A time-space hierarchical framework to model dynamic urban systems

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Abstract

Passenger transport modeling has evolved from the study of aggregate flows between origin to destination zones to increasingly more disaggregate behavioral models. Classical approaches were based on the spatial interaction approach (entropy based models), aggregate networks, random utility methods (logit and probit models) and multi-commodity traffic assignment. More recently, attention has been paid to the temporal process, particularly on the activities and their spatial-temporal sequence under activity-based and micro-simulation models. A common feature of most of these approaches is the combinatorial problem that arises from the complexity associated to the treatment of time and spatial scales, which lead to two different strategies: aggregate approaches- from meso to macro- compromise on the level of detail in order to calculate micro economic equilibrium based models; on the other extreme, simulation approaches compromise on the equilibrium paradigm in order to develop Montecarlo based models with detailed analysis of humans choices.

In this paper, a new paradigm for urban system modeling is proposed, which is based on the structural elements observed on the dynamics of natural complex systems (for example the ecology) that can be used for describing social and economic dynamic systems, generally named as the Panarchy approach[3]. This approach explicitly identifies multiple *hierarchies*, with two predominant examples in the urban system: time and geography, as well as the more classical differences on individuals' behavior. Daily (or weekly) activities for each individual is associated here with a time scale, from long term (e.g. residence location and job choices), to medium term (weekly shopping and leisure), to short term choices (grocery buying and transport mode and route choice). For each temporal scale, an appropriate and subordinate geographical scale is identified. The complex process of sequencing daily choices into a trip trajectory is also conceived here regarding the underlying hierarchical temporal and geographical choice process, combined with micro-economic equilibrium conditions at individual and system levels as well.

Dynamics systems are also subject to shocks whose impact on the system is confined to the relevant temporal and geographical scales. Memory on the choice making process is natural in ecological and human processes along with the non-linear effect introduced by hysteresis (forward trajectories are different to backward ones). Moreover, we introduce the issue of the cost and cognitive limitations of humans in the choice search process.

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The hierarchical structure, together with shocks, memory and human limitations, yield a more realistic as well as simplified choice approach, leading to operational models with less compromises in representing the choice process. In this paper we propose and discuss a set of axioms derived from dynamic systems and microeconomic theory, in order to produce on the one hand, a consistent and interacting time and space choice making and a dynamic equilibrium in the urban system on the other. This approach in modular in that subsystems dynamics, defined by a given range of time scales, can be analyzed consistently and explicitly conditional to other longer term scales. We present the theoretical model and a conclusions derived from its application to activities, trips demand and traffic assignment models.

1. Introduction

The development of appropriate methodologies for modeling urban transport processes is of paramount importance since they are utilized as planning instruments that support decision-making. Models are applied within planning units and they serve for the analysis of effects of infrastructure or transport demand management measures and help to monitor the performance of transportation systems in major cities. To this purpose, models have to deal with complex urban environments and processes on different spatial and temporal scales. In this sense, one would expect that urban transport models are able to handle different challenges: be responsive to changes that influence transport demand on a long-term, like new job opportunities or changes in land-use and to be sensitive to adjustments on the short-term, like daily activity-scheduling in response to e.g. new shopping or leisure opportunities.

Some of the well-known strategic urban transport equilibriums models (such as the 4-step models ESTRAUS, EMME-3, or VISUM) and land use models (MEPLAN, MUSSA, TRANUS), highly simplify the complexity of urban systems in order to represent reality in a single scale for each of the most relevant dimensions: time, space and population clusters. At the same time, they are trip-based and often consider a small number of travel purposes, distinguish not more than three, usually one or two time periods and divide the demand side in a limited number of user classes. Today these models are popular to support strategic decision-making concerning environmental and fare policies as well as projects of road and public transport infrastructure. Despite the simplifying assumptions mentioned, their pay-off is that equilibrium conditions can be imposed, which yields analytical models, accepted level of accuracy on estimates of congestion levels and the framework is consistent with microeconomic theory.

A direction to generate more detailed models in urban land-use has been to develop system simulations (see for example Waddell, 2002; Hunt and Abraham, 2007), which by eliminating equilibrium conditions reduce complexity with the benefit of an increase in modeling details. The gains are particularly on the spatial representation of the system as well as in the disaggregation of agents, maintaining the single scales in geography and time. Further disaggregation of time scales and users, has been introduced in a family of models based on the concept of activity-based analysis (Bhat et al., 2008; Bowman and Ben-Akiva, 2000; Rilett, 2001), which understand travel demand as derived from interdependent activities and trips chains throughout a day. Their main contribution is the increased detail on the chain of activities and destinations and the interdependency among them. Again, the more realism provided by this models is at the cost of departing from the equilibrium paradigm.

Thus, recent models have increased the spatial detail (urban simulators) and the time dependence on activities (activity based) (for an overview of transportation and land-use models under development and in application see Davidson, et al., 2007; Iacono et al., 2008; Buliung and Kanaroglou, 2007). A common problem of these models is the difficulty on handling the combinatorial number of choices and the dependency among them that arise from activities, spatial location, travel options and scheduling activities for all consumers.

However, this technical problem of estimating demand in a complex space of options has a fundamental counterpart on the assumption about consumers' behavior: the classical assumption that consumers have complete information about their options. Although this is plausible in some contexts, like mode choice, in others the assumption is hardly justifiable due to the huge amount of information required to be storage and analyzed, which becomes inconsistent with human capacity and also with the concept of rationality itself, because it is inefficient to expend such an enormous amount of intellectual resources in making every single decision. Such inconsistency has been pointed in several eco-systems (Gunderson and Holling, 2002) leading to the conclusion that a efficient behavior is a hierarchical in nature, particularly referred to choices in geography and time.

In this paper, we develop a methodology and analyze first empirical results aimed at introducing different scales of time in choice making and different scales on geography in the search for spatial options. Scales leads naturally to a hierarchical structure in each these dimensions, time and geography,

Our approach is to design a multi-scale model of the urban system, applying classical concepts of hierarchical processes drawn from the literature of dynamic systems that were originally observed in ecological systems and subsequently extended to economic and social systems (Gunderson and Holling, 2002). In all these systems, the complex combinatorial process as a result of the huge amount of potential individual choices is simplified by means of a hierarchical approach, which describes a natural order of systems dynamics.

The practical benefit of the approach is the efficiency gain in the calculation of demand in an urban context, including engagement in activities and transport decisions, and their sequence, which introduce also more realism in the assumption of human behavior. This proposed approach impact on the way we model the location and travel demand model concerning users, space, time, activities (trip purposes) and travel modes, as well as on sequencing activities and trips. It also impacts the interpretation of the perceived and estimated value of time.

