VALUATIONS OF TRAVEL TIME VARIABILITY IN SCHEDULING VERSUS MEAN-VARIANCE MODELS

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Abstract

The standard method to estimate valuations of travel time variability for use in appraisal is to estimate the parameters of a reduced-form utility function, where some measure of travel time variability (such as the standard deviation) is included. A recently discovered problem with this approach is that the obtained valuation will in general depend on the standardized travel time distribution, and hence cannot be transferred from one context to another. Instead, we test another recently suggested approach: estimating a scheduling model and then deriving an implied reduced-form expression, which can be used for appraisal. The valuations implied by the scheduling model turn out to deviate substantially from a reduced-form model estimated on the same sample. We conclude that the scheduling model – in the way it is usually interpreted and estimated – is not able to capture the entire disutility of travel time variability. Hence, although it can be shown that scheduling and reduced-form models are ”theoretically equivalent”, they are apparently not ”empirically equivalent”. We hypothesize that the derivation of reduced-form models from an underlying scheduling model omits two essential features: first, the notion of an exogenously fixed “preferred arrival time” neglects the fact that most activities can be rescheduled given full information about the travel times in advance, and second, disutility may be derived from uncertainty as such, in the form of anxiety, decisions costs or costs for having contingency plans. Finally, we report our best estimates of the valuation of travel time variability for public transit trips, for use in applied appraisal.

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1 INTRODUCTION

In recent years, the reliability of transport systems has attracted more and more attention, both by researchers and policy-makers. There is a rapidly growing body of research literature on measuring and valuing reliability, and enhancing reliability seems to be an increasingly higher priority for policy-makers. For example, the Netherlands has introduced "reliability" as a goal for its transport policies, and the Swedish Government recently declared that more efforts and resources need to be spent on reliability, at the expense of funding for completely new investments. The importance of reliability is perhaps most apparent in the public transit system: it is now standard that transit operators publish reliability measures regularly.

There are two principally distinct approaches used in the analysis of travel time variability: the scheduling approach, where the traveller's departure time choice is explicit in the model, and the reduced-form approach, where some measure of the variability is introduced directly in a reduced-form indirect utility function. While a scheduling approach is the natural choice for forecasting purposes (modelling departure time choices etc.), the reduced-form approach is usually the only feasible alternative for appraisal and evaluation purposes. As a measure of the variability in the reduced-form function, most studies have used either the standard deviation of the travel time or the average delay relative to scheduled arrival time, although some studies include both (Batley et al., 2007), while some studies use percentiles of the travel time distribution (Lam and Small, 2001). When the standard deviation is used as the variability measure, the approach is usually termed the "mean-variance" approach.

Reduced-form approaches can be derived from an underlying scheduling model, assuming that travel times follow a known random distribution and that travellers choose their departure time optimally to account for this randomness. This connection between the two approaches has provided a theoretical underpinning for the use of reduced-form approaches in applied appraisal. However, the reduced-form approach has two potentially important drawbacks. First, and most important, an estimated valuation from a reduced-form model will (in general) depend on the *specific* travel time distribution – not just on its general properties such as its standard deviation[1]. This means that a value obtained in one circumstance, using a particular distribution, cannot easily be transferred to another context where the standardized travel time distribution is different. This relationship has been properly characterized only recently, in Fosgerau and Karlström (2010), even if the insight had been pointed out at earlier (Bates et al., 2002). Second, estimating reduced-form models rests on the possibility to present travel time distributions to respondents in a stated choice experiment, and then having them indicate which they would prefer. The problem of presenting travel time distributions comprehensibly to respondents has proven difficult – especially since the

[1] However, there are some scheduling models that result in reduced-form models that are independent of the standardized travel time distribution. The Vickrey (1973) model analyzed by Fosgerau and Engelson (2010) and used in this paper is an example of this.

valuation will depend on the shape of the tail of the distribution, so it is important to convey enough information about the "far end" of the distribution to the respondent.

These two arguments have recently led to a discussion among researchers about the possibility to replace the current practice of how valuations of travel time variability are estimated. Rather than estimating marginal valuations in a reduced-form expression directly, one should perhaps estimate a scheduling model first, and then *derive* a reduced-form valuation that can be used for appraisal, *given* the relevant travel time distribution in each applied context (see e.g. Bates, 2009, for a review of these discussions). This solves, in principle, the two problems with the reduced-form approach – its dependence on the specific travel time distribution, and the need to present travel time distributions to respondents – while maintaining its main strength: that one does not need to model travellers' choices explicitly, nor use information about the distribution of preferred arrival times. The possibility to stringently derive a reduced-form approach from a scheduling model for any given travel time distribution was made possible by a groundbreaking paper by Fosgerau and Karlström (2010). This was followed up by Fosgerau and Engelson (2010), who give an analogous derivation using a different scheduling model.

Testing this transferability between the two approaches is one of the main purposes of this paper. We have collected stated choice data where respondents answer two stated choice experiments, one for estimating a scheduling model and one for estimating a reduced-form model. Theoretically, the two models should be possible to "translate" into each other: the estimated parameters of the scheduling model imply specific values of a reduced-form function, which should then, in principle, coincide with the parameters estimated directly in the second stated choice experiment. *If* that is indeed the case, then this opens up a way to circumvent the problems with the reduced-form approach. On the other hand, if the two estimated models are *not* consistent, then this means that something is missing from the underlying derivation of the reduced-form expression, and one needs to understand why and how our theory deviates from how travellers actually make travel choices.

The possibility that scheduling models might not be able to capture the entire disutility of travel time uncertainty has been observed before. Noland et al. (1998) hypothesize that the pure nuisance of not being able to plan activities precisely could, in addition to scheduling costs, play a role in understanding aversion to travel time uncertainty. They denote this possible additional cost a "planning cost". However, they do not investigate the precise relationship between "scheduling" and "uncertainty" costs, primarily because the necessary theoretical framework was not available at the time.

We estimate two different scheduling models, one introduced by Vickrey (1969) and subsequently Small (1982) (called the "step" model since the marginal utility of being at the destination has a "step" at the preferred arrival time), and one introduced by Vickrey (1973) and subsequently Tseng and Verhoef (2008) (called the "slope" model since the marginal utilities of being at the origin and destination, respectively, change linearly over time). These two scheduling models imply different reduced-form expressions: the “step” model results in an expression including the standard devation, as shown by Fosgerau and Karlström (2010), while the “slope” model gives an expression including the variance, as shown by Fosgerau and Engelson (2010).

For model estimation, we use two different sets of binary stated choice experiments. One set of choices is designed to estimate scheduling models and the other set was deigned to estimate reduced form models. The stated choice experiment is related to an observed public transit trip.

The question of consistency between scheduling and reduced-form models goes beyond just the idea to derive valuations from an estimated scheduling model. Historically, different measures of travel time variability were introduced for appraisal purposes in a rather ad-hoc manner, without any proper theoretical motivation. The theoretical motivation, that variability measures such as standard deviation or variance can be formally derived from scheduling models, is a fairly recent insight[2]. However, to our knowledge there have been no investigations into whether the two approaches are really “empirically consistent”, i.e. whether the valuation implied by a certain scheduling model coincides with the valuation obtained from the corresponding reduced-form model. If not, then this is a signal that either model captures something that the other does not, and it may be important to understand what this might be.

In addition to the investigation of the consistency between scheduling and reduced-form approaches, we also present valuations of travel time variability for use in applied appraisal of public transport investments and policy measures, and we analyse what factors affect travellers’ valuations of delays.

The paper is organized as follows. Section 2 is devoted to explaining the relation between the scheduling and reduced-form approaches more precisely, hence making the research question more precise. In section 3, we present the survey data. The main estimation results are presented in section 4, followed by a discussion of their implications in section 5. Section 6 concludes.

2 THEORY

We referred earlier to two approaches to valuation of travel time variation: the “scheduling approach” and the “reduced-form approach”. In our analysis, we draw on two particular scheduling models, each with its own assumptions about what conditions and constraints travellers are facing. For each of these, we use recent theoretical findings that connect these to the corresponding reduced-form expressions. We refer to the two scheduling models as the “step model” and the “slope model”, in reference to the shape of the marginal utilities of spending time at the origin and the destination.

