in DMU			-0.112	-2.60		
No. of Graduate Education Degrees in DMU	0.066	2.14				
No. of Vehicles in DMU	0.102	3.44	0.311	4.40		
No. of Jobs in the DMU			-0.050	-1.43		
No. of Parking Spots Available for DMU			-0.036	-2.70		
No. of Children					0.023	1.23
No. of Members with College Degree or Higher					0.022	1.10
DMU Anuual Income /100					0.193	2.37
Average Price of Fleet in DMU	-0.045	-4.17			-0.042	-3.61
Average Cost of Depreciation of DMU			-0.106	-3.27		
Copula Dependency Parameter	4.858	30.91				
Likelihood at Convergence	-4059.96					

Different education levels were tested in the competing hazard transaction decision model while only two of them were found to be significant in the models. Graduate education level was found to be statistically significant in the trade model entailing that highly educated people trade their vehicles slower than households with fewer highly educated members. The second education related variable found to be significant is the number of members with education level higher than college. A higher level of education was also found to postpone the disposal decision. Therefore, it can be concluded from the education pertained variables that higher education may defers the transaction decision. Number of vehicles in the household was found to be positively significant in trade and acquisition models, implying that as the DMU's fleet size increases the chance of making these types of transactions diminishes. Another policy-sensitive explanatory variable used in this study represents the availability of parking spot at the residence. This variable represents, in a sense, the characteristics of the neighborhood in which the DMU resides. Households with more available parking spots tend to purchase more vehicles as the sign of this explanatory variable is negative in the acquisition model. *Income* was found to be only significant in the dispose model with a negative sign, meaning that wealthier households dispose of their cars later. *Income* was not found to be significant in the other transaction decision models. A DMU with higher income is able to purchase a newer car model, which in turn leads to the delay in their disposal decision. Less affluent DMUs tend to purchase used or older vehicles, an influencing factor in their decision to keep vehicles for a longer period of time. Finally, the presence of children in a DMU postpones disposing a car. This finding can be rationalized by the fact that families with children, particularly younger children, are more reluctant to change their lifestyle while households without children are more flexible with regards to potential changes.

Households with more valuable vehicles are more likely to dispose of or trade their cars earlier than households with cheaper cars in their fleet. Furthermore, two variables that represent vehicle holding costs were found to be statistically significant in the models. These variables are policy sensitive and should be of interest to policy makers. First, maintenance cost is an important factor in motivating decision makers to dispose of their vehicles or acquire a new vehicle. Average depreciation cost was also found to be positively influential on acquisition decisions.

Finally, the Gumbel copula dependency parameter as it was expected from the fundamentals of the copula theory was found to be statistically significant and greater than one in both Weibull and log-logistic models.

Validation and Policy Analysis

In order to compliment the statistical goodness-of-fit analysis of the competing hazard formulation that was discussed earlier, further analyses were performed to explain the effectiveness and applicability of the model.

One simple but critical measure for evaluating a discrete choice model is its capability of replicating the decision makers' decisions. The estimated model of this study was found to replicate 77% of the trade decisions, 96% of the acquisition decisions and 96% of the disposal decisions correctly. This level of accuracy seems to be promising for such a complicated system of equations. However, it was found that the shortage in accurately predicting trade decisions has been connected to almost the same amount of redundancy in overestimating the total number of disposal decision. In other words, some of the trade decisions have been substituted by disposal decisions in the simulated results while the acquisition decisions have been accurately replicated by the model to a great extent. Therefore, it can be concluded that the potential correlation between disposal and trade decisions is likely to be greater than the correlation among these two choices and the acquisition decision. Thus, application of a weighted interdependency function might be useful and potentially more appropriate for modeling the household transaction behavior.

The second analysis on the developed competing hazard model was conducted to introduce some applications of the proposed model. A selection of the covariates utilized in the modeling practice of this study was picked out and a set of policy scenarios were examined by changing those covariates. Two contraction and two expansion scenarios were examined to find the impact of the selected covariates on the DMU's final decision. Table 5 shows the results of this analysis by showing the percentage of people who are likely to change their transaction decision if the value of a covariate is increased/decreased by 10 or 20 percent. For example, if people's tendency for purchasing vehicles with higher fuel efficiency is increased, this can result in higher probability of acquisition and less trade decisions. Nonetheless, the change in transaction decision would not happen when the increase in MPG is less than 10 percent. Furthermore, the price of the purchased vehicle was also examined. It was found that any increase in the price of the purchased vehicle can result in more trade decisions while it may reduce the number of acquisition and disposal decisions. In this regard, it can also be recognized that the increase in disposal decision is greater than the increase of the acquisition decision. In other words, if decision makers purchase more expensive vehicles, that can be positively correlated with the number of transaction decisions and negatively correlated with the number of acquisition and disposal decision.