We particularly concentrate on time and spatial choices made on two scales levels, called micro and macro scales, since the extension to multiple scales follows naturally. A third and complementary scale refers to the aggregation of the population, where individual's and families' may be represented as at different aggregation levels. We develop the approach to describe long term decisions, like residential and job location choices, and short term activities, like shopping and errands, including trips performed in a single. Additionally, we introduce the concept of optimal behavior in a multi-scales setting, and the dynamics associated to that behavior.

In the next section we first define a notation for activity and trips chains; then we specify a theoretical and hierarchical decision model based on the microeconomic theory of consumers behavior which we simulate in a theoretical context to compare the effect of the hierarchical approach compared with the classical static equilibrium model. The macro-micro approach is currently under implementation for the case of Santiago City, combining information provided by the existing macro scale models of land-use and transport (MUSSA and ESTRAUS) with detailed information of consumers' choices obtained from Santiago's Travel and Household Survey (EOD).

2. The Hierarchical Model

2.1. The hierarchical structure of choices

A basic assumption of the methodology is that choices for activities, and their related destinations and travel modes, are modeled as they were made hierarchically. Individuals first choose an activity plan, which the set of activities and their order and timing. After that they choose a trip path, including the location of activities and the transport mode. The daily activity plan, including its associated trip path is called the individual's program. The individual is assumed to perform a search and evaluation process for an optimal program. This process is extremely complex for human beings due to the huge amount of options that defines the feasible seat: the combination of ordered activities, locations, transport modes and the allocation of time. Such complexity means also high requirements for advanced computers to simulate such a process. The number of options increases with the number of ways in which time can be allocated –due to its divisible nature- and the set and order of activity plans performed. On the spatial dimension, the number of options increases with the square of the number of location points that describe origin-destinations alternatives; thus, a crucial parameter in the magnitude of the combinatorial set of options is the level of detail that describes the geography, i.e. the zoning system. Additionally, the individual search needs to consider the limited amount of resources available. In order to introduce a rational strategy to reduce complexity, the information can be organized into a hierarchical structure of space and time.

This structure, common to several complex systems both in natural sciences and in social organizations, is a plausible strategy for consumers as long as we assume that alternatives are fundamentally different in their contribution to happiness and/or in their consumption of resources that constrain the consumer's rational choice domain.

The hierarchical structure is adequate to model the differentiated dynamics of subsystems at each temporal scale: at a micro level the speed of change of –secondary- decisions is higher (small time windows) and the amount of resources involved is lower, than those at a macro scale –or primary decisions-, while changes at the macro scale have dominant impacts on the micro scale. For example, a secondary activity, such as daily shopping, takes place in a time window of the order of one hour and choices of their location usually occur in the vicinity and dependent on the location of a primary activity conducted before (like home or work location). Conversely, the work activity generally consumes a large part of the time budget and changes in job locations usually happen in a time window of years in the context of the whole city. An example of the time-hierarchy in a city is shown in the following Table 1, which of course contains assignments of activities to cells subject to analysis in specific contexts.

Table 1: Time hierarchy for activities

TIME-WINDOWS	TIME-UNITS	CONSUMERS	SUPPLIERS	REGULATORS
Very long-term	Centuries	Cultural factors	Cities building and network structure New cities	
Long-term	Decades to Years	Family structure Education Car-Ownership Residence Job	Infrastructure: Buildings Roads/Tracks Bridges Technology	Plans: Regulations Incentive Policies Infrastructure Plans
Short to very short-term	Years to Months	Time and location of leisure activities Transport Modes	Operations: Level-of-Service	
Very short-term	Days to Hours	Route Choice Local Transport Modes Walking Destinations	Adjustment of Operations: Stops and Delays	

Within each time scale, geographical search for location choices is simplified under a spatial hierarchical strategy. First, searching takes place on the macro level, i.e. among a small number of zones, and second, the search is extended to a further disaggregated space, a micro spatial level, but at every time limited to the area defined by the chosen macro zones. The macro level is usually spatially characterized by macro zones, municipalities or Traffic Analysis Zones (TAZ), while the micro level is a disaggregated zone system, for example Census track or block levels. In this context a direct search at a city block or micro level implies, for Santiago city 2.5 million points, while two levels search represents 360.000 macro zones plus, on average, 2,500 micro zone points.

In order to develop a model framework we start introducing some axioms. Each decision has to be made on the appropriate scale for the individual's choice, for what we consider the following rules:

1. *Hierarchies of scales:* the geographical scale is subordinated to the temporal scale. Is the rational behavior is to maximize satisfaction in an ample sense, then it is on the time scale where the fundamental choice of -long and short term- resources allocation into activities are made, which essentially defines the level of satisfaction; hierarchical search the geographic scale contributes only marginally to that purpose.

2. *The resources structure:* The scale of each process or activity choice is directly associated with the amount of resources required.

3. *Self–contained subsystems:* Every process (constraint) that exceeds the dimensions of the micro scale has to take place and be defined at the macro level. Conversely, every process (constraint) that is fully embedded in a micro scale has to be described at the micro scale.

4. *Macro–Micro temporal dependency:* The dynamic of a micro scale system is constrained by the slow moving variables at the macro level because It defines available resources. Conversely, micro level fast moving variables influence future macro variables by means of memory.

The temporal scale defines the long-term effects of macro scale choices, both in terms of the consumption of resources and the level of utility attained. The appropriate geographical scale is one that provides efficiency for a complex search processes, saving efforts and resources, but does not affect fundamentally the long-term quality of life. Therefore, the rule that subordinates the geography scale to the temporal scale implies that the formed may be univocally defined by the latter.

Macro level processes consume and produce a larger amount of the individual scarce resources than micro level ones. The larger amount of resources consumption - time and wealth - are defined at the macro level, including the savings left to be spent at the micro level. Hence, it is more profitable for the consumer to invest time on searching in a larger context of activities and space at the macro scale, than to identify detailed differences at a micro scale. The amount of information required to make informed choices at each sub-system is bounded to the subsystem hierarchy, which is consistent with the existence of a maximum amount of feasible effort made by human beings in a search process, be that physically or economically defined.

The implication is that by assuming consumers' are efficient by using hierarchical strategies we reproduce the real behavior and save computing resources. Saving computing resources becomes crucial when demand model enter equilibrium processes where demand calculations increase exponentially.