[2] The way theory has trailed application here is rather similar to the way the widely used “gravity models” introduced were theoretically motivated only after a long period of practical application, through the work by Wilson (1967, 1970), Snickars and Weibull (1977) and several others.

For both the step model and the slope model we apply a schedule-based and a reduced-form approach. A key difference is that in the schedule-based model, we specify exactly what departure and arrival times the traveller will experience in each alternative, while in the reduced-form version, we give the traveller information about the random distribution, and then assume that the resulting choice is based on an optimal choice of departure time.

2.1 The "Step Model"

The first scheduling model draws from Vickrey (1969) and Small (1982), as adapted by Fosgerau and Karlström (2010). Assume that the traveller faces the problem of choosing the optimal departure time from the origin, given that there is some preferred arrival time (PAT) at the destination. Let the PAT be 0, $-D$ the departure time and T the travel time. Assume that the traveller has a utility function of the form:

$$u = \alpha D + \omega T + \beta(T - D)^{+} + \lambda c \qquad (1.)$$

where the notation $(x)^{+}$ denotes the positive part of x, i.e. $\max(0, x)$. Hence, the traveller has a marginal utility of α of being at the origin, a marginal utility of ω of being on trip, a marginal utility normalized to 0 of being at the destination before the PAT, and a marginal utility of β of being at the destination after the PAT[3]. The travel cost c has a marginal utility of λ.

We refer to this model as the "step" model, and we demonstrate it graphically in Figure 1. Marginal utilities for the activities at the origin and destination are constant, with the exception that the marginal utility of being at the destination changes from 0 to β at time 0 (the PAT). In the example shown, the headstart D is greater than the travel time T, so the traveller arrives before the PAT. The figure is drawn assuming that $\omega<0<\alpha<\beta$, which may not necessarily be the case. The shaded area is the achieved total utility.

[3] One of these four marginal utilities has to be normalized: we have chosen to normalize time at the destination before the PAT to 0. Other authors have chosen other normalizations, but this does not change the model.

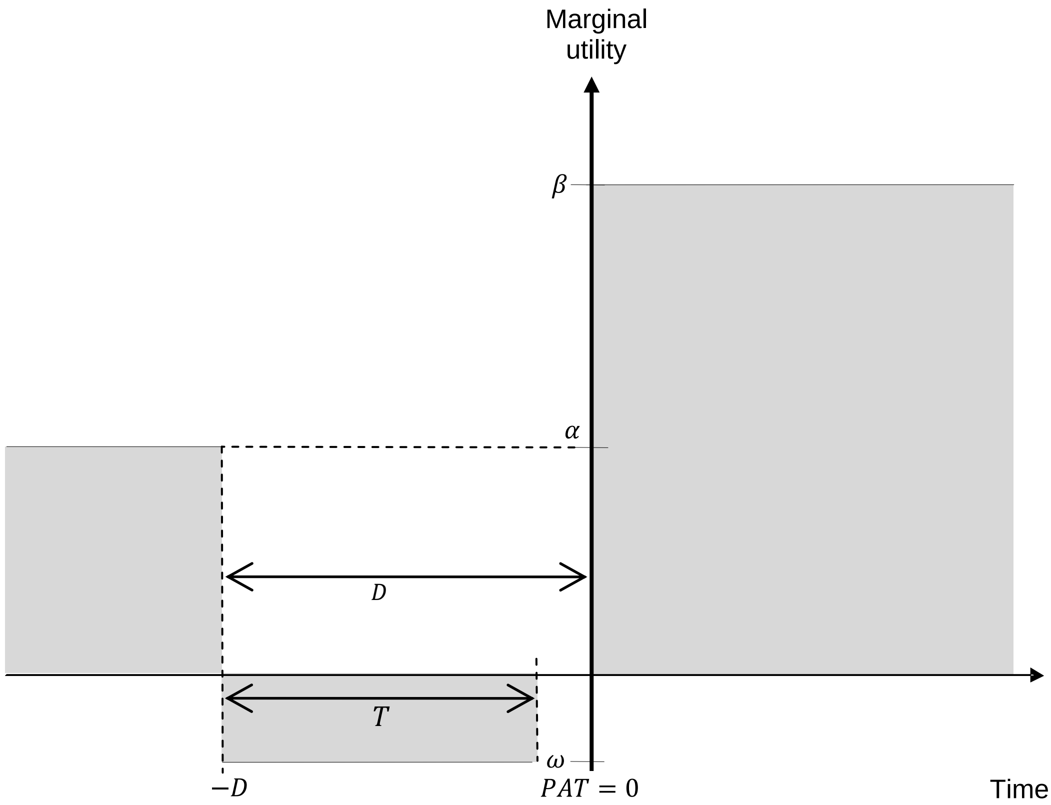

Figure 1. The "Step" Model of Scheduling Preferences

In the case of random travel time, assume that T is distributed as $T \sim \mu + \sigma X$, where X is a standardized random variable with mean 0, standard deviation 1, density ϕ and cumulative distribution Φ. If the traveller can choose head-start D freely and the distribution of T is independent of D, then it can be shown (Fosgerau and Karlström, 2010) that the optimal head-start D^* is:

$$D^* = \mu + \sigma\Phi^{-1}(1 - \alpha/\beta), \tag{2.}$$

and the expected optimal utility is:

$$u^* = E[\max_D\{\alpha D + \omega T + \beta(T-D)^+ + \lambda c\}]$$
$$= (\alpha+\omega)\mu + \beta\sigma \int_{1-\frac{\alpha}{\beta}}^{1} \Phi^{-1}(s)ds + \lambda c \equiv (\alpha+\omega)\mu + \beta H\left(\Phi, \frac{\alpha}{\beta}\right)\sigma + \lambda c \tag{3.}$$

It is the last formula that provides the bridge from the scheduling model (1) to the reduced-form expression (3.). Given a set of scheduling parameters (α, β, ω) and a standardized travel time distribution Φ, the resulting reduced-form expression is the so-called "mean-variance" expression (a slightly misleading name, since it is the standard deviation, not the variance, that enters the expression). Note that the valuation of the standard deviation will depend not only on the parameters β and α, but also on the standardized distribution Φ (more specifically on the tail of Φ^{-1} beyond $1 - \alpha/\beta$).

Estimating the Step Model

The estimated "step" scheduling model is specified in (1). Respondents are asked to choose between two alternative trips with different departure, arrival and travel times, which allows us to estimate the parameters of (1) directly.

The corresponding reduced-form model corresponds to (3), and is simply a standard mean-variance expression. We estimate the parameters (θ_1, θ_2) in the utility function:

$$u^* = \theta_1 \mu + \theta_2 \sigma + \lambda c. \tag{4.}$$

If the step model correctly describes travel behaviour, then we should get $\theta_1 = \alpha + \omega$ and $\theta_2 = \beta H(\Phi, \alpha/\beta)$. One of the main purposes of the paper is to investigate whether this prediction holds, i.e. whether we get the predicted relation between the parameters of a scheduling model and a mean-variance model.

In this paper, we will use a particularly simple travel time distribution, namely the binary distribution where the travel time is t with probability $1-p$ and $t+L$ with probability p. In other words, there is a delay of length L with probability p. In this case, u^* takes the form (see appendix 1):

$$u^* = \begin{cases} (\alpha+\omega)t + (\beta+\omega)pL + \lambda c & \alpha > \beta p \quad \text{(Case I)} \\ (\alpha+\omega)t + (\alpha+\omega p)L + \lambda c & \alpha < \beta p \quad \text{(Case II)} \end{cases}. \tag{5.}$$

For an individual, we see one of these two cases depending on whether $\alpha < \beta p$. In Case I, a risk-taking individual will maximize utility by choosing a later departure, while in Case II, a risk-averse individual will optimize by choosing an earlier departure.