Table 5 The percentage increase/decrease in the total number of DMUs' decisions of any transaction type based on changes in the values of a covariates

	10% Increase			20% Increase			10% Decrease			20% Decrease		
Variable Used for Policy Analysis	Tra	Acq	Dis	Tra	Acq	Dis	Tra	Acq	Dis	Tra	Acq	Dis
Purchased Vehicle MPG	0.00	0.00	0.00	-0.49	1.23	0.00	0.73	-1.85	0.00	0.73	2.47	0.47
Purchased Vehilce Pirce	2.20	-1.23	-3.29	3.90	-2.47	-5.63	-2.93	1.23	4.69	-6.10	4.32	8.45
Purchased Vehicle Age	0.24	0.00	-0.47	0.73	-0.62	-0.94	-0.49	0.00	0.94	-0.73	0.00	1.47
Disposed Vehicle Age	-2.68	0.00	5.16	-3.41	-0.62	7.04	1.95	1.23	-4.69	2.93	2.47	-7.51
Disposed Vehicle MPG	2.20	-1.85	-2.82	4.39	-3.70	-5.63	-2.44	3.09	2.35	-7.07	9.88	6.10
No. of Parkings Available for DMU	0.00	0.62	-0.47	0.00	0.00	0.00	0.24	-0.62	0.00	0.73	-1.85	0.00
DMU Anuual Income /100	0.24	0.00	-0.47	2.20	0.62	-4.69	-0.73	0.00	1.41	-1.46	0.00	2.82

As shown in the table, total number of trade transaction decisions can increase by an increase in the age of the purchased vehicle while this number decreases for acquisition and disposal decisions. Therefore, it appears that people are likely to replace a vehicle with another one if they are buying an older vehicle. Similarly, if they own older vehicles, they prefer to purchase a vehicle and then dispose of the other one. Interestingly, it was found that increasing the number of available parking spots beyond a specific number (e.g. here 20%) does not change the DMU's transaction decision while decreasing its value can only affect trade and acquisition decisions. Another intuitive finding of the policy analysis that is presented in Table 5 concerns the income fluctuation. It appears that people would not purchase a car if their income is not considerably raised. However, when their income raise reaches an acceptable point that can trigger the decision to increase the vehicle fleet size. Similarly, if a reduction in income is imposed, it can immediately and negatively affects the acquisition decision. It was also found that decision maker's income is negatively correlated with the disposal decision.

Conclusion

This study introduced a competing duration risk model in which the interdependencies among the error terms were formulated by a copula multivariate distribution. A copula distribution approximates the multivariate joint probability density function with a closed-form function. In this study, the competing transaction decisions were formulated in discrete time intervals of one month. More specifically, the discrete competing hazard formulation presented by Han and Hausman (1990) was extended to the case of three competing decisions and the utilized bivariate normal distribution of the error terms was substituted with an Archimedean Gubmel copula trivariate distribution which resulted in a closed-form competing choice model. A retrospective survey data spanning nine years from the Toronto area was utilized for the modeling practice of this study. The joint competing model was then compared against the independent non-competing scenario and was shown to significantly enhance the non-competing model. This implies that a competing behavior can be considered for the case of vehicle transaction behavior which should be also included in the formulation. It was also shown that such a formulation could make the models more complicated. Two baseline hazard functions namely, Weibull and log-logistic were tested in the competing hazard models to examine whether the monotonic characteristic of the Weibull hazard is problematic or not. It was found that the Weibull function outperformed the log-logistic formulation, although the parameters of the model under both assumptions were found to be very close and statistically significant. Nonetheless, a more robust conclusion about the priority of these two baseline hazards relies on further detailed sensitivity analyses. The copula parameter was also found to be statistically different from zero and consistent with the assumption that it should be greater than one in the Gumbel distribution. Finally, it was shown that the model can acceptably replicate the behavior of the transaction decision maker. Further, a detailed policy analysis of the sensitivity of the utilized covariates was presented and proved the applicability of the developed model in replicating decision makers' vehicle transaction behavior. The analysis showed that the model is sensitive to a range of policy scenarios including changes to the values of covariates and as expected the extent of the sensitivity was supported by the intuitive expectations.

Further improvements to the competing duration transaction model can include three major categories:

1- Inclusion of other types of copulas
2- Comparison of the effectiveness of copula to multivariate distributions
3- Comparison between the presented competing duration model and a joint MNL-duration model

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