2.2. The hierarchical theoretical model: Notation

The notation is defined in order to provide complete flexibility to specify the activity-travel demand model in multiple scales. The temporal scale defines primary, secondary and other levels of activities while the geographical scale defines the levels relating to the spatial context, for example macro and micro locations.

k : Index denoting the temporal scale for activities, *k=1,…,K;* we use the convention that *k=1* is the largest time window and *k=K* the smallest time window. In the bi-scales model, *k=1* is the macro scale and *k=2* is the micro scale; *K*=2. We also assume that time windows are well defined, i.e. a macro time window at level *k* is disaggregated into an integer number of micro time windows at level *k'=k+1*, hence a micro time window does not overlap over two macro time windows.

g : Index denoting the geographical or spatial scale, *g=1,…,G;* we apply the convention that *g<g'* implies geography *g* is more aggregate than *g'*. In the bi-scales model, *g=1* is the macro scale and *g=2* is the micro scale in the geographical dimension; *G*=2. We assume that the geography is also well defined, i.e. a macro zone at level *g* is disaggregated into an integer number of micro zones at level *g+1,* hence no micro zone overlaps over two macro zones.

C : Set of individuals' socio-economic classes at a given level of clustering.

Z^g : Set of location zones defined at the geographical level *g.* The map $M(z\in Z^g, z\in Z^{g+1})$ defines the partition of each zone at geographical level *g* into the level *g+1*; we denote $\tilde{Z}^{g+1/g}=\left\{z_j^{g+1}\in Z^{g+1} / M(z\in Z^g, z\in Z^{g+1})\right\}$. Under map *M* every micro zone is completely contained in a macro zone.

A^k : Set of activities associated at the temporal level *k,* with $A^k=\bigcup_{k'<k}\tilde{A}^{k'}$, with $\tilde{A}^k$ the set of activities decided at the temporal window *k* . Thus, A^k is the aggregated set of activities decided at temporal scale *k* and at all temporal scales $k'\in(1,...,k-1)$, then $A^k\subset A^{k+1}$.

$n=(h,i), h\in H, i\in Z^g$: Index denoting an individual, identified by socio-economics *h* and located at zone *i.*

Activity plans: $\zeta_n^k = \left\{ a_{e/n}^k ; a \in A^k ; e = 1,..,e_n^k \right\}$ is the tuplex denoting the individual n^{th}*'s* daily sequence of activities, described by the sub-set of activities at temporal scale *k* scheduled in the temporal sequence defined by index *e; e_s* is the number of daily activities.

Example: Consider the following activity sets: $\tilde{A}^1 = (H,W)$ and $\tilde{A}^2 = (S)$, with *H* for home and *W* for work, the set of primary activities (*k=1*) and *S* for shopping the secondary activity (*k*=2). An individual *n*'s activity plan is $\zeta_n^{k=2} = \left(H,W,S,H\right)$, with four activities, i.e. $e_n^{k=2} = 4$; this is represented at the macro temporal scale as $\zeta_n^{k=1} = \left(H,W,H\right)$, with three activities, i.e. $e_n^{k=1} = 3$. Thus, it follows that $e_n^k < e_n^{k+1}$.

This notation allows us to combine the hierarchical structure of activities in the temporal scale with the sequential representation of the activity plan at any given temporal scale. It is particularly relevant to emphasize that the engagement in each activity is associated to the appropriate time window and remains unchanged during that time window. Additionally, notice that activities plan at time scale *k* includes activities of the reference temporal scale (*k*) as well as of all other higher levels (*k'<k)*. This is despite the fact that all these activities are performed within the same time period τ_0 (say a day).

Trip paths: $v_{\zeta_n^k}^g = \left\{ v_{e/\zeta_n^k}^g ; e = 1,..,e_n^k \right\}$ is the tuplex defining the *trip stages* in the trip path. There is one stage per activity visited, ordered in the sequence of the activity plan. Trip paths can be described at any geographical scale for trips. Each trip stage $v_{e/\zeta_n}^g = \left(l_{q/e,\zeta_n}^g ; q = 1,...,q_g \right)$ is a set of triplexes identifying the *trip links* (each one describing the origin and destination zones denoted by $d_{q/e-1}$ and $d_{q/e}$ respectively) and mode-route of transport denoted by $\omega_{q/e}$; then $l_{q/e,\zeta_n}^g = \left(d_{q/e-1}^g, d_{q/e}^g, \omega_{q/e}^g ; d \in Z^g, \omega \in \Omega^g \right)_{e,\zeta_n}$.

The number of links is $q_g=2g-1$. To explain this, consider a geography represented in a single spatial level *(g=1)*, then a trip stage contains only one link, which is the minimum number of links. Consider now a given trip stage represented in a hierarchical spatial structure with *g* =2, called macro and micro zones as shown in Figure 1. It represents the trip from the micro zone *i* of macro zone *I,* to the micro zone *j* of the macro zone *J,* using the following sequence of transport modes-routes: m^- (departure local mode-route), M (long distance mode-route) and m^+ (arrival local mode-route). This trip stage is denoted with $v_{e/\zeta_n}^{g=2} = \left((i,I,m^-),(I,M,J),(J,j,m^+) \right)$ for the micro scale and $v_{e/\zeta_n}^{g=1} = \left((I,M,J) \right)$ for the macro scale.

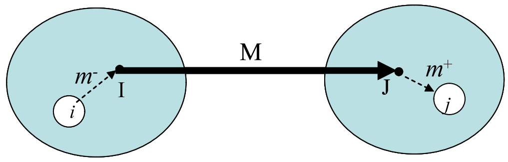

Figure 1: The micro-macro-micro trip link

Under this notation, in a trip path defined at geography level *g* we represent the destination zone at all geography levels *g.* At *g'=g* the destination zone of the trip stage *e* is $d^g_{q_g/e}$ (represented by the micro zone $d^g_{q=3/e}$ =*j* in the example), and the same destination is represented at the next aggregated geography level (*g'=g-1*) by the destination zone $d^g_{q_g-1/e}$ (the macro zone $d^g_{q=2/e}$ =*J* in the example).

2.3. The hierarchical utility maximization problem

The consumer's behavior is modeled assuming the classical utility maximization paradigm. Following De Serpa (1971) and followers, utility is assumed to be yield by the use of the time budget and wealth expenditure on goods and services. We expand this perspective as we innovate through the specification of quality of the activity performed as differentiator of the utility level attained by the consumer. Time includes its allocation to the activity (t_e) and travel duration (tv_e) at each stage *e*. The quality is assumed to depend on the environment at the activity location $d_e = d^g_{q_g/e}$, described by the vector of land use $l_e = l(d_e)$, and the level of consumption assigned to the activity, denoted by vector x_e.