For a population, we estimate the parameters in (5.) by simplifying to a single utility function:

$$u^* = \theta_1 t + \theta_2(p)pL + \lambda c, \tag{6.}$$

To interpret the shape of $\theta_2(p)$, consider the case where β is distributed across the population. For small p, θ_2 will converge towards $\beta + \omega$, while for large p, θ_2 will converge toward an upper bound of $\alpha/p + \omega$. For moderate values of p, the estimated value of θ_2 will depend on what mixture of the population belongs to each of the two cases in (5.). We estimate $\theta_2(p)$ non-parametrically by estimating a dummy parameter for each risk level p.

2.2 The "Slope Model"

Another scheduling model is specified in Vickrey (1973), and has been used by Tseng and Verhoef (2008) and Jenelius et al. (2010). Fosgerau and Engelson (2010) derive the corresponding reduced form. Instead of constant marginal utilities at the origin and destination, marginal utilities are assumed to be affine functions, increasing or decreasing with clock time, while the marginal utility of travel time is normalized to zero. We refer to this model as the “slope” model.

Let β_1 be the rate of change of the marginal utility of spending time at the origin, and γ_1 be the rate of change of the marginal utility of spending time at the destination (see Figure 2). Assume that the marginal utilities intersect at some point (otherwise, the traveller would spend all his time at either the origin or the destination); we normalize the point of intersection to $(0, \beta_0)$. This means that if travel time T was zero, the traveller would depart and arrive at time 0, and at this point in time, the marginal utility of time is β_0.

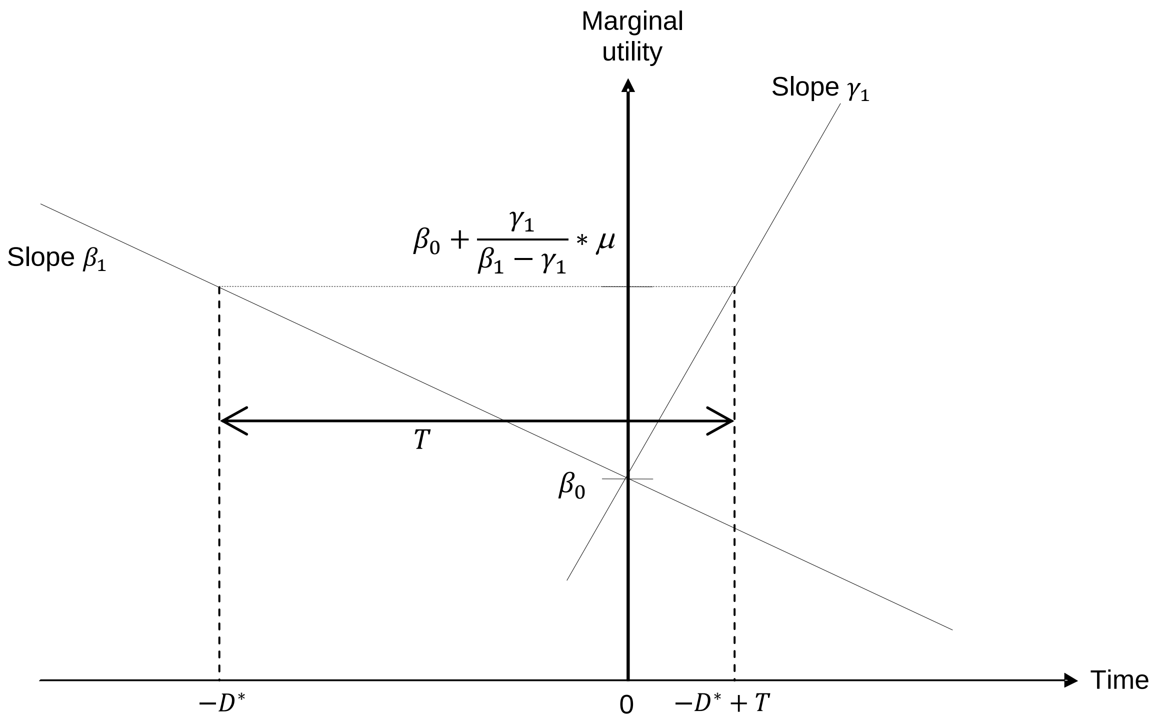

Figure 2. The “Slope”-Model of Scheduling Preferences. Derivation of optimal departure time.

As with the step model, the slope model implies a specific reduced-form expression, as shown by Fosgerau and Engelson (2010). Let T be a random variable with $E(T) = \mu$ and $Var(T) = \sigma^2$. The traveller’s optimal departure time D^* is given by the condition that the marginal utility of time at the origin at the time of departure must be the same as the marginal utility of time at the destination at the time of arrival:

$$\beta_1 D^* = \gamma_1(D^* + \mu) \Rightarrow D^* = \gamma_1 \mu / (\beta_1 - \gamma_1) \quad (7.)$$

Note that this quantity is independent of σ, so the optimal departure time D^* is the same whether T is random or not.

The expected maximal utility, assuming optimal choice of departure time, then becomes:

$$u^* = -\beta_0\mu - \frac{\beta_1\gamma_1}{[2(\beta_1-\gamma_1)]}\mu^2 - \frac{\gamma_1}{2}\sigma^2 + \lambda c \qquad (8.)$$

Note that u^* does not depend on the full distribution of T, only on its mean and variance. The reduced-form expression implied by slope model differs from that of the step model two ways: it is the variance, not the standard deviation, that enters the expression, and the value of travel time is not constant, but increases with travel time (the value of travel time becomes $\beta_0 + \frac{\beta_1\gamma_1}{[2(\beta_1-\gamma_1)]}\mu$).

Estimating the Slope Model

As with the step model, we estimate the slope model using both a schedule-based and a reduced-form approach. Estimating these models are less straight-forward than the step models, however, since the formulation of this model includes the travel time of a reference trip (the observed trip of the respondent). To define estimable models, let d be a deviation from the observed departure time (D), and let a be a deviation from the observed arrival time. Assume that the respondent has chosen arrival and departure time optimally, so the observed travel time equals μ in the slope model expression.

Suppose that d is negative and a is positive; then the traveller loses activity time at both the origin and the destination. The total loss of utility is shown in Figure 3 by the shaded areas. Similarly, if activity time had been gained at the origin or the destination, the equivalent shaded area would represent a gain in utility.

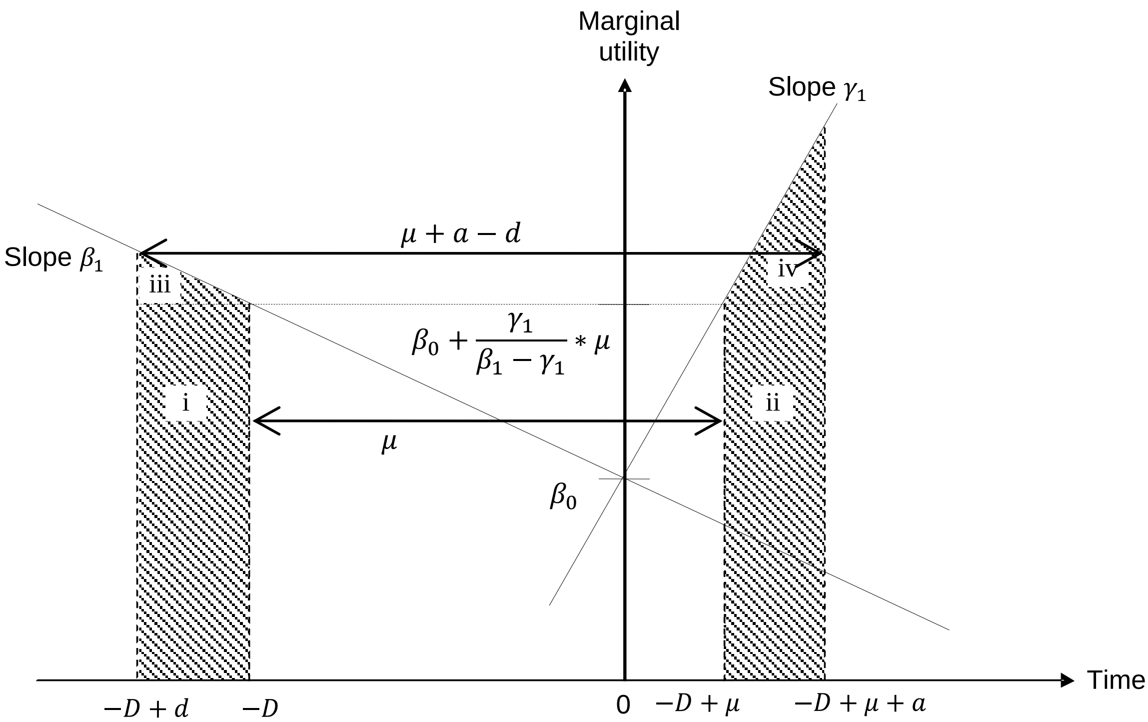

Figure 3. Variations in Utility in the "Slope"-Model

The difference in utility due to these changes in trip schedule compared to the observed trip $(D, D+\mu)$, assuming no change in cost, is:

$$\Delta u \ = \ \left(\beta_0 + \frac{\gamma_1}{\beta_1 - \gamma_1} * \mu\right) * (d+a) + \frac{\beta_1}{2} * d^2 \ + \frac{\gamma_1}{2} * a^2. \tag{9.}$$

The first component corresponds to the rectangular change in total utility shown as areas i and ii in Figure 3; this corresponds to a fixed marginal utility of $(\beta_0 + \gamma_1/(\beta_1 - \gamma_1) * \mu)$. Yet with each additional loss of activity time, the marginal cost increases according to β_1 and γ_1. The last two terms in (9.) account for these additional costs, which are given in Figure 3 as areas iii and iv. Note that (9.) is valid irrespective of the signs of d and a.

When estimating the slope model, the utility function of each alternative in the binary choice experiment becomes:

$$u = (\beta_0 + \beta_2 * \mu) * (d+a) + \frac{\beta_1}{2} * d^2 \ + \frac{\gamma_1}{2} * a^2 + \lambda c. \tag{10.}$$

The parameters β_0, β_1, β_2 and γ_1 can be estimated directly in the scheduling-model approach. Theoretically, we should have $\beta_2 \ = \ \gamma_1/(\beta_1 - \gamma_1)$. However, the estimation of β_2 will rely on inter-individual variation only, since μ only varies across respondents. If β_0 varies between individuals and is correlated with μ, β_2 picks up intra-individual variation in β_0. The utility of each alternative u does not reflect the absolute utility level in this model, but rather a utility difference as compared to the observed trip. This has no effect on the estimation, however, since (10.) is linear in μ, and only the utility difference between the alternatives in each binary choice is relevant.

Just as before, we will use a binary travel time distribution where a delay of length L occurs with probability p. This implies $\mu = t + pL$ and $\sigma^2 = p(1-p)L^2$, and the reduced-form expression (8.) becomes

$$u \ = \ \theta_1(t+pL) + \ \theta_2(t+pL)^2 \ + \ \theta_3 p(1-p)L^2 + \lambda c, \tag{11.}$$

After estimating the three θ-parameters, we can compare these to the estimated parameters from the scheduling model. Theoretically, we should get $\theta_1 = \ -\beta_0$, $\theta_2 = \beta_2 = -\frac{\beta_1\gamma_1}{[2(\beta_1-\gamma_1)]}$ and $\theta_3 = -\gamma_1/2$.

The value of expected delay will be $\theta_1 + \theta_2(1\text{-}p)L + \theta_3(pL + 2t)$; hence, the value of delay time will depend on p, L and t. Note in particular that the disuility of delay is proportional to the square of delay length. The value of scheduled travel time, θ_1 in the step model, corresponds to $\theta_1 + 2\theta_3 pL$ in the slope model, which depends on the average delay.

3 THE DATA

3.1 Survey method

The data originate from a stated choice study administered to travellers on the metro and commuter trains going toward and from the centre of Stockholm city during the morning and afternoon peak periods. The respondents were recruited at the stations Monday-Thursday 7-9 am and 4-6 pm during one week in October 2009. The respondents received questionnaires along with a pen and a stamped envelope. They were asked to fill in the questionnaire on the journey or shortly afterwards and to mail-back the survey. In total, 3200 questionnaires were distributed, and the final data set comprised 1260 respondents, giving a response frequency of 39%.

The questionnaire first listed some questions about the observed trip (travel time, start time, constraints at origin/destination, transfers, safety margins, frequency of delays etc.), partly to remind respondents about the circumstances of this particular trip.

The second part of the questionnaire comprised two different sets of binary stated choice experiments related to their actual journey. The first experiment was designed to estimate a scheduling model. The alternatives differed in three dimensions: fare, start time and travel time. As a consequence of the last two variables the alternatives also differed with respect to arrival time. The second choice experiment was designed for estimating a reduced-form model, where the binary choices differed in delay length, delay risk, scheduled travel time (without delays) and fare. The option to cancel the trip was also offered. Examples of the binary choices are shown in Figure 4 and Figure 5. Each questionnaire contained four binary choices from each of the two experiments.

	Departure 1	**Departure 2**
Start Time	**25 min later** than today	**5 min later** than today
Travel Time	**15 min longer** than today	**45 min longer** than today
Ticket Price	**€0.70 higher** than today	**€0.40 lower** than today
Arrival Time	*(40 min later than today)*	*(50 min later than today)*
I choose	☐ 1	☐ 2
	Cancel the trip ☐ 3	

Figure 4: Survey question, choice experiment 1

	Departure 1	**Departure 2**

Delay (if you made this trip every day):	Once **every other month**, the train is **45 min** delayed. All other trips are on-time.	Once **every other week,** the train is **10 min** delayed. All other trips are on-time**.**
Travel time according to the timetable:	**3 min shorter** than today	**10 min shorter** than today
Ticket price	**€0.20 higher** than today	**€1.00 higher** than today
I choose	☐ 1	☐ 2
	Cancel the trip ☐ 3	

Figure 5: Survey question, choice experiment 2

3.2 Experimental Design

An orthogonal pivot design was used. The difference between each of the factors in the binary choice alternatives was constructed using an orthogonal design tables with 16 rows and one column for each factor. Remember that the first experiment included three factors and the second four factors. In both experiments, all factor differences took four levels. The absolute level of each factor facing the respondents took many more levels, but this is irrelevant in the estimation. Simulation over a wide range of model specifications and parameter values, which also included the parameter values that were achieved in the pilot and the main study, was undertaken. This guarantees sufficient efficiency in parameter estimates and that the design retrieves properties of the data assuming different underlying model specifications.

Eight different questionnaires with eight choice situations were constructed. Each of these were also mirrored, such that the left and right hand alternatives were switched, to investigate if there is a greater tendency to choose the left hand alternative. No such tendency was found in the analysis. The choice alternatives are summarized in Table 1.

Table 1: Summary of the stated choice design. Departure, travel, arrival and travel time is given in [min] and fare in [€][4]

	Min.	**Mean**	**Max.**
Departure time, exp. 1	-45	-0.2	47
Travel time, exp. 1	-6	9	45
Fare, exp. 1 [€]	-1	-0.1	1.5
Arrival time, exp. 1	-49	9	50
Travel time, exp. 2	-10	3.5	22
Fare, exp. 2 [€]	-0.5	0.4	1.2
Delay length, exp. 2	5	25	70
Delay risk, exp. 2	0.025	0.08	0.2

[4] In this table and throughout the paper we have converted SEK to Euro using an approximate conversation rate of 10 SEK/€.

3.3 Sample Statistics

Table 2 and Table 3 summarize the descriptive statistics of the sample. The purpose at the destination was ‘work’ for 61 percent of the respondents and ‘home’ for 17 percent. The remaining 22 percent had some other purpose at the destination. 49 percent state that they had a time constraint at the destination. The mean door-to-door travel time was 46 minutes, and respondents having a constraint at the destination had a safety margin of on average 15 minutes.