This means that a time schedule program, which includes an activity plan and a trip path, must be defined at a given temporal scale, denoted Γ^k . This program contains the following decisions: what activities to perform (ζ^k_n), including their duration (t_e), sequence (*e*), location (l_e) and time spent on traveling (tv_e); how much goods and services to consume (x_e). The utility derived from a program by individual *n* is: $U_n(\Gamma^k) = U_n(t_e, tv_e, l^g_e, x_e; \forall e = 1,....,e^k_n)_{\zeta^k_n, v^g}$.

The utility maximization problem must allocate time and monetary resources constrained to the feasible domain regarding the set of resources wealth (*S*) and time (τ). At each temporal level *k* the feasible domain is defined by net wealth and time made available from longer term choices, and what is spent and saved for smaller term time window decisions. Additionally, following De Serpa we assume that associated to the consumption of goods, a minimum of time is required. We specify the following domain:

$$\sum_{a^k \in \zeta^k_n} y^a_n - p \cdot x^a_n + S^k_n = S^{k-1}_n \ , \quad \forall k = 1,...,K$$

$$\sum_{a^k \in \zeta_n^k} t_n^a + tv_n^a + \tau_n^k = \tau_n^{k-1} \ , \quad \forall k = 1,...,K$$

$$t_n^a = f(x_n^a) \quad \forall a \in \zeta_n^k, k = 1,...,K \tag{1}$$

where each activity is assumed to potentially yield income (y^a) and induce expenditure depending on consumption and the prices *p*. At any time window *k,* the income disposable is S^{k-1} and the time disposable is τ^{k-1}. Here resources are defined by unit of time period (say 24 hours). At the beginning of any long term decision period *(*i.e. *k=1)* each individual has an initial wealth S_n^0 and all individuals have the same time period budget τ^0 (24 hours). At the most shorter term *k=K,* we define that $S_n^K = 0$ and $\tau_n^K = 0$, which ensures that all resources are consumed under the assumption that consumers behave ; alternatively we may allow inter-temporal transference of resources such that at the end of any given time window the consumer may save resources for the next time window.

In a multi-scales model with temporal and spatial hierarchies the utility optimization problem that yields the individuals rational program choice may be specified in several ways, depending on a spatial and temporal hierarchical structure.

Assumption 1: The geographical scale is subordinated to the temporal scale, denoted by g(k).

This means that the allocation of time to activities to the temporal scale if fundamental while the spatial search is instrumental, under the argument that it is on the time scale where the consumer assigns the relevant resources to activities (e.g. income and time). This does not ignore that the spatial scale influences the consumer's behavior since the individual's cognitive map of a space is hierarchical in nature and represents an individual's selective search strategy in a complex space. Contrarily, decisions on the temporal scale involve a lump amount of resources whose magnitude and time window for the next adjustment is exogenous to the individual. Thus, the temporal scale is substantial to the individual's behavior, while the spatial representation is an auxiliary tool.

Assumption 2: The optimal program specified in a temporal scale K, can be decomposed into a hierarchical structure of conditional optimal sub-programs $k \leq K$*, each one conditional on the optimal sub-programs k'<k and parametric on the set of the individual's total resources.*

The rationale of Assumption 2 is that every program at temporal scale *k* can only adjust activities associated to that level while activities decided at levels *k'<k* are fixed and impose a set condition. Additionally, at level *k* the individual has to save resources to make choices on micro levels *k'>k*, hence decisions are also conditional on the savings.

Under Assumptions 1 and 2 we propose the following optimization problem, called P_K for a temporal scale span specified in K levels.

$$P_K)\quad \underset{a^{k=1}\in A^{k=1}}{Max}\left[\underset{\substack{l^k\in Z^{g(k)}\\ \omega\in\Omega^{g(k)}\\ t_a\in[0,T]\\ x_a\in R(S^k)}}{Max}\; U_n(\Gamma^{k=1})\right] \;\ldots.\; \underset{a_e^k\in A^k}{Max}\left[\underset{\substack{l\in Z^{g(k)}\\ \omega\in\Omega^{g(k)}\\ t_a\in[0,\tau^k]\\ x_a\in R(S^k)}}{Max}\; U_n\left(\Gamma^k\right)\right]\Bigg|_{\zeta_n^{k-1},\, v_{\zeta_n^{k-1}}^{g(k-1)}} \;\ldots.$$

$$\underset{a^K\in A^K}{Max}\left[\underset{\substack{l\in Z^{g(K)}\\ \omega\in\Omega^{g(K)}\\ t_a\in[0,\tau^K]\\ x_a\in R(S^K)}}{Max}\; U_n\left(\Gamma^K\right)\right]\Bigg|_{\zeta_n^{K-1},\, v_{\zeta_n^{k-1}}^{g(K-1)}}$$

The solution of problem P_K should be in the domain defined by equations (1) which is imposed by the domain of time and consumption variables in the maximization process. This hierarchical optimization problem assumes that individuals first decide the long term activities of an activity plan and the trip path, then he/she decides the next level (shorter term) of activities-trips choices conditional on the longer term activity-trip choices and so on. It follows that at each temporal window k the optimization process is conditional on the solution of all longer temporal scales $k'<k$. This sequential optimization process is linked by the allocation of the common resources of time and income.

For the bi-levels spatial-temporal model (*K=2)* the optimal program is the solution of:

*P*2)

$$
\left[\begin{array}{ll}
\underset{a^1\in A^1}{Max}\left[\underset{\substack{z\in Z^1\\ \omega\in\Omega^1\\ t,x}}{Max}U_n(\Gamma^1)\right] & \\
s.t.\quad \sum_{a^1\in\zeta_n^1} y_n^a - p\cdot x_n^a + S_n^1 = S_n^0 & \\
\sum_{a^1\in\zeta_n^1} t_n^a + tv_n^a + \tau_n^1 = \tau^0 & \left[\begin{array}{l}\underset{a^2\in A^2}{Max}\left[\underset{\substack{z\in Z^2\\ \omega\in\Omega^2\\ t,x}}{Max}U_n\left(\Gamma^2\right)\right]\Bigg|_{\zeta_n^1, v_{\zeta_n^1}^1}\\ \sum_{a^2\in\zeta_n^2} y_n^a - p\cdot x_n^a = S_n^1 \\ \sum_{a^2\in\zeta_n^2} t_n^a + tv_n^a = \tau^1 \\ t_n^a = f(x_n^a) \quad \forall a\in\zeta_n^2\end{array}\right] \\
t_n^a = f(x_n^a) \quad \forall a\in \zeta_n^1 &
\end{array}\right]
$$

with *k=1* (*g=1*) a macro temporal (spatial) level and *k=2* (*g=2*) a micro temporal (geographical) level .