Table 4 shows experiences of frequency of delays stated by respondents who make regular trips. The tabulated frequencies correspond to the levels of risk of delay included in the stated choice experiment, which according to the table, are realistic. Table 4 shows also that about half of the respondents do not have a connection from the subway/commuting train to reach their final destination. The vast majority of the respondents having a connection have a connection with high frequency.

Table 2: Sample statistics for categorical variables.

Shares of the total sample population		**Shares of purposes at destination**	
Travellers recruited on the subway	0.45	Work	0.61
Travellers with constraint at the origin	0.20	School	0.06
Travellers with constraint at the destination	0.49	Business	0.02
Women	0.65	Shopping	0.03
Children <13 years in the household	0.29	Recreation	0.03
Employed	0.81	Other	0.08
		Home trip	0.17

Table 3: Sample statistics for continuous variables.

	Min.	**1st**	**Mean**	**3rd**	**Max.**	**NA's**
Door-to-door travel time? [min]	0	30	46	55	580	39
Safety margin, travellers with constraint at destination [min]	0	10	15	20	100	703
Age [years]	18	34	44	55	87	69
Monthly income before tax [€]	500	2250	2850	3750	7000	7700

Table 4: Stated frequency of delays and headway of connection to final destination. Shares of the total sample.

Stated frequency of delays for regular trips			**Headway of connection**	
	Subway	Commuting train		
Once per week	0.14	0.19	< 5 min	0.17
Every other week	0.13	0.15	10 min	0.15
Once per month	0.18	0.26	15 min	0.13
Every other month / less	0.30	0.25	20 min	0.04
Not a regular trip	0.26	0.15	1 hour or more	0.02
			Don’t know	0.03
			Don’t take a connection	0.45
Sum	1.00	1.00		1.00

4 ESTIMATION RESULTS

In this section, estimation results are presented for the two scheduling models and the two corresponding reduced-form models. As explained in Section 2, we denote the two scheduling models the “step model” and the “slope model”. We estimate binary logit models. The left-hand alternative is chosen if:

$$u_L - u_R + \varepsilon \geq 0$$

where ε is a logistic error term, and u_L and u_r are the measurable utilities of the left- and right-hand alternatives.

4.1 Experiment 1: Scheduling models

The step model specification was given in (1.). In our estimation, the preferred arrival time was taken to be the actual arrival time of the respondent. Respondents were also asked whether they had to be in time for anything specific at the destination, and if so, how much margin they had before that time. However, replacing the actual arrival time with this time yielded considerably worse estimation results.

To estimate the “slope” model, we use (10.). Remember that we assumed that the observed departure time is the optimal departure time D so that observed arrival time is $D + \mu$.

Estimation results for the two models are found in Table 5.

Table 5: Estimation Results Using a Scheduling Approach

	Step Model		**Slope Model**	
	Value	*t-stat*	Value	*t-stat*
λ	-0.0805	*-14.5*	-0.0962	*-16.1*
α	-0.0631	*-17.4*		
ω	-0.0267	*-4.4*		
β	-0.122	*-18.1*		
β_0			-0.115	*-18.9*
β_2			-0.000007	*-0.4*
$\beta_1/2$			-0.00121	*-11.7*
$\gamma_1/2$			-0.00028	*-2.3*
Observations	3413		3413	
Final log(L)	-1768.2		-1719.4	
D.O.F.	4		5	
Rho²(0)	0.253		0.273	
Rho²(c)	0.253		0.273	
All values below are evaluated at a 15 min. change and expressed in €/hour[5]				
Value of earlier departure time	$(\alpha+\omega)/\lambda$ = 6.7		$(\beta_0+15\beta_1/2)/\lambda$ = 8.3	
Value of later departure time	$(\alpha+\omega)/\lambda$ = 6.7		$(\beta_0-15\beta_1/2)/\lambda$ = 6.1	
Value of earlier arrival time	ω/λ = 2.0		$(\beta_0-15\gamma_1/2)/\lambda$ = 6.9	
Value of later arrival time	$(\beta+\omega)/\lambda$ = 11.1		$(\beta_0+15\gamma_1/2)/\lambda$ = 7.5	

All important parameters are significant and have the expected sign. The slope model fits the data better than the step model. The models' common cost parameter is similar in size in the two models.

There is no evidence that the value of time increases with travel time – the parameter β_2 is not significant, and the equality $\beta_2 = \gamma_1/(\beta_1 - \gamma_1)$, predicted by the theoretical model, does not hold empirically. The estimation of β_2 relies on inter-individual variation only, since μ only varies across respondents, not across each binary stated choices. If β_0 varies between individuals and is correlated with μ, β_2 picks up intra individual variation in β_0. If, for instance, travellers with lower β_0 had longer travel times, then the impact of travel time on the marginal utility of time would be cancelled out. This could be a reason for the insignificance of β_2.

The values of scheduling time in the models are compared in Table 5 and Figure 6. The step model gives a lower value of early arrival and higher value of late arrival, compared to the slope model. This is mainly due to the

[5] The cost parameters have units in SEK, but the values of time are converted to € assuming 1 € = 10 SEK.

restriction of constant marginal utility of time at the origin in the step model, whereas the slope model shows that the marginal utility of time at origin is larger before than after the observed departure time. The step model's restriction in marginal utility of time at the origin affects the other estimates, since early departure is correlated with early arrival and late departure is correlated with late arrival. Since the disutility of early departure is underestimated by the step model, so is the utility of early arrival; since the utility of late departure is overestimated, so is the disutility of late arrival arriving late. In fact, according to the slope model, the marginal disutility of early departure is in fact higher than the marginal disutility of late arrival.

Interestingly, we could not find any differences in scheduling parameters between respondents with constraints at origin/destinations. We could interpret this finding as that most individuals have constraints to a varying degree, but that this is not always obvious to them if this is rigid 'constraint' or not, which would strengthen the support for the slope model.

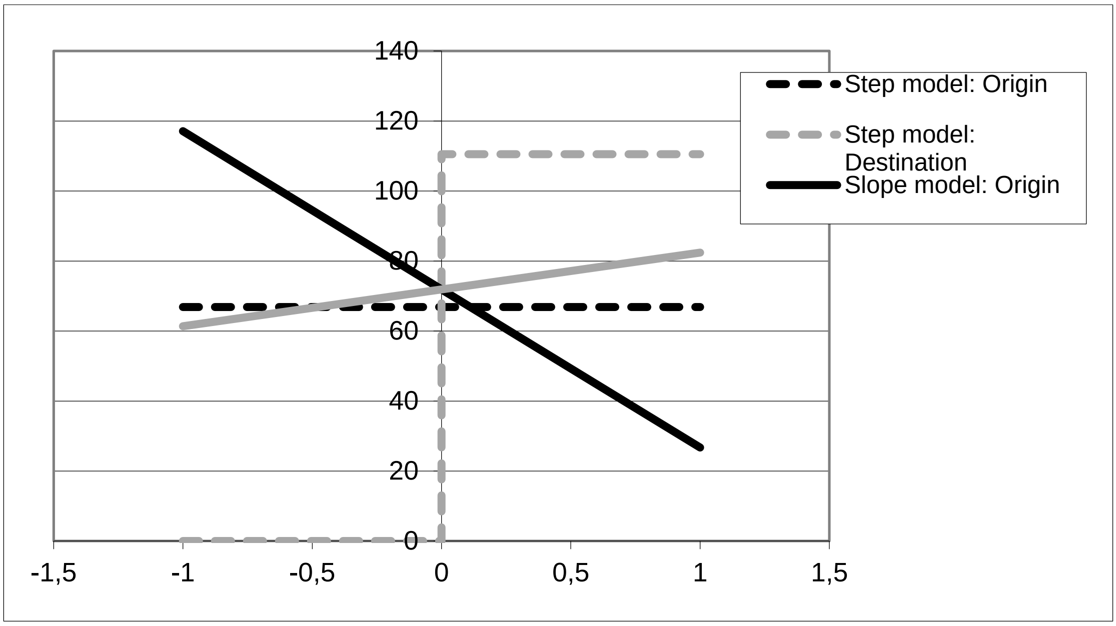

Figure 6. Comparison of Estimated Values of Time at the Origin and Destination in the Step and Slope Models

4.2 Experiment 2: Reduced-form models

As explained in Section 2, the two scheduling models imply different reduced-form model defined in (6.) and (11.). Estimation results are found in Table 6.

Table 6: Estimation Results Using a Reduced-Form Approach

	Step Model		**Slope Model**	
	Value	*t-stat*	Value	*t-stat*
λ	-0.112	*-7.4*	-0.143	*-13.0*
θ_1	-0.111	*-5.8*	-0.119	*-8.4*
θ_2			-0.0086	*-5.3*
θ_2(p=.025)	-0.742	*-5.3*		

θ_2(p=.05)	-0.595	*-6.6*		
θ_2(p=.10)	-0.648	*-7.5*		
θ_2(p=.20)	-0.547	*-8.8*		
θ_3			-0.00298	*-9.3*
Observations	3129		3172	
Final log(L)	-1863.79		-1863.57	
D.O.F.	6		4	
Rho²(0)	0.1407		0.1524	
Rho²(c)	0.1406		0.1524	
Value of scheduled travel time (€/h)	6.0		5.5	*p=0.08, L=25 (design means)*
Value of delay time (€/h)				*L=25, T=46 (design and sample means)*
p=.025	39.8		25.3	
p=.05	31.9		25.3	
p=.10	34.8		24.9	
p=.20	29.4		24.3	

By definition, the step model predicts that the disutility of a possible delay of length L should be linear in L. Intuitively, this might be unexpected – but in fact, this is supported by the estimation results. Trying various piecewise linear functions, we find no evidence that the marginal value of a “delay minute” changes with L.

The two reduced-form models have similar goodness-of-fit, although the slope model has fewer parameters. The values of travel time are very similar. The θ_2 dummy parameters in the step model get the theoretically expected relative magnitude (higher for higher risk levels), except for θ_2 and θ_3, but the difference between these two parameters is not significant.

The slope model produces lower values of average delay time than the step model, especially for high risk levels. The relatively low value of average delay in the slope model arises from the fact that the coefficient for pL is restricted to equal the coefficient for t in this model. According to the step model, the parameters for pL are considerably larger than the parameter for t. The third term in the utility function of the slope model in (11.) also includes both pL and t and cannot increase the relative weight on pL as compared to t. The fourth term includes the square of L; the finding that the value of expected delay is linear in L, according to the step model, explains why this parameter also becomes relatively small. Still, at larger delay lengths, L, and travel times, t, the third and fourth terms of the utility function will increase the value of average delay in the slope model. For instance, at $L = 35$ and $t = 60$ the value of average delay in the slope model will lie in the same range as the step model (31-32 €/h, depending on the risk level).

Just as in the scheduling models, the valuation of average delay is not different for respondents with arrival or departure time constraints.

Note that in the step model, we must have $\theta_1 \leq p\theta_3(p)$ for theoretical reasons[6], with the "gap" getting smaller for increasing risk p. If the difference in direct utility between travelling and being at the destination before the PAT (ω in the scheduling model) is negligible, then equality is possible. It turns out that the estimated model passes this consistency test, with equality obtained for $p = 0.20$. This means that it is, in fact, theoretically possible to extrapolate the value of delay for higher risk levels, since $\theta_3(p)$ must be equal to θ_1/p – if we trust the theoretical model, naturally.

4.3 Comparison of scheduling and reduced-form models

With the estimation results above, it is now possible to compare the parameters of the estimated reduced-form models with those of the scheduling models. The comparison is shown in Table 7.

Table 7: Parameter Comparisons Between Scheduling and Reduced-Form Specifications

	Step Model		Slope Model	
Theoretical prediction	Scheduling specification	Reduced-form specification	Scheduling specification	Reduced-form specification
Value of travel time: $(\alpha + \omega) \sim \theta_1$	$\frac{\alpha + \omega}{\lambda} = 6.7$	$\theta_1/\lambda = 6.0$		
Value of expected delay: $(\beta + \omega) \sim \theta_2(p)(for\ small\ p)$	$(\beta + \omega)/\lambda = 11.1$	$\theta_2(p = 0.025)/\lambda = 39.8$		
Baseline value of time: $\beta_0 \sim \theta_1$			$\frac{\beta_0}{\lambda} = 7.2$	$\theta_1/\lambda = 5.0$
Value of mean travel time squared: $\frac{\beta_1\gamma_1}{[2(\beta 1-\gamma 1)]} \sim \theta_3$			$\frac{\beta_1\gamma_1}{[2(\beta_1 - \gamma_1)]}/\lambda = 0.014$	$\frac{\theta_3}{\lambda} = 0.12$
Slope of destination marginal utility = value of travel time variance: $\gamma_1/2 \sim \theta_2$			$\frac{\frac{\gamma_1}{2}}{\lambda} = 0.02$	$\frac{\theta_2}{\lambda} = 0.36$

Evidently, the reduced-form models give much higher valuations of delays. While valuations of travel time are comparable in size (although it differs quite considerably in the slope model), delay valuations differ by a factor 4 and 20, respectively.

4.4 Other remarks on estimation results

From the extensive testing of model specification that was carried out, the following observations can be noted:

[6] This follows from the condition that $\alpha < \beta p$ in the derivation of the reduced-form expression.

- There are no significant differences in scheduling parameters between morning and afternoon trips, or between different trip purposes.
- There are only small differences in preferences between metro and commuter train travellers. The only significant difference is that commuting train travellers value time at origin higher. A likely reason for this is that commuting trains are more crowded; the traveller's door-to-door travel times do not differ between the models. This difference is only significant in the first experiment. This was not included in the final model, however.
- People who stated that they had constraints on their arrival or departure time did in fact not have significantly larger scheduling parameters than the rest of the respondents. Nor did people with constraint on their arrival have a higher valuation of expected delay.
- The model using the observed arrival time as preferred arrival times give better model fit and response scale than models adjusting the arrival time with respondents' stated "safety margin" (if any).
- Having a connecting trip at the end of the recruitment trip did not affect valuations.
- Higher income attenuates the cost parameter.
- Once income differences are controlled for, no gender differences are found.
- Parents, both men and women, with children 12 years or younger have a higher valuation of time but not a higher valuation of expected delay or larger scheduling parameters. A reason might be that there are very few trips with purpose "fetch the children from day-care" in the sample.
- Employed persons have a higher valuation of lateness.
- The experiment also contained an "abstain from the trip" alternative. The experiments were also estimated as nested models with the choice of whether to accept any of the alternatives at the upper level and the choice between the right-hand and left-hand side at the lower level. This did not change parameter estimates for the scheduling model, while it almost doubled the value of delay time in the reduced-form models. This seemed to be because some respondents abstained from the trip once there were high delay risks. Due to a number of internal consistency problems with these model specifications, "abstain" answers were eventually dropped from the estimations.
- There is no significant constant for the left-hand alternative in any of the experiments. Hence, no tendency was found that respondents "cheated" on a difficult by task by always selecting the same side.

4.5 Mixed logit models

Mixed logit (MXL) models have been estimated, to ensure that the results are robust and to take panel effects into account. In the first MXL model of the first experiment, the β, ω, α, and γ parameters were assumed to be normally distributed. The valuations, computed using the mean values of the parameters, remained robust as compared to the MNL model. There was no significant variation between individuals in the α parameter. The largest random variation was found in the β parameter.

Assuming normally distributed parameters, fairly large parts of the mass of the distribution of the parameters were positive. To remedy this problem, the parameters were assumed symmetrically triangular and constrained to be negative. This model, on the other hand, turned out unsatisfactory, resulting in very high valuations. Presumably, there is a significant mass close to zero, with are better accounted for by using the normal distribution. The option of using log-normally distributed parameters was not tested, since the valuations are normally sensitive to assumptions about the long and flat tail where the data do not cover the support of the distribution.

Mixed logit models estimated for experiment 2 give the same results as for experiment 1. The heterogeneity is largest for the expected delay parameters. The valuations are robust assuming normally distributed parameters. The triangular distributions give very high valuations that would be due to a misspecification.