Observations:

1. *Time coordination.* The above multi-scales model considers that all activities decided at a given time level *k* are decided instantaneously at the beginning of the time window *k*, and remain constant along the time window. In this context, when the individual optimizes choices at the end of a time window of level *k*, all faster choices made at *k'>k* can be adjusted simultaneously, hence the optimization problem at level *k* is only conditional on slower variables decided at *k'<k.* It follows that the utility supreme of the activities programs is attained at the end of a time window of level *k=1,* when all variables are adjusted simultaneously.
2. *The indirect utility*: At any time level, the optimization problem is conditional on the budgets of time and wealth and on the program (activities and path) decided at all long term temporal levels. Hence the indirect utility is denoted by $V_n^{k_0} = V_n(\Gamma^{k_0};\Gamma^{k-1}, S^k, \tau^k; \forall e = 1,...,e_n^k)$, with $\Gamma^k = (t_e^k, tv_e^k, l_e^{g(k)}, x_e^k; \forall e)$ the set of decision variables at each time window *k*. Notice that function V_n^k is recursive on macro level programs leading to the direct consequence that current behavior on short term choices is conditional on all longer term choices. Another consequence is that individuals which are equal in all socio-economic characteristics may be observed choosing different sets of activity-trips because their set of choices made in the longer term activities are different. This conditional choice process also includes durable goods, such as car ownership

or housing, which affect short term decisions in the transport system: the choices for travel modes, routes and locations of short term activities.

3. *Resource dependency*: We remark that the hierarchical optimization problem at time scale *k* and the corresponding indirect utility function are explicitly dependent on the set of resources saved for activities to be decided at *k'>k.* Indeed, observe that at *k=1* in P_2 the optimum choice is to exhaust all resources making S^1 and τ^1 equal to zero which is not a reasonable outcome because the arbitrary time scale chosen for a given type of analysis define the way in which resources are allocated. Thus, dependency is both ways, with times scales of longer and smaller terms.
4. *Memory*. The above framework with short term choices being dependent on all long term choices introduces a limited form of memory in the decision process. Indeed, decisions made at any temporal level take into account the decisions at all longer term levels but this does not represent the memory on the history of decisions. In fact, at the end of time level *k=1* all decisions are optimized simultaneously without memory. The introduction of full memory can be introduced as an extension of this model.

3. The random utility multi scales approach

In order to develop a decision model that considers idiosyncratic variability we assume that consumers face shocks on their decisions. Shocks are assumed to be temporally and spatially dependent on the associated scales, such that long-term choices are subject to shocks at the macro scale affecting the level of resources (time and income) in the order of magnitude of the associated scale.

We propose the following hierarchical random utility optimization problem considering a time scale span *K* (denoted by RUP_k):

$$RUP_K)\quad \underset{a^{k=1}\in A^{k=1}}{Max}\left[\underset{\substack{z\in Z^{g(k)}\\ \omega\in\Omega^{g(k)}}}{Max}\, U_n(\Gamma^{k=1})+\varepsilon^{k=1}\right]\; \ldots\; \underset{a^k\in A^k}{Max}\left[\underset{\substack{z\in Z^{g(k)}\\ \omega\in\Omega^{g(k)}}}{Max}\, U_n(\Gamma^k)+\varepsilon^k\right]\Bigg|_{\zeta_n^{k-1},\,v_{\zeta_n^{k-1}}^{g(k-1)}} \ldots\; \underset{a^K\in A^K}{Max}\left[\underset{\substack{z\in Z^{g(K)}\\ \omega\in\Omega^{g(K)}}}{Max}\, U_n(\Gamma^K)+\varepsilon^K\right]\Bigg|_{\zeta_n^{K-1},\,v_{\zeta_n^{k-1}}^{g(K-1)}}$$

$$\sum_{a^k\in\zeta_n^k} y_n^a - p\cdot x_n^a + S_n^k = S_n^{k-1}\ ,\quad \forall k=1,\ldots,K$$

$$\sum_{a^k\in\zeta_n^k} t_n^a + tv_n^a + \tau_n^k = \tau_n^{k-1}\ ,\quad \forall k=1,\ldots,K$$

$$t_n^a = f(x_n^a)\quad \forall a\in\zeta_n^k, k=1,\ldots,K$$

Previous to the analysis of this problem we illustrate the above model by the following example: the choice for the activity shopping can be subject to shocks, like weather conditions that disrupt the daily schedule affecting shopping time and the location choice. We may observe that the effect of this shock vanishes in the time window of a week because rescheduling takes place and expenditure is readjusted. It seems also reasonable that a shorter term shock like a minute delay in a congested junction does not alter the daily scheduling since in the next hour or so such a delay vanishes by readjustments of times. A longer term shock like a sudden acceptance in a new job alters all shorter term decisions, including shopping, because the new job implies a specific commitment to time and a different income. This example shows that shocks at time scale *k* affects all decisions at scales *k'*>*k*; conversely, it does not affect decisions at scale *k'*<*k*.

The above argument implies the following: $Cov(\varepsilon^k, \varepsilon^{k'}) \begin{cases} = 0 & if \quad k > k' \\ \neq 0 & if \quad k \leq k' \end{cases}$, then the covariance matrix is assumed to be triangular superior. This implies that the solution of the problem is a joint probability of the following form:

$$P_n(\Gamma^K) = P_n(\Gamma^1) \cdot P_n(\Gamma^2)_{\Gamma^1} \cdot \ldots \cdot P_n(\Gamma^K)_{\Gamma^{K-1}} \quad (2)$$

where each choice probability is independent on the choice probabilities of activities decided at smaller time scales (*k'*>*k*), but conditional on the choice probabilities of activities decided at larger time scales (*k'*>*k*). Notice that (2) implies the following decomposition or multiplicative property: $P_n(\Gamma^K) = P_n(\Gamma^{K-1}) \cdot P_n(\Gamma^K)_{\Gamma^{K-1}}$.

This property is valid for any time scale *k*<*K* under the condition that the allocation of total resources is consistent with the activities to be decided at all time scales. In other words, a program $\Gamma^{k<K}$ must be a subprogram of a full time scales program Γ^K because the latter provides the feasibility conditions on resources at all scales. Notice that this condition links all decisions associated to a scale *k'*>*k* in a way that imposes that all these decisions should be taken simultaneously. However, in the dynamic choice process described above these is feasible and necessary. To understand why, notice that at the end of the time window in scale *k*, all time windows at scales *k'*<*k* also elapse because fast moving variables are readjusted any time a slower moving variable is modified. Therefore, we conclude that choices at a given time window are conditional on the resources available from longer term choices and are decided simultaneously with decisions made at smaller time scales.

Dependency on the geographical scale

Within a temporal scale *k*, the problem RUP_k includes location decisions for all activities decided at that scale (conditional on activities and locations decided at longer term scales). Thus, let us consider the following example: the individual is searching an optimal location for shopping. From time scales *k-1* he/she has saved a limited amount of time and income, say three hours and 50 dollars, which limits the maximum return travel time to say 2 hours (one hour on each direction). This limit defines a suitable geography scale *g* such that: i) the time spent in mental searching is reasonable compared to the travel and activity duration time (say a couple of minutes); ii) it fits with the limited information handled by the consumer to make such decision. For a one hour travel radius the individual search strategically at two levels, the macro zone TAZ-level for a subset of most attractive city sub-centers and then at a micro zone block level for a specific store option. Shocks may affect the location choice at the TAZ level, for example because the car has few gasoline reducing the maximum distance, and at a block level for example because congestion in a given street has blocked access to park nearby the store.

Let us define the marginal probability of performing a given activity-trip at stage *e* as $P_n(e \in \Gamma^k) = P_n(t_e^k, tv_e^k, l_e^{g(k)}, x_e^k)_{\Gamma^{k-1}}$. Assuming that the activity, say shopping, is decided prior to the associated transport and location related choices, then we decompose $P_n(e \in \Gamma^k) = P_n^a(t_e^k, x_e^k)_{\Gamma^{k-1}} \cdot P_n^t(tv_e^k(\omega, l), l_e^{g(k)})_{\Gamma^{k-1}, t_e, x_e}$ where the travel time is dependent on the mode, route and location choice. Notice that this decomposition imposes that the activity choice marginal probability (P^a) and the corresponding trip choice as a conditional probability (P^t) on the activity choice are jointly feasible. For example, one will not choose to go for shopping if the closest location option is beyond the time and budget available at the time scale of shopping. Secondly, notice that P^t is conditional on P^a, then on t_e and x_e. These two considerations make the decomposition assumption plausible though not sufficient because it is possible to identify activities where activity and trip choices are taken jointly. Nevertheless, since the trip choice probability concentrates and isolates the geographical choice, it is useful to analyze the choice process in the spatial scale. Similarly to time shocks, in the shopping example above we observe that in the geographical scale shocks at scale *g* do not affect spatially wider decisions at *g'<g*, and the decisions at level *g* are conditional on shocks at *g'<g.* This implies the following spatial multiplicative property:

$$P_n^t(\omega, l)^g = P_n^t(\omega, l)^{g-1} \cdot P_n^t(\omega, l^g)_{(\omega, l)^{g-1}}$$

with $P_n^t(\omega, l)^g = P_n^t(tv_e^k(\omega, l), l_e^{g(k)})_{\Gamma^{k-1}, t_e, x_e}$.

We emphasize that, although the multiplicative property applies to both the time and geographical scales it is valid in a more general context for the time scale.

The above hierarchical approach provides the base structure to develop a family of urban models, each of them specific on the set of decisions, the spatial-temporal scales and the way shocks are modeled. The multi scales approach is modular because it permits that each of these models represent a sub-problem of a larger one, which can be modeled by extending the model by allowing more scale levels.

4. Application of the proposed approach

In this section we describe the potential use of the proposed hierarchical framework to solve different problems involving decisions made at different scales. Special attention is devoted to the interpretations and modeling coming from the application of the theoretical scheme to trips demand and assignment of activities, time use and value and traffic assignment models. We have explored via simulation as well as surveys' data how the decision making process can become simpler and more realistic when decisions are made at different scales conditioning the results of options at lower levels by upper options; the method is illustrated by the aforementioned three potential applications in different contexts as explained next.

4.1. Microeconomic model of activities and trip demand

First, let us briefly describe how the model works in case of being applied to the trip demand modeling in a dynamic decision making process. In such a case, the hierarchical structure of the process comes directly from the inherent classification of the activities (work, education, shopping, leisure, and so on) and their scope in terms of assignment of resources. Thus, in this context we can classify the activities decided by a decision maker as primary and secondary activities, the former ones dominating and conditioning the second ones.

Based on the individual choice for an activity plan (number and sequence of activities) we are able to calculate complete activity trip trajectories, denominated as trip paths. Each path represents the interdependency of activities and trips and includes choices for locations and modes (routes were not considered in this case). We are currently developing a methodology to reduce the spatial search for secondary activity locations and constrain trip paths against daily travel time budgets. The application is illustrated for an activity plan of type 'Home-Work-Shopping-Home'. As main information sources we use an existent trip demand model to reproduce the long-term (macro level) decisions for work and education locations and the Santiago's Travel and Household Survey to describe the characteristics of conditional secondary activities (micro level). In recent publications (Martínez et al, 2009a; Justen et al, 2010) we show results of the calculations of trip paths and validate them against the empirical database for the area of Greater Santiago, Chile.

In these works we basically discover that the geographical scale is subordinated to the temporal scale; for example, the decision whether or not to buy a car is a long-term (temporal scale) decision that once realized influences decisions of location choice (geographical scale), particularly with regards to the search space for shopping. Hence, decisions associated to the long-term temporal scale define the effects that last for the long term, both in terms of the consumption of resources and the level of utility attained. The hierarchy among time is also highlighted, in the sense that the time spent on long-term decisions compared with short-term ones is consistent with the corresponding scales. This means that time spent for a long-term related decision such as the work duration, ascertains durations of secondary activities. Finally, it is also relevant the hierarchy among activities locations: long-term decisions for home or work locations determine any subordinated decision of a short to very short term decision, such as the location of a shopping or leisure facility, because they depend on the residential and work locations, or on the route between them. This typology is related to the argument of limited available resources where the scales between activities coincide with the amount of resources associated with long and short term decisions.

Illustrative results from the approach are illustrated in Justen et al (2010), showing how work location, time allocation and shopping location are linked at different levels, highlighting the main conclusions under the influence of short and long-term shocks. Ongoing research is oriented to the development of an analytical framework to treat the conditional probabilities for computing joint probability structures for modeling decision making process at different scales in this context; we expect to calibrate such expressions with the data collected and processed in current analyses.