5 DISCUSSION

Comparing the results of Experiment 1 and Experiment 2, it is apparent that they are inconsistent, in the sense that the estimated scheduling parameters in Experiment 1 cannot explain the value put on travel time variability in Experiment 2. In the "step" model, the valuation of travel time variability in Experiment 2 is almost four times higher than implied by the parameters of Experiment 1. In the "slope" model, the difference is even larger.

Clearly, the disutility of being subject to a risk of a delay exceeds what the scheduling models predict that the disutility should be. There seems to be some disutility associated with the uncertainty in Experiment 2, over and above the mere disutility of "late arrival" that is captured by the scheduling model in Experiment 2 – where the arrival time is certain. In other words, being "delayed" seems to be worse than just being "late", i.e. to arrive after one's preferred arrival time.

Interestingly, the respondents do in fact answer the two experiments consistently in a specific sense: the respondents that have the highest disutility of late arrival in Experiment 1 are also the ones with the highest disutility of delay risk in Experiment 2. This was established using a bootstrap estimation, where the valuations of repeated sub-samples were estimated. This gave a correlation of 0.25 between the valuation of late arrival in Experiment 1 and the valuation of average delay in Experiment 2. On the other hand, no correlations were found for the cost or the (scheduled) travel time parameters. Apparently, the individual-specific variation in these parameters is swamped by random variation. This is consistent with the finding from the mixed-logit estimations, that the variations in the cost and time parameters are small compared to the variation in the value of expected delay.

So, it seems as if the discrepancy between the scheduling and reduced-form models cannot be explained merely by randomness. Instead, there may be several possible explanations of this phenomenon.

Timing of information about the actual arrival time

One of the differences between the two experiments is the point in time at which the traveller gets information about her actual arrival time. In experiment 1, the traveller is given her arrival time in advance, and hence knows in advance whether she will be late with respect to the preferred arrival time (PAT). In experiment 2, only the probability and length of a delay is given, so the traveller does not know if she will actually be delayed when making a choice – in other words, information about the actual arrival time comes later than in experiment 1. Under the theory of the scheduling model, this does not matter: the disutility of arriving after the PAT only depends on how late the traveller arrives – not how long in advance the actual travel time is known.

In reality, it is often the case that given ample warning, travellers can respond by adjusting their plans, thereby minimizing any lateness penalty they may incur by arriving late. In experiment 1, the traveller is told that she will arrive after the PAT, and she can take measures accordingly, such as rescheduling activities in advance. In experiment 2, if a delay actually occurs, activities have to be rescheduled with much shorter notice, if at all possible. Expressed in the terminology of the scheduling model, this means that the lateness penalty depends on how long in advance the traveller knows that he or she will be late. If information about the actual arrival time comes "early" (as in experiment 1), the lateness penalty will be lower than if the information comes "late" (as, potentially, in experiment 2). Put somewhat differently, "rescheduling costs" are bigger the later information comes.

Another way of putting it is that the notion of an exogenously fixed preferred arrival time may be untenable. For many travellers, the PAT may in fact be a wide interval, in which all arrival times are equally preferred – as long, that is, the traveller knows in advance when she will arrive, and can plan accordingly. If this is the case, the "*preferred*" arrival time will be a function of when the traveller *expects* to arrive.

A natural question is then whether there is any way to present experiment 1 in such a way that the traveller does not get informed about the arrival time in advance (and thus can adjust her PAT). However, this is virtually impossible: one would have to ask respondents to rank alternatives that are essentially variants of "you would be delayed X minutes *without knowing it in advance*" – and at the same time, we want to assume that travellers choose the optimal departure time given an *unknown* travel time (knowing just its distribution). This is obviously not realistic. It is simply not possible to ask respondents whether they would prefer A or B – pretending at the same time that they would not know whether they had "chosen" A or B!

The disutility of uncertainty *per se*

In the scheduling model, there is no direct disutility of travel time uncertainty *per se*. Disutility only arises from the risk of arriving late (and hence the need to start the trip earlier than otherwise). But due to the higher parameters estimated parameters in Experiment 2, it seems as if many people dislike uncertainty *as such*. This dislike can either be interpreted as an “anxiety cost”, or as the “decision cost” needed to solve the optimal departure time problem. From other sources, it is well established that information about the length of a delay is valued by travellers. This may be because it facilitates rescheduling of activities, but it may also be that the uncertainty in itself is a concern for many travellers.

Another interpretation is that uncertainty creates a need for “contingency plans”, if the traveller should be late. The need to arrange a contingency plan, should one be delayed, is a cost created by the uncertainty in itself, over and above the mere value of the (expected) delay.

In practice, anxiety costs, decisions costs and contingency planning costs are virtually inseparable from each other, Separating them is not necessary, on the other hand: what is important is that they all may cause a disutility exceeding the value of the actual, average delay.

Policy bias and focus bias

In all stated preference experiments, there is a risk that valuations are overestimated due to policy bias or focus bias. “Policy bias” refers to respondents’ tendency to answer in a way that they think will cause certain desirable effects – for example overstating their valuation of delays to make the transit authority try harder to decrease delays. “Focus bias” refers to respondents’ tendency to forget other important characteristics of a trip, and hence unconsciously overstate their valuation of a certain aspect – in this case delays. Both these phenomena may be present in experiment 2, since this dealt with delay risks explicitly.

6 CONCLUSIONS

The standard way to include the cost of travel time variability in appraisal is to estimate the parameters of a reduced-form expression including some measure of travel time variability – usually the standard deviation or the average delay. Different assumptions about the underlying scheduling model result in different forms of the reduced-form expression. The problem with such reduced-form approaches is that the valuation of variability will usually depend on the standardized travel time distribution. Since this distribution is likely to vary between contexts, a valuation estimated in one context cannot (consistently) be applied in another context.

One way to overcome this problem would be to estimate a scheduling model, and then derive the valuation in the reduced-form expression. This would also overcome the need to present travel time distributions to respondents in stated choice experiments, which has turned out to be a difficult pedagogical task.

This approach has been tested in the present paper through comparing the results from two scheduling models and the corresponding reduced-form expressions. It turns out that the valuations in the reduced-form models are much higher than the valuations implied by the scheduling models. The differences are so large that one may question the validity of the approach of motivating the standard reduced-form models from underlying scheduling models of the usual type. It seems unlikely that the valuations from the two models can be reconciled by mere tweaks of the models.

Instead, we are inclined to believe that the basic assumptions used when deriving a reduced-form model from an underlying scheduling model are not realistic – they do not capture enough of the essential features of the choice situation that travellers face. We hypothesize that there are two factors that are important to take into account.

First, the parameters of scheduling models, as they are usually interpreted and estimated, refer to a situation where the traveller gets information about his or her actual arrival time in advance (since respondents need to be aware of the attributes of the alternatives to make stated choice questions meaningful). This means that activities can be rescheduled to some extent, if necessary, to fit the actual arrival time. In contrast, when the travel time is random, the actual arrival time is not known until immediately before the arrival time. This makes rescheduling much more onerous, if possible at all, and hence the "lateness penalty" will be higher. Phrased differently, the parameters of the scheduling model, including the preferred arrival time, may depend on when traveller gets information about the actual arrival time.

Second, there might be a disutility connected to the uncertainty of the travel time *per se*. This disutility can either be interpreted as an "anxiety" cost, as a "decision cost" to solve the optimal departure time problem, or as a cost for making up contingency plans for the event that a delay occurs.

In a historical perspective, this is a half step backwards. Reduced-form expressions, where some measure of travel time variability was introduced in an indirect utility function, were originally introduced and motivated in a rather ad-hoc manner. The theoretical underpinning of reduced-form expressions came only later, when it was shown that such models could be derived from scheduling models in a microeconomically consistent way. Our results in this paper, however, cast some doubt on whether this derivation really captures the whole story. It seems as if the derivation of the reduced-form model from a scheduling model omits certain features essential to the understanding, analysis and valuation of travel time variability. Even if this derivation is useful for proving the microeconomic consistency of suggested reduced-form expression, and for showing the theoretical equivalence between scheduling models and reduced-form models, the *empirical* equivalence of these models turns out to be a different matter. It seems as if the "late arrival" concept in scheduling models does not fully capture the "delay risk" central to reduced-form models. Or in other words, it seems as if being “delayed” is considerably worse than just being “late”.