Another observation from the survey is that time spent on secondary activities is dependent on the time spent on primary ones. Indeed, in activity patterns such as home-work-shopping-home, the time spent in shopping depends on the departure time from home and the working hours, both macro scale decisions.

4.2. A time-hierarchical approach for time use and value

In recent years, various models have been designed that attempt to explain people's behavior, either the consumption of discrete or continues goods, the choice of feasible activities within a system or the use of time for their consumption. Most microeconomic models of the consumer's behavior assume that the individuals maximize the utility yield by the consumption of goods, imposing time and budget constraints; these models yield a strong theory supporting that the value of time depends on the activity performed, composed by a common component that values the time as a scarce resource and other components that depends on whether the activity is work (generates income) or leisure (consumes income and time). The above research line focused on the value of time has in common the assumption that all choices are optimized instantaneously. In the line of the approach presented above, we explore the implications of relaxing this assumption introducing the time scale of activities under the otherwise same approach. Our argument is that activities performed by an individual can be clearly differentiated by the time window that the choice last, and that such time window is directly related with the proportion of available resources (time and income) consumed or produced by the activity. For example, work and study involve many years of commitment to a given choice and a mayor proportion of daily time and the monetary resources, while other activities, like shopping or leisure choices, can be modified within a day or week. This is represented by a consumer's problem that optimizes utility by choosing a set of activities, i.e. both time duration and goods consumption. The model explicitly considers a hierarchical structure, based on the time windows or the frequency on which choices are made, to represent the speed of change and the amount of resources consumed.

Using the hierarchical multi time scale structure we seek to analyze the dynamic that exists in the individual's behavior and, particularly, to study the effects that random shocks in the economy or long term choices may have on the set of decisions taken and on the interpretation of the value of time when is derived from choices in the micro scale.

In this two scales framework we consider the following interactions. The micro scale choice of activities is conditioned by the choices made at the macro scale, because the latter cannot be adjusted as frequently. Therefore, at the micro scale the set of feasible activities depends on the durables, goods and services, available as a result of choices made at the macro scale. Finally, the resources, time and wealth, available to decide on micro scale activities are constrained by what is left after deducting the expenditure on macro activities. Additionally, we assume that consumers may behave differently according to their perception of risk, which we represent by weighting the utility yield by each scale on the total utility obtained in the time horizon considered.

In the application shown in (Martínez et al, 2009b) the overall utility of time horizon is then calculated following the standard approach of adding separable utilities along time periods weighted by a discount factor. In that application we demonstrate that the value of time, as a resource, can be generalized to all activities, including those that do not generate income, extending the results of the static models. Another interesting result is that the approach clearly justifies that value of time associated to micro scale activities is dependent on the set of choices made at the macro scale, including expenditure in durables. Then the value of time estimated from the econometric calibration of transport mode choice models is bound to be different according to the individual's choice of housing, car(s), education level, job, as well as the location of macro scale activities, like work and study. This macro-micro direction of dependency among activities' choices is not explicit in classic models where all activities belong to the same hierarchy level. In the next figure, we see the expected relation between value of time at macro and micro scales, noting that if economy improves for example, the value of time determined from micro scale choices underestimates consumers' benefits. Assuming that goods prices decrease along time, the straight line represent change in the value of time is the consumer optimize all choices at every time; here the positive slope is explained by the equivalent income increase yield by decreasing prices. The macro value of time (Macro VT) is revealed by long term choices, and the micro values of time (Micro VT) is revealed by micro choice which the value revealed by mode choice studies. The figure indicated the gap between these values.

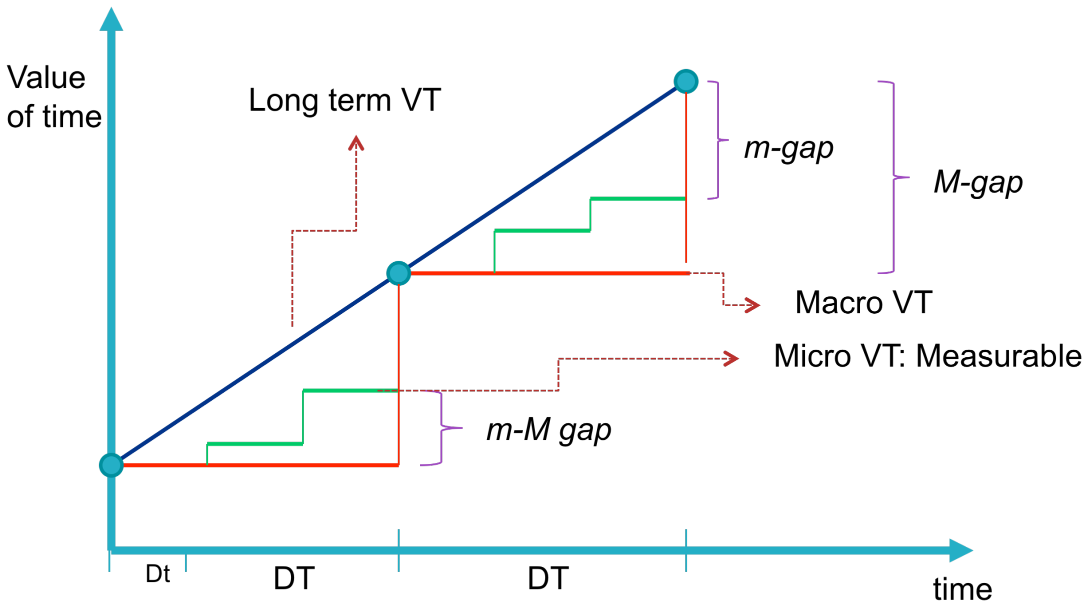

Figure 2: Expected relation of macro and micro scales for the value of time

4.3. Dynamic multi-user traffic model for general urban networks

One important issue with regard to the planning process of most urban transport systems is the capability of forecasting the traffic flow conditions that are expected to happen on the corresponding urban networks in a future time horizon. In the case of private cars, when the objective is to study strategic transport plans, the most utilized tool by planners is to find conditions leading to a traffic flow equilibrium, based on the equivalent optimization problem that allow finding the static equilibrium of the private traffic assignment problem on large-scale urban networks (macroscopic approach). In such a problem, the decision variables are the aggregated flows of vehicles on each link within the modeled network. Moreover, each link is characterized by a traffic-flow delay performance function, which normally depends on the flow on such a specific link. Total travel time along each route (connecting an origin-destination pair) is computed as the summation of the travel time of each link along the chosen route. One limitation of such a static approach is that this modeling scheme does not recognize that urban networks are utilized by different type of users, with different driving habits and other features that clearly affect the motorists' behavior when sharing the infrastructure with other motorists (social, economical, cognitive, and so on).

Such an important limitation can be overcome by recognizing the existence and interaction across time of different motorists. This differentiation of users will allow us treating more realistically the interrelations that occur on the network while travelers are moving from origins to destinations within the urban street network.

From such a motivation, and following the same hierarchical structure, we are currently developing a multi-user dynamic formulation of the private traffic problem for a generic urban network, considering processes occurring at multiple scales. For example, once the route decision has been made, the lower level will be the process the individual follows in order to reach his/her destination (i.e. how motorists drive through the network streets, for a given route chosen to fulfill the journey). Of course, if a lot of congestion is coming up at this lower process, at a certain time the user will evaluate the alternative of changing the original route decision.

In this formulation, at the lower level we are recognizing different behavioral patterns of individuals sharing the network infrastructure, which will result in link performance functions also differentiated by user type. At a more aggregate (or upper level), we will define the equilibrium conditions for the route choice, if that exists. The model will have the capability of considering negative externalities (caused by congestion) on links and nodes, also differentiated by user type, and characterized by the interaction between different users with different behavior.

One first stage of the developments was to formulate the routing decision by following a hierarchical structure (Muñoz et al., 2009). All users play the game over a common board (urban street system), however not all of them have the same knowledge (learning process). The decision of defining a hierarchical structure was in order to make the routing process more realistic on one hand, and to speed up the solution of a real problem in a reasonable computation time on the other.

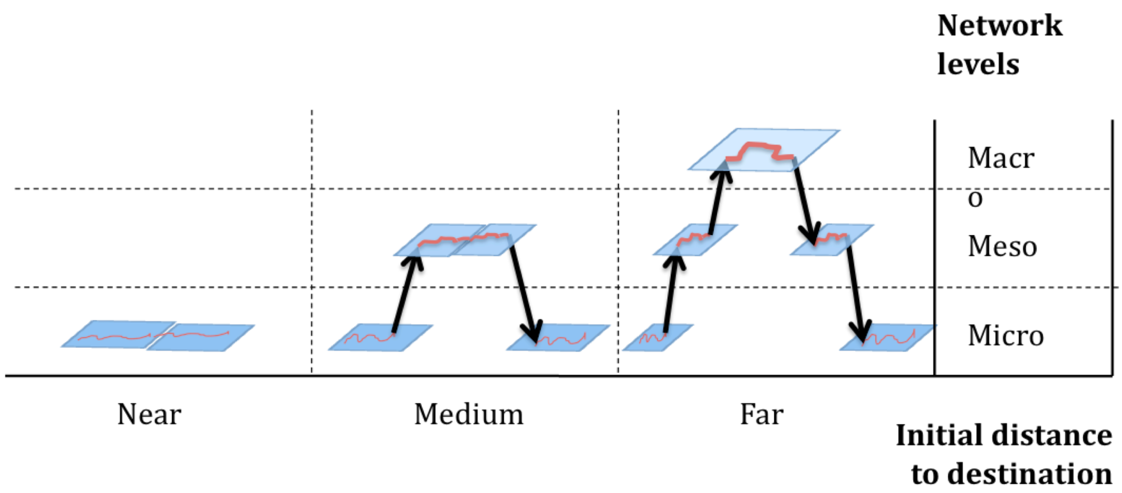

Figure 3: Hierarchical structure of the routing individual decision

After taking such initial path decisions, and depending on the dynamical structure of the trip progression, we postulate a marginal change of routing decisions based on the same type of hierarchical structure. In such a case, the individual decides the links at which he(she) jumps from one network stage to the next, with the condition of performing this change only from adjacent stages at any marginal change in routing (as schematically depicted in Figure 2).

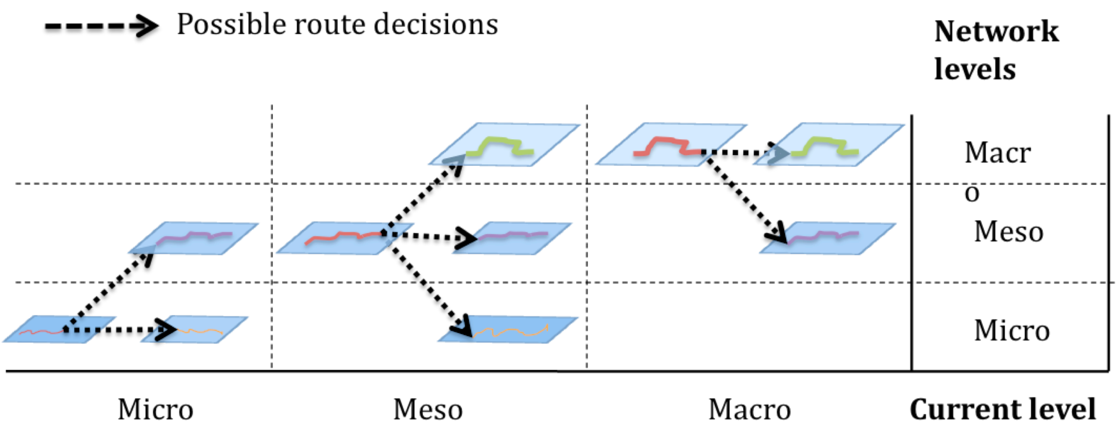

Figure 4: Possible changes between stages in dynamic changes of individual routing decisions

The driving force behind route-change decision is the congestion experimented on the current path followed by the user, because each trip generated has a purpose and a time horizon to accomplish it.

5. Conclusions and Outlook

Regarding the attempts of bringing the theoretical framework into practice the results achieved so far are promising. The methodology and its main property of hierarchical decision-making in time and space have been implemented in several sub-models.

The approach is supported by empirical evidence and does provide theoretical support for them. It also offers a consistent and unified theoretical approach to all decisions made in the urban context by their inhabitants. This structural consistency provides further benefits. A more advanced understanding of the elusive concept of the value of time has been advanced, an equilibrium approach for activities and travel demand with time sequence and bounded search is under construction, and a more complex model of route choice and traffic equilibrium in under development. All this is conceived in modular system to study certain levels of choices, time scales and geographic contexts.

Above all these issues, the model seeks to explain individuals' behavior in a unified way, such that full consistency is imposed in the set of choices he/she makes. This should reduce of the current number of parameters currently used in travel modeling, but it is also aimed at integrating other processes, such as the goods consumption, information and social networks. Although this is the same aim of micro-simulation models, in our model we add the potential of deriving system equilibrium conditions such as prices and agents interactions.

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