7 ACKNOWLEDGMENTS

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APPENDIX 1: DERIVATION OF THE REDUCED-FORM EXPRESSION FOR A BINARY DISTRIBUTION

Assume that the travel time is t with probability $(1-p)$, and $t+L$ with probability p. The reduced form expression corresponding to the "step" scheduling model can be derived directly, or using Fosgerau & Karlström's (2010) general formula. To derive it directly, note that a traveller departing at time $-D$ before the PAT will get the expected utility:

$$EU = E\big(U(D)\big) = E(\alpha D + \omega T + \beta(T-D)^{+} = \alpha D + \omega(t+pL) + \beta p(t+L-D)^{+} + \beta(1-p)(t-D)^{+}, \tag{12.}$$

where the cost parameters α, β and ω are all negative. Let D^{*} be the optimal departure time and U^{*} the maximal achieved utility, i.e. $D^{*} = \mathrm{argmax}(EU)$ and $U^{*} = \max(EU)$. Assuming $\alpha/\beta < 1$, i.e. that the cost of late arrivals is greater than the cost of early departures, we always have $D^{*} \geq t$ (otherwise, the traveller is always late), so the last term is always zero. Moreover, we always have $D^{*} \leq t+L$, since $\beta < 0$ and $\alpha < 0$. This means that we can rewrite the expression as:

$$EU = (\alpha - \beta p)D + \omega(t - pL) + \beta p(t+L),\ t \leq D \leq t+L \tag{13.}$$

Depending on the sign of $(\alpha - \beta p)$, we get one of two cases:

$$\begin{array}{llll} \alpha - \beta p < 0: & D^{*} = t, & U^{*} = (\alpha+\omega)t + (\beta+\omega)pL & \text{(Case I)} \\ \alpha - \beta p > 0: & D^{*} = t+L, & U^{*} = (\alpha+\omega)t + (\alpha + p\omega)L & \text{(Case II)} \end{array} \tag{14.}$$

In case I, the expected lateness penalty is so small that the traveller will still depart just to be in time if no delay occurs. In case II, the expected lateness penalty is so large that the traveller starts early enough to always be on time.

The same expression can be derived using the general Fosgerau-Karlström formula. Let μ be the mean travel time and σ be the standard deviation of the travel time. For the case of a binary travel time, we have:

$$T = \begin{cases} t & \text{with probability } 1-p \\ t+L & \text{with probability } p \end{cases} \tag{15.}$$

$$\mu = t(1-p) + (t+L)p = t + pL \tag{16.}$$

$$\sigma = L\sqrt{p(1-p)} \tag{17.}$$

To express the reduced-form utility in terms of scheduling parameters, we use the approach presented by Fosgerau & Karlström (2010). We start by taking the standardized distribution of travel times:

$$X = \frac{T-\mu}{\sigma} \tag{18.}$$

$$X = \begin{cases} a & \text{for } 1-p \\ b & \text{for } p \end{cases} \tag{19.}$$

where:

$$a \equiv \frac{t-\mu}{\sigma} = \frac{t-t(1-p)-(t+L)p}{\sigma} = -\frac{pL}{\sigma} \tag{20.}$$
$$b \equiv \frac{t+L-\mu}{\sigma} = \frac{t+L-t(1-p)-(t+L)\,p}{\sigma} = \frac{(1-p)L}{\sigma} \tag{21.}$$

Note that we leave a and b in terms of σ, since the standard deviation will cancel out at later stages. (The same derivation can be made without standardizing the travel time distribution by σ, but we leave it in here for consistency with earlier literature.)

Now, define the cumulative distribution of the standardized travel time distribution as $\Phi(x)$. With the binary distribution given above, this distribution follows a step function:

$$\Phi(x) = \begin{cases} 0 & \text{for } x < a \\ 1-p & \text{for } a \le x < b \\ 1 & \text{for } x \ge b \end{cases} \tag{22.}$$

We invert this to find the quantile function, $\Phi^{-1}(r)$:

$$\Phi^{-1}(r) = \begin{cases} a & \text{for } 0 < r \le 1-p \\ b & \text{for } 1-p < r \le 1 \end{cases} \tag{23.}$$

In the setting used by Fosgerau & Karlström (2010), the optimal departure time, in terms of the mean and standard deviation, is given by:

$$D^* = \mu + \sigma * \ \Phi^{-1}\left(1-\frac{\alpha}{\beta}\right) \tag{24.}$$

Because of the discontinuous nature of the quantile function $\Phi^{-1}(r)$, we can define its value in terms of where in the range $0 < 1-p < 1$ lies the quantity $1-\alpha/\beta$ using this to find the optimal head-start depending on α/β:

Case I $$1-\frac{\alpha}{\beta} < 1-p \rightarrow \frac{\alpha}{\beta} > p \ \rightarrow \Phi^{-1}\left(1-\frac{\alpha}{\beta}\right) = a \tag{25.}$$
$$D^* = \mu + \sigma\, a = t + pL - \ pL = t$$

Case II $$1-\frac{\alpha}{\beta} > 1-p \rightarrow \frac{\alpha}{\beta} < p \ \rightarrow \Phi^{-1}\left(1-\frac{\alpha}{\beta}\right) = b$$
$$D^* = \mu + \sigma\, a = t + pL + (1-p)L = t + L$$

As a consequence of these optimal departure times with respect to the two cases, we can compute disutility using the formula for mean lateness given by Fosgerau & Karlström (2010):

$$H\left(\Phi, \frac{\alpha}{\beta}\right) = \ \int_{1-\frac{\alpha}{\beta}}^{1} \Phi^{-1}(s)ds \tag{26.}$$
$$EU^* = (\alpha+\omega)\mu + \beta H\left(\Phi, \frac{\alpha}{\beta}\right)\sigma \tag{27.}$$

Under Case I, where a risk-taking traveller chooses a later departure and risks being delayed, we have:

$$H\left(\Phi,\frac{\alpha}{\beta}\right) = a\left(\frac{\alpha}{\beta} - p\right) + bp = -\frac{pL}{\sigma}\left(\frac{\alpha}{\beta} - p\right) + \frac{(1-p)L}{\sigma}p = \frac{pL}{\sigma}\left(1 - \frac{\alpha}{\beta}\right) \quad (28.)$$

$$EU^* = (\alpha+\omega)\mu + \beta H\left(\Phi,\frac{\alpha}{\beta}\right)\sigma = (\alpha+\omega)(t+pL) + \beta\frac{pL}{\sigma}\left(1-\frac{\alpha}{\beta}\right)\sigma = (\alpha+\omega)t + (\beta+\omega)pL \quad (29.)$$

The first term has the scheduled travel time weighted by a value of time $(\omega+\alpha)$, and the second term has expected delay weighted by the value of delays $(\omega+\beta)$.

Under Case II, where a risk-averse traveller chooses an earlier departure time and avoids the possibility of delay, we have:

$$H\left(\Phi,\frac{\alpha}{\beta}\right) = b\frac{\alpha}{\beta} = \frac{(1-p)L}{\sigma}\frac{\alpha}{\beta} \quad (30.)$$

$$EU^* = (\alpha+\omega)\mu + \beta H\left(\Phi,\frac{\alpha}{\beta}\right)\sigma = (\alpha+\omega)(t+pL) + \beta\frac{(1-p)L}{\sigma}\frac{\alpha}{\beta}\sigma = (\alpha+\omega)t + (\alpha+p\omega)L \quad (31.)$$

That is, the first term for the scheduled travel time weighted by $(\omega+\alpha)$, and the second term for the delay has mixed weights, where the cost of the certain headway is fully α, but the cost of extra travel time, $p\omega$, is contingent on the delay actually occurring (which has probability p).